Research Article

Decision Analysis of Advertising and Price for Bilateral Competing Supply Chain

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The outcome of centralized equilibrium, prisoner’s dilemma equilibrium, and decentralized equilibrium under different decision models has been provided with regards to bilateral competing supply chain system, either side of which is composed of one manufacturer and one retailer. Theoretical analysis indicates a positive correlation between price and one’s own advertising investment level and a negative correlation between price and the opponent’s advertising investment level. Through analysis of numerical examples, the results reveal a first mover advantage that leads to prisoner’s dilemma in the system as well as the impact that price and advertising competition intensity has on the supply chain’s choice of decision model.

1. Introduction

With the rocketing development of economic globalization, the competitions among enterprises have to some extent gradually evolved into fierce competitions among chains, with advertising war and price war being the common competitive approach. In order to enhance public awareness and seize market share, the sellers usually invest in massive advertisement propaganda. This helps the customers to understand fully the corporate culture, the product brands, postsale services, and so on, and is benefit for exploiting potential purchasing power market.

The response function of demand to advertising is generally thought to have two possible situations, namely, the S-shape and concave shape (see Jones [1]). Since the so-called threshold value of S-shape function is hard to determine, the concave response function is usually adopted [2]. For concave response function, increase in advertising investment is able to trigger the increase in demand, yet the increase rate declines as the cost grows. Dant and Berger [3] investigated the advertising cooperation within the supply chain for which the manufacturer share advertising cost with the seller. Also, Li et al. [4] based their analysis of transaction efficiency in manufacturer-retailer cooperative advertising system on their review of the change in market structure that the retailing power shifts from manufacturers to retailers. Afterwards Huang et al. [5] placed particular emphasis on the research of optimal advertising strategy selection in cooperative games of manufacturers and retailers and then conducted a similar investigation in sequential games to make a comparative analysis. The results indicate that the former kind of game is more profitable than the latter one. Wang et al. [6] then expanded Huang et al.’s model [5] to further study the vertical advertising cooperation when retailers are in competition. Existing studies concerning advertising mainly concentrate on unilateral supply chain system composed of one manufacturer and one retailer. A few scholars extended their study to incorporate the supply chain system with one manufacturer and multiple competing retailers.

When multiple retailers are involved in competition and their respective products are substitutable, price war is inevitable. Yao et al. conducted a subsequent research on Padmanabhan and Png’s model [7] to analyze the influence that returns contracts impose on the price war between two retailers with stochastic demands [8] and explored
afterwards that the revenue-sharing contract may improve the performance of the supply chain with two retailers in price competition [9]. In addition, Atkins and Zhao [10] internalized competition with utility theory and inspected the asymmetric supply chain equilibrium structure based on price and service competitions, while Bernstein and Federgruen [11] explored the industry equilibrium features of price and service competitions in the context of demand uncertainty and inventory endogenesys.

In case of competition between two supply chains, whether each supply chain adopts cooperative or decentralized decision-making will have a direct impact on the competitive equilibrium of supply chains. Recently, Wang et al. [12] found that the stronger the bias in the utility function, the higher the level of public cooperation. However, if the coordination process is disturbed, network reciprocity fails, resulting in the total collapse of cooperation [13]. Perc et al. [14, 15] analyzed in detail how the prisoner's dilemma game influences the evolution of cooperation. Moreover, as social dilemma, games governed by pairwise interactions were recently also found to be susceptible to the "wisdom of group" effect [16]. Szolnoki et al. [17] introduced a spatial ultimatum game with discrete strategies and reported the occurrence of traveling waves and cyclic dominance, where one strategy in the cycle can be an alliance of two strategies. Recent advances point towards the fact that the evolution of strategies alone may be insufficient to fully exploit the benefits offered by cooperative behavior.

With respect to the situation that the demand of every supply chain is correlated with advertising and price, this essay is an effort to analyze, on basis of the above studies, the decision equilibrium results of bilateral competing supply chain. And this essay aims to reveal the changes in decision members' rational choices and supply chain performance in competitive environment so as to provide managers with some theoretical supports and decision reference.

