Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 420420, 13 pages
http://dx.doi.org/10.1155/2013/420420

*Research Article*

# Exponential Stability in Mean Square of Singular Markovian Jump System with Saturating Actuators and Time-Varying Delay

**Fangqing Ding[1,2] and Xianfa Jiao[1]**

[1] *School of Mathematics, Hefei University of Technology, Hefei 230009, China*
[2] *Department of Mathematics and Physics, Hefei University, Hefei 230601, China*

Correspondence should be addressed to Xianfa Jiao; xfjiao@126.com

Received 12 March 2013; Revised 15 June 2013; Accepted 4 July 2013

Academic Editor: Jian Li

Copyright © 2013 F. Ding and X. Jiao. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper investigates exponential stability in mean square of singular Markovian jump systems with saturating actuators and time-varying delay. The statistical property of the Markov process is fully used to derive the differential of the function. By using a delay decomposition method, a mode-dependent Lyapunov-Krasovskii function is established. A sufficient condition is proposed for exponential stability in mean square of the system designing the memoryless state feedback. A numerical example shows that the approach proposed is effective.

## 1. Introduction

Markovian jump systems are a special class of hybrid systems with both modes and state variables, which is described by a set of time-delayed linear systems with the transitions among models determined by a Markov chain in a finite mode set. In recent years, Markovian jump systems have received much attention, and many problems have been solved, including stability analysis, state feedback, and output feedback controller design [1, 2]. Both time-delay and saturating controls are commonly encountered in various engineering systems, which often lead to poor performance and instability of a control system. Therefore, much attention has been paid to the study on the stability problem of systems with saturating actuator. The problem of robust stabilization of uncertain time-delay systems with a saturating actuator was addressed by Niculescu et al. [3]. Liu has obtained the condition of stability for a class of time-varying delay systems with saturating actuators according to the usage of the linear matrix inequality and Leibniz-Newton formula [4]. For the systems with time-varying delays, the reported results are generally based on the assumption that the derivative of time-varying delays is less than one [3, 4]. Such restriction is very conservative. Furthermore, singular system model is a natural presentation of dynamic systems and can describe a larger class of systems than regular ones, such as large-scale systems, power systems, and constrained control systems, electrical circuits, power systems, and economics [5]. For a singular time-delay system, it is important to develop conditions which guarantee that the given singular system is not only stable but also regular and impulse free. Wu et al. discussed the stability problem on uncertain singular Markovian jump time-delay systems [6, 7]. Ma et al. investigated the robust stochastic stability problem for discrete-time uncertain singular Markov jump systems with actuator saturation [8]. Boukas et al. established an LMI condition for the singular time-delay systems with Markovian jump to be regular, impulse free, and stochastically stable. However, the result of [9] is delay independent. Generally speaking, delay-independent conditions are more conservative than delay-dependent ones, especially for small time delays. To the best of our knowledge, the robust stability for singular Markov jump systems with actuator saturation and time-varying delay has not been investigated in the literature; this problem is important in both theory and practice.

In this paper, we are concerned with stability of singular Markovian jump system with saturating actuators and time-varying delay. A new Lyapunov function can be constructed by using a delay decomposition method and considering the statistical property of Markovian process. In terms of

LMI approach, stability conditions are proposed to guarantee the considered systems to be regular, impulse free, and exponentially stable in mean square. Free weighting matrix and reduction method are not introduced in the derivation of the stability criterion. Thus, the results reduce conservation.

## 2. Problem Formulation and Assumption

Consider the singular markovian jump system with saturating actuators and time-varying delay as follows:

$$\begin{aligned} E\dot{x} &= A(r_t)x(t) + B(r_t)x(t-\tau(t)) \\ &\quad + C(r_t)\ \mathrm{Satu}\ (t-\tau(t)), \quad t \geq 0, \\ x(t) &= \phi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{1}$$

where $x(t) \in R^N$ and $u(t) \in R^M$ are the system states and inputs, respectively, $u_j(t)$, $j = 1, \ldots, M$ is the $j$th component of $u(t)$, and $\phi(t) \in R^N$ is a continuous vector-valued initial function defined on the interval $[-\tau, 0]$. The matrix $E \in R^{N\times N}$ may be singular, and rank $E = r < n$. $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ is the probability space. $A(r_t)$, $B(r_t)$, and $C(r_t)$ are known real constant matrices with appropriate dimension. As $r_t = i$, for the sake of simplicity, $A_i$, $B_i$, and $C_i$ denote the matrixes, respectively. $\tau(t)$ is the time-varying delay satisfying

$$0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq \mu, \tag{2}$$

and $h$, $\mu$ are real constant scalars.

The saturating function is defined as follows:

$$\mathrm{Sat}(u(t)) = \left[\mathrm{Sat}(u_1(t))\ \ \mathrm{Sat}(u_2(t))\ \ \cdots\ \ \mathrm{Sat}(u_m(t))\right]^T, \tag{3}$$

$$\mathrm{Sat}(u_j(t)) = \begin{cases} U_j, & \text{if } u_j(t) \leq -U_j,\ u_j(t) \leq -U_j, \\ u_j(t), & \text{if } -U_j < u_j(t) < U_j, \\ U_j, & \text{if } u_j(t) \geq U_j,\ u_j(t) \geq U_j, \end{cases} \tag{4}$$

where $U_j > 0\ (j = 1, \ldots, m)$ are known real constants.

*Assumption 1.* The markovian jumping parameter $r_t$ is right-continuous markovian process and takes values in finite set $S = \{1, 2, \ldots, N\}$, and is defined by

$$P(r_{t+\Delta} = j \mid r_t = i) = \begin{cases} \rho_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \rho_{ij}\Delta + o(\Delta), & i = j, \end{cases} \tag{5}$$

where $\Delta > 0$, $\lim_{\Delta\to\infty} o(\Delta)/\Delta = 0$, $\rho_{ij}$ is the transition rate from $i$ to $j$ and

$$\rho_{ii} = -\sum_{j=1, j\neq i}^{S} \rho_{ij}, \quad \forall i, j \in S\ \left(\rho_{ij} \geq 0, j \neq i\right). \tag{6}$$

*Definition 2.* The infinitesimal generator of the solution to system (1) is defined as

$$LV(x_t, r_t) = \lim_{\Delta\to 0^+} \frac{E\left[V(x_{t+\Delta}, r_{t+\Delta}) \mid x_t, r_t\right] - V(x_t, r_t)}{\Delta}. \tag{7}$$

*Definition 3.* (1) The singular Markovian jump system with saturating actuators and time-varying delay

$$\begin{aligned} E\dot{x}(t) &= A(r_t)x(t) + A_d(r_t)x(t-\tau(t)), \\ x(t) &= \phi(t), \quad t \in [-\tau, 0] \end{aligned} \tag{8}$$

is said to be regular and impulse free, if the pairs $(E, A(r_t))$ are singular and impulse free for every $r_t \in S$.

(2) The singular Markovian jump system with saturating actuators and time-varying delay (8) is said to be exponential stable in mean square if there exist scalars $\alpha > 0$, $\beta > 0$, such that

$$E\left\{\|x(t)\|^2\right\} \leq \alpha e^{-\beta t}\|\phi(t)\|_\tau^2, \quad t > 0. \tag{9}$$

(3) The singular Markovian jump system with saturating actuators and time-varying delay (8) is said to be mean-square exponentially admissible, if it is regular, impulse free, and exponentially stable in mean square.

**Lemma 4** (see [7]). *Suppose that a positive continuous function $f(t)$ satisfies*

$$E\{f(t)\} \leq \varsigma_1 E\left\{\sup_{t-\tau\leq s\leq t} f(s)\right\} + \varsigma_2 e^{-\varepsilon t}; \tag{10}$$

*then*

$$E\{f(t)\} \leq e^{-\varepsilon t}\|f(s)\|_\tau + \frac{\varsigma_2 e^{-\varepsilon t}}{1 - \varsigma_1 e^{\varepsilon\tau}}, \tag{11}$$

*where $\varepsilon > 0$, $0 < \varsigma_1 < 1$, $0 < \varsigma_1 e^{\varepsilon\tau} < 1$, $\varsigma_2 > 0$ and $d > 0$.*

**Lemma 5** (see [10]). *$D$ is an $M \times M$ diagonal matrix whose entries are either 1 or 0. Note that there are $2^M$ elements in $D$. Suppose that each element in $D$ is labeled as $D_s$, $s \in [1, 2^M]$, and let $D_s^- = I - D_s$; it is obvious that $D_s^- \in D$, if $D_s \in D$. Given matrix $F, H \in R^{M\times N}$, one has*

$$Sat(Fx) \in co\left\{D_s Fx + D_s^- Hx : s \in \left[1, 2^M\right]\right\}, \tag{12}$$

*where co denotes convex hull.*

**Lemma 6** (see [11]). *Given $v > 0$, for a vector $\omega(s)$ and a positive-definite matrix $Z$ with appropriate dimensions, and the integrations concerned are well defined, then*

$$v\int_0^v \omega(s)^T Z\omega(s)\,ds \geq \left(\int_0^v \omega(s)\,ds\right)^T Z\left(\int_0^v \omega(s)\,ds\right). \tag{13}$$

