Research Article

A Dynamic Interval-Valued Intuitionistic Fuzzy Sets Applied to Pattern Recognition

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We present dynamic interval-valued intuitionistic fuzzy sets (DIVIFS), which can improve the recognition accuracy when they are applied to pattern recognition. By analyzing the degree of hesitancy, we propose some DIVIFS models from intuitionistic fuzzy sets (IFS) and interval-valued IFS (IVIFS). And then we present a novel ranking condition on the distance of IFS and IVIFS and introduce some distance measures of DIVIFS satisfying the ranking condition. Finally, a pattern recognition example applied to medical diagnosis decision making is given to demonstrate the application of DIVIFS and its distances. The simulation results show that the DIVIFS method is more comprehensive and flexible than the IFS method and the IVIFS method.

1. Introduction

Professor Zadeh's paper on fuzzy sets [1] has influenced many researchers and has been applied to many application fields, such as pattern recognition [2], fuzzy reasoning [3], portfolio selection [4], and decision making [5]. In 1986, Atanassov [6] introduced membership function, nonmembership function, and hesitancy function and launched the concept of intuitionistic fuzzy sets (IFS), which generalized the FS theory. In the research field of IFS, Yager [7] discussed its cut set characteristics, some scholars (such as [8–13]) applied it to medical diagnosis, Dengfeng and Chuntian [14] and Li et al. [15] applied it to pattern recognition, and Xu [16–20] and Wei [21] applied it to decision making. In 1989, Atanassov and Gargov [22] proposed interval-valued intuitionistic fuzzy sets (IVIFS) based on the comparative analysis of interval-valued fuzzy sets and IFS. Thereafter, many researchers have investigated IVIFS: some researchers discussed the cut set characteristics of IVIFS [23] based on IFS, some applied it to pattern recognition [16, 19, 20, 24], others applied it to decision making analysis [18, 20, 25–29], and Zhang et al. [29] studied interval-valued intuitionistic fuzzy reasoning. However, there are few references related to the construction

of IVIFS from IFS at present. Thus, in this paper, the main objective is to study the construction and application of IVIFS from IFS.

First, we introduce a series of definitions and construction methods of DIVIFS and propose some DIVIFS models according to IFS and IVIFS. Then, on the basis of the traditional distance measures, DIVIFS is applied to pattern recognition on medical diagnosis. The simulation results show that the method introduced in this paper is more comprehensive and flexible than the IFS method and the IVIFS method. Thus, this paper can provide valuable conclusion for the application research of IFS and IVIFS. The model of DIVIFS is also useful for the generalization from fuzzy reasoning [3] and intuitionistic fuzzy reasoning to interval-valued intuitionistic fuzzy reasoning [29].

2. Construction of DIVIFS

Definition 1. An IFS A in universe X is formulated as follows [6, 22]:

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}, \quad (1)$$

where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. $\mu_A(x) \in [0, 1]$ and $\nu_A(x) \in [0, 1]$ denote a degree of membership and a degree of nonmembership of x to A , respectively. For each IFS in X , we call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ a degree of hesitancy of x to A , $0 \leq \pi_A(x) \leq 1$ for each $x \in X$.

Theorem 2. Let A be an IFS as mentioned above; then

$$\mu_A(x) - \mu_B(x) + \nu_A(x) - \nu_B(x) + \pi_A(x) - \pi_B(x) = 0. \quad (2)$$

Based on Definition 1, one has Theorem 2.

Definition 3. An IVIFS A in universe X is stated as follows [22]:

$$A = \{ \langle x, M_A(x), N_A(x) \rangle \mid x \in X \}, \quad (3)$$

where $M_A(x) = [\mu_A^-(x), \mu_A^+(x)]$, $N_A(x) = [\nu_A^-(x), \nu_A^+(x)]$, and $H_A(x) = [\pi_A^-(x), \pi_A^+(x)]$ with the conditions $M_A(x) \subseteq [0, 1]$, $N_A(x) \subseteq [0, 1]$, and $H_A(x) \subseteq [0, 1]$. The interval $M_A(x)$, $N_A(x)$, and $H_A(x)$ denote a degree range of membership, a degree range of non-membership, and a degree range of hesitancy of x to A , respectively. For each IVIFS in X , we call $\pi_A^-(x) = 1 - \mu_A^+(x) - \nu_A^+(x)$ lower boundary of x to A and $\pi_A^+(x) = 1 - \mu_A^-(x) - \nu_A^-(x)$ upper boundary of hesitancy of x to A , respectively, where $0 \leq \pi_A^-(x) \leq \pi_A^+(x) \leq 1$ for each $x \in X$.

Theorem 4. Let A be an IVIFS as mentioned above; then

$$\begin{aligned} \mu_A^+(x) - \mu_B^+(x) + \nu_A^+(x) - \nu_B^+(x) + \pi_A^-(x) - \pi_B^-(x) &= 0, \\ \mu_A^-(x) - \mu_B^-(x) + \nu_A^-(x) - \nu_B^-(x) + \pi_A^+(x) - \pi_B^+(x) &= 0. \end{aligned} \quad (4)$$

Based on Definition 3, one has Theorem 4.

Definition 5. X is a universe of discourse. A DIVIFS A in X is an object having the following form:

$$A = \{ \langle x, M_A(x), N_A(x) \rangle \mid x \in X \}, \quad (5)$$

where $M_A(x)$, $N_A(x)$, and $H_A(x)$ are the same as Definition 3. Let $\mu_A^+(x) = \mu_A^-(x) + \alpha_A(x)$, $\nu_A^+(x) = \nu_A^-(x) + \beta_A(x)$. Obviously, we have $\alpha_A(x) + \beta_A(x) = \pi_A^+(x) - \pi_A^-(x)$, $\alpha_A(x) \geq 0$, $\beta_A(x) \geq 0$. Thus, DIVIFS is equivalent to IVIFS, and IFS is a special case of DIVIFS when $\alpha_A(x) = \beta_A(x) = 0$.

From Definition 3, let all sample data be divided into three parts, $\mu_A^-(x)$ being the firm support party of event A , $\nu_A^-(x)$ representing the firm opposition party of event A , and $\pi_A^+(x)$ showing all the absent party. In the absent party, $\pi_A^-(x)$ is the firm absent party, and $\pi_A^+(x) - \pi_A^-(x)$ is the convertible absent party, in which each sample may become either the support party or the opposition party. If there is $\alpha_A(x)$ sample supporting event A and $\beta_A(x)$ sample opposing event A , we have DIVIFS based on Definition 5. If the proportion of the absent party converted to the support party is $\lambda_{A1}(x)$ and the proportion of the absent party converted to the opposition party is $1 - \lambda_{A1}(x)$, then the model will become a DIVIFS

with single variable, where $\alpha_A(x) = \lambda_{A1}(x)(\pi_A^+(x) - \pi_A^-(x))$, $\beta_A(x) = (1 - \lambda_{A1}(x))(\pi_A^+(x) - \pi_A^-(x))$. If the firm absent party is $\pi_A^-(x) = (1 - \lambda_{A0}(x))\pi_A^+(x)$, then the convertible absent party is $\pi_A^+(x) - \pi_A^-(x) = \lambda_{A0}(x)\pi_A^+(x)$, and then we will get the other DIVIFS definition as follows.

Definition 6. X is a universe of discourse. DIVIFS A in X is an object having the following form

$$A = \{ \langle x, M_A(x), N_A(x) \rangle \mid x \in X \}, \quad (6)$$

where $M_A(x)$, $N_A(x)$, and $H_A(x)$ are the same as Definition 5. For each $\lambda_{A0}(x) \in [0, 1]$ and for each $\lambda_{A1}(x) \in [0, 1]$, we assume that $\mu_A^+(x) = \mu_A^-(x) + \lambda_{A0}(x)\lambda_{A1}(x)\pi_A^+(x)$, $\nu_A^+(x) = \nu_A^-(x) + \lambda_{A0}(x)(1 - \lambda_{A1}(x))\pi_A^+(x)$ and that $\pi_A^-(x) = (1 - \lambda_{A0}(x))\pi_A^+(x)$. Evidently, the DIVIFS model is also equivalent to IVIFS. If $\lambda_{A0}(x) = 0$, then DIVIFS is IFS. And if $\mu_A^-(x) = \mu_A(x)$, $\nu_A^-(x) = \nu_A(x)$, $\pi_A^+(x) = \pi_A(x)$, then we have a special DIVIFS which is derived from IFS.