2. Model Assumptions and Notations

Consider a bilateral competing model composed of two supply chains that each consists of one manufacturer $M_i$ and one retailer $R_i$. The two supply chains face the same market and the products of the two manufacturers are substitutable. Customers choose products according to the influence of retail price $p_i$ ($i = 1, 2$) and manufacturers’ advertising investment $A_i$ ($i = 1, 2$). Supply chain members are only fully aware of the observable retail price information. We use $D_i(A, p)$ to express the demand for products with advertising investment being $A$ and price vector being $p$

$$D_i(A, p) = \theta - ap_i + bp_j + \alpha \sqrt{A_i} - \beta \sqrt{A_j},$$

$$i, j = 1, 2, j = 3 - i.$$  \hspace{1cm} (1)

Hereinto, parameters $a$ and $b$, respectively, represent the response of customer demand to product price and competing product price. Parameter $\alpha$ reflects product demand's dependent degree on advertisements, while $\beta$ is the influence coefficient of competing advertisements to demand. $a, b, \alpha, \beta \geq 0$ and $a > b, \alpha > \beta$. $A_i$ is manufacturer $M_i$'s advertising investment, whose influence on sales volume is $\alpha \sqrt{A_i}$. This hypothesis agrees with empirical rules: the demand increases with the increase of advertisements, while the return of advertising investment shows a decline (see Xie and Wei [2]). We use $\theta$ to represent the basic market demand of corresponding supply chain. Assume that the basic market demands of the two chains are the same, thus the basic demand of the entire market is $2\theta$. We also assume that the cost price and wholesale price of the two manufacturers are, respectively, $c$ and $w$, so as to concentrate our analysis on the price and advertising competition of the two chains.

3. Model Formulation and Solution

The game sequence of the advertising and price competition between two supply chains for market share is as follows: in view of decentralized supply chain structure, downstream enterprises choose profit-dominant retail price for themselves; then upstream enterprises determine their advertising investment level and horizontal competitive market emerges. A comprehensive analysis shows that three competition models will probably be formed: (1) belonging to the same management body, the upstream and downstream enterprises of each chain work in joint forces to decide variables $A_i$ and $p_j$ so as to compete with the opponent chain. This model is hereinafter referred to as centralized decision model (i.e., model CC). (2) The downstream enterprise of one chain competes with the other chain in price, while according to price response function the upstream enterprise competes with the other chain in advertising. Such situation is hereinafter referred to as hybrid decision model (i.e., model DC). (3) The downstream enterprises of the two chains initiate price competition. Afterwards the upstream enterprises compete in advertising investment. This model is called the decentralized decision model (i.e., model DD). The respective profits of manufacturer $M_i$ and retailer $R_i$ are

$$\Pi_{M_i}(A_i, p_i) = (w - c) \times \left( \theta - ap_i + bp_j + \alpha \sqrt{A_i} - \beta \sqrt{A_j} \right) - A_i,$$

$$\Pi_{R_i}(A_i, p_j) = (p_i - w) \left( \theta - ap_i + bp_j + \alpha \sqrt{A_i} - \beta \sqrt{A_j} \right).$$

(2)

Herein, $i, j = 1, 2, j = 3 - i$.

3.1. Model CC. The manufacturers and retailers of the two supply chains open up competitions between chains by joint decision on price and advertising investment. Inasmuch as the decision-maker knows the opponent's selling price, the game sequence begins with the competition between supply chains in $p_i$, then advertising investment level $A_i$ is determined. The optimization problem of supply chain
decision is
\[
\max \Pi_i(A_i, p_i) = (p_i - c) \\
\times \left( \theta - ap_i + bp_j + \alpha \sqrt{A_i} - \beta \sqrt{A_j} \right) - A_i
\]
s.t. \(D_i(A, p) = \theta - ap_i + bp_j + \alpha \sqrt{A_i} - \beta \sqrt{A_j} \geq 0\)
\[
A_i, A_j \geq 0, \quad i, j = 1, 2, \quad j = 3 - i.
\]

The constraint conditions ensuring that the channel demands and advertising investments are nonnegative, this problem converts to
\[
\min f\left(\sqrt{A_i}, p_i\right) = (p_i - c) \\
\times \left( -\theta + ap_i - bp_j - \alpha \sqrt{A_i} + \beta \sqrt{A_j} \right) \\
+ A_i
\]
s.t. \(\theta - ap_i + bp_j + \alpha \sqrt{A_i} - \beta \sqrt{A_j} \geq 0\)
\[
A_i, A_j \geq 0, \quad i, j = 1, 2, \quad j = 3 - i.
\]

On account that the objective's Hessian matrix \(\nabla^2 f(\sqrt{A_i}, p_i) = \left( \frac{a}{2a^2} \right)\) is positive definite matrix, consequently there exists optimal value for the objective function.

