Research Article

Sliding Sector-Based Variable Structure Control of Continuous-Time Markov Jump Linear Systems Subject to Unknown Transition Rates

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Received 8 August 2013; Accepted 5 October 2013

Academic Editor: Tao Li

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Based on sliding sector technique, the variable structure control for a class of uncertain continuous-time Markovian jump linear systems (MJLS) is investigated. The elements in the transition rate matrix include completely known, boundary known, and completely unknown ones. First, the related notions about sliding sector for continuous-time Markov jump linear systems are given; then based on linear matrix inequalities (LMIs) technique, sufficient conditions for the design of the sliding sector are provided. Second, a variable structure control law is presented to guarantee the mean-square quadratic stability of the closed-loop system in spite of the effects of the existing uncertainties and unknown/uncertain transition rates. Finally, an example is given to verify the validity of the theoretical results.

1. Introduction

The subject of this paper is variable structure control of continuous-time Markov jump linear uncertain systems. Control design problems of Markov jump systems are motivated by the fact that these systems represent a class of important stochastic systems which is popular in modeling practical engineering systems subject to failures or changes in structures and parameters. Such as in [1, 2], Markovian switching technique has been used to model the switching action of mobile manipulators. In addition, some important theoretical results have also been published so far; see, for example, [3–9] and the references therein. It is worth noticing that these results require that the transition probabilities/rates must be known precisely. In fact, the likelihood of obtaining the perfect information on all transition probabilities/rates is questionable, and the cost might be expensive in some areas. As a result, the study of control synthesis for Markov jump systems with uncertain or unknown transition probabilities/rates becomes interesting, and some important results have been established. For instance, the robust control designs for MJLS subject to the norm-bounded or polytopic

uncertainties in the transition rates were considered in [10, 11]. However, they both require to know the structure and “nominal” terms of the uncertain transition rates. Furthermore, control designs of MJLS with partly unknown transition rates were developed in [12–14], in which the transition rate matrix included two kinds of elements: completely known and completely unknown ones. In fact, besides the two kinds of elements, it might also own boundary known elements in practice, and we will consider this more general case for reducing the conservatism of control design in this paper.

On the other hand, variable structure control is a well-known robust control technique against external disturbances, parameter uncertainties, and unmodeled dynamics [15–17], and thus there are also some remarkable results published on variable structure control of continuous-time MJLS recently [18–22]. In [21], the robust stabilization of variable structure control for continuous-time MJLS with matched uncertainties was first investigated, and then in [22], a singular system approach was introduced for systems with parameter uncertainties and unknown nonlinear function. It is worth mentioning that these works require the transition rates to be

precisely known *a priori*, and to the best of our knowledge, no results have been reported to variable structure control of continuous-time MJLS with unknown transition rates. In addition, one of the disadvantages about variable structure control is that chattering phenomenon might occur on the neighbor of the sliding manifold which is harmful to actuator devices, and thus it is a barrier for the use of the variable structure control strategy [23–25]. In order to avoid the occurrence of chattering phenomenon, it is worth noting that for a class of linear deterministic systems, a variable structure control-based sliding sector is first introduced in [26]. The main feature of the sliding sector-based variable structure control scheme is that zero control input can be used in the sliding sector of the state space and then the chattering phenomenon is avoided.

Motivated by the aforementioned discussion, for a class of continuous-time uncertain MJLS subject to unknown transition rates, a novel robust variable structure control-based sliding sector technique is investigated. The main contribution of this paper is summarized as follows. First, related notions of the sliding sector for MJLS, $P_i R_i$ -sliding sector, simplified $P_i R_i$ -sliding sector, and inner sector and outer sector are given. Second, sufficient conditions for the design of the simplified $P_i R_i$ -sliding sector for the considered Markov jump linear uncertain system are presented in terms of LMI technique. Finally, a variable structure control law is established to drive the state trajectory to converge to the inner sector of the $P_i R_i$ -sliding sector and guarantee mean square quadratic stability of the closed-loop system in spite of the effect of unknown transition rates and system uncertainties.

The rest of this paper is organized as follows. The problem statement and preliminaries are presented in Section 2. The main results are given in Section 3. In Section 4, a numerical example is presented to illustrate the effectiveness of the results, and the conclusions are drawn in Section 5.

Throughout this paper, the following notations are used. $\mathbb{R}^n$ denotes the n -dimensional Euclidean space; A^T stands for the transpose of matrix A ; I and 0 represent the identity matrix and zero matrix in appropriate dimension, respectively. In addition, the symbol $\text{He}(X)$ is used to represent $X + X^T$. $P > 0$ ($P \geq 0$) means that P is real symmetric and (semi-) positive definite. $|\cdot|$ denotes the absolute value of a scalar, the standard Euclidean norm of a vector, or the induced norm of a matrix, respectively. In symmetric block matrices, the notation $*$ is used to represent a term that is induced by symmetry.