Definition 7. A DIVIFS derived from IFS is given as follows:

$$A = \{ \langle x, M_A(x), N_A(x) \rangle \mid x \in X \}, \quad (7)$$

where $M_A(x)$, $N_A(x)$, and $H_A(x)$ are the same as Definition 5 and $\mu_A^-(x) = \mu_A(x)$, $\nu_A^-(x) = \nu_A(x)$, $\pi_A^+(x) = \pi_A(x)$, $\mu_A^+(x) = \mu_A(x) + \lambda_{A0}(x)\lambda_{A1}(x)\pi_A(x)$, $\nu_A^+(x) = \nu_A(x) + \lambda_{A0}(x)(1 - \lambda_{A1}(x))\pi_A(x)$, $\pi_A^-(x) = (1 - \lambda_{A0}(x))\pi_A(x)$, for each $\lambda_{A0}(x) \in [0, 1]$ and for each $\lambda_{A1}(x) \in [0, 1]$. This DIVIFS model is also an extension of the IFS model.

3. Conventional Distance Measures of IFS and IVIFS

Derived from the definitions of FS, IFS, and IVIFS, some distance measures are discussed (Figure 1). Let $d = 1 - S$, $\Delta\mu = \mu_A - \mu_B$, $\Delta\nu = \nu_A - \nu_B$, and $\Delta\pi = \pi_A - \pi_B$. According to Definition 1 and Theorem 2, $\Delta\mu + \Delta\nu + \Delta\pi = 0$.

According to fuzzy sets [1], the distance measure of FS is composed of membership degree:

$$d_{Za}(A, B) = \left(\sum_{i=1}^n w_i |\Delta\mu|^p \right)^{1/p}. \quad (8)$$

In [6], Atanassov introduced a distance measure (9) of IFS from membership degree and non-membership degree:

$$d_{At}(A, B) = \left(\sum_{i=1}^n w_i (|\Delta\mu|^p + |\Delta\nu|^p) \right)^{1/p}. \quad (9)$$

Considering the degree range of hesitancy, Szmidt and Kacprzyk presented a novel distance measure (10) according to Definition 1 and Theorem 2 in 2000 [30]:

$$d_{S,K}(A, B) = \left(\sum_{i=1}^n w_i (|\Delta\mu|^p + |\Delta\nu|^p + |\Delta\mu + \Delta\nu|^p) \right)^{1/p}. \quad (10)$$

FIGURE 1: Distance between each patient and each disease on IFS calculated by Xu's methods.

In [14], Dengfeng and Chuntian proposed a novel distance model as follows:

$$d_{L,C}(A, B) = \left(\sum_{i=1}^n w_i |\Delta\mu - \Delta\nu|^p \right)^{1/p}. \quad (11)$$

At the same time, Grzegorzewski [31] and Hung and Yang [32] proposed a Hausdorff distance measure of IFS, respectively:

$$d_{P,H}(A, B) = \sum_{i=1}^n w_i \max(|\Delta\mu|, |\Delta\nu|). \quad (12)$$

In 2005, Wang and Xin [33] presented a new distance measure which is made up of d_{At} and $d_{P,H}$:

$$d_W(A, B) = \sum_{i=1}^n w_i \frac{|\Delta\mu| + |\Delta\nu| + 2 \max(|\Delta\mu|, |\Delta\nu|)}{4}. \quad (13)$$

In 2005, Liu [34] introduced some similar distance formulas as mentioned above.

In Szmidt and Kacprzyk [8–10], the ratio-based measure of similarity is defined as follows:

$$d_{RB}(A, B) = \frac{d(A, B)}{d(A, B^C)}. \quad (14)$$

If B is a set $(\mu_B(x), \nu_B(x), \pi_B(x))$, Szmidt and Kacprzyk describe B^C to be a set $(\nu_B(x), \mu_B(x), \pi_B(x))$. For example, if B is $(\mu_B(x) = 0.7, \nu_B(x) = 0, \pi_B(x) = 0.3)$, then B^C is $(0, 0.7, 0.3)$, and so on. And d denotes a kind of distance measure about A and B .

In 2010, Xia and Xu [20] presented some weighted distance formulas, where $k \geq 0$ and $\lambda \geq 0$ denote the weights:

$$d_{D_k}(A, B) = \left[\sum_{i=1}^n w_i (|(1-k)\Delta\mu - k\Delta\nu|^p) \right]^{1/p}, \quad (15)$$

$$\begin{aligned} d_{F_{k,\lambda}}(A, B) &= \left[0.5 \sum_{i=1}^n w_i (|(1-k)\Delta\mu - k\Delta\nu|^p + |(1-\lambda)\Delta\nu - \lambda\Delta\mu|^p \right. \\ &\quad \left. + |(1-k-\lambda)(\Delta\mu + \Delta\nu)|^p) \right]^{1/p}, \\ &k + \lambda \leq 1, \end{aligned} \quad (16)$$

$$\begin{aligned} d_{G_{k,\lambda}}(A, B) &= \left[0.5 \sum_{i=1}^n w_i (|k\Delta\mu|^p + |\lambda\Delta\nu|^p + |k\Delta\mu + \lambda\Delta\nu|^p) \right]^{1/p}, \\ &k + \lambda \leq 1, \end{aligned} \quad (17)$$

$$\begin{aligned} d_{H_{k,\lambda}}(A, B) &= \left[0.5 \sum_{i=1}^n w_i (|k\Delta\mu|^p + |\lambda\Delta\mu - (1-\lambda)\Delta\nu|^p \right. \\ &\quad \left. + |(k-\lambda)\Delta\mu + (1-\lambda)\Delta\nu|^p) \right]^{1/p}, \\ &k + \lambda \leq 1, \end{aligned} \quad (18)$$

$$\begin{aligned} d_{H_{k,\lambda}^*}(A, B) &= \left[0.5 \sum_{i=1}^n w_i (|k\Delta\mu|^p + |k\lambda\Delta\mu - (1-\lambda)\Delta\nu|^p \right. \\ &\quad \left. + |(1-\lambda)(k\Delta\mu + \Delta\nu)|^p) \right]^{1/p}, \\ &k + \lambda \leq 1, \end{aligned} \quad (19)$$

$$\begin{aligned} d_{J_{k,\lambda}}(A, B) &= \left[0.5 \sum_{i=1}^n w_i (|(1-k)\Delta\mu - k\Delta\nu|^p + |\lambda\Delta\nu|^p \right. \\ &\quad \left. + |(1-k)\Delta\mu + (\lambda-k)\Delta\nu|^p) \right]^{1/p}, \\ &k + \lambda \leq 1, \end{aligned} \quad (20)$$

$$\begin{aligned} d_{J_{k,\lambda}^*}(A, B) &= \left[0.5 \sum_{i=1}^n w_i (|(1-k)\Delta\mu - k\lambda\Delta\nu|^p + |\lambda\Delta\nu|^p \right. \\ &\quad \left. + |(1-k)(\Delta\mu + \lambda\Delta\nu)|^p) \right]^{1/p}, \\ &k + \lambda \leq 1. \end{aligned} \quad (21)$$

Let $\Delta\mu^- = \mu_A^- - \mu_B^-$, $\Delta\mu^+ = \mu_A^+ - \mu_B^+$, $\Delta\nu^- = \nu_A^- - \nu_B^-$, $\Delta\nu^+ = \nu_A^+ - \nu_B^+$, $\Delta\pi^- = \pi_A^- - \pi_B^-$, $\Delta\pi^+ = \pi_A^+ - \pi_B^+$. According to Definition 3 and Theorem 4, $\Delta\mu^- + \Delta\nu^- + \Delta\pi^- = 0$, $\Delta\mu^+ + \Delta\nu^+ + \Delta\pi^+ = 0$. Similar to the distance measures of IFS, some scholars define some distance measures made up of $\Delta\mu^-$, $\Delta\mu^+$, $\Delta\nu^-$, $\Delta\nu^+$, $\Delta\pi^-$, $\Delta\pi^+$.

4. Construction of Distance Measures on DIVIFS

First, we will define a novel essential ranking property of the distance measure on IFS and IVIFS.

Definition 8. Assume that A and B are IFSs, and then we have the following:

$A = B$ if and only if $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\pi_A(x) = \pi_B(x)$.

$A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$.

$A \subset B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) > \nu_B(x)$, or $\mu_A(x) < \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$.

Definition 9. Assume that A and B are IVIFSs, and then we obtain the following:

$A = B$ if and only if $\mu_A^-(x) = \mu_B^-(x)$, $\mu_A^+(x) = \mu_B^+(x)$, $\nu_A^-(x) = \nu_B^-(x)$, $\nu_A^+(x) = \nu_B^+(x)$, $\pi_A^-(x) = \pi_B^-(x)$, $\pi_A^+(x) = \pi_B^+(x)$.

$A \subseteq B$ if and only if $\mu_A^-(x) \leq \mu_B^-(x)$, $\mu_A^+(x) \leq \mu_B^+(x)$, $\nu_A^-(x) \geq \nu_B^-(x)$, $\nu_A^+(x) \geq \nu_B^+(x)$.