Price response function is derived from simultaneous equilibrium equations \(\partial \Pi_i / \partial p_i = 0, \partial \Pi_j / \partial p_j = 0\),
\[
p_i' = \left(2a\theta + 2a^2 + 2a\alpha \sqrt{A_i} - 2a\beta \sqrt{A_j} + b\theta + cba \right) \\
+ ba \sqrt{A_i} - b\beta \sqrt{A_j} \times (4a^2 - b^2)^{-1}.
\]

Substitute \(p_i'\), \(p_j'\) into \(\Pi_1(A_1, p_1), \Pi_2(A_2, p_2)\) and use backwards induction on basis of the profit maximization principle. That is, from simultaneous equations \(\partial \Pi_1(A_1, p_1') / \partial A_1 = 0, \partial \Pi_2(A_2, p_2') / \partial A_2 = 0\); therefore, in CC model, the optimal equilibrium solution of chain 1 and chain 2's advertising investment and price is derived as
\[
\sqrt{A_i^*} = \frac{a (2a\alpha - b\beta) \left( ac - bc - \theta \right)}{\left(2a^2 + ab\right) \left( a\beta - 2b \right) + 2a^2 (4a - a^2) - ab\beta^2 + b^3},
\]
\[
p_i^* = \frac{\theta + a\alpha + (\alpha - \beta) \sqrt{A_i^*}}{2a - b}.
\]

Accordingly, optimal profit of supply chain is \(\Pi_i(A_i^*, p_i^*)\) with \(i = 1, 2\).

### 3.2. Model DC.

Without loss of generality, we assume that \(M_2\) and \(R_3\) reach supply chain coordinating contract and make joint decisions, while \(M_1\) and \(R_1\) make decentralized decision. In that way the response function of the price competition between \(R_1\) and the other supply chain (centralized decision) is
\[
p_i'' = \left(2a\theta + 2a^2 \sqrt{w} + (2a\alpha - b\beta) \sqrt{A_1} - (2a\beta - b\alpha) \sqrt{A_2} \right) \\
\times \left(4a^2 - b^2\right)^{-1},
\]
\[
p_j'' = \left(2a\theta + 2a^2 + (2a\alpha - b\beta) \sqrt{A_1} - (2a\beta - b\alpha) \sqrt{A_2} \right) \\
\times \left(4a^2 - b^2\right)^{-1}.
\]

Substitute price function \(p_i'', p_j''\) into \(\Pi_{M1}(A_1)\), \(\Pi_{M2}\), the Manufacturer \(M_1\)'s optimal effort level is found by solving
\[
\max \Pi_{M1}(A_1) = (w - c) \\
\times \left( \theta - ap_1'' + bp_2'' + \alpha \sqrt{A_1} - \beta \sqrt{A_2} \right) \\
- A_1
\]
s.t. \(D_1(A_1, A_2) = \theta - ap_1'' + bp_2'' + \alpha \sqrt{A_1} - \beta \sqrt{A_2} \geq 0\), \(A_1 \geq 0, \quad A_2 \geq 0\)
\[
\max \Pi_{M2}(A_2) = (p_2'' - c) \\
\times \left( \theta - ap_2'' + bp_1'' + \alpha \sqrt{A_2} - \beta \sqrt{A_1} \right) \\
- A_2
\]
s.t. \(D_2(A_1, A_2) = \theta - ap_2'' + bp_1'' + \alpha \sqrt{A_2} - \beta \sqrt{A_1} \geq 0\), \(A_1 \geq 0, \quad A_2 \geq 0\).

Taking into account (9) and (11), from simultaneous equations
\[
\frac{\partial \Pi_{M1}(A_1, p_1'')}{\partial A_1} = 0, \quad \frac{\partial \Pi_2(A_2, p_2'')}{\partial A_2} = 0,
\]
\[
\frac{\partial \Pi_{M1}(A_1, p_1'')}{\partial A_2} = 0, \quad \frac{\partial \Pi_2(A_2, p_2'')}{\partial A_1} = 0,
\]
\[
\frac{\partial \Pi_{M1}(A_1, p_1'')}{\partial p_1} = 0, \quad \frac{\partial \Pi_2(A_2, p_2'')}{\partial p_2} = 0.
\]
we immediately obtain the optimal advertising investment of $M_1$ and chain 2, which are expressed by
\[
\sqrt{A_{1}^{**}} = \frac{a(w-c)(2\alpha a - b\beta)}{2(4a^2 - b^2)},
\]
\[
\sqrt{A_{2}^{**}} = \left(a(2\alpha a - b\beta) \times \left[2a(ac-\theta) - b(aw+bc+\theta) + (2a\beta - ba)\sqrt{A_{1}^{**}} \right] \times \left(4a^2 - b^2 \right) \right)^{-1}.
\]