## 3. Main Results

Considering the system (1), for each region $r_t = i \in S$, the system (1) becomes

$$E\dot{x}(t) = A_i x(t) + B_i x(t-\tau(t)) + C_i\ \mathrm{Satu}(t-\tau(t)). \tag{14}$$

Taking the memoryless state feedback control for the systems (14), one has

$$u(t-\tau(t)) = K_i x(t-\tau(t)). \tag{15}$$

Substituting (15) into (14), then the corresponding closed-loop systems become

$$E\dot{x}(t) = A_i x(t) + B_i x(t-\tau(t)) + C_i \mathrm{Sat}\left(K_i x(t-\tau(t))\right). \tag{16}$$

**Theorem 7.** *For given constants $\tau$, $\mu$ and two positive integers $k$, $l$, The system* (16) *is exponentially stable in mean square. If there exist symmetric positive matrices $P_i = P_i^T > 0$, $N_i = N^T{}_i > 0$, $R_{mi} = R_{mi}^T > 0$ $(m = 1, \ldots, l)$, $M_{ni} = M_{ni}^T > 0$ $(n = 1, \ldots, k)$, $T_1 = T_1^T > 0$, $T_2 = T_2^T > 0$, $Q_i = Q_i^T > 0$, where*

$$Q_i = Q_i^T = \begin{bmatrix} Q_{11i} & \cdots & Q_{1ki} \\ \vdots & \cdots & \vdots \\ Q_{1ki}^T & \cdots & Q_{kki} \end{bmatrix}, \tag{17}$$

*for every $i \in S$, such that*

$$E^T P_i = P_i E, \tag{18a}$$

$$\Sigma = \begin{bmatrix}
\Sigma_{11} & Q_{12i} + M_{1i} - M_{2i} & Q_{13i} + M_{2i} - M_{3i} & \cdots & Q_{1k-1i} + M_{k-2i} - M_{k-1i} & Q_{1ki} + M_{k-1i} - M_{ki} & M_{ki} \\
* & Q_{22i} - Q_{11i} & Q_{23i} - Q_{12i} & \cdots & Q_{2k-1i} - Q_{1k-2i} & Q_{2ki} - Q_{1k-1i} & -Q_{1ki} \\
* & * & Q_{33i} - Q_{22i} & \cdots & Q_{3k-1i} - Q_{2k-2i} & Q_{3ki} - Q_{2k-1i} & -Q_{2ki} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & \cdots & Q_{k-1k-1i} - Q_{k-2k-2i} & Q_{k-1ki} - Q_{k-2k-1i} & -Q_{k-2k-1i} \\
* & * & * & \cdots & * & Q_{kki} - Q_{k-1k-1i} & -Q_{k-1ki} \\
* & * & * & \cdots & * & * & -Q_{kki} - \frac{1}{l}T_2 \\
* & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
* & 0 & 0 & \cdots & 0 & 0 & *
\end{bmatrix}$$

$$\begin{bmatrix}
2E^T T_2 E & 0 & 0 & \cdots & 0 & 0 & \Sigma_{1k+l+1} \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & -\frac{1}{l}E^T T_2 E \\
-2E^T T_2 E - \left(1 - \frac{\mu}{l}\right) R_{i1} & E^T T_2 E & 0 & \cdots & 0 & 0 & 0 \\
* & -2E^T T_2 E - \left(1 - \frac{2\mu}{l}\right) R_{i2} & E^T T_2 E & \cdots & 0 & 0 & 0 \\
0 & * & -2E^T T_2 E - \left(1 - \frac{3\mu}{l}\right) R_{i3} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & -2E^T T_2 E - \left(1 - \frac{l-2}{l}\right) R_{il-2} & E^T T_2 E & 0 \\
0 & 0 & 0 & \cdots & * & -2E^T T_2 E - \left(1 - \frac{l-1}{l}\right) R_{il-1} & T_2 \\
0 & 0 & 0 & \cdots & 0 & * & \Sigma_{k+l+1k+l+1}
\end{bmatrix}$$

$$< 0, \tag{18b}$$

*where*

$$\Sigma_{11} = A_i^T P_i + P_i A_i + \sum_{j \in S} \rho_{ij} E^T P_i + \sum_{m=1}^{l} R_{mi} + \frac{\tau^2}{k} A_i^T T_1 A_i + \frac{\tau^2}{l} A_i^T T_2 A_i + \frac{\tau^2}{k} T_1 + \frac{\tau^2}{l} T_2 - M_{1i} - E^T T_2 E + Q_{11i},$$

$$\Sigma_{1k+l+1} = B_i^T P_i + K_i^T D_q^T C_i^T P_i + H_i^T D_q^{-T} C_i^T P_i + \frac{\tau^2}{k} B_i^T T_1 A_i + \frac{\tau^2}{k} K_i^T D_q^T C_i^T T_1 A_i$$

$$+\frac{\tau^2}{k}H_i^T D_q^{-T} C_i^T T_1 A_i + \frac{\tau^2}{l}B_i^T T_2 A_i$$
$$+\frac{\tau^2}{l}K_i^T D_q^T C_i^T T_2 A_i + \frac{\tau^2}{l}H_i^T D_q^{-T} C_i^T T_2 A_i,$$
$$\Sigma_{k+lk+l} = \left[B_i + C_i D_q K_i + C_i D_q^- H_i\right]^T \left(\frac{\tau^2}{k}T_1 + \frac{\tau^2}{l}T_2\right) \times \left[B_i + C_i D_q K_i + C_i D_q^- H_i\right] - \frac{1}{l}E^T T_2 E - (1-\mu) R_{il},$$
$$\begin{bmatrix} -\frac{1}{h}M_{1i} & 0 & A_i^T M_{ni} + \sum_{j\in S}\rho_{ij}E^T M_{nj} \\ 0 & -E^T T_2 E & \left[B_i + C_i D_q K_i + C_i D_q^- H_i\right]^T M_{ni} \\ * & * & -\tau T_1 \end{bmatrix} \le 0,$$
$$\begin{bmatrix} h\Gamma_1 & 0 & N_i \\ 0 & h\Gamma_2 & -N_i \\ * & * & \sum_{j\in S}\rho_{ij}\left(N_j + Q_j\right) \end{bmatrix} \le 0,$$
$$\left[\sum_{j\in S}\rho_{ij}R_{mj} - \tau T_2\right] \le 0. \tag{18c}$$

*Proof.* According to the definition of the saturating function and Lemma 5, we have

$$\mathrm{Sat}K_i x(t-\tau(t)) = D_q K_i x(t-\tau(t)) + D_q^- H_i x(t-\tau(t)) \quad q \in \left[1, 2^M\right]. \tag{19}$$

Substituting (19) into (16) gives

$$E\dot{x}(t) = A_i x(t) + \left[B_i + C_i D_q K_i + C_i D_q^- H_i\right] x(t-\tau(t)). \tag{20}$$

Since rank $E = r < n$, there exist nonsingular matrices $G$ and $\Upsilon$, such that

$$GE\Upsilon = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{21}$$

Let

$$GA_i\Upsilon = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}, \qquad G^{-T}P_i\Upsilon = \begin{bmatrix} \overline{P}_{i1} & \overline{P}_{i2} \\ \overline{P}_{i3} & \overline{P}_{i4} \end{bmatrix}. \tag{22}$$

According to (18a), it is easy to obtain that $\overline{P_{i2}} = 0$ for every $i \in S$. By pre- and postmultiplying $\Sigma_{11i} < 0$ by $\Upsilon^T$ and $\Upsilon$, respectively, we have $A_{i4}^T\overline{P}_{i4} + \overline{P}_{i4}^T A_{i4} < 0$, which implies that $A_{i4}$ are nonsingular for every $i \in S$. Thus, the pairs $(E, A_i)$ are regular and impulse free for every $i \in S$. Thus, by Definition 2, the system (8) is regular and impulse free for any time-varying delay $\tau(t)$ satisfying (2).

