Research Article

Research on Stochastic Resonance Signal’s Recovery

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Using stochastic resonance to detect weak periodic signals has been widely used in various fields of science, which attracts much attention of researchers due to its advantages of revealing recessive periodic laws. This paper utilized this method to seek the underlying rule of setting weather index, so we can find that how to obtain the accurate expression of original periodic law by further investigation. This paper deals with the noise-contained signal restoring on the basis of the established system coupling the inversion system and bistable system. The simulation shows that this signal recovery method inversion effect is better and the application range is wider.

1. Introduction

As a new type of insurance project, weather index insurance is a hot issue in the current weather economy research. However, there still exists much difficulty in the weather index insurance research; particularly, the setting of weather index is closely related to the hidden rule obtained by using advanced scientific technology and method.

The setting of weather index cannot do without plenty of meteorological data. Especially mining the potential rules of meteorological data helps to define the critical point of meteorological index, which contributes to find reasonable effective method for setting the meteorological index. The application of stochastic resonance method [1] provides a new idea for exploring the underlying rule of data.

Since the concept of stochastic resonance (SR) has been proposed, it has attracted researchers’ interests, and it has been studied in physics, chemical, biological, economic, and other fields [2–7]. The ideas and methods of SR have broken the old mode of thinking that noise was harmful and profitless in the processing of signal. It identified the unknown weak periodic signal by using the synergies between the noise, signal, and nonlinear systems [8]. Some periodic laws are hidden from meteorological data, which are hard to be found due to various complicated natural factors. Even if the acquisition equipment precision is improved, only the disturbance of noise is reduced, and the reliability of data is enhanced. Via stochastic resonance method, it is possible to mine the potential rule of meteorological data.

In the process of meteorological data signal acquisition, due to the complex and volatile natural factors and the inevitable noise disturbance, it is sure that there will appear signal loss or distortion. How to identify and recover the meteorological data signal? On one hand, we can improve the accuracy of the collection equipment, reduce noise, and amplify data signal. On the other hand, we can improve the signal processing method; for example, according to stochastic resonance, the disturbed data signal recovery can be enhanced by multifactor synergism. This contributes to the identification of the unknown weak periodic weather signal.

Initially, we are committed to enhance unknown weak periodic signal by adding noise in the study of SR. Obviously, this method has strong contingency. In 1992, Anishchenko et al. [9] found that SR can also be achieved by changing the system parameters, and the range of signal recognition was expanded. Then, Bulsara and Gammatoni [10] also proved the importance of adjust system parameters on the application of SR. Professor Leng et al. [11] proposed a theory of SR that based on parameter adjustment, analyzed the law of adjusting the parameters of SR, and believed that the two methods of SR are the same essentially.

In the process of the realization of the SR, affected by the nonlinearity of the system, signal waveform is distorted at the inflection point in the system potential function, and because
SR will amplify the signal, we cannot get the accurate amplitude of the signal, and the research of output waveform has been limited. Therefore, it usually identified the frequency characteristics of periodic input signal in the frequency domain by using the stochastic resonance technology.

However, such identification methods cannot meet the requirements in dealing with nonperiodic signal or when you need to obtain more valuable information, a recovery processing of the signal in the time domain is needed. In [12], an inversion formula which is based on the bistable nonlinear systems was proposed to recover output waveform in the time domain. In [13], a detailed and reasonable explanation of the periodic signal inversion principle was given by the particle Kinematic. In [14–16], Professor Leng et al. studied the reasons of the signal distortion in inversion process, respectively, the law of resonance and inversion of signal with different signal amplitude and system parameters, and then proposed two inversion methods for different signals. With the deepening of the study, the combination of the inversion system and bistable system will become a new signal detection method. In [17–19], it analyzed the influence of system parameters, the intensity of noise, the signal amplitude, and frequency to the SR phenomenon in the nonlinear bistable system SR model. Wu et al. proposed an adaptive strategy to adjust the parameters of the system [20]. Whether the selected system parameters can reach the optimal state of SR, which will affect the effect of signal recovery. Therefore, how to choose the system parameters becomes one of the important topics of study SR.