(13)

Substitute (13) into $p_i^{**}$, $p_j^{**}$. Then optimal retail price for chains 1 and 2, $p_1^{***}$, $p_j^{***}$, and optimal profit of supply chain $\Pi_i(A_{i}^{***}, p_i^{***})$ is derived.

3.3. Model DD. The two supply chains are both inharmonious that each supply chain makes decentralized decisions. Initially, the retailers compete in $p_i$ with price response function as
\[
\begin{equation}
 p_i^{***} = \left(2a\theta + 2a^2 w + 2a\alpha\sqrt{A_i} - 2a\beta\sqrt{\sqrt{A_i} + b\theta + wba}
+ b\alpha\sqrt{A_i - b\beta\sqrt{A_i}} \times \left(4a^2 - b^2 \right) \right)^{-1}.
\end{equation}
\]

(14)

Given the retailers’ response function (14), the manufacturers solve
\[
\max \text{ } \Pi_{M_i}(A_i) = (w-c) \times \left(\theta - ap_i^{***} + bp_j^{***} + \alpha\sqrt{A_i} - \beta\sqrt{A_j} \right) - A_i
\]

s.t. $D_i(A_i) = \theta - ap_i^{***} + bp_j^{***} + \alpha\sqrt{A_i} - \beta\sqrt{A_j} \geq 0,$

$A_i, A_j \geq 0, \text{ } i, j = 1, 2, j = 3 - i$

(15)

to determine the advertising investment, which are expressed by
\[
\sqrt{A_{i}^{***}} = \frac{a(w-c)(2\alpha a - b\beta)}{2(4a^2 - b^2)}.
\]

(16)

Because for function (14), we have
\[
p_i^{***} = \frac{\theta + aw + (\alpha - \beta)\sqrt{A_{i}^{***}}}{2a-b}.
\]

(17)

Proposition 1. If $a > b, \alpha > \beta$,

(i) retail price positively correlates with advertising investment level (i.e., $\frac{\partial p_i^{*}}{\partial \sqrt{A_i}} > 0$);

(ii) retail price negatively correlates with opponent’s advertising investment level (i.e., $\frac{\partial p_j^{*}}{\partial \sqrt{A_j}} < 0$);

(iii) for manufacturers that make decisions independently, if wholesale price $w$ is endogenous variable, then $w$ increases with the increase in advertising investment.

Proof. (i)-(ii) For price response function $p_i^{*}$ (i.e., $p_i', p_j'', p_j'''$) in all three decision models, there is
\[
\frac{\partial p_i^{*}}{\partial \sqrt{A_i}} = \frac{2a\alpha - b\beta}{4a^2 - b^2}, \text{ } \frac{\partial p_j^{*}}{\partial \sqrt{A_j}} = \frac{b\alpha - 2a\beta}{4a^2 - b^2}.
\]

(18)

Since $a, b, \alpha, \beta \geq 0$ and $a > b, \alpha > \beta$, if market demand is nonnegative, we find
\[
\frac{2a}{b} > \frac{\alpha}{\beta} > \frac{b}{2a}.
\]

(19)

Therefore,
\[
\frac{2a\alpha - b\beta}{4a^2 - b^2} > 0, \text{ } \frac{b\alpha - 2a\beta}{4a^2 - b^2} < 0
\]

(20)
is tenable.

(iii) ‘The manufacturers’ wholesale price with independent decision can be expressed as
\[
w \left(\sqrt{A_i} \right) = \frac{2(4a^2 - b^2)}{a(2a\alpha - b\beta)} \sqrt{A_i} + c.
\]

(21)

Since
\[
\frac{\partial w \left(\sqrt{A_i} \right)}{\partial \sqrt{A_i}} = \frac{2(4a^2 - b^2)}{a(2a\alpha - b\beta)} > 0,
\]

(22)
it is easy to prove that the conclusion is correct.

On one hand, Proposition 1 demonstrates that advertising and price are significantly correlated and shows that the level of advertising investment and price directly affects the products’ absolute market occupation rate, competitiveness, and even the overall performance and profit of the enterprises. On the other hand, it reflects that the higher the manufacturers invest in advertising, the higher the wholesale price they offer to retailers will be.

