Research Article

H_2 Control for the Continuous-Time Markovian Jump Linear Uncertain Systems with Partly Known Transition Rates and Input Quantization

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For a class of continuous-time Markovian jump linear uncertain systems with partly known transition rates and input quantization, the H_2 state-feedback control design is considered. The elements in the transition rates matrix include completely known, boundary known, and completely unknown ones. First, an H_2 cost index for Markovian jump linear uncertain systems is introduced; then by introducing a new matrix inequality condition, sufficient conditions are formulated in terms of linear matrix inequalities (LMIs) for the H_2 control of the Markovian jump linear uncertain systems. Less conservativeness is achieved than the result obtained with the existing technique. Finally, a numerical example is given to verify the validity of the theoretical results.

1. Introduction

Recently, much attention has been devoted to the study of the stochastic stability for the Markovian jump systems, and many important results have been published [1–4]. This is because the Markovian jump systems have been widely employed to model many practical systems, such as manufacturing systems, the power systems, and the economic systems in which they may experience abrupt changes in their structures and parameters [5, 6]. It is worth noticing that these results require that the transition probabilities/rates must be known *a priori*. However, in many practical engineering applications, the likelihood for obtaining the perfect information on all transition probabilities/rates elements is questionable, and the cost might be expensive in some cases. Therefore, the study of the stabilization of the Markovian jump systems with partly known transition probabilities/rates becomes interesting, and some well-known results have been published. The idea for the stochastic stability of the Markovian jump linear uncertain systems with partly known

transition probabilities/rates is developed in Zhang et al. [7]. It is then applied to the H_∞ control design in Zhang and Boukas [8]. In those papers, the feature of the information about the transition probabilities/rates matrix considered includes two kinds of elements: completely known and completely unknown ones. As a matter of fact, the transition probabilities matrix might involve completely known, completely unknown, and boundary known elements. In Shen and Yang [9], an H_2 state-feedback controller design method is proposed for the continuous-time Markovian jump linear uncertain systems with the three kinds of transition rates matrix elements. In order to yield the design condition for analysis and synthesis, a matrix inequality is introduced to present LMIs conditions. In this paper, less conservative design conditions will be formulated by introducing a new relaxed matrix inequality condition.

On the other hand, in many modern engineering practices, all kinds of information processing devices, such as analog-to-digital and digital-to-analog converters, have been widely used. By the utilization of such information processing

devices, some advantages have been brought, for example, lower cost, reduced weight and power, simple installation, and maintenance. However, some new phenomena have also been induced, which might cause server deterioration of system performance or even lead to system instability. Signal quantization is one of the important aspects that should be fully considered in such cases, which always exists in computer-based control systems. Nowadays, many well-known results have been published on quantized feedback control. For example, the feedback stabilization problem is considered by utilizing dynamic quantizers [10–13] and static quantizers [14–17]. In addition, the filter design [18] and the H_∞ control design [19] are also investigated. Specially, quantization errors have adverse effects on the network control systems which can often be modeled as Markovian jump systems. In Xiao et al. [20], the stabilization problem for single-input discrete Markovian jump linear uncertain systems via mode-dependent quantized state-feedback is addressed, but the transition rates are assumed to be completely known.

To the best of our knowledge, no result has been presented for the control design of the continuous-time Markovian jump linear uncertain systems with partly known transition rates and input signal quantization. In this paper, the H_2 control for a class of continuous-time Markovian jump linear uncertain systems with respect to partly known transition rates and input signal quantization is addressed. The structure of the controller consists of two parts: the nonlinear part is provided to eliminate the effect of input quantization, and the linear part is obtained by solving LMIs for achieving the H_2 performance against unknown transition rates and model uncertainties. In comparison with the design utilizing the LMIs technique in Shen and Yang [9], the design method has less conservativeness by introducing a relaxed inequality condition.

The rest of this paper is organized as follows. The problem statement and preliminaries are presented in Section 2. The main results are given in Section 3. In Section 4, a numerical example is presented to illustrate the effectiveness of the results, and the conclusions are drawn in Section 5.

Notations. Throughout this paper, the following notations are used. R^n denotes the n -dimensional Euclidean space; A^T denotes the transpose of matrix A ; I and 0 represent the identity matrix and a zero matrix in appropriate dimensions, respectively; $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation operator; $X > Y$ ($X \geq Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive definite (positive semi-definite); $|x|_p$ denotes the p -norm of the vector x ; that is, $|x|_p = (|x_1|^p + |x_2|^p + \dots + |x_n|^p)^{1/p}$, where $p \geq 1$. When $p = \infty$, $|x|_\infty = \max_{1 \leq i \leq n} |x_i|$. For matrix $X \in R^{m \times n}$, $|X|_p$ is used to present the matrix p -norm: $|X|_p = \sup_{y \neq 0} (|Xy|_p / |y|_p)$. The notation $|\cdot|$, in particular denotes the absolute value of a scalar, the standard Euclidean norm of a vector, and the induced norm of a matrix, respectively. In symmetric block matrices, an $*$ is used to represent a term that is induced by symmetry. Finally, the symbol $He(X)$ is used to represent $X + X^T$.

2. Problem Statement and Preliminaries

Consider a class of the continuous-time Markovian jump linear uncertain systems in the following probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{aligned} \dot{x}(t) &= (A(r_t) + \Delta A(r_t))x(t) + (B(r_t) + \Delta B(r_t))q(u(t)), \\ x_0 &= x(0), \quad r_0 = r(0), \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the system state and $u(t) \in R^m$ is the control input. $\{r_t, t \geq 0\}$ is a continuous-time Markovian process with right continuous trajectories taking values in the finite set $\mathcal{S} = \{1, 2, \dots, \mathbb{N}\}$. It governs the switching among the different system modes with the following mode transition probabilities:

$$\Pr\{r_{t+h} = j \mid r_t = i\} = \begin{cases} \lambda_{ij}h + o(h), & \text{if } j \neq i, \\ 1 + \lambda_{ii}h + o(h), & \text{if } j = i, \end{cases} \quad (2)$$

where $h > 0$, $\lim_{h \rightarrow 0} (o(h)/h) = 0$, and $\lambda_{ij} \geq 0$ ($i, j \in \mathcal{S}; j \neq i$) denote the switching rate from mode i at time t to mode j at time $t + h$ and that $\lambda_{ii} = -\sum_{j=1, j \neq i}^{\mathbb{N}} \lambda_{ij}$ for each $i \in \mathcal{S}$.

