Research Article

A Mathematical Modeling of Resonances of the Nonlinear Tilted Support Spring System under Harmonic Excitation

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A novel nonlinear tilted support spring damping system is proposed in this paper. The nonlinear dynamic equation is developed under the foundation displacement excitation and the resonance frequency of the system is also analyzed. Furthermore, different support angles affecting the vibration amplitude and the natural frequency of the system are researched. The model of the tilted support spring damping system is analyzed by ANSYS, and modal analysis and harmonic response analysis are made under harmonic excitation. The study shows that the supporting angle makes a great impact on the amplitude of the system and the response amplitude of the system is also reduced with the decrease of support angle. Through the analysis and comparison of the example, it provides a theoretical basis of the analysis for the tilted support spring damping system support angle's optimal design.

1. Introduction

The nonlinear tilted support spring damping system has been used in packaging transportation since 1960s. The tilted spring system possesses self-nonlinear characteristics which provide good vibration performance of the precision instruments. The existing researches on tilted support spring damping system focus on the qualitative analysis, and material nonlinear analysis, undamped system analysis [1, 2].

The products are prone to damage during the transportation suffered by shock and vibration. So, the study has a significant meaning to improve the vibration performance of the product under the shock and vibration. To characterize the damage potential of shocks and dropping impact on damage of packaged products, the three-dimensional shock response spectrum and damage boundary surface were proposed by Wang et al. [3, 4]. Then, the response spectrum concept was applied into the tilted support spring damping in a rectangular pulse excitation by Chen [5]. The kinetic equation of a tilted support spring system is established by Wu, and a conclusion that the tilted support spring system has a better damping effect in the vertical direction than the straight support spring system by using functional relationship between the system natural frequency and φ (the spring inclination)

is also made [6]. However, the structure exhibits nonlinear characteristics which need further analysis. So, a novel nonlinear tilted support spring damping system is selected as research object, and the relationship between support angles of the tilted support spring and system vibration characteristics is studied. This research will provide an important analysis basis for the design of the tilted support spring system.

2. Titled Spring Damping System Dynamic Model and Dynamic Equation

The tilted spring system has self-nonlinear characteristics. An analysis of system performance shows that it provides better protection performance for the products than a linear system, which is widely utilized in shock absorber of the motor and packaging engineering [1–3]. We are interested in the resonant frequencies which might occur during the operation and should be avoided during designing the tilted spring system. It is very important to investigate the resonant frequency. The model of the tilted spring system is shown in Figure 1, where the product is supported by four springs which own the same stiffness k and length l_0 , φ_0 is the angle of primary support position, m is the mass of the product, and c is the damping coefficient of the system. The approximate

FIGURE 1: The model of tilted spring system.

natural vibration dynamic equation of system can be written as [7–9]

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + 4k \left(a_0 x + \frac{b_0}{l_0} x^2 + \frac{c_0}{l_0^2} x^3 \right) = 0. \quad (1)$$

The dynamic equation is in the same form as the symmetrical three-spring oscillator [4], where

$$a_0 = \sin^2 \varphi_0, \quad (2)$$

$$b_0 = -\frac{3}{2} \sin \varphi_0 \cos^2 \varphi_0, \quad (3)$$

$$c_0 = \frac{1}{2} (1 - 6 \sin^2 \varphi_0 + 5 \sin^4 \varphi_0). \quad (4)$$

If the foundation displacement excitation is in the form of [6]

$$x_1 = a \sin(\Omega t). \quad (5)$$

The nonlinear vibration equation of the tilted spring system under foundation displacement excitation can be expressed as follows [6]

$$\frac{d^2 y}{dt^2} + 2\xi \frac{dy}{dt} + \omega_0^2 y + b_0 \omega^2 y^2 + c_0 \omega^2 y^3 = d \Omega^2 \sin(\Omega t), \quad (6)$$

