Research Article

Exact Solutions of Equation Generated by the Jaulent-Miodek Hierarchy by (G'/G) -Expansion Method

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The (G'/G) -expansion method is proposed for constructing more general exact solutions of the nonlinear $(2 + 1)$ -dimensional equation generated by the Jaulent-Miodek Hierarchy. As a result, when the parameters are taken at special values, some new traveling wave solutions are obtained which include solitary wave solutions which are based from the hyperbolic functions, trigonometric functions, and rational functions. We find in this work that the (G'/G) -expansion method give some new results which are easier and faster to compute by the help of a symbolic computation system. The results obtained were compared with tanh method.

1. Introduction

In the present paper, we obtained exact solutions of the nonlinear $(2 + 1)$ -dimensional equation (1) generated by the Jaulent-Miodek Hierarchy [1] in the form

$$w_t = -\frac{1}{4}(w_{xx} - 2w^3)_x - \frac{3}{4}\left(\frac{1}{4}\partial_x^{-1}w_{yy} + w_x\partial_x^{-1}w_y\right), \quad (1)$$

where y is a spatial variable, and $w(x, y, t)$ is the analytic function with respect to x , y , and t . In addition, ∂_x^{-1} is the inverse of ∂_x with $\partial_x\partial_x^{-1} = \partial_x^{-1}\partial_x = 1$ and $(\partial_x^{-1}f)(x) = \int_{-\infty}^x f(t)dt$, under the decaying condition at infinity. This equation was studied by Liu and Yan by using qualitative analysis method [2]. It is clear that searching for explicit solutions of nonlinear evolution equation (NLEEs) by using various method has become the main aim for many authors. Many powerful methods have been created and developed to obtain analytic solutions of (NLEEs), such as the the tanh-coth method [3–5], sine-cosine method [6], homogeneous balance method [7], exp-function method [8, 9], Hirota bilinear transformation [10, 11], Jacobi elliptic function method [12], and solitary wave ansatz method [13]. One of the most effective direct method to build traveling wave solution of (NLEEs) is the (G'/G) -expansion method, which was first

proposed by Wang et al. [14]. It is assumed that the traveling wave solutions can be expressed by a polynomial in (G'/G) , where $G = G(\zeta)$ satisfies the following second-order linear ordinary differential equation $G''(\zeta) + \lambda G'(\zeta) + \mu G(\zeta) = 0$, where $\zeta = x - ct$, and λ , μ , and c are constants. Next, Bekir applied the method to some nonlinear evolution equations earning some traveling wave solutions [15]. After that, Zhang et al. have improved and generalized the (G'/G) -expansion method to seek exact solutions of other nonlinear evolution equations [16]. Moreover, Zhang et al. have further extended the method to deal with evolution equations with variable coefficients [17]. In fact the (G'/G) -expansion method has been successfully applied to obtain exact solution for a variety of (NLEEs) [12, 18–35]. In this paper, the (G'/G) -expansion method is proposed for constructing more general exact solutions of the above nonlinear $(2 + 1)$ -dimensional equation generated by the Jaulent-Miodek Hierarchy. Our paper is organized as follows: in Section 2, the description of the (G'/G) -expansion method is provided; in Section 3, we spell out the applications of this method to the nonlinear $(2 + 1)$ -dimensional equation; in Section 4, application of the tanh method on the nonlinear $(2 + 1)$ -dimensional equation and finally in Section 5, some conclusions are given.

2. Description the (G'/G) -Expansion Method

In this section we describe the (G'/G) -expansion method for finding traveling wave solutions of (NLEEs). Suppose that a nonlinear evolution equation, say in three independent variables x , y , and t , is given by

$$p(u, u_t, u_x, u_y, u_{xt}, u_{yt}, u_{tt}, u_{xx}, u_{yy}, \dots) = 0, \quad (2)$$

where $u = u(x, y, t)$ is an unknown function, and p is a polynomial in $u = u(x, y, t)$ and its various partial derivatives, in which highest-order derivatives and nonlinear terms are involved. To determine the (G'/G) -expansion method, we take the following six steps.