Choose a Lyapunov-Krasovskii function candidate as

$$V(x_t, r_t, t) = V_1(x_t, r_t, t) + V_2(x_t, r_t, t) + V_3(x_t, r_t, t) + V_4(x_4, r_t, t) + V_5(x_t, r_t, t) + V_6(x_t, r_t, t), \tag{23}$$

where

$$V_1(x_t, r_t, t) = x(t)^T E^T P(r_t) E x(t),$$
$$V_2(x_t, r_t, t) = \int_{t-h}^{t} \xi(s)^T Q(r_t)\xi(s)\,ds,$$
$$V_3(x_t, r_t, t) = 2x(t)^T \sum_{n=1}^{k} E^T M_n(r_t) \int_{t-nh}^{t-(n-1)h} x(s)\,ds,$$
$$V_4(x_t, r_t, t) = \int_{t-h}^{t} ds \int_{t-h}^{t} \xi(s)^T N(r_t)\xi(\theta)\,d\theta,$$
$$V_5(x_t, r_t, t) = \sum_{m=1}^{l} \int_{t-m\sigma(t)}^{t} x(s)^T R_m(r_t) x(s)\,ds,$$
$$V_6(x_t, r_t, t) = h\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}(s)^T E^T T_1 E\dot{x}(s) + x(s)^T T_1 x(s)\,ds\,d\theta + \sigma\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}(s)^T E^T T_2 E\dot{x}(s) + x(s)^T l T_2 x(s)\,ds\,d\theta, \tag{24}$$
$$\xi^T(t) = [x(t), x(t-h), \ldots, x(t-(k-1)h)]^T,$$
$$\sigma(t) = \frac{\tau(t)}{l}, \qquad h = \frac{\tau}{k}, \qquad \sigma = \frac{\tau}{l}.$$

Moreover, the action of infinitesimal generator (7) on each term function (23) could be expressed as

$$LV_1(x_t i, t) = 2x(t)^T E^T P_i \dot{x}(t) + x(t)^T \sum_{j\in S}\rho_{ij}E^T P_j x(t),$$
$$LV_2(x_t, i, t) = \xi(t)^T Q_i \xi(t) - \xi(t-h)^T Q_i \xi(t-h) + \int_{t-h}^{t} \xi(s)^T \sum_{j\in S}\rho_{ij}Q_j\xi(s)\,ds,$$
$$LV_3(x_t, i, t) = 2\dot{x}(t)^T \sum_{n=1}^{k} E^T M_{ni} \int_{t-nh}^{t-(n-1)h} x(s)\,ds - 2x(t)^T \sum_{n=1}^{k} M_{ni}\left[x(t-(n-1)h) - x(t-nh)\right]$$

$$+ 2x(t)^T \sum_{n=1}^{k} \sum_{j\in S} \rho_{ij} E^T M_{nj} \int_{t-nh}^{t-(n-1)h} x(s)\, ds,$$

$$LV_4(x_t, i, t) = 2\left[\xi(t) - \xi(t-h)\right]^T N_i \int_{t-h}^{t} \xi(s)\, ds + \int_{t-h}^{t} \int_{t-h}^{t} \xi(s)^T \sum_{j\in S} \rho_{ij} N_j \xi(\theta)\, ds\, d\theta,$$

$$LV_5(x_t, i, t) = x(t)^T \sum_{m=1}^{l} R_{mi} x(t) - \sum_{m=1}^{l} \left(1 - m\,\dot{\sigma}(t)\right) \times x(t - m\sigma(t))^T R_{mi} x(t - m\sigma(t)) + \sum_{m=1}^{l} \int_{t-m\sigma(t)}^{t} x(s)^T \sum_{j\in S} \rho_{ij} R_{mj} x(s)\, ds,$$

$$\begin{aligned} LV_6(x_t, i, t) = {} & h\tau\, \dot{x}(t)^T E^T T_1 E\, \dot{x}(t) \\ & + h\tau x(t)^T T_1 x(t) \\ & - h \int_{t-\tau}^{t} \dot{x}(s)^T E^T T_1 E x(s) \\ & + x(s)^T T_1 x(s)\, ds \\ & + \sigma\tau\, \dot{x}(t)^T E^T T_2 E\, \dot{x}(t) \\ & + \sigma\tau x(t)^T l T_2 x(t) \\ & - \sigma \int_{t-\tau}^{t} \dot{x}(s)^T E^T T_2 E\, \dot{x}(s) \\ & + x(s)^T l T_2 x(s)\, ds. \end{aligned} \tag{25}$$

According to (20), (23), and (25), we have

$$\begin{aligned} LV(x_t, i, t) \le {} & x(t)^T \Bigg[ A_i^T P_i + P_i A_i + \sum_{j\in S} \rho_{ij} E^T P_j + \sum_{m=1}^{l} R_{im} \\ & + h\tau A_i^T T_1 A_i + \sigma\tau A_i^T T_2 A_i \\ & + h\tau T_1 + \sigma\tau T_2 \Bigg] \\ & \times x(t) + 2x(t)^T \left[ P_i B_i + P_i C_i D_q K_i \right. \\ & + P_i C_i D_q^- H_i \\ & + h\tau A_i^T T_1 B_i \\ & + h\tau A_i^T T_1 C_i D_q K_i \\ & + h\tau A_i^T T_1 C_i D_q^- H_i \\ & + \sigma\tau A_i^T T_2 B_i \\ & + \sigma\tau A_i^T T_2 C_i D_q K_i \\ & \left. + \sigma\tau A_i^T T_2 C_i D_q^- H_i \right] \\ & \times x(t - \tau(t)) \\ & + x(t - \tau(t))^T \left[ B_i + C_i D_q K_i + C_i D_q^- H_i \right] \\ & \times \left( h\tau T_1 + \sigma\tau T_2 \right) \left[ B_i + C_i D_q K_i + C_i D_q^- H_i \right] \\ & \times x(t - \tau(t)) + \xi(t)^T Q_i \xi(t) \\ & - \xi(t-h)^T Q_i \xi(t-h) \\ & + \int_{t-h}^{t} \xi(s)^T \sum_{j\in S} \rho_{ij} Q_j \xi(s)\, ds \\ & + \sum_{n=1}^{k} \int_{t-nh}^{t-(n-1)h} \left[ x(t) \;\; x(t-\tau(t)) \;\; x(s) \right]^T \\ & \times \begin{bmatrix} -\frac{1}{h} M_{1i} & 0 & A_i^T M_{ni} + \sum_{j\in S} \rho_{ij} E^T M_{nj} \\ 0 & 0 & \left[ B_i + C_i D_q K_i + C_i D_q^- H_i \right]^T M_{ni} \\ * & * & -hT_1 \end{bmatrix} \\ & \times \begin{bmatrix} x(t) \\ x(t - \tau(t)) \\ x(s) \end{bmatrix} \\ & - x(t)^T M_{1i} x(t) + 2x(t)^T M_{i1} x(t-h) \\ & - 2x(t)^T \sum_{n=2}^{k} M_{ni} \left[ x(t - (n-1)h) - x(t - nh) \right] \\ & + \int_{t-h}^{t} \left[ \xi(t) - \xi(t-h) \right]^T N_i \xi(s) \\ & + \xi(s) N_i \left[ \xi(t) - \xi(t-h) \right] \\ & + \xi(s)^T \sum_{j\in S} \rho_{ij} h N_j \xi(s)\, ds \\ & - \sum_{m=1}^{l} \left(1 - m\,\dot{\sigma}(t)\right) x(t - m\sigma(t))^T \\ & \times R_{mi} x(t - m\sigma(t)) \\ & + \sum_{m=1}^{l} \int_{t-m\sigma(t)}^{t} x(s)^T \sum_{j\in S} \rho_{ij} R_{mj} x(s) \\ & - x(s)^T \tau T_2 x(s)\, ds \\ & - h \int_{t-\tau}^{t} \dot{x}(s)^T E^T T_1 E\, \dot{x}(s)\, ds \\ & \times \sigma \int_{t-\tau}^{t} \dot{x}(s)^T E^T T_2 E\, \dot{x}(s)\, ds, \end{aligned} \tag{26}$$

$$\xi(t)^T Q_i \xi(t) - \xi(t-h)^T Q_i \xi(t-h) = \zeta(t)^T \Xi \zeta(t),$$

$$\zeta(t)^T = \left[\xi(t)^T, x(t-\tau)^T\right]^T,$$

$$\Xi = \begin{bmatrix} Q_{11i} & Q_{12i} & Q_{13i} & Q_{14i} & \cdots & Q_{1k-1i} & Q_{1ki} & 0 \\ * & Q_{22i}-Q_{11i} & Q_{23i}-Q_{12i} & Q_{24i}-Q_{13i} & \cdots & Q_{2k-1i}-Q_{1k-2i} & Q_{2ki}-Q_{1k-1i} & -Q_{1ki} \\ * & * & Q_{33i}-Q_{22i} & Q_{34i}-Q_{23i} & \cdots & Q_{3k-1i}-Q_{2k-2i} & Q_{3ki}-Q_{2k-1i} & -Q_{2ki} \\ * & * & * & Q_{44i}-Q_{33i} & \cdots & Q_{4k-1i}-Q_{3k-2i} & Q_{4ki}-Q_{3k-1i} & -Q_{3ki} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & * & Q_{k-1k-1i}-Q_{k-2k-2i} & Q_{k-1ki}-Q_{k-2k-1i} & -Q_{k-2ki} \\ * & * & * & * & * & * & Q_{kki}-Q_{k-1k-1i} & -Q_{k-1ki} \\ * & * & * & * & * & * & * & -Q_{kki} \end{bmatrix}, \tag{27}$$

$$\begin{aligned} &-l\sigma \int_{t-\tau}^{t} x(s)^T T_2 x(s)\, ds \\ &\quad \le -l\sigma \int_{t-\tau(t)}^{t} x(s)^T T_2 x(s)\, ds \\ &\quad \le -\sigma \int_{t-\tau(t)}^{t} x(s)^T T_2 x(s)\, ds \\ &\qquad -\sigma \int_{t-(l-1)\sigma(t)}^{t} x(s)^T T_2 x(s)\, ds - \cdots \\ &\qquad -\sigma \int_{t-2\sigma(t)}^{t} x(s)^T T_2 x(s)\, ds \\ &\qquad -\sigma \int_{t-\sigma(t)}^{t} x(s)^T T_2 x(s)\, ds \\ &\quad = -\sigma \sum_{m=1}^{l} \int_{t-m\sigma(t)}^{t} x(s)^T T_2 x(s)\, ds, \end{aligned} \tag{28}$$