Signal waveform will generate pulse distortion at the inflection point of the system in the inversion system [12–16]. To restore the signal, firstly we need to find the inflection point, and to process the waveform of the inflection point, then to take the nonlinear interpolated in serious distortion parts; finally, polynomial fitting should be done for the signal. This method is very complex, and too many parameters need to be adjusted. Therefore, it is difficult to carry out, and the estimate of the signal amplitude is not accurate.

In this paper, according to the reason of the waveform distortion in the processing of bistable SR and signal inversion, a bistable recovery system combines the inversion system, and bistable system is established directly to recovery denoise signal. At the same time, it is utilized the relationship between the bistable system parameters $b$ and noise variance, and used an adaptive optimization method. Firstly, to sample noisy periodic signal and estimate the noise variance and the signal frequency and then regard maximum SNR as the optimization target to determine the optimal system parameters, at last, a cascade recovery system was used to further filtering signal; the simulation results show that the effect is good.

2. The Bistable Stochastic Resonance Systems and the Inversion System

SR can be considered as a process that amplified excitation signal containing noise $H(t)$ by the bistable system with potential function $U(x)$, the system equation is as follows:

$$\ddot{x} + \delta \dot{x} = -\frac{dU(x)}{dx} + H(t), \ \ \ \ (1)$$

where $\delta$ is an infinitesimal and was ignored, and set $\delta = 1$, so the Langevin equation is as follows:

$$\dot{x} = -\frac{dU(x)}{dx} + H(t). \ \ \ \ (2)$$

In the classical model of the nonlinear bistable stochastic resonance, an excitation signal containing noise $H(t)$ consists of a random noise $n(t)$ and external periodic signal $s(t); n(t)$ is a Gaussian white noise with zero mean and autocorrelation $E(n(t)) = 0, n(t) = \sigma \varepsilon(t); \varepsilon(t)$ is a white noise with zero mean and its variances equal to 1. The external periodic signal is a sine signal that $s(t) = A \sin(2\pi ft)$.\n
Assuming that the nonlinear bistable system $U(x)$ would have the following bistable potential function:

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4, \ \ \ (3)$$

where $a$, $b$ are considered as the system parameters, and the Langevin equation can be simplified as follows:

$$\dot{x} = ax - bx^3 + s(t) + n(t). \ \ \ \ (4)$$

SR is mainly used for the small parameter signal, so that the amplitude of the periodic input signal and noise intensity are required to be much less than 1. When the signal does not meet the parameters required, we can process after that the signal is converted to a small parameter by the subsampling method [21]. So it is assumed that the noise is far less than 1.

Because the frequency of the signal is very small, $\dot{x}(t)$ can be approximated to zero, at this time, (4) is satisfied:

$$ax(t) - bx^3(t) + s(t) + n(t) = 0. \ \ \ \ (5)$$

The above equation can be simplified to

$$s(t) + n(t) \approx -ax(t) + bx^3(t). \ \ \ \ (6)$$

It is averaged on both sides of (6), because the means of the noise $E[n(t)] = 0$, so we can obtain the following equation:

$$E [s(t)] = -a (E [x(t)]) + b (E [x(t)])^3. \ \ \ \ (7)$$

Signal inversion equation obtained from the statistical average sense [12] is

$$s(t) = -ax(t) + bx^3(t). \ \ \ \ (8)$$

In [12], it proposed an inversion system in accordance with the above inversion equation:

$$h(x) = -ax + bx^3. \ \ \ \ (9)$$

Figure 1(a) is a weak sine signal $s(t) = 0.3 \sin(2\pi \times 0.01t)$ containing a white noise with zero mean, and its variance is equal to 0.6, which is unable to distinguish a sine signal by the noise. SR of the measured noisy signal is generated in the bistable nonlinear system, which system parameters $a = 1, b = 1$, and the output signal is shown in Figures 1(b) and 1(c);
it is shown that SR enhances the signal amplitude, so that the weak signal is prominent, and the frequency of the sine signal has been detected, but the time domain waveform of the sine signals has become a trapezoid wave, and the waveform is distorted the waveform cannot accurately show the input signal.