$A \subset B$ if and only if $A \subseteq B$ and $A \neq B$.

In general, a distance measure should conform to the following Theorem 10 according to its properties.

Theorem 10. A , B , and C are IFSs; the distances among them must satisfy the following:

(P1) $d(A, B) \geq 0$ and $d(A, B) = 0$ if and only if $A = B$.

(P2) $d(A, B) = d(B, A)$.

(P3) If $C \subseteq B \subseteq A$, then $d(A, C) \geq d(A, B)$ and $d(A, C) \geq d(B, C)$.

If $C \subset B \subset A$, then $d(A, C) > d(A, B)$ and $d(A, C) > d(B, C)$.

(P4) If $D \subseteq C \subseteq B \subseteq A$, then $d(A, D) \geq d(B, C)$, and $d(A, D) = d(B, C)$ if and only if $A = B$ and $C = D$.

If $D \subset C \subset B \subset A$, then $d(A, D) > d(B, C)$.

In fact, property (P4) is derived from property (P3). According to (P3), one has the following:

If $D \subseteq C \subseteq B \subseteq A$, then $d(A, D) \geq d(A, C) \geq d(B, C)$, $d(A, D) \geq d(B, D) \geq d(B, C) \rightarrow d(A, D) \geq d(B, C)$.

If $D \subset C \subset B \subset A$, then $d(A, D) > d(A, C) > d(B, C)$, $d(A, D) > d(B, D) > d(B, C) \rightarrow d(A, D) > d(B, C)$.

Obviously, for all the distance formulas above, (P1), (P2), and (P3) can be obtained. However, most of them do not satisfy the ranking property (P4).

Example 11. Let $A = (0.5, 0.4, 0.1)$, $B = (0.5, 0.5, 0)$, $C = (0.4, 0.4, 0.2)$, and $D = (0.4, 0.5, 0.1)$. For IFS A , we obtain $\mu_A(x) = 0.5$, $\nu_A(x) = 0.4$, and $\pi_A(x) = 0.1$. In the following, we denote $\mu_A = \mu_A(x) = 0.5$, $\nu_A = \nu_A(x) = 0.4$, and $\pi_A = \pi_A(x) = 0.1$.

According to Definition 3, if $\mu^-(x) = \mu^+(x) = \mu(x)$, $\nu^-(x) = \nu^+(x) = \nu(x)$, $\pi^-(x) = \pi^+(x) = \pi(x)$, then IVIFS is equivalent to IFS. Correspondingly, we can define some IVIFSs as follows, respectively:

$$\begin{aligned} A &= \langle x, [0.5, 0.5], [0.4, 0.4] \rangle, \\ B &= \langle x, [0.5, 0.5], [0.5, 0.5] \rangle, \\ C &= \langle x, [0.4, 0.4], [0.4, 0.4] \rangle, \\ D &= \langle x, [0.4, 0.4], [0.5, 0.5] \rangle. \end{aligned} \quad (22)$$

If $X = \{x_1, x_2, \dots, x_n\}$ is a universe of discourse, we define

$$\begin{aligned} A &= \{ \langle x_1, [0.5, 0.5], [0.4, 0.4] \rangle, \\ &\quad \dots, \langle x_n, [0.5, 0.5], [0.4, 0.4] \rangle \}; \\ B &= \{ \langle x_1, [0.5, 0.5], [0.5, 0.5] \rangle, \\ &\quad \dots, \langle x_n, [0.5, 0.5], [0.5, 0.5] \rangle \}; \\ C &= \{ \langle x_1, [0.4, 0.4], [0.4, 0.4] \rangle, \\ &\quad \dots, \langle x_n, [0.4, 0.4], [0.4, 0.4] \rangle \}; \\ D &= \{ \langle x_1, [0.4, 0.4], [0.5, 0.5] \rangle, \\ &\quad \dots, \langle x_n, [0.4, 0.4], [0.5, 0.5] \rangle \}. \end{aligned} \quad (23)$$

Thus we have $\mu_A(x) = \mu_B(x) > \mu_C(x) = \mu_D(x)$, $\nu_A(x) = \nu_C(x) < \nu_B(x) = \nu_D(x)$. According to Definitions 8 and 9, we have $D \subset C \subset B \subset A$. Therefore, we obtain the necessary condition for distance measure on IFS and IVIFS: $d(A, D) > d(B, C)$.

Obviously, according to A , B , C , and D , we obtain

$$\begin{aligned} \Delta\mu(A, D) &= \mu_A - \mu_D = 0.1, \\ \Delta\nu(A, D) &= \nu_A - \nu_D = -0.1, \\ \Delta\pi(A, D) &= \pi_A - \pi_D = 0, \\ \Delta\mu(B, C) &= \mu_B - \mu_C = 0.1, \\ \Delta\nu(B, C) &= \nu_B - \nu_C = 0.1, \\ \Delta\pi(B, C) &= \pi_B - \pi_C = 0.2. \end{aligned} \quad (24)$$

From formulas (8)–(14), we have

$$\begin{aligned} d_{Za}(A, D) &= \left(\sum_{i=1}^n w_i 0.1^p \right)^{1/p} = d_{Za}(B, C), \\ d_{At}(A, D) &= \left(\sum_{i=1}^n w_i (0.1^p + 0.1^p) \right)^{1/p} = d_{At}(B, C), \\ d_{s,K}(A, D) &= \left(\sum_{i=1}^n w_i (0.1^p + 0.1^p) \right)^{1/p} \\ &< \left(\sum_{i=1}^n w_i (0.1^p + 0.1^p + 0.2^p) \right)^{1/p} = d_{s,K}(B, C), \\ d_{L,C}(A, D) &= \left(\sum_{i=1}^n w_i 0.2^p \right)^{1/p} > 0 = d_{L,C}(B, C), \end{aligned}$$

$$\begin{aligned}
d_{p,H}(A, D) &= \sum_{i=1}^n w_i \max(0.1, 0.1) = d_{p,H}(B, C), \\
d_W(A, D) &= \sum_{i=1}^n w_i \frac{0.1 + 0.1 + 2 \max(0.1, 0.1)}{4} = d_W(B, C), \\
d_{RB}(A, D) &= +\infty > 1 = d_{RB}(B, C).
\end{aligned} \tag{25}$$

Only (11) and (14) satisfy property (P4).

Considering (15), we get

$$\begin{aligned}
d_{D_k}(A, D) &= \left(\sum_{i=1}^n w_i (0.1^p) \right)^{1/p} \\
&\geq \left(\sum_{i=1}^n w_i (|1 - 2k| 0.1^p) \right)^{1/p} \\
&= d_{D_k}(B, C).
\end{aligned} \tag{26}$$

If $k \neq 0$ and $k \neq 1$, then we have $d_{D_k}(A, D) > d_{D_k}(B, C)$; otherwise, we have $d_{D_k}(A, D) = d_{D_k}(B, C)$.

From (16), we get

$$\begin{aligned}
d_{F_{k,\lambda}}(A, D) &= \left(0.5 \sum_{i=1}^n w_i (0.1^p + 0.1^p) \right)^{1/p}, \\
d_{F_{k,\lambda}}(B, C) &= \left(0.5 \sum_{i=1}^n w_i 0.1^p (|1 - 2k|^p + |1 - 2\lambda|^p + |1 - k - \lambda|^p) \right)^{1/p}, \\
&\quad k + \lambda \leq 1.
\end{aligned} \tag{27}$$

If $p = 1$, then we have $k + \lambda < 1/2 \rightarrow d_{D_k}(A, D) < d_{D_k}(B, C)$; $k + \lambda \geq 1/2 \rightarrow d_{D_k}(A, D) \geq d_{D_k}(B, C)$.

From (17), we get

$$\begin{aligned}
d_{G_{k,\lambda}}(A, D) &= \left(0.5 \sum_{i=1}^n w_i (|0.1k|^p + |0.1\lambda|^p + |0.1(k - \lambda)|^p) \right)^{1/p} \\
&\leq \left(0.5 \sum_{i=1}^n w_i (|0.1k|^p + |0.1\lambda|^p + |0.1(k + \lambda)|^p) \right)^{1/p} \\
&= d_{G_{k,\lambda}}(B, C).
\end{aligned} \tag{28}$$

Obviously, we get $d_{D_k}(A, D) \leq d_{D_k}(B, C)$ for each $k \geq 0$, for each $\lambda \geq 0$, and for each $P > 0$.