$$\begin{aligned} &-h \int_{t-\tau}^{t} x(s)^T T_1 x(s)\, ds \\ &\quad = -h \sum_{n=1}^{k} \int_{t-nh}^{t-(n-1)h} x(s)^T T_1 x(s)\, ds, \end{aligned} \tag{29}$$

$$\begin{aligned} &-h \int_{t-\tau}^{t} \dot{x}(s)^T E^T T_1 E \dot{x}(s)\, ds \\ &\quad = -h \int_{t-h}^{t} \dot{x}(s)^T E^T T_1 E \dot{x}(s)\, ds \\ &\qquad -h \int_{t-2h}^{t-h} \dot{x}(s)^T E^T T_1 E \dot{x}\, ds - \cdots \\ &\qquad -h \int_{t-(k-1)h}^{t-(k-2)h} \dot{x}(s)^T E^T T_1 E \dot{x}(s)\, ds \\ &\qquad -h \int_{t-\tau}^{t-(k-1)h} \dot{x}(s)^T E^T T_1 E \dot{x}(s)\, ds \\ &\quad \le -\left[\int_{t-h}^{t} \dot{x}(s)\, ds\right]^T E^T T_1 E \left[\int_{t-h}^{t} \dot{x}(s)\, ds\right] \\ &\qquad -\left[\int_{t-2h}^{t-h} \dot{x}(s)\, ds\right]^T E^T T_1 E \\ &\qquad \times \left[\int_{t-2h}^{t-h} \dot{x}(s)\, ds\right] - \cdots - \left[\int_{t-\tau}^{t-(k-1)h} \dot{x}(s)\, ds\right]^T \\ &\qquad \times E^T T_1 E \left[\int_{t-\tau}^{t-(k-1)h} \dot{x}(s)\, ds\right] \\ &\quad = -[x(t) - x(t-h)]^T E^T T_1 E [x(t) - x(t-h)] \\ &\qquad -[x(t-h) - x(t-2h)]^T \\ &\qquad \times E^T T_1 E [x(t-h) - x(t-2h)] \\ &\qquad - \cdots - [x(t-(k-1)h) - x(t-\tau)]^T \\ &\qquad \times E^T T_1 E [x(t-(k-1)h) - x(t-\tau)] \\ &\quad = \xi(t)^T \Gamma_1 \xi(t) + \xi(t-h)^T \Gamma_2 \xi(t-h), \end{aligned} \tag{30}$$

$$\Gamma_1 = \begin{bmatrix} -E^T T_1 E & E^T T_1 E & 0 & \cdots & 0 & 0 \\ * & -E^T T_1 E & E^T T_1 E & \cdots & 0 & 0 \\ 0 & * & -E^T T_1 E & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -E^T T_1 E & E^T T_1 E \\ 0 & 0 & 0 & \cdots & E^T T_1 E & -E^T T_1 E \end{bmatrix}, \tag{31}$$

$$\Gamma_2 = \begin{bmatrix} -E^T T_1 E & 0 & 0 & \cdots & 0 & 0 \\ 0 & -E^T T_1 E & 0 & \cdots & 0 & 0 \\ 0 & 0 & -E^T T_1 E & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -E^T T_1 E & 0 \\ 0 & 0 & 0 & \cdots & 0 & -E^T T_1 E \end{bmatrix}, \tag{32}$$

$$\begin{aligned} &-\sigma \int_{t-\tau}^{t} \dot{x}(s)^T E^T T_2 E \dot{x}(s)\, ds \\ &\quad = -\sigma \int_{t-\sigma(t)}^{t} \dot{x}(s)^T E^T T_2 E \dot{x}(s)\, ds \\ &\quad - \int_{t-2\sigma(t)}^{t-\sigma(t)} \dot{x}(s)^T E^T T_2 E x(s)\, ds \\ &\quad - \cdots - \sigma \int_{t-\tau}^{t-\tau(t)} \dot{x}(s)^T E^T T_2 E x(s)\, ds \\ &\quad \le -\sigma(t) \int_{t-\sigma(t)}^{t} \dot{x}(s)^T E^T T_2 E \dot{x}(s)\, ds \\ &\quad - \sigma(t) \int_{t-2\sigma(t)}^{t-\sigma(t)} \dot{x}(s)^T E^T T_2 E \dot{x}(s)\, ds \\ &\quad - \cdots - \frac{\sigma}{\tau}(\tau - \tau(t)) \\ &\quad \times \int_{t-\tau}^{t-\tau(t)} \dot{x}(s)^T E^T T_2 E \dot{x}(s)\, ds. \end{aligned} \tag{33}$$

Similar to (33), by Lemma 6, it is easy to obtain

$$\begin{aligned} &-\sigma \int_{t-\tau}^{t} \dot{x}(s)^T E^T T_2 E \dot{x}(s)\, ds \\ &\quad \le \zeta_2(t)^T \begin{bmatrix} -E^T T_2 E & 0 & E^T T_2 E & 0 & \cdots & 0 & 0 \\ 0 & -\frac{1}{l} E^T T_2 E & 0 & 0 & \cdots & 0 & \frac{1}{l} E^T T_2 E \\ * & 0 & -2E^T T_2 E & E^T T_2 E & \cdots & 0 & 0 \\ 0 & 0 & * & -2E^T T_2 E & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -2E^T T_2 E & E^T T_2 E \\ 0 & * & 0 & 0 & \cdots & * & -\left(1 + \frac{1}{l}\right) E^T T_2 E \end{bmatrix} \zeta_2(t), \end{aligned} \tag{34}$$

where

$$\zeta_2(t) = \left[ x(t)^T \;\; x(t-\tau)^T \;\; x(t-\sigma(t))^T \;\; x(t-2\sigma(t))^T \;\; \cdots \;\; x(t-(l-1)\sigma(t))^T \;\; x(t-\tau(t))^T \right]. \tag{35}$$

In conclusion, one has

$$\begin{aligned} &LV(x_t, i, t) \\ &\quad \le \sum_{n=1}^{k} \int_{t-nh}^{t-(n-1)h} \left[ x(t)^T \;\; x(t-\tau(t))^T \;\; x(s)^T \right]^T \\ &\quad \times \begin{bmatrix} -\frac{1}{h} M_{i1} & 0 & A_i^T M_{ni} + \sum_{j \in S} \rho_{ij} E^T M_{nj} \\ 0 & -E^T T_2 E & \left[ B_i + C_i D_q K_i + C_i D_q^{-} H_i \right] M_{ni} \\ * & * & -\tau T_1 \end{bmatrix} \\ &\quad \times \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(s) \end{bmatrix} ds \\ &\quad + \int_{t-h}^{t} \left[ \xi(t)^T \;\; \xi(t-h)^T \;\; \xi(s)^T \right] \\ &\quad \times \begin{bmatrix} h\Gamma_1 & 0 & N_i \\ 0 & h\Gamma_2 & -N_i \\ * & * & \sum_{j \in S} \rho_{ij} \left( hN_j + Q_j \right) \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t-h) \\ \xi(s) \end{bmatrix} ds \\ &\quad + \sum_{m=1}^{l} \int_{t-m\sigma(t)}^{t} x(s)^T \left[ \sum_{j \in S} \rho_{ij} R_{jm} - \tau T_2 \right] x(s)\, ds \\ &\quad + \pi(t)^T \Sigma \pi(t), \end{aligned} \tag{36}$$

where

$$\pi(t) = \left[\xi(t)^T, x(t-\tau)^T, x(t-\sigma(t))^T, x(t-2\sigma(t))^T, \ldots, x(t-(l-1)\sigma(t))^T, x(t-\tau(t))^T\right]^T,$$