2. Problem Statement and Preliminaries

Consider the following time-invariant continuous-time linear uncertain system:

$$\dot{x} = (A(r_t) + \Delta A(r_t))x + (B(r_t) + \Delta B(r_t))u, \quad (1)$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}$ are state and control input, respectively. $\Delta A(r_t)$ and $\Delta B(r_t)$ are the system uncertainties. $\{r_t, t \geq 0\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{S} = \{1, 2, \dots, \mathbb{N}\}$. It governs the switching among the different

system modes with the following mode transition probabilities:

$$\text{Prob}\{r_{t+m} = j \mid r_t = i\} = \begin{cases} \lambda_{ij}m + o(m), & \text{if } j \neq i, \\ 1 + \lambda_{ii}m + o(m), & \text{if } j = i, \end{cases} \quad (2)$$

where $m > 0$, $\lim_{m \rightarrow 0} (o(m)/m) = 0$ and $\lambda_{ij} \geq 0$ ($i, j \in \mathcal{S}, j \neq i$) denote the switching rate from mode i at time t to mode j at time $t + m$, and $\lambda_{ii} = -\sum_{j=1, j \neq i}^{\mathbb{N}} \lambda_{ij}$ for each $i \in \mathcal{S}$.

In general, the Markov process transition rate matrix Λ is defined by

$$\Lambda = [\lambda_{ij}]_{\mathbb{N} \times \mathbb{N}}. \quad (3)$$

In this paper, the transition rates of the jumping process are assumed to be partly available; that is, some elements in Λ have been exactly known; some ones are uncertain but with known boundaries; others may have no information to use. For instance, for system (1) with four operation modes, the transition rate matrix might be described by

$$\Lambda = \begin{bmatrix} ? & \lambda_{12} & \lambda_{13} & ? \\ \lambda_{21} & ? & \lambda_{23} & ? \\ ? & a & ? & \lambda_{34} \\ b & ? & \lambda_{43} & \lambda_{44} \end{bmatrix}, \quad (4)$$

where $?$ represents the completely unknown element in the transition rate matrix; parameters a and b represent the elements with known lower and upper bounds. That is, $\underline{a} \leq a \leq \bar{a}$ and $\underline{b} \leq b \leq \bar{b}$, where $\underline{a}$, $\bar{a}$, $\underline{b}$, and $\bar{b}$ are known parameters; λ_{ij} denotes the precisely known element.

For clarity, denote $\mathcal{S} = \mathcal{S}_{k1}^i \cup \mathcal{S}_{k2}^i \cup \mathcal{S}_{uk}^i$, $i = 1, 2, \dots, \mathbb{N}$ by

$$\mathcal{S}_{k1}^i = \{j : \lambda_{ij} \text{ is exactly known}\},$$

$$\mathcal{S}_{k2}^i = \{j : \text{the bounds of } \lambda_{ij} \text{ is known}\}, \quad (5)$$

$$\mathcal{S}_{uk}^i = \{j : \text{there is no information available for } \lambda_{ij}\}.$$

Furthermore, let $\mathcal{S}_k^i = \{j : \underline{\lambda}_{ij} \leq \lambda_{ij} \leq \bar{\lambda}_{ij}\}$; then $\mathcal{S}_k^i = \mathcal{S}_{k1}^i \cup \mathcal{S}_{k2}^i$. Let $\mathcal{S}_k^i = \{\mathcal{K}_1^i, \mathcal{K}_2^i, \dots, \mathcal{K}_{m_k^i}^i\}$. $\mathcal{S}_{uk}^i = \{\mathbb{K}_1^i, \mathbb{K}_2^i, \dots, \mathbb{K}_{m_{uk}^i}^i\}$, where $\mathcal{K}_{l_1}^i$ denotes the l_1 th element in $\mathcal{S}_k^i$ with the index $\mathcal{K}_{l_1}^i$ in the i th row of the matrix Λ . $\mathbb{K}_{l_2}^i$ denotes the l_2 th element in $\mathcal{S}_{uk}^i$ with the index $\mathbb{K}_{l_2}^i$ in the i th row of the matrix Λ . m_k^i and m_{uk}^i represent the number of elements in $\mathcal{S}_k^i$ and $\mathcal{S}_{uk}^i$, respectively.

Remark 1. When $\underline{\lambda}_{ij} = \bar{\lambda}_{ij}$, the transition rates are reduced to the situation in [12]. Obviously, the solving method there can only treat the above known bound case as the completely unknown case, and some conservativeness has been brought in.

For convenience, let $A_i = A(r_i)$, $\Delta A_i = \Delta A(r_i)$, $B_i = B(r_i)$, and $\Delta B_i = \Delta B(r_i)$ for each possible value $r_i = i$, $i \in \mathcal{S}$,

where A_i and B_i are known constant matrices with appropriate dimensions; then system (1) can be described by

$$\dot{x} = (A_i + \Delta A_i)x + (B_i + \Delta B_i)u. \quad (6)$$

Throughout this paper, for each i , suppose that the following assumptions are valid.

Assumption 2. The pair (A_i, B_i) is controllable.

Assumption 3. The uncertainties satisfy $\Delta A_i = D_i E_i(t, x) F_i$, $\Delta B_i = B_i \Theta_i(t)$, where D_i and F_i are known constant matrices with appropriate dimensions, $E_i(t, x)$ is time-varying uncertain matrix satisfying $E_i E_i^T \leq I$, the parameter $\Theta_i(t)$ satisfies $|\Theta_i(t)| \leq \bar{d}_i < 1$, $\bar{d}_i$ is a known positive scalar.

For continuous-time Markov jump linear nominal systems

$$\dot{x} = A_i x + B_i u, \quad (7)$$

some related notions of sliding sector are firstly presented.