In general, the Markovian process transition rates matrix Λ is defined by:

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1\mathbb{N}} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2\mathbb{N}} \\ \vdots & \cdots & \ddots & \vdots \\ \lambda_{\mathbb{N}1} & \cdots & \cdots & \lambda_{\mathbb{N}\mathbb{N}} \end{bmatrix}. \quad (3)$$

In this paper, the transition rates of the jumping process are assumed to be partly available; that is, some elements in matrix Λ have been exactly known, some ones have been merely known with lower and upper bounds, and others may have no information to use. For instance, for the system (1) with four operation modes, the transition rates matrix might be described by

$$\Lambda = \begin{bmatrix} \lambda_{11} & ? & ? & \lambda_{14} \\ ? & ? & \lambda_{23} & ? \\ ? & ? & \beta & ? \\ \alpha & ? & \lambda_{43} & \lambda_{44} \end{bmatrix}, \quad (4)$$

where $?$ represents the completely unknown element of the transition rates matrix and the parameters α and β represent the elements with known lower and upper bounds. That is, $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$, and $\underline{\beta} \leq \beta \leq \bar{\beta}$, where $\underline{\alpha}$, $\bar{\alpha}$, $\underline{\beta}$, and $\bar{\beta}$ are known parameters; λ_{ij} denotes the precisely known element.

For clarity, we denote that $\mathcal{S} = \mathcal{S}_{k1}^i \cup \mathcal{S}_{k2}^i \cup \mathcal{S}_{uk}^i$, $i = 1, 2, \dots, \mathbb{N}$, with

$$\begin{aligned} \mathcal{S}_{k1}^i &= \{j : \lambda_{ij} \text{ is exactly known}\}, \\ \mathcal{S}_{k2}^i &= \{j : \text{the bounds of } \lambda_{ij} \text{ are known}\}, \\ \mathcal{S}_{uk}^i &= \{j : \text{there is no information available for } \lambda_{ij}\}. \end{aligned} \quad (5)$$

Furthermore, let $\mathcal{S}_k^i = \{j : \lambda_{ij} \leq \lambda_j \leq \bar{\lambda}_{ij}\}$; then one can obtain $\mathcal{S}_k^i = \mathcal{S}_{k1}^i \cup \mathcal{S}_{k2}^i$. If $\mathcal{S}_k^i \neq \emptyset$, it can be described as

$$\mathcal{S}_k^i = \{\mathcal{K}_1^i, \mathcal{K}_2^i, \dots, \mathcal{K}_{m_k^i}^i\}. \quad (6)$$

Similarly, if $\mathcal{S}_{uk}^i \neq \emptyset$, let us denote that

$$\mathcal{S}_{uk}^i = \{\mathbb{K}_1^i, \mathbb{K}_2^i, \dots, \mathbb{K}_{m_{uk}^i}^i\}, \quad (7)$$

where $\mathcal{K}_{l_1}^i$ denotes the l_1 th element in $\mathcal{S}_k^i$ with the index $\mathcal{K}_{l_1}^i$ in the i th row of the matrix Λ . $\mathbb{K}_{l_2}^i$ denotes the l_2 th completely unknown element with the index $\mathbb{K}_{l_2}^i$ in the i th row of the matrix Λ . m_k^i and m_{uk}^i represent the number of elements in $\mathcal{S}_k^i$ and $\mathcal{S}_{uk}^i$, respectively. For example, considering the transition rates matrix (4), one can easily check that $\mathcal{S}_k^4 = \{\mathcal{K}_1^4, \mathcal{K}_2^4, \mathcal{K}_3^4\}$ with $\mathcal{K}_1^4 = 1, \mathcal{K}_2^4 = 3, \mathcal{K}_3^4 = 4, \mathcal{S}_{uk}^4 = \{\mathbb{K}_1^4\}$ with $\mathbb{K}_1^4 = 2, m_k^4 = 3$, and $m_{uk}^4 = 1$.

Remark 1. When the lower and upper bounds of the elements in $\mathcal{S}_{k2}^i$ are equal, the transition rates matrix is reduced to the considered case in Zhang et al. [7]. It is obvious that the solving method there can only treat $j \in \mathcal{S}_{k2}^i$ as the completely unknown case which can result in some conservativeness.

For convenience, the notations $A_i = A(r_i)$, $B_i = B(r_i)$, $\Delta A_i = \Delta A(r_i)$, and $\Delta B_i = \Delta B(r_i)$ are used for each possible value $r_i = i, i \in \mathcal{S}$, where A_i and B_i are known constant matrices with appropriate dimensions. Then, the system (1) can be described by

$$\begin{aligned} \dot{x}(t) &= (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)q(u(t)), \\ x_0 &= x(0), \quad r_0 = r(0). \end{aligned} \quad (8)$$

The following assumptions are assumed to be valid.

Assumption 1. The pair (A_i, B_i) is controllable.

Assumption 2. Consider the following: $\Delta A_i = D_i E_i(t) F_i$, $\Delta B_i = B_i M_i \Xi_i(t) N_i$, and $|M_i \Xi_i(t) N_i|_\infty \leq \psi_i$, where D_i, F_i, M_i , and N_i are known constant matrices with appropriate dimensions, $E_i(t)$ and $\Xi_i(t)$ are time-varying uncertain matrices satisfying $E_i E_i^T \leq I$ and $\Xi_i \Xi_i^T \leq I$, and the parameter ψ_i satisfies $0 \leq \psi_i < 1$.

In addition, the quantizer $q(\cdot)$ is defined by an operator function $\text{round}(\cdot)$ that rounds towards the nearest integer; that is,

$$q(u(t)) = \mu \cdot \text{round}\left(\frac{u(t)}{\mu}\right), \quad (9)$$

where $\mu(>0)$ is called a quantizing level of the quantizer. In computer-based control systems, the value of μ depends on the sampling accuracy and is known *a priori*. $q(\cdot)$ is the uniform quantizer with the fixed level μ . Define $e_\mu = q(u(t)) - u(t)$, since each component of e_μ is bounded by the half of the quantizing level μ ; thus, we have $|e_\mu|_\infty \leq \mu/2$.