$$\Sigma = \begin{bmatrix}
\Sigma_{11} & Q_{12i}+M_{i1}-M_{i2} & Q_{13i}+M_{i2}-M_{i3} & \cdots & Q_{1k-1i}+M_{ik-2}-M_{ik-1} & Q_{1ki}+M_{ik-1}-M_{ik} & M_{ik} & E^TT_2E \\
* & Q_{22i}-Q_{11i} & Q_{23i}-Q_{12i} & \cdots & Q_{2k-1i}-Q_{1k-2i} & Q_{2ki}-Q_{1k-1i} & -Q_{1ki} & 0 \\
* & * & Q_{33i}-Q_{22i} & \cdots & Q_{3k-1i}-Q_{2k-2i} & Q_{3ki}-Q_{2k-1i} & -Q_{2ki} & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & \cdots & Q_{k-1k-1i}-Q_{k-2k-2i} & Q_{k-1ki}-Q_{k-2k-1i} & -Q_{k-2k-1i} & 0 \\
* & * & * & \cdots & * & Q_{kki}-Q_{k-1k-1i} & -Q_{k-1ki} & 0 \\
* & * & * & \cdots & * & * & -Q_{kki}-\frac{1}{l}E^TT_2E & 0 \\
* & 0 & 0 & \cdots & 0 & 0 & 0 & -2E^TT_2E-\left(1-\frac{\mu}{l}\right)R_{i1} \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & * \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
* & 0 & 0 & \cdots & 0 & 0 & * & 0
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & \cdots & 0 & 0 & \Sigma_{1k+l+1} \\
0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & -\frac{1}{l}E^TT_2E \\
E^TT_2E & 0 & \cdots & 0 & 0 & 0 \\
-2E^TT_2E-\left(1-\frac{2\mu}{l}\right)R_{i2} & E^TT_2E & \cdots & 0 & 0 & 0 \\
* & -2E^TT_2E-\left(1-\frac{3\mu}{l}\right)R_{i3} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & -2E^TT_2E-\left(1-\frac{(l-2)\mu}{l}\right)R_{il-2} & E^TT_2E & 0 \\
0 & 0 & \cdots & * & -2E^TT_2E-\left(1-\frac{(l-1)\mu}{l}\right)R_{il-1} & E^TT_2E \\
0 & 0 & \cdots & 0 & * & \Sigma_{k+l+1k+l+1}
\end{bmatrix}, \quad (37)$$

$$\Sigma_{11} = A_i^TP_i + P_iA_i + \sum_{j\in S}\rho_{ij}E^TP_i + \sum_{m=1}^{l}R_{im} + \frac{\tau^2}{k}A_i^TT_1A_i + \frac{\tau^2}{l}A_i^TT_2A_i + \frac{\tau^2}{k}T_1 + \frac{\tau^2}{l}T_2 - M_{i1} - E^TT_2E + Q_{11i},$$

$$\Sigma_{1k+l-1} = B_i^TP_i + K_i^TD_q^TC_i^TP_i + H_i^TD_q^{-T}C_i^TP_i + \frac{\tau^2}{k}B_i^TT_1A_i + \frac{\tau^2}{k}K_i^TD_q^TC_i^TT_1A_i + \frac{\tau^2}{k}H_i^TD_q^{-T}C_i^TT_1A_i + \frac{\tau^2}{l}B_i^TT_2A_i + \frac{\tau^2}{l}K_i^TD_q^TC_i^TT_2A_i + \frac{\tau^2}{l}H_i^TD_q^{-T}C_i^TT_2A_i,$$

$$\Sigma_{k+lk+l} = \left[B_i + C_iD_qK_i + C_iD_q^-H_i\right]^T\left(\frac{\tau^2}{k}T_1 + \frac{\tau^2}{l}T_2\right)\left[B_i + C_iD_qK_i + C_iD_q^-H_i\right] - \frac{1}{l}E^TT_2E - (1-\mu)R_{il};$$

using (18b) and (18c), it is easy to obtain

$$LV(x_t, i, t) \le 0. \quad (38)$$

Since $A_{i4}$ are nonsingular for every $i \in S$, we set

$$\hat{G} = \begin{bmatrix} I_r & -A_{i2}A_{i4}^{-1} \\ 0 & A_{i4}^{-1} \end{bmatrix} G. \quad (39)$$

It is easy to get

$$\hat{G}\, E\Upsilon = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad \hat{G}\, A_i\Upsilon = \begin{bmatrix} \hat{A}_{i1} & 0 \\ \hat{A}_{i3} & I \end{bmatrix}, \tag{40}$$

where

$$\hat{A}_{i1} = A_{i1} - A_{i2}A_{i4}^{-1}A_{i3}, \hat{A}_{i3} = A_{i4}^{-1}A_{i3}. \tag{41}$$

Denote that

$$\hat{G}\left[B_i + C_iD_qK_i + C_iD_q^{-}H_i\right]\Upsilon = \begin{bmatrix} A_{id1} & A_{id2} \\ A_{id3} & A_{id4} \end{bmatrix},$$

$$\hat{G}^{-T}\, P_i\Upsilon = \begin{bmatrix} P_{i1} & P_{i2} \\ P_{i3} & P_{i4} \end{bmatrix}, \tag{42}$$

$$\Upsilon^T R_{il}\Upsilon = \begin{bmatrix} R_{1il} & R_{2il} \\ R_{3il} & R_{4il} \end{bmatrix}.$$

Let

$$\overline{x}(t) = \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix} = \Upsilon^{-1}x(t). \tag{43}$$

Then, for every $i \in S$, system (20) is a restricted system equivalent to

$$\begin{aligned} \dot{\overline{x}}_1(t) &= \hat{A}_{i1}\overline{x}_1(t) + A_{id1}\overline{x}_1(t-\tau(t)) + A_{id2}\overline{x}_2(t-\tau(t)), \\ -\overline{x}_2(t) &= \hat{A}_{i13}\overline{x}_1(t) + A_{id3}\overline{x}_1(t-\tau(t)) + A_{id4}\overline{x}_2(t-\tau(t)), \\ \overline{x}(t) &= \varphi(t) = S^{-1}\phi(t), \quad t \in [-\tau, 0]. \end{aligned} \tag{44}$$

To prove the exponential stability in mean square, we define a new function as

$$W(x_t, i, t) = e^{\varepsilon t}V(x_t, i, t), \tag{45}$$

where $\varepsilon > 0$ and then by Dynkin's formula, we find that for every $i \in S$,

$$\begin{aligned} EW(x_t, r_t, t) &\le EW(x_0, r_0, 0) \\ &\quad + E\int_0^t e^{\varepsilon s}\left[\varepsilon V(x_s, r_s, s) - \lambda\|x(s)\|^2\right]ds. \end{aligned} \tag{46}$$

It can be seen from (23), (45), and (46) that if $\varepsilon$ is chosen small enough, a constant $k > 0$ can be found such that for $t > 0$,

$$\begin{aligned} \min_{i\in S}\{\lambda_{\min}(P_{i1})\}E\left\{\|\overline{x}_1\|^2\right\} &\le E\left\{x^T(t)E^TP_ix(t)\right\} \\ &\le ke^{-\varepsilon t}\|\phi(t)\|_\tau^2. \end{aligned} \tag{47}$$

Hence, for any $t > 0$

$$E\left\{\|\overline{x}_1\|^2\right\} \le \alpha e^{-\varepsilon t}\|\phi(t)\|_\tau^2, \tag{48}$$

where $\alpha = (\min_{i\in S}\{\lambda_{\min}(P_{i1})\})^{-1}k$. To study the exponential stability in mean square of $\overline{x}_2(t)$, we apply the similar analysis method of [12] and define

$$e(t) = \hat{A}_{i13}\overline{x}_1(t) + A_{id3}\overline{x}_1(t-\tau(t)). \tag{49}$$

Then, according to (48), we have a constant $m > 0$; it can be found that when $t > 0$,

$$E\left\{\|e(t)\|^2\right\} \le me^{-\varepsilon t}\|\phi(t)\|_\tau^2. \tag{50}$$

We construct a function as

$$J(t) = \overline{x}_2(t)^TR_{4il}\overline{x}_2(t) - \overline{x}_2(t-\tau(t))^TR_{4il}\overline{x}_2(t-\tau(t)). \tag{51}$$

By premultiplying the second equation of (44) with $\overline{x}_2(t)^TP_{i4}^T$, we obtain that

$$0 = \overline{x}_2(t)^TP_{i4}^T\overline{x}_2(t) + \overline{x}_2(t)^TP_{i4}^T\overline{x}_2(t-\tau(t)) + \overline{x}_2(t)^TP_{i4}^Te(t). \tag{52}$$

Adding (52) to (51) yields that

$$\begin{aligned} J(t) &= \overline{x}_2(t)^T\left(P_{i4} + P_{i4} + R_{4il}\right)\overline{x}_2(t) \\ &\quad + 2\overline{x}_2(t)^TP_{i4}^TA_{id4}\overline{x}_2(t-\tau(t)) \\ &\quad - \overline{x}_2(t-\tau(t))^TR_{4il}\overline{x}_2 + 2\overline{x}_2(t)^TP_{i4}^Te(t) \\ &\le \begin{bmatrix} \overline{x}_2(t) \\ \overline{x}_2(t-\tau(t)) \end{bmatrix}^T \begin{bmatrix} P_{i4} + P_{i4}^T + R_{4il} & P_{i4}^TA_{id4} \\ * & -R_{4il} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \overline{x}_2(t) \\ \overline{x}_2(t-\tau(t)) \end{bmatrix} \\ &\quad + \eta_1\overline{x}_2(t)^T\overline{x}_2(t) + \eta^{-1}e(t)^TR_{4il}R_{4il}^Te(t), \end{aligned} \tag{53}$$

where $\eta_1$ is any positive scalar.