Definition 4. The $P_i R_i$ -sliding sector is a subset of $\mathbb{R}^n$ defined as

$$\mathcal{S}_i = \left\{ x \mid x^T \left(A_i^T P_i + P_i A_i + \sum_{j=1}^N \lambda_{ij} P_j \right) x \leq -x^T R_i x, \quad x \in \mathbb{R}^n \right\}, \quad (8)$$

where $P_i \in \mathbb{R}^{n \times n}$ is a positive definite symmetric matrix, $R_i \in \mathbb{R}^{n \times n}$ is a positive semidefinite symmetric matrix, $R_i = C_i^T C_i$, $C_i \in \mathbb{R}^{l_i \times n}$, $l_i \geq 1$, and (C_i, A_i) is an observable pair.

The set of the $P_i R_i$ -sliding sector is nonempty since at least zero state stays inside it.

Obviously, while the state of the plant evolves in the $P_i R_i$ -sliding sector $\mathcal{S}_i$, the P_i -norm $|x|_{P_i} = x^T P_i x$ of the plant (7) decreases with zero control input since

$$\begin{aligned} \dot{L}_i &= x^T \left(A_i^T P_i + P_i A_i + \sum_{j=1}^N \lambda_{ij} P_j \right) x \\ &\leq -x^T R_i x \leq 0, \quad \forall x \in \mathcal{S}_i, \quad x \neq 0, \end{aligned} \quad (9)$$

where $L_i = |x|_{P_i}^2$.

Remark 5. When $A_i = A$, $P_i = P$, $R_i = R$, the $P_i R_i$ -sliding sector in (8) becomes

$$\mathcal{S} = \{x \mid x^T (A^T P + P A) x \leq -x^T R x, \quad x \in \mathbb{R}^n\}. \quad (10)$$

The definition of PR -sliding sector for linear deterministic systems is introduced in [26].

Definition 6. A simplified $P_i R_i$ -sliding sector is a subset of $\mathbb{R}^n$ defined as

$$\mathcal{S}_i = \{x \mid |s_i(x)| \leq \delta_i(x), \quad x \in \mathbb{R}^n\}, \quad (11)$$

where the linear function $s_i(x)$ and the square root $\delta_i(x)$ of the quadratic function $\delta_i^2(x)$ are determined by

$$\begin{aligned} s_i(x) &= S_i x, \quad S_i \in \mathbb{R}^{1 \times n}, \\ \delta_i(x) &= \sqrt{x^T \Omega_i x}, \quad \Omega_i \in \mathbb{R}^{n \times n}, \quad \Omega_i \geq 0 \quad (\Omega_i \neq 0). \end{aligned} \quad (12)$$

Remark 7. Suppose that the following inequalities are satisfied:

$$\begin{aligned} A_i^T P_i + P_i A_i + \sum_{j=1}^N \lambda_{ij} P_j - P_i B_i B_i^T P_i + Q_i &< 0, \\ i &= 1, 2, \dots, N. \end{aligned} \quad (13)$$

Then the parameters in the simplified $P_i R_i$ -sliding sector P_i , R_i , S_i , and Ω_i can be selected as follows:

$$\Omega_i = r_i Q_i, \quad R_i = (1 - r_i) Q_i, \quad S_i = B_i^T P_i, \quad (14)$$

where $0 < r_i < 1$.

Obviously, while the state of the plant evolves in the simplified $P_i R_i$ -sliding sector $\mathcal{S}_i$, the P_i -norm $|x|_{P_i}$ of the plant (7) decreases with zero control input since

$$\begin{aligned} \dot{L}_i &= x^T \left(A_i^T P_i + P_i A_i + \sum_{j=1}^N \lambda_{ij} P_j \right) x \\ &= x^T (P_i B_i B_i^T P_i - Q_i) x \\ &= s_i^2(x) - \delta_i^2(x) - x^T R_i x \\ &\leq -x^T R_i x, \quad \forall x \in \mathcal{S}_i, \quad x \neq 0. \end{aligned} \quad (15)$$

In addition, in order to avoid the chattering phenomenon on the boundary of the $P_i R_i$ -sliding sector, an inner sector $\mathcal{S}_i^{\text{in}}$ and an outer sector $\mathcal{S}_i^{\text{out}}$ are introduced as subsets of the $P_i R_i$ -sliding sector (11):

$$\mathcal{S}_i^{\text{in}} = \{x \mid |s_i(x)| \leq \alpha_i \delta_i(x), \quad x \in \mathbb{R}^n\}, \quad (16)$$

$$\mathcal{S}_i^{\text{out}} = \{x \mid \alpha_i \delta_i(x) < |s_i(x)| \leq \delta_i(x), \quad x \in \mathbb{R}^n\}, \quad (17)$$

where the positive scalar α_i satisfies $0 < \alpha_i < 1$. It is not difficult to see that the sets $\mathcal{S}_i^{\text{in}}$ and $\mathcal{S}_i^{\text{out}}$ constitute a partition of the simplified PR -sliding sector $\mathcal{S}_i$. That is, $\mathcal{S}_i = \mathcal{S}_i^{\text{in}} \cup \mathcal{S}_i^{\text{out}}$ and $\mathcal{S}_i^{\text{in}} \cap \mathcal{S}_i^{\text{out}} = \emptyset$.