where

$$y = \frac{(x - x_1)}{l_0}, \quad (7)$$

$$\xi = \frac{c}{2m}, \quad (8)$$

$$\omega_0^2 = a_0 \omega^2, \quad (9)$$

$$d = \frac{a}{l_0}. \quad (10)$$

Here, y is the dimensionless displacement parameter, ξ is the damping parameter, a and Ω are amplitude and frequency of the foundation excitation, and $\omega = \sqrt{4k/m}$ is the frequency parameter. With initial conditions, ω_0 is natural frequency of the system

$$y(0) = A, \quad \frac{dy(0)}{dt} = 0. \quad (11)$$

3. First Order Approximation Solution

The dynamic equation (6) relates to the angle of primary support position and is geometric nonlinear, it is difficult to get exact result. We are interested in the resonant frequencies which might occur during the operation and should be avoided during designing the tilted spring system. Many effective methods were suggested to solve nonlinear system, such as the variational iteration method and the parameter-expansion method [9]. Being different from the other nonlinear analytical methods, such as perturbation methods, the variational iteration method does not depend on small parameters [10–13], such that it can find wide application in nonlinear problem without linearization or small perturbations. Wang first introduced the VIM into packaging problems and obtained the inner-resonance conditions for nonlinear packaging system [13, 14]. Using the variational iteration method [9, 10], we can construct the following iteration formulas for (6)

$$y_{n+1} = y_n + \frac{1}{\omega_0} \int_0^t \sin[\omega_0(s-t)] R_n(s) ds, \quad (12)$$

where

$$R_n(t) = \frac{d^2 y_n}{dt^2} + 2\xi \frac{dy_n}{dt} + \omega_0^2 y_n + b_0 \omega^2 y_n^2 + c_0 \omega^2 y_n^3 - d \Omega^2 \sin(\Omega t). \quad (13)$$

We choose the initial approximation

$$y_0 = A \cos(\alpha t). \quad (14)$$

This satisfies the initial conditions (11). Substitution initial approximation into (13) results in the following residual:

$$\begin{aligned} R_0(t) = & \left(A \omega_0^2 - A \alpha^2 + \frac{3}{4} c_0 \omega^2 A^3 \right) \cos(\alpha t) \\ & + \frac{1}{2} b_0 \omega^2 A^2 \cos(2\alpha t) + \frac{1}{4} c_0 \omega^2 A^3 \cos(3\alpha t) \\ & - 2\xi A \alpha \sin(\alpha t) - d \Omega^2 \sin(\Omega t) + \frac{1}{2} b_0 \omega^2 A^2. \end{aligned} \quad (15)$$

By the iteration equation (12), we have the following first order approximation solution

$$\begin{aligned} y_1 = & C \cos(\omega_0 t) + D \sin(\omega_0 t) + \frac{-3\omega^2 A^3 c_0}{4(\omega_0^2 - \alpha^2)} \cos(\alpha t) \\ & + \frac{2\xi A \alpha}{\omega_0^2 - \alpha^2} \sin(\alpha t) + \frac{-b_0 \omega^2 A^2}{2(\omega_0^2 - 4\alpha^2)} \cos(2\alpha t) \\ & + \frac{-c_0 \omega^2 A^3}{4(\omega_0^2 - 9\alpha^2)} \cos(3\alpha t) + \frac{d \Omega^2}{\omega_0^2 - \Omega^2} \sin(\Omega t) \\ & + \frac{-b_0 \omega^2 A^2}{2\omega_0^2}, \end{aligned} \quad (16)$$

where C and D are the coefficients of the $\cos(\omega_0 t)$ and $\sin(\omega_0 t)$, respectively,

$$C = A + \frac{b_0 \omega^2 A^2}{2\omega_0^2} + \frac{b_0 \omega^2 A^2}{2(\omega_0^2 - 4\alpha^2)} + \frac{3c_0 \omega^2 A^3}{4(\omega_0^2 - \alpha^2)} + \frac{c_0 \omega^2 A^3}{4(\omega_0^2 - 9\alpha^2)} \quad (17)$$

$$D = -\frac{2\xi A \alpha^2}{\omega_0(\omega_0^2 - \alpha^2)} - \frac{d\Omega^3}{\omega_0(\omega_0^2 - \Omega^2)}.$$

