Research Article

A Decoupling Control Algorithm for Unwinding Tension System Based on Active Disturbance Rejection Control

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This paper presents a new control methodology based on active disturbance rejection control (ADRC) for designing the tension decoupling controller of the unwinding system in a gravure printing machine. The dynamic coupling can be actively estimated and compensated in real time, which makes feedback control an ideal approach to designing the decoupling controller of the unwinding system. This feature is unique to ADRC. In this study, a nonlinear mathematical model is established according to the working principle of the unwinding system. A decoupling model is also constructed to determine the order and decoupling plant of the unwinding system. Based on the order and decoupling plant, an ADRC decoupling control methodology is designed to enhance the tension stability in the unwinding system. The effectiveness and capability of the proposed methodology are verified through simulation and experiments. The results show that the proposed strategy not only realises a decoupling control for the unwinding system but also has an effective antidisturbance capability and is robust.

1. Introduction

Tension control stability is essential to the printing process of the gravure printing machines because the fluctuation and variation of tension have great influence on the precision of the printing register. The unwinding system is vital for generating web tension in the tension control system of the machine. In the process of unwinding, the diameter and rotational inertia of the unwinding roller are time-variant, which creates an indeterminate tension control system that is nonlinear and strong-coupling. Hence, accurate tension control is essential for ensuring adequate printing performance.

Presently, the control synthesis methodology based on the proportional-integral-derivative (PID) algorithm is still the most common control strategy for the tension control system of the gravure printing machines. However, with the development of high-speed and high-precision printing technologies, a control strategy based solely on PID is insufficient for satisfying control requirements. Many researchers have focused their efforts on improving the traditional PID strategy [1–6]. With the development of modern control theories, a number of modern control methods have been applied to web tension

control. Chung et al. [7] applied fuzzy logic control to tension control. Wang et al. [8] achieved decoupling control between tension and speed using neural networks. Abjadi et al. [9] designed a decoupling controller based on nonlinear sliding-mode control for the tension control system. Pagilla et al. [10, 11] proposed fully decentralized state feedback controller and verified the controller by experimental results. A decentralized adaptive controller with model based feedforward was developed and experimentally verified in [12]. As the web processing plants are known as highly resonant systems with strong coupling between the variables to control, several improvements of multivariable controllers based on robust control have been introduced to address the resonant effects and tension control while maintaining performance [13–20]. Although most of these methods resulted in acceptable control performance, they relied too heavily on the system model. However, in industrial practice, it is very difficult to establish a fully accurate mathematical model. Therefore, it was necessary to find a solution using a small amount of model information that could effectively address the nonlinear, time-varying, strong coupling, and uncertain nature of web tension control. One ideal candidate is the ADRC. The

FIGURE 1: Experimental setup for the four-colour gravure printing machine.

FIGURE 2: Schematic diagram of the unwinding system.

essence of ADRC is that the internal dynamics and external disturbances can be estimated and compensated in real time [21–24]. Due to its strong robustness and antidisturbance capability, ADRC has been successfully applied in many fields [25–28]. An initial evaluation of the application of ADRC for the regulation of web tension was performed with a linear transfer function model of the tension loop [29]. ADRC was then applied to design control methodologies for a truly nonlinear model of the tension dynamics in [30, 31]; acceptable performance was observed. Few researchers have addressed the design of tension decoupling control methodology based on ADRC, particularly, with regard to the unwinding system for gravure printing machines.

The objective of this research is to design a tension decoupling control methodology based on ADRC for the unwinding system in gravure printing machines. First, a nonlinear model is established based on the working principle of unwinding system. Next, a decoupling model is constructed in detail and an ADRC decoupling controller is designed to enhance the tension stability of the unwinding system. Last, to test the effectiveness of the ADRC controller, simulations and experiments with PID and ADRC controllers are carried out.

2. Mathematical Modeling

2.1. Experimental Setup. Figure 1 shows the experimental setup for a four-colour gravure printing machine that consists

of an unwinding system, a four-colour printing system, and a rewinding system. All of the driving shafts are driven by Mitsubishi servo motors. The unwinding and rewinding rollers are driven by HF-SP102, and the other rollers are driven by HF-SE52JW1-S100. Tension signals are measured by passive dancer rolls with tension observer and Mitsubishi tension load cells (LX-030TD). The entire system is controlled by a multiaxis controller (Googoltech T8VME). The experimental web material is biaxially oriented polypropylene (BOPP).

The schematic diagram of the unwinding system in the experimental setup is presented in Figure 2. To achieve constant tension control, the unwinding system consists primarily of an unwinding component driven by M_1 and a traction component driven by M_2 . A passive dancer roll is used in the unwinding component to reduce tension fluctuations and simultaneously measure the tension signal $T_1(t)$ by the position θ and tension observer [32]. To simplify the mathematical model, the upstream and downstream tensions of the dancer roller and the idle rollers are considered equal. In the traction component, the value of $T_2(t)$ is measured by the tension load cell. M_3 is the first printing motor whose main task is to maintain a constant speed.

2.2. System Model. The standard model of the relationship between tension and velocity has been presented in [4, 13, 32–34]. The model is constructed from equations that describe the web tension behaviour of two consecutive rolls and

FIGURE 3: Structure of the ADRC decoupling control methodology.