In order to obtain a more realistic signal, Figure 2 shows that an inversion signal obtained the output signal from the bistable stochastic resonance by the inversion system (9); the parameters of inversion system are \( a = 1 \), \( b = 1 \).

Figure 2 shows that the amplitude of inversion signal is reduced to some extent, and the waveform is much closer to the input signal, but the distortion is large, so that the inversion is ineffective.

Assuming that the noise intensity is zero, (6) can be approximated to

\[
s(t) = -ax + bx^3.
\]

(10)

Derivative with respect to \( t \) on both sides of the equation:

\[
\dot{s}(t) = -a \cdot \dot{x}(t) + 3bx^2 \cdot \dot{x}(t).
\]

(11)

The above equation can be simplified to

\[
\dot{x}(t) = \frac{s(t)}{-a + 3bx^2}.
\]

(12)

Obviously, the system output signal \( x(t) \) no longer strictly follows input signal \( s(t) \), and when the signal is in the system inflection \( x = \pm \sqrt{a/3b} \), \( \dot{x} \rightarrow \infty \), the output signal waveform has a pulse distortion.

To make the output signal approximated to the periodic input signal \( s(t) \), both sides of the formula (12) multiply \(-a + 3bx^2\):

\[
\dot{x} \cdot (-a + 3bx^2) = \dot{s}(t).
\]

(13)

Therefore, we find a recovery system:

\[
h(x) = -a_1 + 3b_1x^2.
\]

(14)

Then, the bistable recovery system combines the inversion system, and bistable system is established; the system equation is as follows:

\[
\dot{x} = (ax - bx^3 + s(t) + n(t)) \cdot (-a_1 + 3b_1x^2).
\]

(15)

Based on the foregoing assumptions that noise is far less than 1, so we adjust the parameters of the inversion system being the same as the parameters of bistable system, that is, \( a_1 = a, b_1 = b \). If the noise is too large, we should reduce the noise before recovering the signal.

Figure 3 is the block diagram of bistable recovery system.

Set the parameters of the bistable recovery system \( a = 1 \), \( b = 1 \), the inversion signal is obtained from noisy periodic signal in Figure 1 by bistable recovery system, which is shown in Figure 4.

The graph of comparing the recovery signal with a sinusoidal signal to be measured is shown in Figure 4(a); we can see that the restored signal had a phase lag about 180°. In order to observe whether the recovery waveform of the signal is closer to a sinusoidal signal to be measured, we delay the phase of the sine signal 180° before comparing with recovery signal in Figure 4(b).
Comparing Figure 4(b) with Figure 2, it is obvious that the result of bistable recovery system is much better; the amplitude and waveform of the recovery signal are more closed to the input signal.

3. Adaptive Parameter Adjustment

3.1. The Parameters of SR System. The amplitude and frequency of the signal, noise intensity, and the system parameters may constrain and influence SR phenomenon in Bistable stochastic resonance system. In order to reach the optimum state and take the maximum value of the output SNR, how to adjust the various parameters are essential.

On account of the characteristics of Gaussian white noise \( \mathbb{E}(n(t)) = 0, n(t) = \sigma \epsilon(t) \), the Langevin equation (4) is a transformed substitution, that \( y = x/\sigma, b_1 = b\sigma^2, s_1 = s/\sigma \), and \( s_1 \) is related with SNR. In this case, the Langevin equation becomes

\[
\dot{y} = ay - b_1 y^3 + s_1(t) + \epsilon(t).
\]  

(16)

\( \epsilon(t) \) is a white noise, its mean is 0, variance is 1, and intensity is invariant, so the main factor is the system parameters \( a \) and \( b_1 \).

When \( b_1 = b\sigma^2 \), the bistable system parameters \( b \) and the noise variance \( \sigma^2 \) play the same role to \( b_1 \), which means that adding noise or change the system parameters \( b \) causes the same effect.