Rearranging (16) results in

$$y_1 = B \sin(\omega_0 t + \beta) + \frac{-3\omega^2 A^3 c_0}{4(\omega_0^2 - \alpha^2)} \cos(\alpha t) + \frac{2\xi A \alpha}{\omega_0^2 - \alpha^2} \sin(\alpha t) + \frac{-b_0 \omega^2 A^2}{2(\omega_0^2 - 4\alpha^2)} \cos(2\alpha t) + \frac{-c_0 \omega^2 A^3}{4(\omega_0^2 - 9\alpha^2)} \cos(3\alpha t) + \frac{d\Omega^2}{\omega_0^2 - \Omega^2} \sin(\Omega t) + \frac{-b_0 \omega^2 A^2}{2\omega_0^2}, \quad (18)$$

where

$$B = \sqrt{C^2 + D^2}, \quad \beta = \tan^{-1}\left(\frac{C}{D}\right). \quad (19)$$

Eliminate the secular term which might occur in the next iteration that

$$B = \sqrt{C^2 + D^2} = 0. \quad (20)$$

From which α can be determined

4. Resonance Conditions

(1) Through (15) and (16), the resonance conditions can be simplified as

$$\alpha = \omega_0 \quad (21)$$

$$\alpha = \frac{1}{2} \omega_0 \quad (22)$$

$$\alpha = \frac{1}{3} \omega_0 \quad (23)$$

$$\Omega = \omega_0, \quad (24)$$

where α is determined by (20). One of (21)–(24) is satisfied, and the resonance can happen.

(2) According to (2), (9), and (21), we can get the equation

$$\omega_0 = \sin \varphi_0 \sqrt{\frac{4k}{m}}. \quad (25)$$

It can be seen that, when ω_0 and φ_0 are determined, k is proportional to m and m increases with the increase of k . But the bigger k will cause higher requirement to the spring process and high cost; so, when the mass is defined, the research concentrates on the relationship between ω_0 and φ_0 .

5. The Analysis of the Novel Nonlinear Tilted Support Spring Damping System with Examples in ANSYS

5.1. Establish Modal Analysis and Harmonic Response Analysis for the System in the Vertical Direction. ANSYS is a general purpose finite element analysis software, and it is widely used in various engineering fields, as the project structure in complex geometry suffering considerable load through theoretical analysis often cannot be solved, but it can be a good solution with ANSYS. ANSYS analyzes the actual structure subjected to external loads which occur after displacement and stress and strain response, and according to the response we can know the structure of the state, in order to determine whether the structure meets the requirements. On this basis, create a system model by ANSYS [15] for further optimization.

From the dynamic equation previously mentioned above, it can be seen that the solution of the equation is related to the damping ratio, spring stiffness, spring tilted angle, pulse amplitude, and pulse response time. ANSYS is used in this study to determine the relation between the amplitude of the response and the natural frequency of the system with the changing of spring inclination.

The device was designed as a piece of accurate packaging equipment in rail transportation. According to the information provided by the Academy of China Ministry of Railways, the usual speed, rail interface spacing, and various models of bogies' free vibration frequency (high frequency: 7–8 cycles/sec; low frequency: less than 2 cycles/sec) of the railway are as follows: the no-load running vibration frequency is 7~8 Cycles/sec, the full-load running vibration frequency is 2~4 Cycles/sec. However, the vibration frequency of the front, medium, and rear parts of the carriage is slightly different. Apart from this, another three high-frequency impact vibrations must be considered in the isolation design. They are the impact vibration frequency caused by train marshalling slope, the impact vibration frequency due to changing tractor trailer, and the impact vibration frequency of changing tracks, which are all 6~8 Cycles/sec. Under normal circumstances, the train's vertical acceleration is 0.3~0.4 g. Trains run smoothly when the speed of the horizontal acceleration is 0.2~0.4 g [16].