Step 1. To find the traveling wave solutions of (2), we introduce the wave variable

$$u(x, y, t) = u(\zeta), \quad \zeta = x + y + ct, \quad (3)$$

where c is constant. Substituting (3) into (2), we obtain the following ordinary differential equations (ODE):

$$p(u, cu', u', cu'', c^2u'', u'', \dots) = 0. \quad (4)$$

Step 2. If necessary we integrate (4) as many times as possible and set the constants of integration to be zero for simplicity.

Step 3. We suppose the solution of nonlinear partial differential equation can be expressed by a polynomial in (G'/G) as

$$u(\zeta) = \sum_{i=0}^m a_i \left(\frac{G'}{G} \right)^i, \quad (5)$$

where $G = G(\zeta)$ satisfies the second-order linear ordinary differential equation

$$G''(\zeta) + \lambda G'(\zeta) + \mu G(\zeta) = 0, \quad (6)$$

where $G' = dG/d\zeta$, $G'' = d^2G/d\zeta^2$, and a_i , λ , and μ are real constants with $a_m \neq 0$.

Step 4. The positive integer m can be determined by considering the homogeneous balance between the highest-order derivatives and nonlinear terms appearing in (4) as follows: if we define the degree of $u(\zeta)$ as $D[u(\zeta)] = m$, then the degree of other expressions is defined by

$$D \left[u^r \left(\frac{d^q u}{d\zeta^q} \right)^s \right] = mr + s(q + m). \quad (7)$$

Therefore, we can get the value of m in (5).

Step 5. Substituting (5) into (4), using (6), collecting all terms with the same order of (G'/G) together, and then setting each coefficient of this polynomial to zero yield a set of algebraic equations for a_i , c , λ , and μ .

Step 6. Substituting a_i , c , λ , and μ obtained in Step 5 and the general solutions of (6) into (5), we obtain traveling wave solutions of the nonlinear (PDE).

3. Application the (G'/G) -Expansion Method

3.1. Equation Generated by the Jaulent-Miodek Hierarchy. In this section, the (G'/G) -expansion method will be applied to the equation generated by the Jaulent-Miodek Hierarchy in the form

$$w_t = -\frac{1}{4}(w_{xx} - 2w^3)_x - \frac{3}{4} \left(\frac{1}{4} \partial_x^{-1} w_{yy} + w_x \partial_x^{-1} w_y \right). \quad (8)$$

We first remove the integral term in (8) by introducing the potential

$$w(x, y, t) = u_x(x, y, t), \quad (9)$$

to carry (8) to the equation

$$u_{xt} + \frac{1}{4} u_{xxxx} - \frac{3}{2} u_x^2 u_{xx} + \frac{3}{16} u_{yy} + \frac{3}{4} u_{xx} u_y = 0. \quad (10)$$

Let $u(x, y, t) = u(\zeta)$, $\zeta = x + y + ct$, where c is the wave speed. Substituting the above traveling wave variable $\zeta = x + y + ct$ in to (10) yields

$$cu'' + \frac{1}{4} u'''' - \frac{3}{2} (u')^2 u'' + \frac{3}{16} u'' + \frac{3}{4} u'' u' = 0. \quad (11)$$

Integrating (11) above with respect to ζ once yields

$$16cu' + 4u''' - 8(u')^3 + 3u' + 6(u')^2 = 0. \quad (12)$$

For simplicity, the integration constant is taken as zero. Assume that

$$u' = v. \quad (13)$$

Substituting (13) into (12), we obtain ordinary differential equation as follows:

$$16cv + 4v'' - 8v^3 + 3v + 6v^2 = 0. \quad (14)$$

We suppose that the solution of (14) has the following formal solution:

$$v(\zeta) = \sum_{i=0}^m a_i \left(\frac{G'}{G} \right)^i, \quad (15)$$

where $G = G(\zeta)$ satisfies the second-order linear ordinary differential equation $G''(\zeta) + \lambda G'(\zeta) + \mu G(\zeta) = 0$.