By pre- and postmultiplying $\begin{bmatrix} \Sigma_{11} & \Sigma_{1i+l} \\ * & \Sigma_{k+lk+l} \end{bmatrix} < 0$ by $\begin{bmatrix} \Upsilon & 0 \\ 0 & \Upsilon \end{bmatrix}^T$ and $\begin{bmatrix} \Upsilon & 0 \\ 0 & \Upsilon \end{bmatrix}$, respectively, we get a constant $\eta_2 > 0$, such that

$$\begin{bmatrix} P_{i4} + P_{i4}^T + R_{4il} & P_{i4}^TA_{id4} \\ * & -R_{id4} \end{bmatrix} \le -\begin{bmatrix} \eta_2 I & 0 \\ 0 & 0 \end{bmatrix}. \tag{54}$$

On the other hand, since $\eta_1$ can be chosen arbitrarily, $\eta_1$ is chosen small enough, such that $\eta_2 - \eta_1 > 0$. Then, we can always find a scalar $\eta_3 > 1$, such that

$$R_{4il} - (\eta_2 - \eta_1)I \ge \eta_3R_{4il}. \tag{55}$$

It follows from (51), (53), and (54) that

$$\begin{aligned} &\overline{x}_2(t)^TR_{4il}\overline{x}_2(t) - \overline{x}_2(t-\tau(t))^TR_{4il}\overline{x}_2(t-\tau(t)) \\ &\quad \le -\eta_2\overline{x}_2(t)^T\overline{x}_2(t) + \eta_1\overline{x}_2(t)^T\overline{x}_2(t) \\ &\qquad + \eta^{-1}e(t)^TP_{i4}P_{i4}^Te(t). \end{aligned} \tag{56}$$

It is clear that the above inequalities (55) and (56) imply that

$$\begin{aligned}\eta_3\overline{x}_2(t)^T R_{4il}\overline{x}_2(t) &\le \overline{x}_2(t)^T\left(R_{4il}-(\eta_1-\eta_2)I\right)\\ &\le \overline{x}_2(t-\tau(t))^T R_{4il}\overline{x}_2(t-\tau(t))\\ &\quad+\eta^{-1}e(t)^T P_{i4}P_{i4}^T e(t).\end{aligned}\tag{57}$$

Thus, we get from (57) that

$$\begin{aligned}\overline{x}_2(t)^T R_{4il}\overline{x}_2(t) &\le \eta_3^{-1}\overline{x}_2(t-\tau(t))^T R_{4il}\overline{x}_2(t-\tau(t))\\ &\quad+(\eta_1\eta_3)^{-1}e(t)^T P_{i4}P_{i4}^T e(t).\end{aligned}\tag{58}$$

Let $f(t)=\overline{x}_2(t)^T R_{4il}\overline{x}_2(t)$; from the above inequality and the fact (50) and (58), we have

$$\begin{aligned}E\{f(t)\} &\le \eta_3^{-1}E\left\{\sup_{t-\tau\le s\le t} f(s)\right\}+\gamma e^{-\varepsilon t}\\ &\le \eta_3^{-1}E\left\{\sup_{t-\tau\le s\le t} f(s)\right\}+\gamma e^{-\delta t},\end{aligned}\tag{59}$$

where $0<\delta<\min\{\varepsilon, d_2^{-1}\ln\eta_3\}$, $\gamma=(\eta_1\eta_3)^{-1}\max_{i\in S}\|P_{i4}\|^2\|\phi(t)\|_\tau^2$.

Therefore, applying Lemma 4 to the above inequality yields that

$$\begin{aligned}E\left\{\left\|\overline{x}_2(t)\right\|^2\right\} &\le \lambda_{\min}^{-1}(R_{4il})\lambda_{\max}(R_{4il})e^{-\delta t}\left\|\overline{x}_2(t)\right\|_\tau^2\\ &\quad+\frac{\lambda_{\min}^{-1}(R_{4il})\gamma e^{-\delta t}}{1-\eta_3^{-1}e^{\delta d_2}}.\end{aligned}\tag{60}$$

We can find from (50) and the above inequality that the system (16) is exponentially stable in mean square. □

*Remark 8.* It should be pointed out that sometimes too many free variables cannot reduce the conservatism of the obtained results. In the theorem, the system is exponentially stable in mean square without resorting to the free-weighting matrices method, which is in contrast with [6], where the free-weighting matrices method was used.

*Remark 9.* In the proof of the theorem, the delay interval $[0;\tau]$ is divided into $N$ segments of equal length $\tau/N$, such that the information of delayed states $x(t-i\tau/N)$, $i=1,2,\ldots,N$, are all taken into account. It is clear that the Lyapunov function defined in the theorem is more general than the ones in [7].

## 4. Numerical Examples

In this section, we provide a numerical example to verify the effectiveness of the proposed method. Consider systems (1) in $R^2$ with two regines $r_t\in S=\{1,2\}$. The system parameters are described as follows:

$$\begin{aligned}E&=\begin{bmatrix}1&0\\0&0\end{bmatrix},\quad A_1=\begin{bmatrix}-0.8&0\\0.6&-2.0\end{bmatrix},\\ A_2&=\begin{bmatrix}-1.8&0.5\\0.5&-1.8\end{bmatrix},\quad B_1=\begin{bmatrix}1.7&1.3\\2.5&2.1\end{bmatrix},\\ B_2&=\begin{bmatrix}-0.5&-0.3\\0&-0.2\end{bmatrix},\quad C_1=\begin{bmatrix}0.2&0\\0&0.2\end{bmatrix},\\ C_2&=\begin{bmatrix}0.4&0\\0&0.4\end{bmatrix},\quad D_q=\begin{bmatrix}1&0\\0&1\end{bmatrix},\\ D_q^-&=\begin{bmatrix}0&0\\0&0\end{bmatrix},\quad \varphi(s)=\begin{bmatrix}3\cos(s)\\-\sin(s)-3\cos^2(s)\end{bmatrix},\\ (\pi_{ij})_{2\times2}&=\begin{bmatrix}-40&40\\10&-10\end{bmatrix},\quad D_q^-=\begin{bmatrix}0&0\\0&0\end{bmatrix},\\ (\pi_{ij})_{2\times2}&=\begin{bmatrix}-40&40\\10&-10\end{bmatrix},\quad H_1=\begin{bmatrix}1&0\\0&1\end{bmatrix},\\ H_2&=\begin{bmatrix}1&0\\0&1\end{bmatrix},\quad \tau=1,\quad \mu=3.\end{aligned}\tag{61}$$

As $l=1,k=1$, and

(1) $r_t=1$, by calculating we have

$$\begin{aligned}P_1&=\begin{bmatrix}13.8176&1.3147\\1.3147&27.9707\end{bmatrix},\\ P_2&=\begin{bmatrix}2.3514&-0.2388\\-0.2388&27.9707\end{bmatrix},\\ K_1&=\begin{bmatrix}-9.9593&-6.2795\\-6.2795&-12.1579\end{bmatrix},\\ Q_{111}&=\begin{bmatrix}2.0672&-0.1326\\-0.1326&2.0672\end{bmatrix},\\ Q_{112}&=\begin{bmatrix}1.9317&-0.2833\\-0.2833&1.9317\end{bmatrix},\\ R_{11}&=\begin{bmatrix}2.3514&-0.2388\\-0.2388&2.3514\end{bmatrix},\\ R_{21}&=\begin{bmatrix}0.8620&-1.2919\\-1.2919&0.8620\end{bmatrix},\\ M_{11}&=\begin{bmatrix}3.5860&-0.2001\\-0.2001&3.5860\end{bmatrix},\\ M_{12}&=\begin{bmatrix}1.0724&0.2600\\0.2600&1.0724\end{bmatrix},\quad N_1=\begin{bmatrix}0.9522&0.2605\\0.2605&0.9522\end{bmatrix},\\ N_2&=\begin{bmatrix}-2.4898&0.2895\\0.2895&-2.4898\end{bmatrix},\\ T_1&=\begin{bmatrix}2.8438&1.5469\\1.5469&8.7138\end{bmatrix},\quad T_2=\begin{bmatrix}5.7161&0\\0&5.7161\end{bmatrix};\end{aligned}\tag{62}$$

(2) $r_t = 2$, by calculating we have

$$P_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 9.0478 & 2.2219 \\ 2.2219 & 36.3742 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.1904 & -0.1109 \\ -0.1109 & 0.3192 \end{bmatrix},$$
$$Q_{111} = \begin{bmatrix} 2.0672 & -0.1326 \\ -0.1326 & 2.0672 \end{bmatrix},$$
$$Q_{112} = \begin{bmatrix} 0.1793 & -0.7876 \\ -0.7876 & 0.1793 \end{bmatrix},$$
$$R_{11} = \begin{bmatrix} 2.3514 & -0.2388 \\ -0.2388 & 2.3514 \end{bmatrix},$$
$$R_{21} = \begin{bmatrix} 0.8620 & -1.2919 \\ -1.2919 & 0.8620 \end{bmatrix},$$
$$M_{11} = \begin{bmatrix} 3.5860 & -0.2001 \\ -0.2001 & 3.5860 \end{bmatrix},$$
$$M_{12} = \begin{bmatrix} 1.0724 & 0.2600 \\ 0.2600 & 1.0724 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 0.2895 & -2.4898 \end{bmatrix}, \quad T_1 = \begin{bmatrix} 2.8301 & 1.5499 \\ 1.5499 & 8.7230 \end{bmatrix},$$
$$T_2 = \begin{bmatrix} 5.7161 & 0 \\ 0 & 5.7161 \end{bmatrix}. \tag{63}$$