For (18), we have

$$\begin{aligned}
d_{H_{k,\lambda}}(A, D) &= \left(0.5 \sum_{i=1}^n w_i (|0.1k|^p + |0.1|^p + |0.1(1 - k)|^p) \right)^{1/p}, \\
d_{H_{k,\lambda}}(B, C) &= \left(0.5 \sum_{i=1}^n w_i (|0.1k|^p + |0.1(1 - 2\lambda)|^p \right. \\
&\quad \left. + |0.1(1 + k - 2\lambda)|^p) \right)^{1/p}.
\end{aligned} \tag{29}$$

If $p = 1$ and $k + \lambda \leq 1$, then we have $0 \leq \lambda < 1/3$, $2\lambda < k \rightarrow d_{H_{k,\lambda}}(A, D) < d_{H_{k,\lambda}}(B, C)$; otherwise, $d_{D_k}(A, D) \geq d_{D_k}(B, C)$.

For example, let $p = 1$, $\lambda = 0.2$, and $k = 0.5$, and then $d_{H_{k,\lambda}}(A, D) = 0.5 \sum_{i=1}^n w_i 0.2 < 0.5 \sum_{i=1}^n w_i 0.22 = d_{H_{k,\lambda}}(B, C)$.

Similar to the result from (18), for (20), we have

$$\begin{aligned}
d_{J_{k,\lambda}}(A, D) &= \left(0.5 \sum_{i=1}^n w_i (|0.1|^p + |0.1\lambda|^p + |0.1(1 - \lambda)|^p) \right)^{1/p}, \\
d_{J_{k,\lambda}}(B, C) &= \left(0.5 \sum_{i=1}^n w_i (|0.1(1 - 2k)|^p + |0.1\lambda|^p \right. \\
&\quad \left. + |0.1(1 + \lambda - 2k)|^p) \right)^{1/p}.
\end{aligned} \tag{30}$$

If $p = 1$ and $k + \lambda \leq 1$, then we have $0 \leq k < 1/3$, $2k < \lambda \rightarrow d_{H_{k,\lambda}}(A, D) < d_{H_{k,\lambda}}(B, C)$; otherwise, $d_{D_k}(A, D) \geq d_{D_k}(B, C)$.

For example, let $p = 1$, $k = 0.2$, and $\lambda = 0.5$, and then $d_{H_{k,\lambda}}(A, D) = 0.5 \sum_{i=1}^n w_i 0.2 < 0.5 \sum_{i=1}^n w_i 0.22 = d_{H_{k,\lambda}}(B, C)$.

For (19), we obtain

$$\begin{aligned}
d_{H_{k,\lambda}^*}(A, D) &= \left(0.5 \sum_{i=1}^n w_i (|0.1k|^p + |0.1(1 + k\lambda - \lambda)|^p \right. \\
&\quad \left. + |0.1(1 - \lambda)(1 - k)|^p) \right)^{1/p}, \\
d_{H_{k,\lambda}^*}(B, C) &= \left(0.5 \sum_{i=1}^n w_i (|0.1k|^p + |0.1(1 - k\lambda - \lambda)|^p \right. \\
&\quad \left. + |0.1(1 - \lambda)(1 + k)|^p) \right)^{1/p}, \\
&\quad k + \lambda \leq 1.
\end{aligned} \tag{31}$$

If $p = 1$ and $k + \lambda \leq 1$, then we have $0 \leq \lambda < 1/2 \rightarrow d_{H_{k,\lambda}}(A, D) < d_{H_{k,\lambda}}(B, C)$; otherwise, $d_{D_k}(A, D) \geq d_{D_k}(B, C)$.

Similarly, we have the same results for (21), which will be as follows:

if $p = 1$ and $k + \lambda \leq 1$, then $0 \leq k < 1/2 \rightarrow d_{H_{k,\lambda}}(A, D) < d_{H_{k,\lambda}}(B, C)$; otherwise, $d_{D_k}(A, D) \geq d_{D_k}(B, C)$.

From the analysis to the formulas from Xia and Xu above, only (15) basically meets the requirement of (P4); the others only meet the requirement of (P4) in part. Similarly, most of the conventional distance measures of IVIFS do not satisfy property (P4). Therefore, it is necessary to find a distance measure satisfying the necessary ranking property. Considering the component elements of the distance formula of IFS and IVIFS, we can define some appropriate distance measures of DIVIFS (Figure 2).

According to Definition 7 and Example 11, we have

$$\mu_A^-(x) = 0.5, \nu_A^-(x) = 0.4, \pi_A^+(x) = 0.1, \mu_A^+(x) = 0.5 + \lambda_0\lambda_1 0.1, \nu_A^+(x) = 0.4 + \lambda_0(1 - \lambda_1)0.1, \pi_A^-(x) = (1 - \lambda_0)0.1,$$

$$\mu_B^-(x) = 0.5, \nu_B^-(x) = 0.5, \pi_B^-(x) = 0, \mu_B^+(x) = 0.5, \nu_B^+(x) = 0.5, \pi_B^+(x) = 0,$$

$$\mu_C^-(x) = 0.4, \nu_C^-(x) = 0.4, \pi_C^+(x) = 0.2, \mu_C^+(x) = 0.4 + \lambda_0\lambda_1 0.2, \nu_C^+(x) = 0.4 + \lambda_0(1 - \lambda_1)0.2, \pi_C^-(x) = (1 - \lambda_0)0.2,$$

$$\mu_D^-(x) = 0.4, \nu_D^-(x) = 0.5, \pi_D^+(x) = 0.1, \mu_D^+(x) = 0.4 + \lambda_0\lambda_1 0.1, \nu_D^+(x) = 0.5 + \lambda_0(1 - \lambda_1)0.1, \pi_D^-(x) = (1 - \lambda_0)0.1.$$

Let $\mu^- = \mu^-(x)$, $\mu^+ = \mu^+(x)$, $\nu^- = \nu^-(x)$, $\nu^+ = \nu^+(x)$, $\pi^- = \pi^-(x)$, $\pi^+ = \pi^+(x)$, and we obtain

$$(1) \Delta\pi^-(A, D) = \pi_A^- - \pi_D^- = 0, \Delta\pi^-(B, C) = \pi_B^- - \pi_C^- = -(1 - \lambda_0)0.2 \rightarrow |\Delta\pi^-(A, D)| \leq |\Delta\pi^-(B, C)|;$$

$$(2) \Delta\pi^+(A, D) = \pi_A^+ - \pi_D^+ = 0 < \pi_B^+ - \pi_C^+ = \Delta\pi^+(B, C) = 0.2 \rightarrow |\Delta\pi^+(A, D)| < |\Delta\pi^+(B, C)|;$$

$$(3) \Delta\mu^-(A, D) = \mu_A^- - \mu_D^- = 0.1 = \mu_B^- - \mu_C^- = \Delta\mu^-(B, C) \rightarrow |\Delta\mu^-(A, D)| = |\Delta\mu^-(B, C)|;$$

$$(4) \Delta\nu^-(A, D) = \nu_A^- - \nu_D^- = -0.1 = -1 \times (\nu_B^- - \nu_C^-) = -\Delta\nu^-(B, C) \rightarrow |\Delta\nu^-(A, D)| = |\Delta\nu^-(B, C)|;$$

$$(5) \Delta\mu^-(A, D) - \Delta\nu^-(A, D) = 0.2 > 0 = \Delta\mu^-(B, C) - \Delta\nu^-(B, C) \rightarrow |\Delta\mu^-(A, D) - \Delta\nu^-(A, D)| > |\Delta\mu^-(B, C) - \Delta\nu^-(B, C)|;$$

$$(6) \Delta\mu^+(A, D) = \mu_A^+ - \mu_D^+ = 0.1, \Delta\mu^+(B, C) = \mu_B^+ - \mu_C^+ = 0.1(1 - 2\lambda_0\lambda_1) \rightarrow |\Delta\mu^+(A, D)| \geq |\Delta\mu^+(B, C)|;$$

$$(7) \Delta\nu^+(A, D) = \nu_A^+ - \nu_D^+ = -0.1, \Delta\nu^+(B, C) = \nu_B^+ - \nu_C^+ = 0.1(1 - 2\lambda_0(1 - \lambda_1)) \rightarrow |\Delta\nu^+(A, D)| \geq |\Delta\nu^+(B, C)|;$$

$$(8) \Delta\mu^+(A, D) - \Delta\nu^+(A, D) = 0.2, \Delta\mu^+(B, C) - \Delta\nu^+(B, C) = 0.2\lambda_0(1 - 2\lambda_1), \rightarrow |\Delta\mu^+(A, D) - \Delta\nu^+(A, D)| \geq |\Delta\mu^+(B, C) - \Delta\nu^+(B, C)|.$$