3.2. The Method of Adaptive Adjusting the System Parameters. According to the adiabatic approximation theory [17], we can get the output SNR of formula (4) as follows:

\[
\text{SNR} = \frac{\sqrt{2} a^2 A^2}{b \sigma^4} \cdot e^{-a^2/2b\sigma^2}.
\]  

(17)

Figure 5 shows the curve graph of the output SNR changing with the noise variance \( \sigma^2 \), when taking \( a \) and \( b \) with different values.

Obviously, increasing noise variance, SNR firstly increases and then decreases, which generates peak, and the position of peak (ie, the optimal noise variance) is different when the SR system parameters is changed.

From the energy conversion, the way to increase SNR by adding noise or input periodic signal will increase the energy
of the output signal, which means that the amplitude of the output signal will be greatly increased, so it is unfavourable to restored signal. So that we utilize the adaptive optimization method to change the parameters by the relationship between the bistable system parameters b and noise variance. The flow diagram is shown in Figure 6, it is divided into the following three steps:

1. Set the bistable system parameters \( a = 1 \) and \( b = 1 \), by adding noise to the system to achieve the best state of stochastic resonance (SNR is maximum), then to estimate the total noise variance \( \sigma_1^2 \).

2. Detect the frequency of the signal \( f_0 \) at the state of the optimum SR, and sampling the received signal with the sampling frequency \( f_s = M f_0 \) (generally, \( M = 500 \)).

3. Estimated the noise variance \( \sigma_0^2 \) in the input signal, calculate optimal system parameters \( b = \sigma_1^2 / \sigma_0^2 \), and configured the bistable recovery system parameters \( a = 1 \) and \( b = 1 \), then recover to input signal.

3.3. The Simulation of Adaptive Regulation System. Assuming noisy periodic input signal \( H(t) = A \sin(2\pi ft) + n(t) \), where the amplitude of the sine signal \( A = 0.3 \), the frequency \( f = 0.01 \) Hz, noise \( n(t) \) is a Gaussian white noise with the mean of 0 and variance of 0.6.

The first step is setting the bistable system parameters \( a = 1, b = 1 \), when SNR is the largest, and to estimate noise variance \( \sigma_1^2 \), the second step is detecting the frequency of the signal \( f_0 \) at the state of the optimum SR, and determining the sampling frequency \( f_s = 500 f_0 \). The simulation results are shown in Figure 7, the best noise variance \( \sigma_1^2 = 1.91 \), \( \text{SNR}_{\text{max}} = 23.2712 \), the frequency of the signal \( f_0 = 0.01 \), and the sampling frequency \( f_s = 5 \).

The third step is calculating optimal system parameters \( b = 3.1833 \) in accordance with \( b = \sigma_1^2 / \sigma_0^2 \), configuring the bistable recovery system parameters \( a = 1, b = 3.1833 \). Figure 8 shows the recovery signal by the bistable recovery system.

As shown in Figure 8, although the recovery waveforms of signals had a certain degree of distortion, it is not strictly agreed to the variation law of the sine signal. But the amplitude of the recovery signal is very close to the amplitude of the input signal, so to ignore this slight error, we can extract the amplitude of the signal. Meanwhile, the frequency
of the signal had been measured on the first step, we find the frequency $f$ and amplitude $A$ to the input signal $A \sin(2\pi f t)$, then we can determine this sine signal.

4. Cascade Recovery

It’s known that cascade SR has a good denoise filtering effects [22]. Simultaneously, cascade recovery system can also achieve the effect of denoise filtering. Figure 9 shows the output waveform of three cascaded recovery system. We can find that the previous output is shaped by the recovery systems, so that the final output waveform contour of the system becomes smoother.

5. Conclusion

By establishing the coupling relationship of the inversion system and the bistable system, we can process noisy periodic input signal directly and use the auxiliary adaptive optimization method and the cascade signal recovery method, the recovery signal is much closer to the actual weak periodic signal; the simulation results show that this method has better speed and accuracy. In order to improve the precision of setting weather index, we will convert the meteorological data to the information flow and thereby use this technique to identify and recover the periodic signal hidden from the meteorological data.

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References