The objective of this paper is to design the state-feedback control law

$$u(t) = K_i x + u_{ic}, \quad K_i = K(r_i), \text{ when } r_t = i, \quad (10)$$

such that the resulting closed-loop system is stochastically stable and obtains as small value of the H_2 cost index for the Markovian jump linear uncertain systems given in the following as possible

$$\begin{aligned} \mathcal{J}(t) &= \mathbb{E} \left[\int_t^\infty \{x^T(\tau) Q(r_\tau) x(\tau) \right. \\ &\quad \left. + x^T(\tau) K^T(r_\tau) R(r_\tau) K(r_\tau) x(\tau)\} d\tau \mid r_0 \right], \end{aligned} \quad (11)$$

where $Q(r_t)$ and $R(r_t)$ are positive definite matrices. The non-linear part of the controller u_{ic} is designed against the effect of signal quantization, and the linear part $K_i x$ is proposed to deal with model uncertainties and to unknown transition rates and achieve optimal H_2 performance.

Remark 2. When $Q(r_t) = Q > 0$, $R(r_t) = R > 0$, and $K(r_t) = K$, the above H_2 cost index is reduced to the following, (12) which is given in Yun et al. [17]:

$$\mathcal{J}(t) = \int_t^\infty \{x^T(\tau) Q x(\tau) + x^T(\tau) K^T R K x(\tau)\} d\tau. \quad (12)$$

Some useful lemmas are firstly presented before formulating the main result.

Lemma 3 (see [21]). *Given a symmetric matrix Π and matrices M and N with appropriate dimensions, then $\Pi + MF(t)N + N^T F^T(t)M^T < 0$ for all $F(t)$ satisfying $F^T(t)F(t) \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that the following inequality holds:*

$$\Pi + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0. \quad (13)$$

Lemma 4. *For any given $\lambda_j \geq 0$ and matrices $P_j > 0$ ($P_j \in R^{n \times n}$, $1 \leq j \leq \mathbb{N}$), if there exists $Z_i \geq P_j$, then the following inequality holds:*

$$\sum_{j=1}^{\mathbb{N}} \lambda_j P_j \leq \sum_{j=1}^{\mathbb{N}} \lambda_j Z_i. \quad (14)$$

Proof. According to $Z_i \geq P_j$ and $\lambda_j \geq 0$, one can obtain that $\lambda_j Z_i \geq \lambda_j P_j$, which further imply that inequality (14) holds. $\square$

Remark 5. In Shen and Yang [9], the inequality $\sum_{j=1}^{\mathbb{N}} \lambda_j P_j \leq (\sum_{j=1}^{\mathbb{N}} \lambda_j)(\sum_{j=1}^{\mathbb{N}} P_j)$ is introduced to obtain design conditions. It is clear that the utilization of Lemma 4 will result in less conservativeness since $\max\{P_1, P_2, \dots, P_{\mathbb{N}}\} < \sum_{j=1}^{\mathbb{N}} P_j$ for any $P_i > 0$.

Lemma 6. *For $\alpha, \beta \in R^n$, $p \geq 1$, and $q \geq 1$, the following inequality holds:*

$$|\alpha^T \beta| \leq |\alpha|_p |\beta|_q, \quad p^{-1} + q^{-1} = 1. \quad (15)$$

Lemma 7 (see [22]). *For the symmetric and positive definite matrices P and Q , if $P \geq Q > 0$, then $Q^{-1} \geq P^{-1} > 0$.*

3. Main Results

Theorem 8. For the system (1) subject to Assumptions 1 and 2, suppose that there exist the symmetric positive definite matrices X_i , general matrices Y_i , and positive scalars ε_i , δ_i , and γ such that

$$\begin{bmatrix} \Gamma_{11} & X_i F_i^T & Y_i^T N_i^T & \Gamma_{14} & \Gamma_{15} & X_i & Y_i \\ * & -\varepsilon_i I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\delta_i I & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_{44} & 0 & 0 & 0 \\ * & * & * & * & -Z_i & 0 & 0 \\ * & * & * & * & * & -Q_i^{-1} & 0 \\ * & * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0, \quad (16)$$

$$Z_i \leq X_j, \quad j \in \mathcal{S}_{uk}^i,$$

$$\begin{bmatrix} \Phi_{11} & X_i F_i^T & Y_i^T N_i^T & \Phi_{14} & X_i & Y_i^T \\ * & -\varepsilon_i I & 0 & 0 & 0 & 0 \\ * & * & -\delta_i I & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & -Q_i^{-1} & 0 \\ * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0, \quad (17)$$

$$X_i \leq X_j, \quad j \in \mathcal{S}_{uk}^i,$$

$$\begin{bmatrix} \gamma & x_0^T \\ x_0 & X_i \end{bmatrix} > 0, \quad (18)$$

where

$$\begin{aligned} \Gamma_{11} &= He(A_i X_i) + He(B_i Y_i) + \varepsilon_i D_i D_i^T + \delta_i B_i M_i M_i^T B_i^T \\ &\quad + \bar{\lambda}_{ii} X_i, \\ \Gamma_{14} &= \left[\sqrt{\bar{\lambda}_{i\mathcal{K}_1^i}} X_i, \sqrt{\bar{\lambda}_{i\mathcal{K}_2^i}} X_i, \dots, \sqrt{\bar{\lambda}_{i\mathcal{K}_{m_k^i}^i}} X_i \right], \\ \Gamma_{44} &= \text{diag} \left\{ -X_{\mathcal{K}_1^i}, -X_{\mathcal{K}_2^i}, \dots, -X_{\mathcal{K}_{m_k^i}^i} \right\}, \\ \Gamma_{15} &= \sqrt{-\sum_{j \in \mathcal{S}_k^i} \bar{\lambda}_{ij}} X_i, \\ \Phi_{11} &= He(A_i X_i) + He(B_i Y_i) + \varepsilon_i D_i D_i^T + \delta_i B_i M_i M_i^T B_i^T \\ &\quad - \sum_{j \in \mathcal{S}_k^i} \bar{\lambda}_{ij} X_i, \\ \Phi_{14} &= \left[\sqrt{\bar{\lambda}_{i\mathcal{K}_1^i}} X_i, \sqrt{\bar{\lambda}_{i\mathcal{K}_2^i}} X_i, \dots, \sqrt{\bar{\lambda}_{i\mathcal{K}_{m_k^i}^i}} X_i \right], \\ \Phi_{44} &= \text{diag} \left\{ -X_{\mathcal{K}_1^i}, -X_{\mathcal{K}_2^i}, \dots, -X_{\mathcal{K}_{m_k^i}^i} \right\}. \end{aligned} \quad (19)$$