As $l = 1, k = 2$, and

(1) $r_t = 1$, by calculating we have

$$P_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 8.4878 & 1.2935 \\ 1.2935 & 32.5716 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -9.9593 & -6.2795 \\ -6.2795 & -12.1579 \end{bmatrix},$$
$$Q_{111} = \begin{bmatrix} 2.0672 & -0.1326 \\ -0.1326 & 2.0672 \end{bmatrix},$$
$$Q_{112} = \begin{bmatrix} 1.9317 & -0.2833 \\ -0.2833 & 1.9317 \end{bmatrix},$$
$$R_{11} = \begin{bmatrix} 2.3514 & -0.2388 \\ -0.2388 & 2.3514 \end{bmatrix},$$
$$Q_{121} = \begin{bmatrix} 0.2586 & -0.3256 \\ -0.3256 & 0.2586 \end{bmatrix},$$
$$Q_{221} = \begin{bmatrix} 0.1904 & -0.0521 \\ -0.0521 & 0.1904 \end{bmatrix},$$
$$M_{12} = \begin{bmatrix} 1.0724 & 0.2600 \\ 0.2600 & 1.0724 \end{bmatrix}, \quad R_{12} = \begin{bmatrix} 0.8620 & -1.2919 \\ -1.2919 & 0.8620 \end{bmatrix},$$
$$M_{11} = \begin{bmatrix} 3.5860 & -0.2001 \\ -0.2001 & 3.5860 \end{bmatrix},$$
$$M_{21} = \begin{bmatrix} 0.1332 & -0.0926 \\ -0.0926 & 0.1332 \end{bmatrix},$$
$$M_{22} = \begin{bmatrix} 0.2025 & -0.2003 \\ -0.2003 & 0.2025 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix},$$
$$T_1 = \begin{bmatrix} 2.8438 & 1.5469 \\ 1.5469 & 8.7138 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 5.7161 & 0 \\ 0 & 5.7161 \end{bmatrix}; \tag{64}$$

(2) $r_t = 2$, by calculating we have

$$P_1 = \begin{bmatrix} 80.5898 & 5.2954 \\ 5.2954 & 229.2114 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 2.4898 & -0.2895 \\ -0.2895 & 2.4898 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.1904 & -0.1109 \\ -0.1109 & 0.3192 \end{bmatrix},$$
$$Q_{111} = \begin{bmatrix} 2.0672 & -0.1326 \\ -0.1326 & 2.0672 \end{bmatrix},$$
$$Q_{112} = \begin{bmatrix} 1.9317 & -0.2833 \\ -0.2833 & 1.9317 \end{bmatrix},$$
$$R_{11} = \begin{bmatrix} 2.3514 & -0.2388 \\ -0.2388 & 2.3514 \end{bmatrix},$$
$$R_{21} = \begin{bmatrix} 0.8620 & -1.2919 \\ -1.2919 & 0.8620 \end{bmatrix},$$
$$M_{11} = \begin{bmatrix} 3.5860 & -0.2001 \\ -0.2001 & 3.5860 \end{bmatrix},$$
$$M_{12} = \begin{bmatrix} 1.0724 & 0.2600 \\ 0.2600 & 1.0724 \end{bmatrix},$$
$$M_{21} = \begin{bmatrix} 0.1332 & -0.0926 \\ -0.0926 & 0.1332 \end{bmatrix},$$
$$M_{22} = \begin{bmatrix} 0.2025 & -0.2003 \\ -0.2003 & 0.2025 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 0.2895 & -2.4898 \end{bmatrix}, \quad T_1 = \begin{bmatrix} 2.8438 & 1.5469 \\ 1.5469 & 8.7138 \end{bmatrix},$$
$$T_2 = \begin{bmatrix} 5.7167 & 0 \\ 0 & 5.7161 \end{bmatrix}. \tag{65}$$

As $l = 2, k = 1$, and

(1) $r_t = 1$, by calculating we have

$$P_1 = \begin{bmatrix} 2.4898 & -0.2895 \\ -0.2895 & 2.1898 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 16.5321 & 2.8270 \\ 2.8270 & 18.5944 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -9.9593 & -6.2795 \\ -6.2795 & -12.1579 \end{bmatrix},$$

$$Q_{111} = \begin{bmatrix} 2.0672 & -0.1326 \\ -0.1326 & 2.0672 \end{bmatrix},$$

$$Q_{112} = \begin{bmatrix} 1.9317 & -0.2833 \\ -0.2833 & 1.9317 \end{bmatrix},$$

$$R_{11} = \begin{bmatrix} 2.3514 & -0.2388 \\ -0.2388 & 2.3514 \end{bmatrix},$$

$$R_{21} = \begin{bmatrix} 0.8620 & -1.2919 \\ -1.2919 & 0.8620 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} 3.5860 & -0.2001 \\ -0.2001 & 3.5860 \end{bmatrix},$$

$$M_{12} = \begin{bmatrix} 1.0724 & 0.2600 \\ 0.2600 & 1.0724 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 0.2895 & -2.4898 \end{bmatrix}, \quad T_1 = \begin{bmatrix} 2.8438 & 1.5469 \\ 1.5469 & 8.7138 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} 5.7161 & 0 \\ 0 & 5.7161 \end{bmatrix}. \tag{66}$$

(2) $r_t = 2$, by calculating we have

$$P_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 7.4211 & 1.4322 \\ 1.4322 & 29.4614 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.1904 & -0.1109 \\ -0.1109 & 0.3192 \end{bmatrix},$$

$$Q_{111} = \begin{bmatrix} 2.0672 & -0.1326 \\ -0.1326 & 2.0672 \end{bmatrix},$$

$$Q_{112} = \begin{bmatrix} 0.1793 & -0.7876 \\ -0.7876 & 0.1793 \end{bmatrix},$$

$$R_{11} = \begin{bmatrix} 2.3514 & -0.2388 \\ -0.2388 & 2.3514 \end{bmatrix},$$

$$R_{21} = \begin{bmatrix} 0.8620 & -1.2919 \\ -1.2919 & 0.8620 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} 3.5860 & -0.2001 \\ -0.2001 & 3.5860 \end{bmatrix},$$

$$M_{12} = \begin{bmatrix} 1.0724 & 0.2600 \\ 0.2600 & 1.0724 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 0.2895 & -2.4898 \end{bmatrix}, \quad T_1 = \begin{bmatrix} 2.8438 & 1.5469 \\ 1.5469 & 8.7138 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} 5.7161 & 0 \\ 0 & 5.7161 \end{bmatrix}. \tag{67}$$

As $l = 2, k = 2$, and

(1) $r_t = 1$, by calculating we have

$$P_1 = \begin{bmatrix} 497.5359 & 30.4498 \\ 30.4498 & 728.9455 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 2.4898 & -0.2895 \\ -0.2895 & 2.4898 \end{bmatrix},$$

$$Q_{111} = \begin{bmatrix} 2.0672 & -0.1326 \\ -0.1326 & 2.0672 \end{bmatrix},$$

$$Q_{112} = \begin{bmatrix} 1.9317 & -0.2833 \\ -0.2833 & 1.9317 \end{bmatrix},$$

$$Q_{121} = \begin{bmatrix} 0.2586 & -0.3256 \\ -0.3256 & 0.2586 \end{bmatrix},$$

$$R_{11} = \begin{bmatrix} 2.3514 & -0.2388 \\ -0.2388 & 2.3514 \end{bmatrix},$$

$$Q_{221} = \begin{bmatrix} 0.1904 & -0.0521 \\ -0.0521 & 0.1904 \end{bmatrix},$$

$$R_{21} = \begin{bmatrix} 0.8620 & -1.2919 \\ -1.2919 & 0.8620 \end{bmatrix}, \tag{68}$$

$$M_{11} = \begin{bmatrix} 3.5860 & -0.2001 \\ -0.2001 & 3.5860 \end{bmatrix},$$

$$M_{12} = \begin{bmatrix} 1.0724 & 0.2600 \\ 0.2600 & 1.0724 \end{bmatrix},$$

$$M_{21} = \begin{bmatrix} 0.1332 & -0.0926 \\ -0.0926 & 0.1332 \end{bmatrix},$$

$$M_{22} = \begin{bmatrix} 0.2025 & -0.2003 \\ -0.2003 & 0.2025 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 0.2895 & -2.4898 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 2.8438 & 1.5469 \\ 1.5469 & 8.7138 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 5.7161 & 0 \\ 0 & 5.7161 \end{bmatrix}.$$