Considering the necessary condition (P4), we draw a conclusion as follows. $\Delta\pi^+$ does not meet the requirement of the ranking property, and if $\lambda_0 \neq 1$, $\Delta\pi^-$ does not satisfy (P4)

too. $\Delta\mu^- - \Delta\nu^-$ meets the necessary condition (P4), and $\Delta\mu^+$, $\Delta\nu^+$, and $\Delta\mu^+ - \Delta\nu^+$ do so if $\lambda_k \in (0, 1)$ ($k = 1, 2$). All the calculated results are shown in (32) and (33):

$$|\Delta\pi^-(A, D)| \leq |\Delta\pi^-(B, C)|;$$

$$|\Delta\pi^-(A, D)| = |\Delta\pi^-(B, C)| \iff \lambda_0 = 1;$$

$$|\Delta\pi^+(A, D)| = |\Delta\pi^+(B, C)|;$$

$$|\Delta\mu^-(A, D)| = |\Delta\mu^-(B, C)|;$$

$$|\Delta\nu^-(A, D)| = |\Delta\nu^-(B, C)|;$$

$$|\Delta\mu^-(A, D) - \Delta\nu^-(A, D)| > |\Delta\mu^-(B, C) - \Delta\nu^-(B, C)|;$$

$$|\Delta\mu^+(A, D)| \geq |\Delta\mu^+(B, C)|;$$

$$|\Delta\mu^+(A, D)|$$

$$= |\Delta\mu^+(B, C)|$$

$$\iff \lambda_0 = 0, \quad \text{or} \quad \lambda_1 = 0, \quad \text{or} \quad \lambda_0 = \lambda_1 = 1;$$

$$|\Delta\nu^+(A, D)| \geq |\Delta\nu^+(B, C)|;$$

$$|\Delta\nu^+(A, D)|$$

$$= |\Delta\nu^+(B, C)|$$

$$\iff \lambda_0 = 0, \quad \text{or} \quad \lambda_1 = 1, \quad \text{or} \quad \lambda_0 = 1, \lambda_1 = 0; \quad (32)$$

$$|\Delta\mu^+(A, D) - \Delta\nu^+(A, D)|$$

$$\geq |\Delta\mu^+(B, C) - \Delta\nu^+(B, C)|;$$

$$|\Delta\mu^+(A, D) - \Delta\nu^+(A, D)| \quad (33)$$

$$= |\Delta\mu^+(B, C) - \Delta\nu^+(B, C)|$$

$$\iff \lambda_0 = 1, \lambda_1 = 0, \quad \text{or} \quad \lambda_0 = 1, \lambda_1 = 1.$$

In all, in order to meet the needs of necessary condition (P4), we conclude that the distance measure of DIVIFS should be composed of $\Delta\mu^- - \Delta\nu^-$, $\Delta\mu^+$, $\Delta\nu^+$, and $\Delta\mu^+ - \Delta\nu^+$ and that $\Delta\mu^-$ and $\Delta\nu^-$ can be involved in the distance formula too. However, considering the contradiction between $\Delta\pi$ and (P4), $\Delta\pi^+$ and $\Delta\pi^-$ are not suitable for new distance measures.

In Xu's papers [16, 17, 19, 20], many weighted similarity measures are proposed according to standardized distance measures based on the IFS theory and the IVIFS theory. And Xu presented them by defining the appropriate weights of membership function, non-membership function, and hesitancy function. According to Xu, we can also define

FIGURE 2: Distance between each patient and each disease on DIVIFS.

the corresponding weighted distance measures on IVIFS as follows:

$$\begin{aligned}
 d(A, B) &= \left\{ \sum_{i=1}^n w_i \left[C_1 |\Delta \mu^-(A(x_i), B(x_i))|^p \right. \right. \\
 &\quad + C_2 |\Delta \mu^+(A(x_i), B(x_i))|^p \\
 &\quad + C_3 |\Delta \nu^-(A(x_i), B(x_i))|^p \\
 &\quad + C_4 |\Delta \nu^+(A(x_i), B(x_i))|^p \\
 &\quad + |C_5 \Delta \mu^-(A(x_i), B(x_i)) \\
 &\quad \quad - C_7 \Delta \nu^-(A(x_i), B(x_i))|^p \\
 &\quad + |C_6 \Delta \mu^+(A(x_i), B(x_i)) \\
 &\quad \quad - C_8 \Delta \nu^+(A(x_i), B(x_i))|^p \left. \right] \right\}^{1/p}; \\
 \sum_{i=1}^n w_i &= 1, \quad w_i \geq 0; \\
 C_j &\geq 0, \quad j = 1, 2, 3, 4, 5, 6, 7, 8.
 \end{aligned} \tag{34}$$

It is easy to prove that (11) is equivalent to a special case of (15) when $k = 0.5$. And (15) is also a special case of (34) when $C_1 = C_2 = C_3 = C_4 = C_6 = C_8 = 0$ and $C_5 = C_7 = 1$. To simplify the formula, let $C_5 = C_7$ and $C_6 = C_8$. Assume that A and B are two DIVIFS, and we have

$$\begin{aligned}
 d(A, B) &= \left\{ \sum_{i=1}^n w_i \left[C_1 |\mu_A^-(x_i) - \mu_B^-(x_i)|^p + C_2 |\nu_A^-(x_i) - \nu_B^-(x_i)|^p \right. \right. \\
 &\quad + C_3 |\mu_A^-(x_i) - \mu_B^-(x_i) \\
 &\quad \quad + \lambda_0 \lambda_1 (\pi_A^+(x_i) - \pi_B^+(x_i))|^p \\
 &\quad + C_4 |\nu_A^-(x_i) - \nu_B^-(x_i) \\
 &\quad \quad + \lambda_0 (1 - \lambda_1) (\pi_A^+(x_i) - \pi_B^+(x_i))|^p \\
 &\quad + C_5 |(\mu_A^-(x_i) - \mu_B^-(x_i)) - (\nu_A^-(x_i) - \nu_B^-(x_i))|^p \\
 &\quad + C_6 |(\mu_A^-(x_i) - \mu_B^-(x_i)) - (\nu_A^-(x_i) - \nu_B^-(x_i)) \\
 &\quad \quad + \lambda_0 (2\lambda_1 - 1) (\pi_A^+(x_i) - \pi_B^+(x_i))|^p \left. \right] \right\}^{1/p}, \\
 \sum_{i=1}^n w_i &= 1, \quad w_i \geq 0; \quad C_j \geq 0, \quad j = 1, 2, 3, 4, 5, 6.
 \end{aligned} \tag{35}$$

Comparing (15)–(21) with (34)–(35), we conclude that Xu's methods are different from the DIVIFS method. Xu defined the measures by constructing the weights of membership function, non-membership function, and hesitancy function, respectively. However, we concentrate on the detachment of hesitancy function and present the DIVIFS method. If we use Xu's methods solely, we can define the weights of certainty and uncertainty, but we cannot analyze the variation of the absent party. Moreover, in Xu's research, and even in all the conventional research, there is no research on the ranking condition (P4). Therefore, the distance measures on DIVIFS above are an important supplement to Xu's methods and the others'.

In order to simplify the calculation, we define the following distance measure:

$$\begin{aligned}
 d(A, B) &= \sum_{i=1}^n w_i \left[C_1 |\mu_A^-(x_i) - \mu_B^-(x_i)|^p + C_2 |\nu_A^-(x_i) - \nu_B^-(x_i)|^p \right. \\
 &\quad + C_3 |\mu_A^-(x_i) - \mu_B^-(x_i) \\
 &\quad \quad \quad \left. + \lambda_0 \lambda_1 (\pi_A^+(x_i) - \pi_B^+(x_i))|^p \right. \\
 &\quad + C_4 |\nu_A^-(x_i) - \nu_B^-(x_i) \\
 &\quad \quad \quad \left. + \lambda_0 (1 - \lambda_1) (\pi_A^+(x_i) - \pi_B^+(x_i))|^p \right. \\
 &\quad + C_5 |(\mu_A^-(x_i) - \mu_B^-(x_i)) - (\nu_A^-(x_i) - \nu_B^-(x_i))|^p \\
 &\quad + C_6 |(\mu_A^-(x_i) - \mu_B^-(x_i)) - (\nu_A^-(x_i) - \nu_B^-(x_i)) \\
 &\quad \quad \quad \left. + \lambda_0 (2\lambda_1 - 1) (\pi_A^+(x_i) - \pi_B^+(x_i))|^p \right], \\
 \sum_{i=1}^n w_i &= 1, \quad w_i \geq 0; \\
 C_j &\geq 0, \quad j = 1, 2, 3, 4, 5, 6.
 \end{aligned} \tag{36}$$