Then, the controller designed as

$$u(t) = K_i x(t) + u_{ic}(t) \quad (20)$$

can drive the state trajectory to the origin asymptotically and can obtain the H_2 cost by the minimum of r , where $K_i = Y_i X_i^{-1}$, $P_i = X_i^{-1}$, and $u_{ic} = -(1 + \psi)\mu/2(1 - \psi) \text{sign}(x^T P_i B_i)$.

Proof. Take the Lyapunov function candidate $V = x^T P_i x$; then, along the system trajectory of plant (1), the weak infinitesimal operator $\mathfrak{S}_a^x[\cdot]$ of the process $\{x(t), r_t, t \geq 0\}$ for plant (8) at the point $\{t, x, i\}$ is given by Kushner [23] as follows:

$$\begin{aligned} \mathfrak{S}_a^x[V] &= \dot{x}^T P_i x + x^T P_i \dot{x} + x^T \sum_{j=1}^N \lambda_{ij} P_j x \\ &= \left[(A_i + \Delta A_i) x + (B_i + \Delta B_i) (K_i x + u_{ic} + e_\mu) \right]^T P_i x \\ &\quad + x^T P_i \left[(A_i + \Delta A_i) x + (B_i + \Delta B_i) (K_i x + u_{ic} + e_\mu) \right] \\ &\quad + x^T \sum_{j=1}^N \lambda_{ij} P_j x \\ &= x^T \left[(A_i + \Delta A_i)^T P_i + P_i (A_i + \Delta A_i) \right. \\ &\quad \left. + P_i (B_i + \Delta B_i) K_i + K_i^T (B_i + \Delta B_i)^T P_i \right] x \\ &\quad + 2x^T P_i (B_i + \Delta B_i) (u_{ic} + e_\mu) + x^T \sum_{j=1}^N \lambda_{ij} P_j x. \end{aligned} \quad (21)$$

According to Assumption 2, u_{ic} in Theorem 8, and Lemma 7, one can obtain that

$$\begin{aligned} &2x^T P_i (B_i + \Delta B_i) (u_{ic} + e_\mu) \\ &= 2x^T P_i B_i u_{ic} + 2x^T P_i \Delta B_i (u_{ic} + e_\mu) + 2x^T P_i B_i e_\mu \\ &= 2x^T P_i B_i u_{ic} + 2x^T P_i B_i M_i \Xi_i(t) N_i (u_{ic} + e_\mu) \\ &\quad + 2x^T P_i B_i e_\mu \\ &\leq 2x^T P_i B_i u_{ic} + 2|x^T P_i B_i|_1 |M_i \Xi_i(t) N_i|_\infty \left(|u_{ic}|_\infty + \frac{\mu}{2} \right) \\ &\quad + 2|x^T P_i B_i|_1 \frac{\mu}{2} \\ &= 2x^T P_i B_i u_{ic} + 2|x^T P_i B_i|_1 \psi \left(|u_{ic}|_\infty + \frac{\mu}{2} \right) \\ &\quad + |x^T P_i B_i|_1 \mu \\ &= 2x^T P_i B_i u_{ic} + (1 + \psi) |x^T P_i B_i|_1 \mu \\ &\quad + 2\psi |x^T P_i B_i|_1 |u_{ic}|_\infty \\ &= -\frac{(1 + \psi)\mu}{1 - \psi} |x^T P_i B_i|_1 + (1 + \psi) |x^T P_i B_i|_1 \mu \\ &\quad + \frac{\psi(1 + \psi)\mu}{1 - \psi} |x^T P_i B_i|_1 = 0. \end{aligned} \quad (22)$$

It follows from (21) and (22) that

$$\begin{aligned} \mathfrak{S}_a^x[V] \leq x^T & \left[(A_i + \Delta A_i)^T P_i + P_i (A_i + \Delta A_i) \right. \\ & \left. + P_i (B_i + \Delta B_i) K_i + K_i^T (B_i + \Delta B_i)^T P_i \right] x \\ & + x^T \sum_{j=1}^N \lambda_{ij} P_j x. \end{aligned} \quad (23)$$

Furthermore, consider the following inequality:

$$\begin{aligned} & (A_i + \Delta A_i)^T P_i + P_i (A_i + \Delta A_i) + P_i (B_i + \Delta B_i) K_i \\ & + K_i^T (B_i + \Delta B_i)^T P_i + \sum_{j=1}^N \lambda_{ij} P_j + Q_i + K_i^T R_i K_i < 0. \end{aligned} \quad (24)$$

Since $\Delta B_i = B_i M_i \Xi_i(t) N_i$ and $\Delta A_i = D_i E_i(t) F_i$, the above inequality can be rewritten as

$$\begin{aligned} & A_i^T P_i + P_i A_i + P_i D_i E_i(t) F_i + F_i^T E_i^T(t) D_i^T P_i + P_i B_i K_i \\ & + K_i^T B_i^T P_i + P_i B_i M_i \Xi_i(t) N_i K_i \\ & + K_i^T N_i^T \Xi_i^T(t) M_i^T B_i^T P_i + \sum_{j=1}^N \lambda_{ij} P_j + Q_i + K_i^T R_i K_i < 0. \end{aligned} \quad (25)$$