A hysteresis dead-zone function $\sigma_i(s_i(x), \delta_i(x))$ is further introduced for avoiding the chattering. It is defined as

$$\sigma_i(s_i(x), \delta_i(x)) = \begin{cases} 0, & x \in \mathcal{S}_i^{\text{in}} \\ \text{unchanged}, & x \in \mathcal{S}_i^{\text{out}} \\ 1, & x \notin \mathcal{S}_i. \end{cases} \quad (18)$$

Lemma 8 (see [20]). For any given $\lambda_j \geq 0$ and matrices $P_j > 0$ ($P_j \in \mathbb{R}^{n \times n}$, $1 \leq j \leq \mathbb{N}$), if there exists $Z_i \geq P_j$, then the following inequality holds:

$$\sum_{j=1}^{\mathbb{N}} \lambda_j P_j \leq \sum_{j=1}^{\mathbb{N}} \lambda_j Z_i. \quad (19)$$

Lemma 9 (see [27]). Given a symmetric matrix Π and matrices M , N with appropriate dimensions, then $\Pi + MF(t)N + N^T F^T(t)M^T < 0$ for all $F(t)$ satisfying $F^T(t)F(t) \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that the following inequality holds:

$$\Pi + \varepsilon MM^T + \varepsilon^{-1} N^T N < 0. \quad (20)$$

Lemma 10 (see [28]). For symmetric and positive definite matrices X and Y , if $X \geq Y > 0$, then $Y^{-1} \geq X^{-1} > 0$.

3. Main Results

First, we present the design of the simplified $P_i R_i$ -sliding sector for the considered continuous-time Markov jump linear uncertain system (6). Second, the controller design for mean-square quadratic stability of the continuous-time Markov jump linear uncertain system is developed.

3.1. The Design of the Simplified $P_i R_i$ -Sliding Sector

Theorem 11. For continuous-time Markov jump linear uncertain system (6), there exist symmetric and positive definite matrices X_i , Z_i , and Y_i satisfying the following LMIs:

$$i \in \mathcal{S}_k^i, \left\{ \begin{array}{l} \begin{bmatrix} \Gamma_{11i} & \Gamma_{12i} & \Gamma_{13i} & X_i & X_i F_i \\ * & -\Gamma_{22i} & 0 & 0 & 0 \\ * & * & -Z_i & 0 & 0 \\ * & * & * & -Y_i & 0 \\ * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0, \\ Z_i \leq X_j, \quad j \in \mathcal{S}_{uk}^i, \quad j \neq i \end{array} \right. \quad (21)$$

$$i \in \mathcal{S}_{uk}^i, \left\{ \begin{array}{l} \begin{bmatrix} \Phi_{11i} & \Phi_{12i} & X_i & X_i F_i \\ * & -X_j & 0 & 0 \\ * & * & -Y_i & 0 \\ * & * & * & -\varepsilon_i I \end{bmatrix} < 0, \\ X_i \leq X_j, \quad j \in \mathcal{S}_{uk}^i, \quad j \neq i, \end{array} \right. \quad (22)$$

where

$$\Gamma_{11i} = \text{He} (A_i X_i) + \bar{\lambda}_{ii} X_i - B_i B_i^T + \varepsilon_i D_i D_i^T,$$

$$\Gamma_{12i} = \left[\sqrt{\bar{\lambda}_{i\mathcal{K}_1^i}} X_i, \sqrt{\bar{\lambda}_{i\mathcal{K}_2^i}} X_i, \dots, \sqrt{\bar{\lambda}_{i\mathcal{K}_{m_k^i}^i}} X_i \right],$$

$$\Gamma_{22i} = \text{diag} \left\{ -X_{\mathcal{K}_1^i}, -X_{\mathcal{K}_2^i}, \dots, -X_{\mathcal{K}_{m_k^i}^i} \right\},$$

$$\Gamma_{13i} = \sqrt{-\left(\sum_{j \in \mathcal{S}_k^i, j \neq i} \underline{\lambda}_{ij} + \underline{\lambda}_{ii} \right)} X_i,$$

$$\Phi_{11i} = \text{He} (A_i X_i) - \sum_{j \in \mathcal{S}_k^i, j \neq i} \underline{\lambda}_{ij} X_i - B_i B_i^T + \varepsilon_i D_i D_i^T,$$

$$\begin{aligned} \Phi_{12i} &= \left[\sqrt{\bar{\lambda}_{i\mathcal{K}_1^i}} X_i, \sqrt{\bar{\lambda}_{i\mathcal{K}_2^i}} X_i, \dots, \sqrt{\bar{\lambda}_{i\mathcal{K}_{m_k^i}^i}} X_i \right], \\ \Phi_{22i} &= \text{diag} \left\{ -X_{\mathcal{K}_1^i}, -X_{\mathcal{K}_2^i}, \dots, -X_{\mathcal{K}_{m_k^i}^i} \right\}; \end{aligned} \quad (23)$$

then the parameters of $P_i R_i$ -sliding sector in (14) can be designed as $P_i = X_i^{-1}$, $Q_i = Y_i^{-1}$, $\Omega_i = r_i Y_i^{-1}$, $R_i = (1 - r_i) Y_i^{-1}$, and $S_i = B_i^T X_i^{-1}$.