If $p = 1$, the distance measure is the Hamming distance; if $p = 2$, it is the Euclidean distance. In this paper, the Euclidean distance on DIVIFS will be used in the simulation of pattern recognition:

$$\begin{aligned}
 d(A, B) &= \sum_{i=1}^n \left\{ [\mu_A^-(x_i) - \mu_B^-(x_i)]^2 + [\nu_A^-(x_i) - \nu_B^-(x_i)]^2 \right. \\
 &\quad + [\mu_A^-(x_i) - \mu_B^-(x_i) + \lambda_0 \lambda_1 (\pi_A^+(x_i) - \pi_B^+(x_i))]^2 \\
 &\quad + [\nu_A^-(x_i) - \nu_B^-(x_i) \\
 &\quad \quad \quad \left. + \lambda_0 (1 - \lambda_1) (\pi_A^+(x_i) - \pi_B^+(x_i))]^2 \right\},
 \end{aligned} \tag{37}$$

$$\begin{aligned}
 d(A, B) &= \sum_{i=1}^n \left\{ [(\mu_A^-(x_i) - \mu_B^-(x_i)) - (\nu_A^-(x_i) - \nu_B^-(x_i))]^2 \right. \\
 &\quad + [(\mu_A^-(x_i) - \mu_B^-(x_i)) - (\nu_A^-(x_i) - \nu_B^-(x_i)) \\
 &\quad \quad \quad \left. + \lambda_0 (2\lambda_1 - 1) (\pi_A^+(x_i) - \pi_B^+(x_i))]^2 \right\}, \\
 d(A, B) &= \sum_{i=1}^n \left\{ [\mu_A^-(x_i) - \mu_B^-(x_i)]^2 + [\nu_A^-(x_i) - \nu_B^-(x_i)]^2 \right. \\
 &\quad + [\mu_A^-(x_i) - \mu_B^-(x_i) + \lambda_0 \lambda_1 (\pi_A^+(x_i) - \pi_B^+(x_i))]^2 \\
 &\quad + [\nu_A^-(x_i) - \nu_B^-(x_i) \\
 &\quad \quad \quad \left. + \lambda_0 (1 - \lambda_1) (\pi_A^+(x_i) - \pi_B^+(x_i))]^2 \right. \\
 &\quad + [(\mu_A^-(x_i) - \mu_B^-(x_i)) - (\nu_A^-(x_i) - \nu_B^-(x_i))]^2 \\
 &\quad + [(\mu_A^-(x_i) - \mu_B^-(x_i)) - (\nu_A^-(x_i) - \nu_B^-(x_i)) \\
 &\quad \quad \quad \left. + \lambda_0 (2\lambda_1 - 1) (\pi_A^+(x_i) - \pi_B^+(x_i))]^2 \right\}.
 \end{aligned} \tag{38}$$

5. Application to Medical Diagnosis Based on Pattern Recognition

To make a proper diagnosis D for a patient with given values of symptoms S , a medical knowledge base is necessary that involves elements described in terms of IFS. We adopt the same data as those in [8, 9]: the set of disease diagnoses is $D = \{\text{Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}\}$, and the set of symptoms is a universe of discourse $S = \{\text{temperature, headache, stomach pain, cough, chest pain}\}$. The data are given in Table 1, where each symptom is described as follows: membership function $\mu_A(x)$, non-membership function $\nu_A(x)$, and hesitancy function $\pi_A(x)$. For example, for malaria the temperature is high ($\mu_A(x) = 0.7$, $\nu_A(x) = 0$, $\pi_A(x) = 0.3$), while for chest problem the temperature is low ($\mu_A(x) = 0.1$, $\nu_A(x) = 0.8$, $\pi_A(x) = 0.1$), and so forth.

The set of patients is $P = \{AI, Bob, Joe, Ted\}$. The symptoms are given in Table 2, where we need all three parameters ($\mu_A(x)$, $\nu_A(x)$, $\pi_A(x)$) to describe each symptom (see [8, 9]) as before. We seek a diagnosis for each patient p_j , $j = 1, 2, 3, 4$. In [8, 9], Szmidt and Kacprzyk [8, 9] proposed to solve the problem in the following way: (1) to calculate a distance from a set of symptoms s_i ($i = 1, 2, 3, 4, 5$) (using the normalized Hamming distance, the Euclidean distance, and ratio-based measure of similarity) between each patient p_j ($j = 1, 2, 3, 4$, Table 2) and each diagnosis d_k , ($k = 1, 2, 3, 4, 5$, Table 1); (2) to determine that the shortest distance should be a proper diagnosis.

For (10), the normalized Hamming distance for all symptoms of patient j th from diagnosis k th is (40), and

TABLE 1: Symptoms characteristic of the diagnoses under consideration.

Disease symptoms	Temperature	Headache	Stomach pain	Cough	Chest pain
<i>Viral fever</i>	(0.4, 0.0, 0.6)	(0.3, 0.5, 0.2)	(0.1, 0.7, 0.2)	(0.4, 0.3, 0.3)	(0.1, 0.7, 0.2)
<i>Malaria</i>	(0.7, 0.0, 0.3)	(0.2, 0.6, 0.2)	(0.0, 0.9, 0.1)	(0.7, 0.0, 0.3)	(0.1, 0.8, 0.1)
<i>Typhoid</i>	(0.3, 0.3, 0.4)	(0.6, 0.1, 0.3)	(0.2, 0.7, 0.1)	(0.2, 0.6, 0.2)	(0.1, 0.9, 0.0)
<i>Stomach problem</i>	(0.1, 0.7, 0.2)	(0.2, 0.4, 0.4)	(0.8, 0.0, 0.2)	(0.2, 0.7, 0.1)	(0.2, 0.7, 0.1)
<i>Chest problem</i>	(0.1, 0.8, 0.1)	(0.0, 0.8, 0.2)	(0.2, 0.8, 0.0)	(0.2, 0.8, 0.0)	(0.8, 0.1, 0.1)

TABLE 2: Symptoms characteristic of the patients under observation.

Patient symptoms	Temperature	Headache	Stomach pain	Cough	Chest pain
<i>AI</i>	(0.8, 0.1, 0.1)	(0.6, 0.1, 0.3)	(0.2, 0.8, 0.0)	(0.6, 0.1, 0.3)	(0.1, 0.6, 0.3)
<i>Bob</i>	(0.0, 0.8, 0.2)	(0.4, 0.4, 0.2)	(0.6, 0.1, 0.3)	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)
<i>Joe</i>	(0.8, 0.1, 0.1)	(0.8, 0.1, 0.1)	(0.0, 0.6, 0.4)	(0.2, 0.7, 0.1)	(0.0, 0.5, 0.5)
<i>Ted</i>	(0.6, 0.1, 0.3)	(0.5, 0.4, 0.1)	(0.3, 0.4, 0.3)	(0.7, 0.2, 0.1)	(0.3, 0.4, 0.3)

the normalized Euclidean distance for all symptoms of patient j th from diagnosis k th is (41). From (14), the ratio-based measure of similarity for all symptoms of patient j th from diagnosis k th is (42):

$$d_H(p_j, d_k) = \frac{\sum_{i=1}^5 |\mu_{p_j}^-(s_i) - \mu_{d_k}^-(s_i)| + |\nu_{p_j}^-(s_i) - \nu_{d_k}^-(s_i)| + |\pi_{p_j}^+(s_i) - \pi_{d_k}^+(s_i)|}{10}, \quad (40)$$

$$d_E(p_j, d_k) = \sqrt{\frac{1}{10} \sum_{i=1}^5 (|\mu_{p_j}^-(s_i) - \mu_{d_k}^-(s_i)|^2 + |\nu_{p_j}^-(s_i) - \nu_{d_k}^-(s_i)|^2 + |\pi_{p_j}^+(s_i) - \pi_{d_k}^+(s_i)|^2)}, \quad (41)$$

$$d_{RB}(p_j, d_k) = \frac{d_H(p_j, d_k)}{d_H(p_j, d_k^C)}. \quad (42)$$

Szmidt and Kacprzyk describe d_k^C to be a set $(\nu_A(x), \mu_A(x), \pi_A(x))$. For example, for malaria the temperature d_k is high $(\mu_A(x) = 0.7, \nu_A(x) = 0, \pi_A(x) = 0.3)$, whereas d_k^C is low $(\mu_A(x) = 0, \nu_A(x) = 0.7, \pi_A(x) = 0.3)$, and so on.

From the obtained distance measures and similarity measures, Szmidt and Kacprzyk get the results as Table 3. Three types of distances given in Table 3 are described as follows: the first data is the Hamming distance as in (40), the second is the Euclidean distance as in (41), the third is the ratio-based measure as in (42). For example, $d_H(AI, \text{viral fever})$, $d_E(AI, \text{viral fever})$, and $d_{RB}(AI, \text{viral fever})$ are 0.28, 0.29, and 0.75 respectively. Similarly, we can get the other results. Obviously, *Bob* suffers from *stomach problem* and *Joe* from *typhoid* based on (40)–(42) above. *AI* suffers from *Malaria* and *Ted* from *Viral fever* calculated by the Hamming distance and the Euclidean distance, while *AI* suffers from *Viral fever* and *Ted* from *Malaria* calculated by the ratio-based measure.