According to Step 4 $m = 1$, and therefore (15) become

$$v(\zeta) = a_0 + a_1 \left(\frac{G'}{G} \right), \quad a_1 \neq 0. \quad (16)$$

On substituting (16) along with (6) into (14), collecting all terms with the same powers of (G'/G) , and setting

each coefficient to zero yield a set of simultaneous algebraic equations for a_0 , a_1 , c , λ , and μ as follows:

$$\begin{aligned} \left(\frac{G'}{G}\right)^0 : 16ca_0 + 4a_1\lambda\mu - 8a_0^3 + 3a_0 + 6a_0^2 &= 0, \\ \left(\frac{G'}{G}\right)^1 : 16ca_1 + 4(a_1\lambda^2 + 2a_1\mu) \\ &\quad - 24a_0^2a_1 + 3a_1 + 12a_0a_1 = 0, \\ \left(\frac{G'}{G}\right)^2 : 12a_1\lambda - 24a_0a_1^2 + 6a_1^2 &= 0, \\ \left(\frac{G'}{G}\right)^3 : 8a_1 - 8a_1^3 &= 0. \end{aligned} \quad (17)$$

Solving this set of algebraic equations by Maple or Mathematica, we can obtain the following results:

$$\begin{aligned} \text{first solution set: } a_0 &= \frac{1}{2}\lambda + \frac{1}{4}, \\ a_1 &= 1, \\ c &= -\frac{1}{4}, \\ \mu &= -\frac{1}{16} + \frac{1}{4}\lambda^2, \\ \text{second solution set: } a_0 &= -\frac{1}{2}\lambda + \frac{1}{4}, \\ a_1 &= -1, \\ c &= -\frac{1}{4}, \\ \mu &= -\frac{1}{16} + \frac{1}{4}\lambda^2, \end{aligned} \quad (18)$$

where λ is arbitrary constant. By using (18), expression (16) can be written as

$$\begin{aligned} v(\zeta) &= \frac{1}{2}\lambda + \frac{1}{4} + \left(\frac{G'}{G}\right), \\ v(\zeta) &= -\frac{1}{2}\lambda + \frac{1}{4} - \left(\frac{G'}{G}\right), \end{aligned} \quad (19)$$

where $\zeta = x + y - (1/4)t$.

Equation (19) is the formula of a solution of (8).

3.2. First Traveling Wave Solution Set. Substituting the general solutions of (6) into (19) we have three types of traveling wave solutions of the equation generated by the Jaulent-Miodek Hierarchy as follows.

Case a. When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function solutions of (8)

$$\begin{aligned} v(\zeta) &= \frac{1}{2}\lambda + \frac{1}{4} \\ &\quad + \left[\frac{-\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \right. \\ &\quad \times \left. \left(\frac{c_1 \sinh \left\{ \left(\sqrt{\lambda^2 - 4\mu}/2 \right) \zeta \right\} + c_2 \cosh \left\{ \left(\sqrt{\lambda^2 - 4\mu}/2 \right) \zeta \right\}}{c_1 \cosh \left\{ \left(\sqrt{\lambda^2 - 4\mu}/2 \right) \zeta \right\} + c_2 \sinh \left\{ \left(\sqrt{\lambda^2 - 4\mu}/2 \right) \zeta \right\}} \right) \right], \end{aligned} \quad (20)$$

where λ , c_1 , and c_2 are arbitrary constants, and $\zeta = x + y - (1/4)t$.

In particular, if c_1 and c_2 are taken as special values, the various known results in the literature can be rediscovered; for instance, when setting $c_1 = 0$, $c_2 = 1$ and $\mu = -1/16 + (1/4)\lambda^2$, v becomes

$$v_1(\zeta) = 0.25 + 0.25 \coth \{0.25\zeta\}, \quad (21)$$

which is the solitary wave solution of (8).

If we take $c_2 = 0$, $c_1 = 1$ and $\mu = -1/16 + (1/4)\lambda^2$ above, then v becomes

$$v_2(\zeta) = 0.25 + 0.25 \tanh \{0.25\zeta\}, \quad (22)$$

and more solitary wave solutions of (8) can be obtained.

Biswas and Kara in (2010) found 1-soliton solution of the Jaulent-Miodek equation with power law nonlinearity that is derived by solitary wave ansatz method [13].

Case b. When $\lambda^2 - 4\mu < 0$, we get the trigonometric function solutions of (8) as follows:

$$\begin{aligned} v(\zeta) &= \frac{1}{2}\lambda + \frac{1}{4} \\ &\quad + \left[\frac{-\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \right. \\ &\quad \times \left. \left(\frac{-c_1 \sin \left\{ \left(\sqrt{4\mu - \lambda^2}/2 \right) \zeta \right\} + c_2 \cos \left\{ \left(\sqrt{4\mu - \lambda^2}/2 \right) \zeta \right\}}{c_1 \cos \left\{ \left(\sqrt{4\mu - \lambda^2}/2 \right) \zeta \right\} + c_2 \sin \left\{ \left(\sqrt{4\mu - \lambda^2}/2 \right) \zeta \right\}} \right) \right], \end{aligned} \quad (23)$$

where $\zeta = x + y - (1/4)t$, and λ , c_1 , and c_2 are arbitrary constants.