Proof. For system (6), for all ΔA_i , $i = 1, 2, \dots, \mathbb{N}$, the conditions in (13) are equal to

$$\begin{aligned} & (A_i + \Delta A_i)^T P_i + P_i (A_i + \Delta A_i) \\ & + \sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} P_j + \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} P_j \\ & + \lambda_{ii} P_i - P_i B_i B_i^T P_i + Q_i < 0. \end{aligned} \quad (24)$$

In the following, we consider two cases: $i \in \mathcal{S}_k^i$ and $i \in \mathcal{S}_{uk}^i$ in the proof.

Case I ($i \in \mathcal{S}_k^i$). In this case, the conditions in (24) can be guaranteed when for $i = 1, 2, \dots, \mathbb{N}$, the following inequalities:

$$\begin{aligned} & \text{He} ((A_i + \Delta A_i)^T P_i) + \sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} P_j \\ & + \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} W_i + \lambda_{ii} P_i \\ & - P_i B_i B_i^T P_i + Q_i < 0, \\ & P_j \leq W_i, \quad j \in \mathcal{S}_{uk}^i, \end{aligned} \quad (25)$$

are satisfied. Substituting $\sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} = -\sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} - \lambda_{ii}$ into the first inequality above and utilizing the boundary information of λ_{ij} , one can see that the above inequalities can be guaranteed by

$$\begin{aligned} & \text{He} ((A_i + \Delta A_i)^T P_i) \\ & + \sum_{j \in \mathcal{S}_k^i, j \neq i} \bar{\lambda}_{ij} P_j - \left(\sum_{j \in \mathcal{S}_k^i, j \neq i} \underline{\lambda}_{ij} + \underline{\lambda}_{ii} \right) W_i \\ & + \bar{\lambda}_{ii} P_i - P_i B_i B_i^T P_i + Q_i < 0, \\ & P_j \leq W_i, \quad j \in \mathcal{S}_{uk}^i. \end{aligned} \quad (26)$$

Pre- and postmultiplying P_i^{-1} on both sides of the first inequality in above inequalities and letting $X_i = P_i^{-1}$, one has

$$\begin{aligned} & \text{He} ((A_i + \Delta A_i) X_i) + \sum_{j \in \mathcal{S}_k^i, j \neq i} \bar{\lambda}_{ij} X_i X_j^{-1} X_i \\ & - \left(\sum_{j \in \mathcal{S}_k^i, j \neq i} \underline{\lambda}_{ij} + \underline{\lambda}_{ii} \right) X_i W_i X_i + \bar{\lambda}_{ii} X_i \\ & - B_i B_i^T + X_i Q_i X_i < 0. \end{aligned} \quad (27)$$

Let $Y_i = Q_i^{-1}$, $Z_i = W_i^{-1}$, and applying Schur-complement technique and Lemma 9, one can obtain the first LMI condition in (21).

In addition, by using Lemma 10, one can see that the second inequality in (26) can be converted into the second LMI condition in (21).

Case II ($i \in \mathcal{S}_{uk}^i$). In this case, substituting $\lambda_{ii} = -\sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} - \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij}$ into (24), one has

$$\begin{aligned} & \text{He}((A_i + \Delta A_i)^T P_i) + \sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} P_j \\ & + \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} P_j - \left(\sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} + \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} \right) P_i \quad (28) \\ & - P_i B_i B_i^T P_i + Q_i < 0, \\ & i = 1, 2, \dots, \mathbb{N}. \end{aligned}$$

The conditions in (28) can be guaranteed provided that for $i = 1, 2, \dots, \mathbb{N}$, one has

$$\begin{aligned} & \text{He}((A_i + \Delta A_i)^T P_i) + \sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} P_j \\ & - \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} P_j - P_i B_i B_i^T P_i + Q_i < 0, \quad (29) \\ & P_j \leq P_i, \quad j \in \mathcal{S}_{uk}^i, \quad j \neq i. \end{aligned}$$

Taking use of the boundary information of λ_{ij} , one can see that the above inequalities can be guaranteed by

$$\begin{aligned} & \text{He}((A_i + \Delta A_i)^T P_i) + \sum_{j \in \mathcal{S}_k^i, j \neq i} \bar{\lambda}_{ij} P_j \\ & - \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \bar{\lambda}_{ij} P_j - P_i B_i B_i^T P_i + Q_i < 0, \quad (30) \\ & P_j \leq P_i, \quad j \in \mathcal{S}_{uk}^i, \quad j \neq i. \end{aligned}$$

Pre- and postmultiplying P_i^{-1} on both sides of the first inequality in above inequalities and letting $X_i = P_i^{-1}$ and $Y_i = Q_i^{-1}$, one has

$$\begin{aligned} & \text{He}((A_i + \Delta A_i) X_i) + \sum_{j \in \mathcal{S}_k^i, j \neq i} \bar{\lambda}_{ij} X_i X_j^{-1} X_i \\ & - \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \bar{\lambda}_{ij} X_i - B_i B_i^T + X_i Q_i^{-1} X_i < 0. \quad (31) \end{aligned}$$

Applying Schur-complement technique and Lemma 9 again, the above inequality (31) can be converted into the first LMI condition in (22).