Using Lemma 3, it is equivalent to

$$\begin{aligned} & A_i^T P_i + P_i A_i + \varepsilon_i P_i D_i D_i^T P_i + \frac{1}{\varepsilon_i} F_i^T F_i + P_i B_i K_i + K_i^T B_i^T P_i \\ & + \delta_i P_i B_i M_i M_i^T B_i^T P_i + \frac{1}{\delta_i} K_i^T N_i^T N_i K_i \\ & + \sum_{j=1}^N \lambda_{ij} P_j + Q_i + K_i^T R_i K_i < 0. \end{aligned} \quad (26)$$

That is,

$$\begin{aligned} & He(A_i^T P_i) + He(P_i B_i K_i) + \varepsilon_i P_i D_i D_i^T P_i + \frac{1}{\varepsilon_i} F_i^T F_i \\ & + \delta_i P_i B_i M_i M_i^T B_i^T P_i + \frac{1}{\delta_i} K_i^T N_i^T N_i K_i + \sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} P_j \\ & + \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} P_j + \lambda_{ii} P_i + Q_i + K_i^T R_i K_i < 0. \end{aligned} \quad (27)$$

Two cases will be considered.

Case 1 ($\lambda_{ii} \in \mathcal{S}_k^i$). In this case, using Lemma 4, one can see that

$$\begin{aligned} & He(A_i^T P_i) + He(P_i B_i K_i) + \varepsilon_i P_i D_i D_i^T P_i \\ & + \frac{1}{\varepsilon_i} F_i^T F_i + \delta_i P_i B_i M_i M_i^T B_i^T P_i + \frac{1}{\delta_i} K_i^T N_i^T N_i K_i \\ & + \sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} P_j + \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} W_i + \lambda_{ii} P_i \\ & + Q_i + K_i^T R_i K_i < 0, \end{aligned} \quad (28)$$

$$P_j \leq W_i, \quad j \in \mathcal{S}_{uk}^i.$$

Substituting $\sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} = -\sum_{j \in \mathcal{S}_k^i, j \neq i} \lambda_{ij} - \lambda_{ii}$ into the first inequality in (22) and using the boundary information of the elements of the transition rates matrix, one can achieve that

$$\begin{aligned} & He(A_i^T P_i) + He(P_i B_i K_i) + \varepsilon_i P_i D_i D_i^T P_i \\ & + \frac{1}{\varepsilon_i} F_i^T F_i + \delta_i P_i B_i M_i M_i^T B_i^T P_i + \frac{1}{\delta_i} K_i^T N_i^T N_i K_i \\ & + \sum_{j \in \mathcal{S}_k^i, j \neq i} \bar{\lambda}_{ij} P_j - \left(\sum_{j \in \mathcal{S}_k^i, j \neq i} \underline{\lambda}_{ij} + \lambda_{ii} \right) W_i \\ & + \bar{\lambda}_{ii} P_i + Q_i + K_i^T R_i K_i < 0, \end{aligned} \quad (29)$$

$$P_j \leq W_i, \quad j \in \mathcal{S}_{uk}^i.$$

Case 2 ($\lambda_{ii} \in \mathcal{S}_{uk}^i$; namely, it is completely unknown). In such case, let us take $\lambda_{ii} = -\sum_{j \in \mathcal{S}_k^i} \lambda_{ij} - \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij}$ into (21); then, one can see that

$$\begin{aligned} & A_i^T P_i + P_i A_i + \varepsilon_i P_i D_i D_i^T P_i + \frac{1}{\varepsilon_i} F_i^T F_i + P_i B_i K_i \\ & + K_i^T B_i^T P_i + \delta_i P_i B_i M_i M_i^T B_i^T P_i \\ & + \frac{1}{\delta_i} K_i^T N_i^T N_i K_i + \sum_{j \in \mathcal{S}_k^i} \lambda_{ij} P_j + \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} P_j \\ & + \left(-\sum_{j \in \mathcal{S}_k^i} \lambda_{ij} - \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} \right) P_i + Q_i + K_i^T R_i K_i < 0. \end{aligned} \quad (30)$$

Then, the stochastic stability can be guaranteed when

$$\begin{aligned} & He(A_i^T P_i) + He(P_i B_i K_i) + \varepsilon_i P_i D_i D_i^T P_i + \frac{1}{\varepsilon_i} F_i^T F_i \\ & + \delta_i P_i B_i M_i M_i^T B_i^T P_i + \frac{1}{\delta_i} K_i^T N_i^T N_i K_i + \sum_{j \in \mathcal{S}_k^i} \bar{\lambda}_{ij} P_j \\ & - \sum_{j \in \mathcal{S}_k^i, j \neq i} \underline{\lambda}_{ij} P_j + Q_i + K_i^T R_i K_i < 0, \end{aligned} \quad (31)$$

$$P_j \leq P_i, \quad j \in \mathcal{S}_{uk}^i.$$

For Case 1, pre- and postmultiplying P_i^{-1} in the first inequality in (27) and using Lemma 7 in the second inequality in (21), one can get that

$$\begin{aligned} & He(P_i^{-1}A_i^T) + He(B_iK_iP_i^{-1}) + \varepsilon_i D_i D_i^T + \frac{1}{\varepsilon_i} P_i^{-1} F_i^T F_i P_i^{-1} \\ & + \delta_i B_i M_i M_i^T B_i^T + \frac{1}{\delta_i} P_i^{-1} K_i^T N_i^T N_i K_i P_i^{-1} \\ & + \sum_{j \in \mathcal{S}_k^i, j \neq i} \bar{\lambda}_{ij} P_i^{-1} P_j P_i^{-1} - \sum_{j \in \mathcal{S}_k^i} \underline{\lambda}_{ij} P_i^{-1} W_i P_i^{-1} \\ & + \bar{\lambda}_{ii} P_i^{-1} + P_i^{-1} Q_i P_i^{-1} + P_i^{-1} K_i^T R_i K_i P_i^{-1} < 0, \\ & W_i^{-1} \leq P_j^{-1}, \quad j \in \mathcal{S}_{uk}^i. \end{aligned} \quad (32)$$

Let $X_i = P_i^{-1}$, $Y_i = K_i P_i^{-1}$, and $Z_i = W_i^{-1}$; then, we have

$$\begin{aligned} & He(A_i X_i) + He(B_i Y_i) + \varepsilon_i D_i D_i^T + \frac{1}{\varepsilon_i} X_i F_i^T F_i X_i \\ & + \delta_i B_i M_i M_i^T B_i^T + \frac{1}{\delta_i} Y_i^T N_i^T N_i Y_i + \sum_{j \in \mathcal{S}_k^i, j \neq i} \bar{\lambda}_{ij} X_i X_j^{-1} X_i \\ & - \sum_{j \in \mathcal{S}_k^i} \underline{\lambda}_{ij} X_i Z_i^{-1} X_i + \bar{\lambda}_{ii} X_i + X_i Q_i X_i + Y_i^T R_i Y_i < 0, \\ & Z_i \leq X_j, \quad j \in \mathcal{S}_{uk}^i. \end{aligned} \quad (33)$$

Applying the Schur complement formula, one can get (16).