On the other hand, when setting $c_1 = 0$ and $c_2 = 1$, the solution of (23) can be written as

$$v_3(\zeta) = 0.25 + 0.25i \cot \{0.25i\zeta\}. \quad (24)$$

Setting again $c_2 = 0$ and $c_1 = 1$, the solution of (23) can be written as

$$v_4(\zeta) = 0.25 + 0.25i \tan \{0.25i\zeta\}. \quad (25)$$

Case c. When $\lambda^2 - 4\mu = 0$, the rational function solutions of (8) are as follows:

$$v(\zeta) = \frac{1}{2}\lambda + \frac{1}{4} + \left(\frac{c_2}{c_1 + c_2\zeta} \right) - \frac{\lambda}{2}. \quad (26)$$

Then (26) becomes

$$v_5 = 0.25 + \frac{c_2}{c_1 + (c_2x + c_2y - c_2t)}. \quad (27)$$

By $v = du/d\zeta \rightarrow u = \int v$ and $w = \partial u/\partial x$, we find the all traveling wave solutions of (8) as follows:

$$\begin{aligned} u_1 &= 0.25(x + y - 0.25t) \\ &\quad - 0.5 \ln(\coth\{0.25(x + y - 0.25t)\} - 1.) \\ &\quad - 0.5 \ln(\coth\{0.25(x + y - 0.25t)\} + 1.), \\ u_2 &= 0.25(x + y - 0.25t) \\ &\quad - 0.5 \ln(\tanh\{0.25(x + y - 0.25t)\} - 1.) \\ &\quad - 0.5 \ln(\tanh\{0.25(x + y - 0.25t)\} + 1.), \\ u_3 &= 0.25(x + y - 0.25t) \\ &\quad + 0.5 \ln(\coth\{0.25(x + y - 0.25t)\} - 1.) \\ &\quad + 0.5 \ln(\coth\{0.25(x + y - 0.25t)\} + 1.), \\ u_4 &= 0.25(x + y - 0.25t) \\ &\quad + 0.5 \ln(\tanh\{0.25(x + y - 0.25t)\} - 1.) \\ &\quad + 0.5 \ln(\tanh\{0.25(x + y - 0.25t)\} + 1.), \end{aligned} \quad (28)$$

then we found

$$\begin{aligned} w_1 &= 0.25 - \frac{0.5(0.25 - 0.25 \coth\{-0.25x - 0.25y + 0.0625t\}^2)}{-\coth\{-0.25x - 0.25y + 0.0625t\} - 1} \\ &\quad - \frac{0.5(0.25 - 0.25 \coth\{-0.25x - 0.25y + 0.0625t\}^2)}{-\coth\{-0.25x - 0.25y + 0.0625t\} + 1}, \end{aligned} \quad (29)$$

$$\begin{aligned} w_2 &= 0.25 - \frac{0.5(0.25 - 0.25 \tanh\{-0.25x - 0.25y + 0.0625t\}^2)}{-\tanh\{-0.25x - 0.25y + 0.0625t\} - 1} \\ &\quad - \frac{0.5(0.25 - 0.25 \tanh\{-0.25x - 0.25y + 0.0625t\}^2)}{-\tanh\{-0.25x - 0.25y + 0.0625t\} + 1}, \end{aligned} \quad (30)$$

$$\begin{aligned} w_3 &= 0.25 + \frac{0.5(0.25 - 0.25 \coth\{-0.25x - 0.25y + 0.0625t\}^2)}{-\coth\{-0.25x - 0.25y + 0.0625t\} - 1} \\ &\quad - \frac{0.5(0.25 - 0.25 \coth\{-0.25x - 0.25y + 0.0625t\}^2)}{-\coth\{-0.25x - 0.25y + 0.0625t\} + 1}, \end{aligned} \quad (31)$$

$$\begin{aligned} w_4 &= 0.25 + \frac{0.5(0.25 - 0.25 \tanh\{-0.25x - 0.25y + 0.0625t\}^2)}{-\tanh\{-0.25x - 0.25y + 0.0625t\} - 1} \\ &\quad - \frac{0.5(0.25 - 0.25 \tanh\{-0.25x - 0.25y + 0.0625t\}^2)}{-\tanh\{-0.25x - 0.25y + 0.0625t\} + 1}. \end{aligned} \quad (32)$$

Similarly a second traveling wave solution set can be obtained by following the above procedure.