Applying Lemma 10, one can see that the second inequality in (30) can be converted into the second condition in (22). By the design of the parameters of PR-sliding sector in (14), the proof of Theorem 11 is completed. $\square$

3.2. Variable Structure Control Design

Theorem 12. Considering the simplified $P_i R_i$ -sliding sector $\mathcal{S}_i$ (11) with the inner sector $\mathcal{S}_i^{\text{in}}$ (16) and the outer sector $\mathcal{S}_i^{\text{out}}$ (17), the following variable structure controller is designed to ensure that the state trajectories of the plant (6) enter into the inner sector from the outside of $P_i R_i$ -sliding sector and obtain mean-square quadratic stability of the closed-loop system. Consider

$$u = u_{i1} + u_{i2}, \quad (32)$$

where

$$u_{i1} = -\sigma_i(s_i(x), \delta_i(x)) (S_i B_i)^{-1} (S_i A_i x + K_i s_i(x)), \quad (33)$$

$$\begin{aligned} u_{i2} = & -\frac{1}{1 - \bar{d}_i} \sigma_i(s_i(x), \delta_i(x)) \\ & \times (S_i B_i)^{-1} \text{sign}(s_i(x)) \rho_i, \end{aligned} \quad (34)$$

$\rho_i = |S_i| |D_i| |F_i| |x| + \bar{d}_i |S_i A_i| |x| + \bar{d}_i K_i |s_i(x)|$ and the parameter $K_i > \max\{S_i B_i/2, H_i\}$ satisfies the following inequality:

$$2H_i \alpha_i^2 r_i Q_i + S_i^T S_i A_i + A_i^T S_i^T S_i > 0. \quad (35)$$

Proof. It is assumed that the initial state of the plant lies outside the $P_i R_i$ -sliding sector. In this case, we have $\sigma_i(s_i(x), \delta_i(x)) = 1$ and $u_{i1} = -(S_i B_i)^{-1} (S_i A_i x + K_i s_i(x))$. It can be easily checked that

$$\begin{aligned} s_i(x) \dot{s}_i(x) = & -K_i s_i^2(x) + s_i(x) S_i \Delta A_i x \\ & + s_i(x) S_i B_i u_{i2} + s_i(x) S_i \Delta B_i u. \end{aligned} \quad (36)$$

Noting that $\Delta B_i = B_i \Theta_i$ and $|\Theta| \leq \bar{d}_i < 1$, we have

$$\begin{aligned} s_i(x) \dot{s}_i(x) \leq & -K_i s_i^2(x) + s_i(x) S_i B_i u_{i2} \\ & + |s_i(x)| |S_i| |\Delta A_i| |x| + \bar{d}_i |s_i(x)| |S_i B_i| |u|. \end{aligned} \quad (37)$$

In view of $u = u_{i1} + u_{i2}$, it is easy to check that

$$\begin{aligned} s_i(x) \dot{s}_i(x) \leq & -K_i s_i^2(x) + s_i(x) S_i B_i u_{i2} \\ & + |s_i(x)| |S_i| |\Delta A_i| |x| + \bar{d}_i |s_i(x)| \\ & \times |S_i B_i| |u_{i1}| + \bar{d}_i |s_i(x)| |S_i B_i| |u_{i2}|. \end{aligned} \quad (38)$$

Substituting $u_{i1} = -(S_i B_i)^{-1} (S_i A_i x + K_i s_i(x))$ into the above inequality and noting that $S_i B_i = B_i^T P_i B_i > 0$, one can see that

$$\begin{aligned} s_i(x) \dot{s}_i(x) \leq & -\left(1 - \bar{d}_i\right) K_i s_i^2(x) + \left(1 - \bar{d}_i\right) s_i(x) S_i B_i u_{i2} \\ & + |s_i(x)| |S_i| |\Delta A_i| |x| + \bar{d}_i |s_i(x)| |S_i A_i| \\ & \times |x| + \bar{d}_i s_i(x) S_i B_i u_{i2} + \bar{d}_i |s_i(x)| |S_i B_i| |u_{i2}|. \end{aligned} \quad (39)$$

Taking use of u_{i2} in (34) where $\rho_i = |S_i||D_i||F_i||x| + \bar{d}_i|S_iA_i||x| + \bar{d}_iK_i|s_i(x)| > \bar{d}_i|S_iA_i||x| + \bar{d}_iK_i|s_i(x)|$, one can see that

$$s_i(x) \dot{s}_i(x) \leq -K_i s_i^2(x) + \bar{d}_i s_i(x) S_i B_i u_{i2} + \bar{d}_i |s_i(x)| |S_i B_i| |u_{i2}|. \quad (40)$$

According to the design of u_{i2} again, one can easily observe that

$$\begin{aligned} & s_i(x) S_i B_i u_{i2} + |s_i(x)| |S_i B_i| |u_{i2}| \\ & \leq -\frac{1}{1-\bar{d}_i} |s_i(x)| \rho_i + \frac{1}{1-\bar{d}_i} |s_i(x)| \\ & \quad \times |S_i B_i| |(S_i B_i)^{-1}| |\text{sign}(s_i(x))| \rho_i = 0. \end{aligned} \quad (41)$$

It follows from (40) and (41) that

$$s_i(x) \dot{s}_i(x) \leq -K_i s_i^2(x). \quad (42)$$

Thus the state trajectory of the closed-loop system will be driven into the inner sector of the $P_i R_i$ -sliding sector.