Let $p = 5$ and $w_i = 1/n$ for each i , and k and λ are constants. Calculated by (15)–(21), Table 4 can be obtained.

From Table 4, we conclude that *Bob* suffers from *stomach problem*, *Joe* from *typhoid*, *Ted* from *viral fever*, and *AI* from *viral fever* according to most of Xu's methods. But on the basis of $D_{J0.1,0.6}$, *AI* can be diagnosed as a *malaria sufferer*. Using Xu's methods of IVIFS from [20], we can get the same results as Table 4. The results above are also the same as those from [35] by Wei et al.

Let $\mu_A^-(x) = \mu_A(x)$, $\nu_A^-(x) = \nu_A(x)$, and $\pi_A^+(x) = \pi_A(x)$. Using (37), we will get Table 5.

The distances given in Table 5 are described as follows: for example, from the first data of *viral fever* and *malaria*, we can describe the distance measure between *AI* and *viral fever*, and the distance measure between *AI* and *malaria* from (37), respectively. Similarly, we can get all the results (43), and obtain Table 5 from all (43):

$$d(AI, \text{Viral fever}) = 1.06 - 0.24\lambda_0 - 0.14\lambda_0\lambda_1 + 0.31\lambda_0^2 - 0.62\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(AI, \text{Malaria}) = 1.08 - 0.2\lambda_0 + 0.2\lambda_0\lambda_1 + 0.1\lambda_0^2 - 0.2\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(AI, \text{Typhoid}) = 1.6 - 0.18\lambda_0 - 0.04\lambda_0\lambda_1 + 0.2\lambda_0^2 - 0.4\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(AI, \text{Stomach problem}) = 5.28 - 0.42\lambda_0 + 0.56\lambda_0\lambda_1 + 0.14\lambda_0^2 - 0.28\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(AI, \text{Chest problem}) = 6.44 - 0.36\lambda_0 + 0.44\lambda_0\lambda_1 + 0.14\lambda_0^2 - 0.28\lambda_0^2\lambda_1(1 - \lambda_1),$$

TABLE 3: Distance description between each patient and each disease based on IFS.

Patient/disease	<i>Viral fever</i>	<i>Malaria</i>	<i>Typhoid</i>	<i>Stomach problem</i>	<i>Chest problem</i>
<i>AI</i>	(0.28, 0.29, 0.75)	(0.24 , 0.25 , 1.19)	(0.28, 0.32, 1.31)	(0.54, 0.53, 3.27)	(0.56, 0.58, ∞)
<i>Bob</i>	(0.40, 0.43, 2.1)	(0.50, 0.56, 3.73)	(0.31, 0.33, 1.10)	(0.14 , 0.14 , 0.35)	(0.42, 0.46, ∞)
<i>Joe</i>	(0.38, 0.36, 0.87)	(0.44, 0.41, 1.52)	(0.32 , 0.32 , 0.46)	(0.50, 0.52, 2.61)	(0.55, 0.57, ∞)
<i>Ted</i>	(0.28 , 0.25 , 0.95)	(0.30, 0.29, 0.77)	(0.38, 0.35, 1.67)	(0.44, 0.43, ∞)	(0.54, 0.50, 2.56)

$$d(\text{Bob}, \text{Viral fever}) = 3.38 - 0.86\lambda_0 + 1.34\lambda_0\lambda_1 \\ + 0.19\lambda_0^2 - 0.38\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Bob}, \text{Malaria}) = 6.12 - 0.62\lambda_0 + 1.12\lambda_0\lambda_1 \\ + 0.06\lambda_0^2 - 0.12\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Bob}, \text{Typhoid}) = 2.04 - 0.52\lambda_0 + 0.84\lambda_0\lambda_1 \\ + 0.1\lambda_0^2 - 0.2\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Bob}, \text{Stomach problem}) = 0.28 + 0.02\lambda_0 - 0.16\lambda_0\lambda_1 \\ + 0.06\lambda_0^2 - 0.12\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Bob}, \text{Chest problem}) = 3.96 - 0.46\lambda_0 + 0.64\lambda_0\lambda_1 \\ + 0.14\lambda_0^2 - 0.28\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Joe}, \text{Viral fever}) = 1.7 - 0.34\lambda_0 - 0.18\lambda_0\lambda_1 \\ + 0.43\lambda_0^2 - 0.86\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Joe}, \text{Malaria}) = 3.12 - 0.64\lambda_0 + 0.6\lambda_0\lambda_1 \\ + 0.34\lambda_0^2 - 0.68\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Joe}, \text{Typhoid}) = 1.12 - 0.36\lambda_0 - 0.24\lambda_0\lambda_1 \\ + 0.48\lambda_0^2 - 0.96\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Joe}, \text{Stomach problem}) = 4.76 + 0.38\lambda_0 - 1.36\lambda_0\lambda_1 \\ + 0.3\lambda_0^2 - 0.6\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Joe}, \text{Chest problem}) = 6 + 0.28\lambda_0 - 1.24\lambda_0\lambda_1 \\ + 0.34\lambda_0^2 - 0.68\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Ted}, \text{Viral fever}) = 0.92 - 0.12\lambda_0 - 0.08\lambda_0\lambda_1 \\ + 0.16\lambda_0^2 - 0.32\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Ted}, \text{Malaria}) = 1.46 - 0.4\lambda_0 + 0.54\lambda_0\lambda_1 \\ + 0.13\lambda_0^2 - 0.26\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Ted}, \text{Typhoid}) = 2.06 - 0.42\lambda_0 + 0.46\lambda_0\lambda_1 \\ + 0.19\lambda_0^2 - 0.38\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Ted}, \text{Stomach problem}) = 3.42 - 0.16\lambda_0 + 0.02\lambda_0\lambda_1 \\ + 0.15\lambda_0^2 - 0.3\lambda_0^2\lambda_1(1 - \lambda_1),$$

$$d(\text{Ted}, \text{Chest problem}) = 4.54 - 0.44\lambda_0 + 0.5\lambda_0\lambda_1 \\ + 0.19\lambda_0^2 - 0.38\lambda_0^2\lambda_1(1 - \lambda_1). \quad (43)$$

We aim to seek the minimum distance among $d(AI, \text{disease})$ s. Therefore, the minimum among all these $d(AI, \text{disease})$ s is the pattern we need. From Table 5, it is clear that if $\lambda_0 = 0$, *AI*, *Bob*, *Joe*, and *Ted* are identified as having *viral fever*, *stomach problem*, *typhoid*, and *viral fever*, respectively; if $\lambda_0 > 0$, *Bob*, *Joe*, and *Ted* are also identified as having *stomach problem*, *typhoid*, and *viral fever*, respectively, which is the same as Xu's results in [20] and Wei's results in [35], but *AI* is different from the above. When $d(AI, \text{viral fever}) > d(AI, \text{malaria})$, *AI* is identified as *malaria*; otherwise, *AI* is *viral fever*. All in all, we have the following results: if $0.02 + 0.04\lambda_0 + 0.34\lambda_0\lambda_1 - 0.21\lambda_0^2 + 0.42\lambda_0^2\lambda_1(1 - \lambda_1) < 0$, we can infer that *AI* suffers from *malaria*, and on the contrary, *AI* from *viral fever*. From Table 5, we conclude that *AI* suffers from either *malaria* or *viral fever*, *Bob* from *stomach problem*, *Joe* from *typhoid*, and *Ted* suffers from *viral fever*. The pattern recognition results are somewhat different from those of the dynamic intuitionistic fuzzy sets (DIFS) method in [26–28], in which we only proposed a DIFS method derived from IFS. However, in this paper, we obtain a series of DIVIFS methods derived from IFS and IVIFS, which are the further study in deep from [26–28].

The experimental results above show that there is a slight difference between the recognition results of DIVIFS and those of IFS. Conventional IFS method is simple, but its recognition results are fixed on the basis of the conventional distance formulas. Therefore, it is difficult to reveal the potential law from all available information when the IFS method is used, whereas the results of the DIVIFS method are flexible, which can be adjusted to sample data with the variables. Furthermore, if the fixed pattern recognition results are different from those of the sample data, the IFS method will fail. However, when using the DIVIFS method, we can meet the requirements of the sample data by adjusting variables to appropriate values. All the results above show that the DIVIFS method is more comprehensive and flexible than the IFS method.

6. Simulation to Medical Diagnosis

In the following, we will compare the DIVIFS method with the conventional method in medical diagnosis. We use a sample of 77 women of embryos transferred, which is from Laboratory of Reproductive Medicine, The First Affiliated

TABLE 4: Distance between each patient and each disease on IFS calculated by Xu's methods.