4. Tanh Method

In this paper, we used tanh method in its standard form presented by Malfliet [36, 37], and the outline of tanh method is known. For more details, refer to [38].

For computation the exact traveling wave solutions of equation generated by the Jaulent-Miodek Hierarchy. We used the ansatz

$$S(Y) = \sum_{i=0}^m a_i Y^i, \quad (33)$$

where $Y = \tanh\{\mu\zeta\}$.

Substituting (33) in the ordinary differential equation of the equation generated by the Jaulent-Miodek Hierarchy, we found

$$\begin{aligned} 16cS + 4 \left[\mu^2 (1 - Y^2) \left(-2Y \frac{ds}{dy} + (1 - Y^2) \frac{d^2s}{dy^2} \right) \right] \\ - 8S^3 + 3S + 6S^2 = 0. \end{aligned} \quad (34)$$

To determine the parameter m , we usually balance the linear term of highest order in the resulting equation (34) with the highest order nonlinear terms, and we find $m = 1$. This means that

$$v(\zeta) = S(Y) = a_0 + a_1 Y. \quad (35)$$

Substituting $S(Y)$, $S'(Y)$, $S''(Y)$, $S^2(Y)$, and $S^3(Y)$ from (35) into (34) yields the system of algebraic equations for a_0 , a_1 , c , and μ as follows:

$$\begin{aligned} Y^0 : &= (16c + 3)a_0 - 8a_0^3 + 6a_0^2 = 0, \\ Y^1 : &= (16c + 3)a_1 - 8\mu^2 a_1 - 24a_0^2 a_1 + 12a_0 a_1 = 0, \\ Y^2 : &= -24a_0 a_1^2 + 6a_1^2 = 0, \\ Y^3 : &= -8\mu^2 a_1 - 8a_1^3 = 0. \end{aligned} \quad (36)$$

FIGURE 1: 3D plots of (a) (29), (b) (30), (c) (31), and (d) (32), for the parameter values $t = 0.1$ and $c = 1/4$.

Solving the system, we obtain

$$\begin{aligned}
 a_0 &= \frac{1}{4}, \\
 a_1 &= \pm \frac{1}{4}, \\
 c &= -\frac{1}{4}, \\
 \mu &= \pm \frac{1}{4}.
 \end{aligned}
 \tag{37}$$

In view of (37), we obtain the solitary wave solutions

$$\begin{aligned}
 v(x, y, t) &= 0.25 + 0.25 \tanh \{0.25\zeta\}, \\
 v(x, y, t) &= 0.25 - 0.25 \tanh \{-0.25\zeta\},
 \end{aligned}
 \tag{38}$$

where $\zeta = x + y - 0.25t$.

Equation (38) is the same as the result obtained by the (G'/G) -expansion method.

We obtain the behaviour of exact traveling wave solution of (8).

5. Conclusion

In this work, we have established the traveling wave solutions of the nonlinear equation generated by the Jaulent-Miodek Hierarchy. It appears that three types of traveling wave solutions have been successfully found by using the (G'/G) -expansion method. The general traveling wave solutions are hyperbolic function solutions, trigonometric function solutions, and rational function solutions. The solitary wave solutions are derived from these functions when the parameters are taken as special values. The solutions of the equation via (G'/G) -expansion method are exactly the same as those obtained by tanh method if the condition $c_2 = 0$ is satisfied, this means that the tanh method is a special case of (G'/G) -expansion method. The (G'/G) -expansion method is quite efficient and practically well suited for use in finding the exact solutions for the equation generated by the Jaulent-Miodek Hierarchy. Some solutions given in this paper are new solutions which have not been reported yet. Three-dimensional plots (Figure 1) of some of the investigated solutions are also drawn to visualize the underlying dynamics of such results. The method which we have proposed in this work is also a standard, direct, and computerizable method.

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