In the following, we will prove that the P_i -norm keeps decreasing. Consider

$$\begin{aligned} \dot{L}_i(t) &= x^T \left\{ (A_i + \Delta A_i)^T P + P(A_i + \Delta A_i) + \sum_{j=1}^N \lambda_{ij} P_j \right\} x \\ &\quad + 2x^T P_i (B_i + \Delta B_i) u \\ &\leq s_i^2(x) - \delta_i^2(x) - x^T R_i x + 2x^T P_i (B_i + \Delta B_i) u. \end{aligned} \quad (43)$$

Noting that $\Delta B_i = B_i \Theta_i$ and $S_i = B_i^T P_i$, one can see that

$$\begin{aligned} \dot{L}_i(t) &\leq s_i^2(x) - \delta_i^2(x) - x^T R_i x + 2s_i(x) u_{i1} \\ &\quad + 2s_i(x) u_{i2} + 2s_i(x) \Theta_i (u_{i1} + u_{i2}). \end{aligned} \quad (44)$$

In the following, we first prove that

$$s_i(x) u_{i2} + \bar{d}_i |s_i(x)| |u_{i1} + u_{i2}| \leq 0. \quad (45)$$

Substituting (33) and (34) into the left side of above inequality, one has

$$\begin{aligned} & s_i(x) u_{i2} + \bar{d}_i |s_i(x)| |u_{i1} + u_{i2}| \\ &= -\frac{1}{1-\bar{d}_i} (S_i B_i)^{-1} |s_i(x)| \rho_i + \bar{d}_i |s_i(x)| \\ &\quad \times \left| (S_i B_i)^{-1} S_i A_i x + K_i (S_i B_i)^{-1} s_i(x) \right. \\ &\quad \left. + \frac{1}{1-\bar{d}_i} (S_i B_i)^{-1} \text{sign}(s_i(x)) \rho_i \right|. \end{aligned} \quad (46)$$

Noting that $S_i B_i = B_i^T P_i B_i > 0$ and $\bar{d}_i < 1$, we have

$$\begin{aligned} & s_i(x) u_{i2} + \bar{d}_i |s_i(x)| |u_{i1} + u_{i2}| \\ &\leq -(S_i B_i)^{-1} |s_i(x)| \rho_i + \bar{d}_i (S_i B_i)^{-1} |s_i(x)| \\ &\quad \times |S_i A_i x + K_i s_i(x)|. \end{aligned} \quad (47)$$

Since $\rho_i = |S_i||D_i||F_i||x| + \bar{d}_i|S_iA_i||x| + \bar{d}_iK_i|s_i(x)|$, it is easy to check that

$$\rho_i \geq \bar{d}_i |S_i A_i x + K_i s_i(x)|. \quad (48)$$

It follows from (47) and (48) that (45) is established. Thus we have

$$\dot{L}_i(t) \leq s_i^2(x) - \delta_i^2(x) - x^T R_i x + 2s_i(x) u_{i1}. \quad (49)$$

Substituting (33) into the above inequality (49) and noting that $K_i > (S_i B_i/2)$, $2K_i \alpha_i^2 r_i Q_i + S_i^T S_i A_i + A_i^T S_i^T S_i > 0$ and $s_i^2(x) \geq \alpha_i \delta_i^2(x)$, one can see that

$$\begin{aligned} \dot{L}_i(t) &\leq s_i^2(x) - \delta_i^2(x) - x^T R_i x + 2s_i(x) \\ &\quad \times \left(-(S_i B_i)^{-1} S_i A_i x - K_i (S_i B_i)^{-1} s_i(x) \right) \\ &= -(S_i B_i)^{-1} x^T \left(2K_i \alpha_i^2 r_i Q_i + S_i^T S_i A_i + A_i^T S_i^T S_i \right) x \\ &\quad - x^T R_i x \\ &< -x^T R_i x. \end{aligned} \quad (50)$$

After being forced into the inner sector of the $P_i R_i$ -sliding sector and before moving out of the $P_i R_i$ -sliding sector, the control input $u(t)$ is set equal to zero with $\sigma_i(s_i(x), \delta_i(x)) = 0$. In this case, $|s_i(x)| \leq \delta_i(x)$ and

$$\begin{aligned} \dot{L}_i(t) &= x^T \left\{ (A_i + \Delta A_i)^T P + P(A_i + \Delta A_i) + \sum_{j=1}^N \lambda_{ij} P_j \right\} x \\ &\leq s_i^2(x) - \delta_i^2(x) - x^T R_i x \leq -x^T R_i x. \quad \forall x \in \mathcal{S}_i. \end{aligned} \quad (51)$$

Once the state trajectory of the plant runs out of the $P_i R_i$ -sliding sector with zero control input, the control input (32)–(34) will make it move back to the inside of the inner sector again, while the P_i -norm $|x|_{P_i}$ keeps decreasing.