In this paper, the mass (m) of the selected precision instrument is 800 kg. Vertical acceleration is 0.3 g according to the parameters. Damping ratio c is 0.3. To meet the system dynamics of the situation, the spring stiffness is calculated as 10^5 N/m through theory and example analysis.

Taking the system resonance into consideration, the system resonance frequency is calculated from 15 to 23 times per second according (25). As the high-frequency vibration in our nation railway packaging transportation is

FIGURE 2: The model of tilted support spring damping system.

FIGURE 3: Relationship between ω_0 and φ_0 .

7~8 Cycles/sec, the system resonance rarely happen in railway transportation.

From the above parameters, the tilted support spring damping system is constructed in ANSYS, and modal analysis and harmonic response analysis are conducted. The system model is shown in Figure 2.

The specific modeling parameters are as follows.

In this paper, the selected quantity unit is 3D mass 21 (i.e., 16,800 kg), spring unit is Spring-damper 14 (i.e., 11,200 kg), the weight of the quantity unit is 800 kg, the stiffness coefficient of the spring element is 10^5 N/m, and damping ratio is set as 0.3.

The changing rule of spring damping system natural frequency and amplitude under different titled angles is shown in Figure 3.

From Figure 3, ω_0 increases with the increase of φ_0 . It is consistent with the previous theory analysis result. It shows that the titled spring angle has a great impact on system natural frequency. And the response amplitude under different titled angles is also analyzed in ANSYS. Due to the impact actor of excitation, the analysis is performed under nonload running train and full-load running separately. The frequency of nonload is 8 Cycles/sec and the full-load is 2 Cycles/sec, as shown in Figures 4 and 5.

From Figure 4, we can see that, in the situation of low excitation frequency, the amplitude of tilted spring damping system decreased with the increase of system tilted angle. So, vertical spring system has a better shock absorber than tilted spring system in this situation, but generally, the excitation frequency of 2 Cycles/sec is a limited value, and the excitation less than 2 Cycles/sec cannot be achieved. Also, the natural

FIGURE 4: Amplitude varies with different tilted angles under full-load running.

FIGURE 5: Amplitude varies with different tilted angles under nonload running.

frequency of spring damping system is 3~4 Cycles/sec. So the amplitude caused in low frequency is no practical value.

From Figure 5, it is obviously indicated that, under the excitation frequency of 8 Cycles/sec, the response amplitude of the tilted spring damping system increased with the increase of system tilted angle. The decreased tilted angle of the system can improve the shock absorber. And the tilted spring damping system has a better shock absorber than the vertical spring damping system, but when the tilted angle is decreased to a small angle, the spring system will exhibit as soft spring. Simultaneously, soft spring results in the low stiffness and poor vibration performance. So, the research priority concentrates on the selection of the tilted spring angle.

5.2. Optimization Analysis. According to the analysis of Figures 4, 5, and 6, it is indicated that, during the accuracy equipment transportation by railway, the tilted angle at 45° is the best in theory, but when the tilted angle of the tilted spring damping system is too small, the spring stiffness on vertical direction decreases, the spring will change to soft spring, and the requirements of spring stiffness are about to increase.

It can be found from Figure 6, the amplitude response of the spring decreases with the increase of excitation frequency, but when the spring support angle is around 75° , the amplitude response of the system changes little, and the growth flattens. By theoretical analysis and further experimental verification, it can be found that the tilted angle of tiled spring damping system among 70° and 80° has the best vibration absorber for precision instruments. Above all, take the factor of natural frequency, economy, and reliability into account and the titled support spring system has a better capability of vibration absorber than linear spring.

FIGURE 6: Relationship between excitation and φ_0 .

6. Conclusion

The nonlinear shock response for tilted support spring damping system is studied under the foundation displacement excitation, and the resonance frequency of the system is obtained and compared with the results obtained by FEM. The study shows that the supporting angle makes a great impact on amplitude of the system and the response amplitude of the system is also reduced with the decrease of support angle. The research provides a theoretical basis of the analysis for the tilted support spring damping system support angle's optimal design.

Conflict of Interests

The authors declare no conflict of interests.

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