For Case 2, pre- and postmultiplying P_i^{-1} in the first inequality in (28) and applying Lemma 7 to the second inequality in (28), one can obtain that

$$\begin{aligned} & He(P_i^{-1}A_i^T) + He(B_iK_iP_i^{-1}) + \varepsilon_i D_i D_i^T + \frac{1}{\varepsilon_i} P_i^{-1} F_i^T F_i P_i^{-1} \\ & + \delta_i B_i M_i M_i^T B_i^T + \frac{1}{\delta_i} P_i^{-1} K_i^T N_i^T N_i K_i P_i^{-1} \\ & + \sum_{j \in \mathcal{S}_k^i} \bar{\lambda}_{ij} P_i^{-1} P_j P_i^{-1} - \sum_{j \in \mathcal{S}_k^i} \underline{\lambda}_{ij} P_i^{-1} + P_i^{-1} Q_i P_i^{-1} \\ & + P_i^{-1} K_i^T R_i K_i P_i^{-1} < 0, \\ & P_i^{-1} \leq P_j^{-1}, \quad j \in \mathcal{S}_{uk}^i. \end{aligned} \quad (34)$$

Let $X_i = P_i^{-1}$ and $Y_i = K_i P_i^{-1}$; then, one can see that

$$\begin{aligned} & He(A_i X_i) + He(B_i Y_i) + \varepsilon_i D_i D_i^T + \frac{1}{\varepsilon_i} X_i F_i^T F_i X_i \\ & + \delta_i B_i M_i M_i^T B_i^T + \frac{1}{\delta_i} Y_i^T N_i^T N_i Y_i \\ & + \sum_{j \in \mathcal{S}_k^i} \bar{\lambda}_{ij} X_i X_j^{-1} X_i - \sum_{j \in \mathcal{S}_k^i} \underline{\lambda}_{ij} X_i \\ & + X_i Q_i X_i + Y_i^T R_i Y_i < 0, \\ & X_i \leq X_j, \quad j \in \mathcal{S}_{uk}^i. \end{aligned} \quad (35)$$

Thus, the LMIs in (17) are derived by using the Schur complement formula.

From the above proof, one can see that

$$\mathfrak{F}_a^x[V] < -x^T Q_i x - x^T K_i^T R_i K_i x. \quad (36)$$

It follows from Kushner [23] that

$$\begin{aligned} & \mathbb{E}[V(t, x, i) | r_0] - V(x_0, r_0) \\ & \leq -\mathbb{E}\left[\int_0^t \{x^T(\tau) Q_i x(\tau) - x^T(\tau) K_i^T R_i K_i x(\tau)\} d\tau\right]. \end{aligned} \quad (37)$$

Since $\mathbb{E}[V(t, x, i) | r_0] \geq 0$, by some simple calculation, one can achieve that

$$\mathcal{J}(t) \leq \mathcal{J}(0) \leq V(x_0, r_0) = x_0^T P_i x_0 \leq \gamma. \quad (38)$$

Therefore, the minimum cost can be obtained by minimizing γ . Thus, the proof is achieved. $\square$

Remark 9. The merit of the proposed results lies in that the transition rates of the jumping process are assumed to be more general, which means that some elements in the transition rates matrix have been exactly known, some ones have been merely known with lower and upper bounds, and others may have no information to use. Dealing with the unknown transition rates, a less conservative method is used. At the same time, the impact of the input signal quantization on the system is also considered. Finally, the controller design conditions are presented in the framework of LMIs.

In order to make comparison with the design method using the LMIs technique in Shen and Yang [9], we present the conditions designed by the utilization of Lemma 2 there.

Proposition 10. For the system (1) subject to Assumptions 1 and 2, suppose that there exist symmetric positive definite matrices X_p , general matrices Y_p , and positive scalars ε_p , δ_p , and γ such that

$$\begin{bmatrix} \Gamma_{11} & X_i F_i^T & Y_i^T N_i^T & \Gamma_{14} & \Gamma_{15} & X_i & Y_i \\ * & -\varepsilon_i I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\delta_i I & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_{44} & 0 & 0 & 0 \\ * & * & * & * & -\Gamma_{55} & 0 & 0 \\ * & * & * & * & * & -Q_i^{-1} & 0 \\ * & * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0, \quad (39)$$

$$Z_i \leq X_j, \quad j \in \mathcal{S}_{uk}^i,$$

$$\begin{bmatrix} \Phi_{11} & X_i F_i^T & Y_i^T N_i^T & \Phi_{14} & X_i & Y_i^T \\ * & -\varepsilon_i I & 0 & 0 & 0 & 0 \\ * & * & -\delta_i I & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & -Q_i^{-1} & 0 \\ * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0, \quad (40)$$