4. Numerical Examples

Since the equilibrium solutions in three decision models are quite complicated, this essay further analyzes the influence that equilibrium results have in competition on supply chain members, customers, and the whole industry and reveals the values of product cost and equilibrium results to supply chain decision in different decision models. In case of disturbance in price sensitive coefficients $a, b$ and
advertising influence coefficients $\alpha$, $\beta$, corresponding results of centralized equilibrium, prisoner’s dilemma equilibrium and decentralized equilibrium under different price and advertising competition intensity are expressed as follows.

Assume that $\theta = 10$, $w = 20$, $c = 10$, $a = 0.3$, $b = 0.2$, $\alpha = 0.5$, and for $\beta < \alpha$, assume a step size of 0.05 and get value from parameter $\beta$. The profit of supply chain, retail price, and the supply chain members’ advertising investments are obtained, as are shown in Figures 1, 3, and 5, respectively.

Let $\alpha = 0.3$, $\beta = 0.2$, $a = 0.5$, and keep values of other parameters $\theta$, $w$, and $c$ unchanged. For $b < a$, assume a step size of 0.05 and get value from parameter $b$. The profit of supply chain, retail price, and the supply chain members’ advertising investments are obtained, as are shown in Figures 2, 4, and 6, respectively.

DC1 and DC2 in the figure, respectively, represent supply chain 1 (decentralized) and supply chain 2 (centralized) in the hybrid decision model, while CC and DD, respectively, represent supply chain 1 (or supply chain 2) in centralized decision model and decentralized decision model.

As is shown in Figures 3–6, in decentralized decision model, the advertising investment level of the manufacturers is the lowest, while the terminal retail price is relatively high, and thus the market demand is the lowest. While in centralized decision model, the overall advertising investment level being the highest, the retail price is the lowest, which greatly triggers the market demand. However, in hybrid decision model, one supply chain initially enters vertical competition with the other decentralized supply chain by joint decision. In this case, the supply chain is advantageous in seizing market and the advertising investment level is the highest of the single chain, elevating the chain’s competitiveness and maximizing the chain’s profit (see Figure 1). Therefore, it is indicated that the joint decision of supply chain at this point is a dominant strategy and this first mover advantage impels every supply chain to adopt joint decision. However, if two supply chains adopt joint decision simultaneously, especially when price or advertising competition intensities enhance, they will probably both suffer losses compared to their profits under decentralized decision (see Figures 1 and 2); namely, there occurs a prisoner’s dilemma.
When a certain supply chain initially adopts joint decision, its profit shows significant increase compared with profit under decentralized decision, while the profit of competitive supply chain presents sharp decline. On the observation of the plummeting decline in profit, it is a necessity for the competitive supply chain to adopt joint decision too, which results in profit increase in one chain and profit decrease in the other chain. Therefore, joint decision of supply chains will lead to prisoner's dilemma. The equilibrium results shown in Figures 1 and 2 indicate that for a certain supply chain, joint decision is its strictly dominant strategy, thus simultaneous adoption of joint decision is the equilibrium solution of this supply chain system.

The results in Figures 3–6 are also indicated as follows: in fierce price competition, the corresponding retail price is relatively high. While the manufacturers’ advertising investment level is the lowest and the terminal retail price is also relatively low when advertising competition intensified. Furthermore, the results reflect the influence of competition intensity on supply chain’s choice of decision models, when price and advertising competition intensities both enhance, which impels a rational tendency of the supply chain to choose decentralized decision. Or else, the market will face vicious competition which is unfavorable for both sides. Hence, decentralized decision is the dominant strategy for supply chain system at this point.

5. Conclusions

This work is an effort to explore the advertising and price decision between two competing supply chains. Previous studies concerning advertising or price competition mainly concentrated on the competition in a certain aspect or within a certain chain. However, this essay extends the study on this basis to include the situation in which advertising and price take part in the competition between two supply chains and draws several different conclusions. The results show the following: The changes in advertising and price competition intensity between chains directly affect the level of advertising investment and price; the relationships of supply chain
members present a dynamic situation that alternates between combination and dispersion; also the first mover advantage of joint decision leads to prisoner’s dilemma phenomenon in supply chain system. Further study is probably needed to investigate the advertising and price competition between multiple supply chains in case of the endogenesis of price and cooperative advertising.

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