Thus, from the above proof, one can see that it guarantees that the state trajectory of the plant moves from the outside of the $P_i R_i$ -sliding sector into the inside of the inner sector and the Lyapunov function $L_i(x) = x^T P_i x$ keeps decreasing in the whole state space; then mean-square quadratic stability of the closed-loop system is established. $\square$

4. Illustrative Example

Consider the Markov jump linear system with four operation modes:

$$\begin{aligned}
 A_1 &= \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}, & A_2 &= \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}, \\
 A_3 &= \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, & A_4 &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, & B_2 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\
 B_3 &= \begin{bmatrix} 2 \\ 1 \end{bmatrix}, & B_4 &= \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \\
 D_1 &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, & D_2 &= \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}, \\
 D_3 &= \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}, & D_4 &= \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, \\
 F_1 &= \begin{bmatrix} 0 & -0.1 \end{bmatrix}, & F_2 &= \begin{bmatrix} 1 & 0.3 \end{bmatrix}, \\
 F_3 &= \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, & F_4 &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}.
 \end{aligned} \tag{52}$$

For simulation, we take $E_1(t, x) = 0.1 \sin(t)$, $E_2(t, x) = 0.1 \sin(2t)$, $E_3(t, x) = 0.5 \sin(t)$, $E_4(t, x) = 0.4 \sin(2t)$, $\Theta_1 = 0.2 \sin(2)$, $\Theta_2 = 0.3 \cos(2t)$, $\Theta_3 = 0.2 \cos(t)$, and $\Theta_4 = 0.2 \sin(2t)$.

The transition rates matrix is

$$\Lambda = \begin{bmatrix} -1.5 & 0.2 & ? & ? \\ ? & ? & 0.5 & 0.9 \\ 1 & ? & -2 & ? \\ 0.1 & \alpha & ? & \beta \end{bmatrix}, \tag{53}$$

where $0.5 \leq \alpha \leq 2$ and $-0.8 \leq \beta \leq -0.1$. Since there exist the completely unknown transition rates “?” and boundary known transition rates α and β , the reported methods in [21, 22] cannot work. Solving the LMIs conditions (21)–(22) in Theorem 11, one can obtain that positive definite matrices are as follows:

$$\begin{aligned}
 P_1 &= \begin{bmatrix} 6.2534 & -6.4840 \\ -6.4840 & 18.4380 \end{bmatrix}, & P_2 &= \begin{bmatrix} 6.8785 & -7.1343 \\ -7.1343 & 20.0458 \end{bmatrix}, \\
 P_3 &= \begin{bmatrix} 8.5811 & -9.6120 \\ -9.6120 & 15.6407 \end{bmatrix}, & P_4 &= \begin{bmatrix} 6.1859 & -1.5374 \\ -1.5374 & 1.1026 \end{bmatrix}, \\
 Q_1 &= \begin{bmatrix} 6.9519 & -7.5088 \\ -7.5088 & 22.4304 \end{bmatrix}, \\
 Q_2 &= \begin{bmatrix} 13.4209 & -14.1627 \\ -14.1627 & 40.0263 \end{bmatrix}, \\
 Q_3 &= \begin{bmatrix} 11.4377 & -11.6823 \\ -11.6823 & 18.2712 \end{bmatrix}, \\
 Q_4 &= \begin{bmatrix} 10.3808 & -2.6205 \\ -2.6205 & 2.0926 \end{bmatrix}.
 \end{aligned} \tag{54}$$

FIGURE 1: The response curves of the system states.

FIGURE 2: The response curve of the control input.

Selecting $\alpha_1 = 0.8$, $\alpha_2 = 0.9$, $\alpha_3 = 0.6$, $\alpha_4 = 0.6$, $r_1 = 0.8$, $r_2 = 0.9$, $r_3 = 0.8$, $r_4 = 0.8$, $K_1 = 20$, $K_2 = 120$, $K_3 = 40$, $K_4 = 50$, then the condition in Theorem 12 is satisfied. For the purpose of simulation, let $\bar{d}_1 = 0.2$, $\bar{d}_2 = 0.3$, $\bar{d}_3 = 0.2$, and $\bar{d}_4 = 0.2$. Using these data, a simulation program has been done in Matlab. The simulation results are presented in Figures 1–3. It can be seen from Figure 1 that the system state can converge to the equilibrium point in less than 5 seconds. Figure 2 shows the response curve of the control input. One can easily see that no chattering phenomenon happens in the whole control process. The transition modes of system are given in Figure 3. It can be seen that the considered continuous-time Markov jump linear uncertain system is stochastic quadratic stable in spite of mismatched uncertainty and the transition rate matrix which covers completely known, boundary unknown, and completely unknown elements.

FIGURE 3: The response curve of the transition modes.

5. Conclusions

In this paper, the problem of designing sliding sector-based variable structure controllers is considered for Markov jump linear uncertain continuous-time time-invariant systems with unknown transition rates. First, the related notions about sliding sector for continuous-time MJLSs are presented and sufficient conditions for the design of the sliding sector are given by LMIs. Second, the variable structure control law is presented to ensure the mean-square quadratic stability of the closed-loop system, and simulation results finally have verified the effectiveness of the proposed method. In our future research, how to design dynamic output feedback variable structure controller based on sliding sector technique and considering the applying of the sliding sector technique in industrial robotic systems are interesting work. Finally, the authors would like to thank the associate editor and all the reviewers for their constructive comments and suggestions for improving this paper.

Acknowledgments

This work was supported in part by the Funds of National Science of China (nos. 11301276, 61104123), the Natural Science Funds of Jiangsu Province (BK20130984, BK20131000), the Scientific Research Foundation of Nanjing University of Information Science & Technology (S8112075001, S8112081001), and Laboratory Open Fund of Nanjing University of Information Science & Technology (N1081005112).

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