$$X_i \leq X_j, \quad j \in \mathcal{S}_{uk}^i,$$

$$\begin{bmatrix} \gamma & x_0^T \\ x_0 & X_i \end{bmatrix} > 0, \quad (41)$$

where

$$\begin{aligned}
\Gamma_{11} &= He(A_i X_i) + He(B_i Y_i) + \varepsilon_i D_i D_i^T + \delta_i B_i M_i M_i^T B_i^T \\
&\quad + \bar{\lambda}_{ii} X_i, \\
\Gamma_{14} &= \left[\sqrt{\bar{\lambda}_{i\mathcal{K}_1^i}} X_i, \sqrt{\bar{\lambda}_{i\mathcal{K}_2^i}} X_i, \dots, \sqrt{\bar{\lambda}_{i\mathcal{K}_{m_k^i}^i}} X_i \right], \\
\Gamma_{44} &= \text{diag} \left\{ -X_{\mathcal{K}_1^i}, -X_{\mathcal{K}_2^i}, \dots, -X_{\mathcal{K}_{m_k^i}^i} \right\}, \\
\Gamma_{15} &= \left[\sqrt{-\sum_{j \in \mathcal{S}_k^i} \underline{\lambda}_{ij}} X_i, \sqrt{-\sum_{j \in \mathcal{S}_k^i} \underline{\lambda}_{ij}} X_i, \dots, \sqrt{-\sum_{j \in \mathcal{S}_k^i} \underline{\lambda}_{ij}} X_i \right], \\
\Gamma_{55} &= \text{diag} \left\{ -X_{\mathbb{K}_1^i}, -X_{\mathbb{K}_2^i}, \dots, -X_{\mathbb{K}_{m_k^i}^i} \right\}, \\
\Phi_{11} &= He(A_i X_i) + He(B_i Y_i) + \varepsilon_i D_i D_i^T + \delta_i B_i M_i M_i^T B_i^T \\
&\quad - \sum_{j \in \mathcal{S}_k^i} \underline{\lambda}_{ij} X_i, \\
\Phi_{14} &= \left[\sqrt{\bar{\lambda}_{i\mathcal{K}_1^i}} X_i, \sqrt{\bar{\lambda}_{i\mathcal{K}_2^i}} X_i, \dots, \sqrt{\bar{\lambda}_{i\mathcal{K}_{m_k^i}^i}} X_i \right], \\
\Phi_{44} &= \text{diag} \left\{ -X_{\mathcal{K}_1^i}, -X_{\mathcal{K}_2^i}, \dots, -X_{\mathcal{K}_{m_k^i}^i} \right\}.
\end{aligned} \tag{42}$$

Then, the controller designed as

$$u(t) = K_i x(t) + u_{ic}(t) \tag{43}$$

can drive the state trajectory to the origin asymptotically and can obtain the H_2 cost by minimizing r , where $K_i = Y_i X_i^{-1}$, $P_i = X_i^{-1}$, and $u_{ic} = -(1 + \psi)\mu/2(1 - \psi) \text{sign}(x^T P_i B_i)$.

Proof. The proof process is similar to that in Theorem 8 but with replacing $P_j \leq W_i$, $j \in \mathcal{S}_{uk}^i$, and $\sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij} W_j$ with $(\sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \lambda_{ij})(\sum_{j \in \mathcal{S}_{uk}^i, j \neq i} P_j)$ in (22). The details are omitted here for space limitation. $\square$

4. Numerical Example

An example is presented to illustrate the effectiveness of the proposed method.

Consider the MJLSs with four operation modes as follows:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 1 & 0.2 \\ 0.4 & -4 \end{bmatrix}, & A_2 &= \begin{bmatrix} -1.5 & 1.2 \\ 1.4 & -5 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix}, & A_4 &= \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \\
B_3 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & B_4 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\end{aligned}$$

TABLE 1: Comparison of optimal γ .

Theorem 8	Proposition 10
1.4518	2.4567

$$\begin{aligned}
D_1 &= \begin{bmatrix} 0 \\ 2 \end{bmatrix}, & D_2 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
D_3 &= \begin{bmatrix} 0.4 \\ 0.8 \end{bmatrix}, & D_4 &= \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}, \\
F_1 &= [1 \ 0], & F_2 &= [0.5 \ 1], \\
F_3 &= [0 \ 0.3], & F_4 &= [0.5 \ 0.8], \\
M_1 &= 0.4, & M_2 &= 0.6, & M_3 &= 0.5, & M_4 &= 1, \\
N_1 &= 0.8, & N_2 &= 0.2, & N_3 &= 0.1, & N_4 &= 0.5.
\end{aligned} \tag{44}$$

The considered transition rates matrix is given as follows:

$$\Lambda = \begin{bmatrix} -1.3 & 0.2 & ? & ? \\ ? & \alpha_1 & 0.3 & 0.3 \\ 0.6 & ? & ? & ? \\ 0.4 & \alpha_2 & ? & \alpha_3 \end{bmatrix}, \tag{45}$$

where $-1.2 \leq \alpha_1 \leq -0.8$, $0.6 \leq \alpha_2 \leq 0.8$, and $-2 \leq \alpha_3 \leq -1.5$. Choosing $x_0 = [1; -0.5]$ and solving (16)–(18) in Theorem 8 and (34)–(38) in Proposition 10, one can obtain the optimal H_2 performance indices shown in Table 1. It can be seen that less conservativeness is obtained by the proposed method in Theorem 8.

For simulation, one can obtain the controller gains by solving the LMIs in Theorem 8 as follows:

$K_1 = [-5.2949 \ -0.5848]$, $K_2 = [-2.4332 \ -1.6826]$, $K_3 = [-4.0556 \ -4.6580]$, and $K_4 = [-14.2188 \ -12.4624]$ and by considering the positive definite matrices

$$\begin{aligned}
P_1 &= \begin{bmatrix} 1.5153 & 0.1052 \\ 0.1052 & 0.1670 \end{bmatrix}, & P_2 &= \begin{bmatrix} 1.1640 & 0.5337 \\ 0.5337 & 0.4595 \end{bmatrix}, \\
P_3 &= \begin{bmatrix} 0.7788 & 0.4269 \\ 0.4269 & 0.4903 \end{bmatrix}, & P_4 &= \begin{bmatrix} 3.6726 & 2.8438 \\ 2.8438 & 2.4925 \end{bmatrix}.
\end{aligned} \tag{46}$$

The following parameters are used in the simulation: $\mu = 0.3$, $E_1(t) = 0.2 \cos(t)$, $E_2(t) = 0.05 \sin(t)$, $E_3(t) = 0.8 \cos(3t)$, $E_4(t) = 0.4 \sin(t)$, $\Xi_1(t) = 0.5 \cos(3t)$, $\Xi_2(t) = 0.6 \cos(2t)$, $\Xi_3(t) = 0.2 \sin(t)$, and $\Xi_4(t) = 0.05 \cos(2t)$. In order to reduce the chattering effects induced by u_{ic} , $\text{sign}(x^T P_i B_i)$ is substituted by $\text{sat}(x^T P_i B_i / \delta_i)$, where $\delta_1 = 0.0015$, $\delta_2 = 0.0004$, $\delta_3 = 0.0015$, and $\delta_4 = 0.0004$.