Distance between patient and disease	$D_D(k = 0.2)$	$D_F(k = 0.2, \lambda = 0.1)$	$D_G(k = 0.7, \lambda = 0.8)$	$D_H(k = 0.8, \lambda = 0.3)$	$D_{H^*}(k = 0.9, \lambda = 0.2)$	$D_I(k = 0.1, \lambda = 0.6)$	$D_{I^*}(k = 0.2, \lambda = 0.7)$
$D(AI, viral\ fever)$	0.26143	0.29072	0.25963	0.26478	0.29445	0.29111	0.27069
$D(AI, malaria)$	0.30533	0.33445	0.26197	0.30561	0.31294	0.27027	0.27086
$D(AI, typhoid)$	0.35839	0.36029	0.28616	0.3306	0.34773	0.3355	0.31541
$D(AI, stomach\ problem)$	0.56074	0.59068	0.45948	0.54019	0.55532	0.49994	0.4901
$D(AI, chest\ problem)$	0.60507	0.61828	0.47761	0.5666	0.58378	0.54117	0.52311
$D(Bob, viral\ fever)$	0.42194	0.52601	0.43156	0.47154	0.48979	0.38652	0.40853
$D(Bob, malaria)$	0.60123	0.64839	0.50394	0.59342	0.60684	0.53092	0.52867
$D(Bob, typhoid)$	0.33646	0.40929	0.33026	0.3696	0.38103	0.30293	0.31836
$D(Bob, stomach\ problem)$	0.14469	0.13263	0.11231	0.12354	0.13796	0.14745	0.13245
$D(Bob, chest\ problem)$	0.52473	0.55204	0.42999	0.50446	0.51855	0.46688	0.45823
$D(Joe, viral\ fever)$	0.35629	0.35212	0.29083	0.32246	0.3462	0.34447	0.32153
$D(Joe, malaria)$	0.46742	0.49471	0.38926	0.45165	0.46599	0.41851	0.41061
$D(Joe, typhoid)$	0.31951	0.31011	0.2846	0.28115	0.31803	0.31742	0.29265
$D(Joe, stomach\ problem)$	0.61639	0.57475	0.43227	0.53105	0.55428	0.55892	0.52068
$D(Joe, chest\ problem)$	0.66501	0.63332	0.48018	0.58484	0.60883	0.60414	0.56562
$D(Ted, viral\ fever)$	0.218	0.23438	0.19472	0.2132	0.22744	0.21415	0.20328
$D(Ted, malaria)$	0.26922	0.33348	0.27321	0.29927	0.31105	0.24789	0.2599
$D(Ted, typhoid)$	0.35577	0.36789	0.29537	0.33285	0.34867	0.32407	0.31266
$D(Ted, stomach\ problem)$	0.45321	0.46272	0.35531	0.42507	0.43656	0.40408	0.39108
$D(Ted, chest\ problem)$	0.48203	0.51701	0.40614	0.47177	0.48604	0.43213	0.42575

TABLE 5: Distance description between each patient and each disease based on DIVIFS.

(AI, Bob, Joe, Ted)	constant	λ_0	$\lambda_0 \lambda_1$	λ_0^2	$\lambda_0^2 \lambda_1 (1 - \lambda_1)$
<i>Viral fever</i>	(1.06 , 3.38, 1.70, 0.92)	(-0.24, -0.86, -0.34, -0.12)	(-0.14, 1.34, -0.18, -0.08)	(0.31, 0.19, 0.43, 0.16)	(-0.62, -0.38, -0.86, -0.32)
<i>Malaria</i>	(1.08 , 6.12, 3.12, 1.46)	(-0.20, -0.62, -0.64, -0.40)	(0.20, 1.12, 0.60, 0.54)	(0.10, 0.06, 0.34, 0.13)	(-0.20, -0.12, -0.68, -0.26)
<i>Typhoid</i>	(1.60, 2.04, 1.12 , 2.06)	(-0.18, -0.52, -0.36, -0.42)	(-0.04, 0.84, -0.24, 0.46)	(0.20, 0.10, 0.48 , 0.19)	(-0.40, -0.20, -0.96, -0.38)
<i>Stomach problem</i>	(5.28, 0.28 , 4.76, 3.42)	(-0.42, 0.02 , 0.38, -0.16)	(0.56, -0.16, -1.36, 0.02)	(0.14, 0.06 , 0.30, 0.15)	(-0.28, -0.12, -0.60, -0.30)
<i>Chest problem</i>	(6.44, 3.96, 6.00, 4.54)	(-0.36, -0.46, 0.28, -0.44)	(0.44, 0.64, -1.24, 0.50)	(0.14, 0.14, 0.34, 0.19)	(-0.28, -0.28, -0.68, -0.38)

TABLE 6: The optimal results of medical diagnosis based on the distances of IFS and DIVIFS.

Equation	Right classification	Wrong classification	Correct recognition rate
Equation (15), $k = 0.5$	57	20	74.026%
Equation (38), $\lambda_0 \geq 0.6, \lambda_1 = 0.5$	64	13	83.117%

TABLE 7: Comparison between classification of DIVIFS and existing classification calculated by similarity ($S = 1 - D$, where D denotes the standardized distance).

i	$A_1(x_i)$ Not successful pregnancy and no symptoms of OHSS	$A_2(x_i)$ Successful pregnancy and no symptoms of OHSS	$A_3(x_i)$ Not successful pregnancy and symptoms of OHSS	$A_4(x_i)$ Successful pregnancy and symptoms of OHSS	DIVIFS classification results	D sets existing classification
1	0.572945016	0.490904912	0.245052718	0.258999945	1	2
2	0.903658468	0.866369075	0.500009121	0.575267057	1	1
3	0.532136793	0.668331297	0.402139818	0.437292429	2	2
4	0.569281981	0.351763437	0.369988299	0.317016112	1	1
5	0.75309088	0.866822163	0.523453221	0.565023619	2	2
6	0.603807663	0.620439622	0.585688802	0.585218663	2	2
7	0.833143565	0.629507422	0.278184995	0.343708098	1	1
8	0.692742485	0.978491415	0.654192881	0.398168922	2	2
9	0.742052675	0.592646858	0.289624287	0.326558445	1	1
10	0.62781314	0.365138462	0.485198477	0.283729817	1	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
70	0.367412382	0.590387248	0.692746959	0.62396661	3	3
71	0.479555791	0.630052699	0.795747193	0.654957234	3	3
72	0.318797905	0.290232035	0.376480055	0.324328504	3	3
73	0.418720261	0.451631428	0.77908381	0.227788025	3	3
74	0.29301287	0.321279436	0.545408461	0.366476514	3	3
75	0.385226266	0.413714892	0.658865834	0.669631158	4	4
76	0.545467491	0.539314756	0.760187187	0.786090732	4	3
77	0.383687857	0.392311966	0.742113723	0.49908387	3	3

Hospital, Southern Medical University. And the set of disease diagnoses is $D = \{\text{Not successful pregnancy and no symptoms of OHSS}, \text{Successful pregnancy and no symptoms of OHSS}, \text{Not successful pregnancy and symptoms of OHSS}, \text{Successful pregnancy and symptoms of OHSS}\}$, and the set of symptoms is a universe of discourse $S = \{\text{Basic Hormone FSH}, \text{Basic Hormone P}, \text{Basic Hormone E2}, \text{Basic Hormone T}, \text{BMI}, \text{Day of HCG E2}, \text{Day of HCG endometrial thickness}\}$, where each symptom is described as follows: membership function $\mu_A(x)$, non-membership function $\nu_A(x)$, and hesitancy function $\pi_A(x)$.

Applying the conventional distance measures on IFS and DIVIFS to medical diagnosis, we have the results in Table 6. From Table 6, we conclude that the most accurate results of medical diagnosis are from Xu's equation (15) when $k = 0.5$, which is also only a special case of (35) when $C_1 = C_2 = C_3 = C_4 = C_6 = C_8 = 0$ and $C_5 = C_7 = 1$. And in 77 individuals, 74% of all the women can be classified correctly. However, using (38), the recognition rate will be 83%. Some results of medical diagnosis have been shown in Table 7.

7. Conclusion

We propose a method for the evaluation of a degree of agreement in a group of individuals by calculating distances between DIVIFS. By analyzing the variation on the degree of hesitancy, we introduce a novel DIVIFS method derived from IVIFS, which is an effective method for us to construct dynamic IVIFS model from IFS and IVIFS. And then we propose a novel ranking condition for the distance measures on IFS, IVIFS, and DIVIFS. According to the new condition, we show the construction of distance measures on DIVIFS in theory. Finally, we apply some distances on DIVIFS to medical diagnosis based on pattern recognition. The experimental results show that the DIVIFS method and its distance measures are more comprehensive and more effective than the conventional distance measures on IFS and IVIFS.

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