(2) $r_t = 2$, by calculating we have

$$P_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 36.1907 & -7.0282 \\ -7.0282 & 101.6875 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.1904 & -0.1109 \\ -0.1109 & 0.3192 \end{bmatrix},$$

$$Q_{111} = \begin{bmatrix} 2.0672 & -0.1326 \\ -0.1326 & 2.0672 \end{bmatrix},$$

$$Q_{112} = \begin{bmatrix} 0.1793 & -0.7876 \\ -0.7876 & 0.1793 \end{bmatrix},$$

$$M_{12} = \begin{bmatrix} 1.0724 & 0.2600 \\ 0.2600 & 1.0724 \end{bmatrix},$$

$$Q_{121} = \begin{bmatrix} 0.2586 & -0.3256 \\ -0.3256 & 0.2586 \end{bmatrix},$$

$$Q_{221} = \begin{bmatrix} 0.1904 & -0.0521 \\ -0.0521 & 0.1904 \end{bmatrix},$$

$$R_{11} = \begin{bmatrix} 2.3514 & -0.2388 \\ -0.2388 & 2.3514 \end{bmatrix},$$

$$R_{21} = \begin{bmatrix} 0.8620 & -1.2919 \\ -1.2919 & 0.8620 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} 3.5860 & -0.2001 \\ -0.2001 & 3.5860 \end{bmatrix},$$

$$M_{12} = \begin{bmatrix} 1.0724 & 0.2600 \\ 0.2600 & 1.0724 \end{bmatrix},$$

$$M_{21} = \begin{bmatrix} 0.1332 & -0.0926 \\ -0.0926 & 0.1332 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.9522 & 0.2605 \\ 0.2605 & 0.9522 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 0.2895 & -2.4898 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 2.8438 & 1.5469 \\ 1.5469 & 8.7138 \end{bmatrix}, \qquad T_2 = \begin{bmatrix} 5.7161 & 0 \\ 0 & 5.7161 \end{bmatrix}. \tag{69}$$

So we can conclude that the condition of the theorem is satisfied; the system (16) is exponentially stable in mean square.

## 5. Conclusions

In this paper, we have investigated the exponential stability in mean square of singular Markovian jump systems with singular Markovian jump systems with saturating actuators and time-varying delay. To guarantee the singular Markovian jump system to be regular, impulse free, and exponentially stable in mean square. Memoryless feedback control has been designed. By exploiting Lyapunov-Krasovskii theory, a new sufficient condition of exponentially stable in mean square is obtained. Finally, the numerical example shows that the approach proposed is effective. To reduce conservatism of the results, our future work focuses on reducing the constraint by an appropriate reduction method.

## Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant no. 11172086 and by the Natural Science Foundation of Hefei University under Grant no. 13KY03ZD.

## References

[1] E. K. Boukas, P. Shi, and K. Benjelloun, "On stabilization of uncertain linear systems with jump parameters," *International Journal of Control*, vol. 72, no. 9, pp. 842–850, 1999.

[2] P. Shi, Y. Xia, G. P. Liu, and D. Rees, "On designing of sliding-mode control for stochastic jump systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 97–103, 2006.

[3] S. I. Niculescu, J. M. Dion, and L. Dugard, "Robust stabilization for uncertain time-delay systems containing saturating actuators," *IEEE Transactions on Automatic Control*, vol. 41, no. 5, pp. 742–747, 1996.

[4] P. L. Liu, "Exponential delay dependent stabilization for time-varying delay systems with saturating actuator," *Journal of Dynamic Systems, Measurement and Control*, vol. 133, no. 1, Article ID 014502, 6 pages, 2011.

[5] E. K. Boukas, *Control of Singular Systems with Random Abrupt Changes*, Communications and Control Engineering Series, Springer, Berlin, Germany, 2008.

[6] Z. G. Wu, H. Y. Su, and J. Chu, "Delay-dependent $H_\infty$ control for singular Markovian jump systems with time delay," *Optimal Control Applications & Methods*, vol. 30, no. 5, pp. 443–461, 2009.

[7] Z. G. Wu, H. Y. Su, and J. Chu, "Robust exponential stability of uncertain singular Markovian jump time-delay systems," *Acta Automatica Sinica*, vol. 36, no. 4, pp. 558–563, 2010.

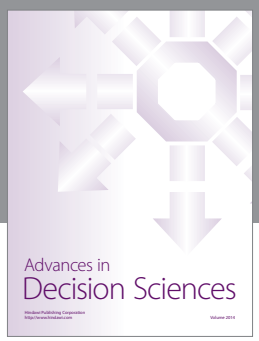
[8] S. Ma, C. Zhang, and S. Zhu, "Robust stability for discrete-time uncertain singular Markov jump systems with actuator saturation," *IET Control Theory & Applications*, vol. 5, no. 2, pp. 255–262, 2011.

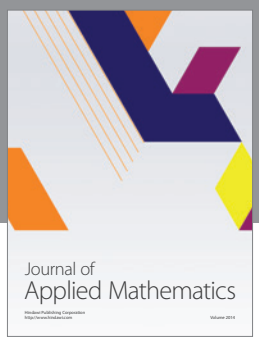
[9] E. K. Boukas, S. Y. Xu, and J. Lam, "On stability and stabilizability of singular stochastic systems with delays," *Journal of Optimization Theory and Applications*, vol. 127, no. 2, pp. 249–262, 2005.

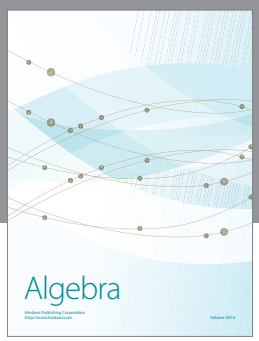
[10] T. Hu, Z. Lin, and B. M. Chen, "Analysis and design for discrete-time linear systems subject to actuator saturation," *Systems & Control Letters*, vol. 45, no. 2, pp. 97–112, 2002.

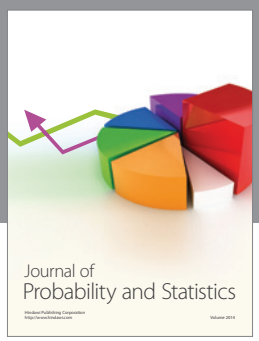
[11] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*, Birkhäauser, Boston, Mass, USA, 2003.

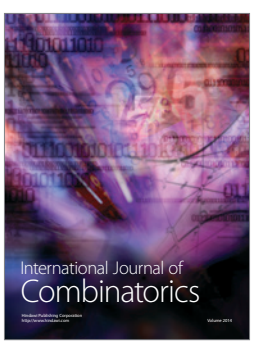
[12] T. Hu and Z. Lin, *Control Systems with Actuator Saturation: Analysis and Design*, Birkhäuser, Boston, Mass, USA, 2001.

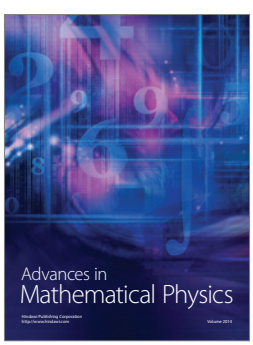


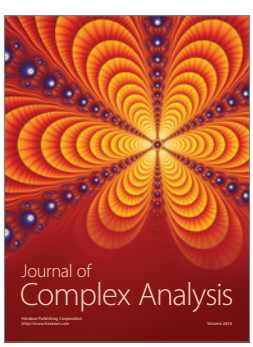



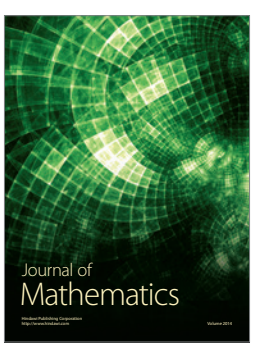



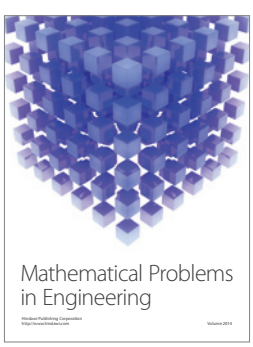



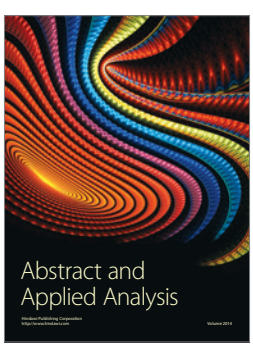


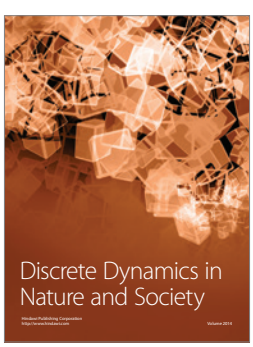



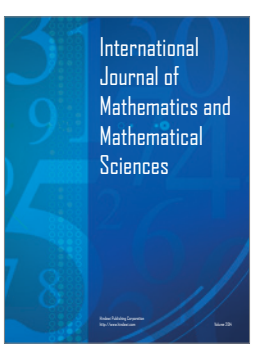



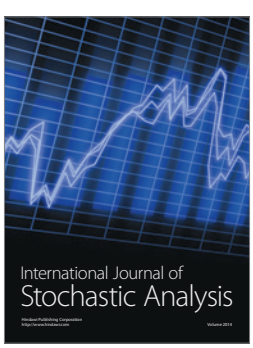





Hindawi

Submit your manuscripts at
http://www.hindawi.com