The switching mode, the control input, and the response curves of the system states are presented in Figures 1, 2, and 3, respectively. Among them, Figure 1 shows a possible system modes evolution which meets the transition rates given in this example. As shown in Figure 1, the system has 4 modes and is in different modes at a different time. Figure 2 shows the curve of the control input $u(t)$. With this controller, Figure 3 depicts the state response curves of the closed-loop system.

FIGURE 1: Evolution of the system mode.

FIGURE 2: The curve of the control input $u(t)$.

It can be seen that the considered continuous-time Markovian jump linear uncertain system is stochastically stable in spite of mismatched uncertainty, the input signal quantization, and the partly known transition rates covering the completely known, the boundary unknown, and the completely unknown elements in the transition rates matrix.

5. Conclusions

H_2 control design for a class of the continuous-time Markovian jump linear uncertain systems with partly unknown transition rates and input quantization has been investigated. The H_2 cost index for the Markovian jump systems is first introduced, and then sufficient conditions of the H_2 control for the Markovian jump linear uncertain systems with unknown transition rates have been presented. The controller is constructed by two parts. The nonlinear part is proposed to eliminate the effect of input quantization. The linear part is designed by solving the LMIs conditions for achieving the H_2

FIGURE 3: The response curve of the state x .

performance against model uncertainties and unknown transition rates. In comparison with the existing result in the literature, less conservativeness has been obtained by introducing new relaxed inequality conditions. Finally, a numerical example is given to show the effectiveness of the proposed design method.

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References

- [1] E.-K. Boukas, *Stochastic Switching Systems: Analysis and Design*, Control Engineering, Birkhäuser, Boston, Mass, USA, 2006.
- [2] S. Ma and E.-K. Boukas, "A singular system approach to robust sliding mode control for uncertain Markov jump systems," *Automatica*, vol. 45, no. 11, pp. 2707–2713, 2009.
- [3] P. Shi, Y. Xia, G. P. Liu, and D. Rees, "On designing of sliding-mode control for stochastic jump systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 97–103, 2006.
- [4] L. Wu, P. Shi, and H. Gao, "State estimation and sliding-mode control of Markovian jump singular systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 5, pp. 1213–1219, 2010.
- [5] W. Li, H. Su, D. Wei, and K. Wang, "Global stability of coupled nonlinear systems with Markovian switching," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 6, pp. 2609–2616, 2012.

- [6] E. F. Costa and J. B. R. do Val, "On the observability and detectability of continuous-time Markov jump linear systems," *SIAM Journal on Control and Optimization*, vol. 41, no. 4, pp. 1295–1314, 2002.
- [7] L. Zhang, E.-K. Boukas, and J. Lam, "Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities," *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2458–2464, 2008.
- [8] L. Zhang and E.-K. Boukas, " H_∞ control of a class of extended Markov jump linear systems," *IET Control Theory & Applications*, vol. 3, no. 7, pp. 834–842, 2009.
- [9] M. Shen and G.-H. Yang, " H_2 state feedback controller design for continuous Markov jump linear systems with partly known information," *International Journal of Systems Science*, vol. 43, no. 4, pp. 786–796, 2012.
- [10] R. W. Brockett and D. Liberzon, "Quantized feedback stabilization of linear systems," *IEEE Transactions on Automatic Control*, vol. 45, no. 7, pp. 1279–1289, 2000.
- [11] D. Liberzon, "Hybrid feedback stabilization of systems with quantized signals," *Automatica*, vol. 39, no. 9, pp. 1543–1554, 2003.
- [12] B.-C. Zheng and G.-H. Yang, "Quantised feedback stabilisation of planar systems via switching-based sliding-mode control," *IET Control Theory & Applications*, vol. 6, no. 1, pp. 149–156, 2012.
- [13] B.-C. Zheng and G.-H. Yang, "Decentralized sliding mode quantized feedback control for a class of uncertain large-scale systems with dead-zone input," *Nonlinear Dynamics*, vol. 71, no. 3, pp. 417–427, 2013.
- [14] M. Fu and L. Xie, "The sector bound approach to quantized feedback control," *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1698–1711, 2005.
- [15] M. Fu and L. Xie, "Quantized feedback control for linear uncertain systems," *International Journal of Robust and Nonlinear Control*, vol. 20, no. 8, pp. 843–857, 2010.
- [16] E. Tian, D. Yue, and X. Zhao, "Quantised control design for networked control systems," *IET Control Theory and Applications*, vol. 1, no. 6, pp. 1693–1699, 2007.
- [17] S. W. Yun, Y. J. Choi, and P. Park, " $\mathcal{H}_2$ control of continuous-time uncertain linear systems with input quantization and matched disturbances," *Automatica*, vol. 45, no. 10, pp. 2435–2439, 2009.
- [18] W.-W. Che and G.-H. Yang, "Quantised H_∞ filter design for discrete-time systems," *International Journal of Control*, vol. 82, no. 2, pp. 195–206, 2009.
- [19] C. Peng and Y.-C. Tian, "Networked H_∞ control of linear systems with state quantization," *Information Sciences*, vol. 177, no. 24, pp. 5763–5774, 2007.
- [20] N. Xiao, L. Xie, and M. Fu, "Stabilization of Markov jump linear systems using quantized state feedback," *Automatica*, vol. 46, no. 10, pp. 1696–1702, 2010.
- [21] I. R. Petersen, "A stabilization algorithm for a class of uncertain linear systems," *Systems & Control Letters*, vol. 8, no. 4, pp. 351–357, 1987.
- [22] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, Cambridge, UK, 1985.
- [23] H. J. Kushner, *Stochastic Stability and Control*, Mathematics in Science and Engineering, vol. 33, Academic Press, New York, NY, USA, 1967.