FIGURE 4: Structure of the PID control methodology.

the velocity of each roll. The relationship between the tension and velocity of the unwinding system can be expressed as

$$\begin{aligned}
 L_i(t) \frac{dT_i(t)}{dt} &= V_i(t) T_{i-1}(t) - V_{i+1}(t) T_i(t) + AE(V_{i+1}(t) - V_i(t)) \\
 &+ (AE - T_i(t)) \frac{dL_i(t)}{dt} \quad i = 1, 2.
 \end{aligned} \quad (1)$$

Equation (1) is a nonlinear tension model that considers variation in web length within a span. As shown in Figure 2, $L_2(t)$ is constant for L_2 . Substituting $V(t) = R(t)\omega(t)$ into (1), we can obtain

$$\begin{aligned}
 L_1(t) \dot{T}_1(t) &= (T_0 - AE) R_1(t) \omega_1(t) \\
 &+ (AE - T_1(t)) R_2 \omega_2(t) \\
 &+ (AE - T_1(t)) \dot{L}_1(t), \\
 L_2 \dot{T}_2(t) &= (T_1(t) - AE) R_2 \omega_2(t) + (AE - T_2(t)) R_3 \omega_3.
 \end{aligned} \quad (2)$$

Assuming that the web does not completely slide on the roll and that the web velocity is equal to roll linear velocity and by applying Newton's law to the rotational movement, (3) can be expressed as

$$\begin{aligned}
 \frac{d}{dt} (J_i(t) \omega_i(t)) &= (T_i(t) - T_{i-1}(t)) R_i(t) \\
 &+ C_i u_i(t) - f \omega_i(t), \quad i = 1, 2.
 \end{aligned} \quad (3)$$

Because $R_1(t)$ and $J_1(t)$ are time-variant parameters for the unwinding roll and decrease with time, (4) can be obtained as follows:

$$\frac{d}{dt} (J_1(t) \omega_1(t)) = \dot{J}_1(t) \omega_1(t) + J_1(t) \dot{\omega}_1(t). \quad (4)$$

In the actual production process of the gravure printing machine, the thickness of the web is very small compared with the radius of the rollers over which the web is wrapped. Because $R_1(t)$ varies gradually over a short period of time, $R_1(t)$ and $J_1(t)$ vary gradually in comparison with the web dynamics. Hence, the differential of $J_1(t)$ can be neglected. As shown in Figure 2, (3) becomes (5) as the following:

$$\begin{aligned}
 J_1(t) \dot{\omega}_1(t) &= (T_1(t) - T_0) R_1(t) + C_1 u_1(t) - f \omega_1(t), \\
 J_2 \dot{\omega}_2(t) &= (T_2(t) - T_1(t)) R_2 + C_2 u_2(t) - f \omega_2(t).
 \end{aligned} \quad (5)$$

As shown in Figure 2, the working process of the dancer roll can cause a change in web length. After analysis, (6) can be obtained as follows:

$$L_1(t) = L_1 - 2L_{D2}\theta(t). \quad (6)$$

FIGURE 5: Step response curves for $\omega_3 = 100$ r/min.

Combining (6) with (2.2), (2.2) can be rewritten as follows:

$$\begin{aligned}
 (L_1 - 2L_{D2}\theta(t))\dot{T}_1(t) &= (T_0 - AE)R_1(t)\omega_1(t) + (AE - T_1(t))R_2\omega_2(t) \\
 &\quad - 2L_{D2}(AE - T_1(t))\dot{\theta}(t), \\
 L_2\dot{T}_2(t) &= (T_1(t) - AE)R_2\omega_2(t) + (AE - T_2(t))R_3\omega_3.
 \end{aligned} \quad (7)$$

According to Newton's laws of motion and eliminating the influence of the weight of the dancer roll, the dynamic equation of the dancer roll is expressed as

$$T_1(t) = \frac{1}{2L_{D2}} (J_{Da}\ddot{\theta}(t) + f_D\dot{\theta}(t) + KL_{D1}^2\theta(t)). \quad (8)$$

Combining (5), (2.2), and (8), the nonlinear mathematical model for the unwinding system is constructed as follows:

$$\begin{aligned}
 J_1(t)\dot{\omega}_1(t) &= (T_1(t) - T_0)R_1(t) + C_1u_1(t) - f\omega_1(t), \\
 J_2\dot{\omega}_2(t) &= (T_2(t) - T_1(t))R_2 + C_2u_2(t) - f\omega_2(t), \\
 2L_{D2}T_1(t) &= J_{Da}\ddot{\theta}(t) + f_D\dot{\theta}(t) + KL_{D1}^2\theta(t), \\
 (L_1 - 2L_{D2}\theta(t))\dot{T}_1(t) &= (T_0 - AE)R_1(t)\omega_1(t) + (AE - T_1(t))R_2\omega_2(t) \\
 &\quad - 2L_{D2}(AE - T_1(t))\dot{\theta}(t), \\
 L_2\dot{T}_2(t) &= (T_1(t) - AE)R_2\omega_2(t) + (AE - T_2(t))R_3\omega_3.
 \end{aligned} \quad (9)$$

The controlled variables $u_1(t)$ and $u_2(t)$ of servo motors M_1 and M_2 , respectively, are the input signals in the unwinding system, and tensions $T_1(t)$ and $T_2(t)$ are the outputs in the unwinding system. Because both $u_1(t)$ and $u_2(t)$ have an

FIGURE 6: Step response curves for $R_1 = 0.2$ m.

impact on $T_1(t)$ and $T_2(t)$, the unwinding system can be characterised as nonlinear, time-varying, multi-input multioutput (MIMO), and coupling.

3. Proposed ADRC Decoupling Controller

3.1. Decoupling Model. According to (2.2) and after calculating the second derivative of $T_1(t)$ and $T_2(t)$, the relationship between the inputs and outputs of the nonlinear model can be represented as follows:

$$\begin{aligned} \ddot{T}_1(t) &= (2L_{D2} [(T_1(t) - AE)\ddot{\theta}(t) + 2\dot{T}_1(t)\dot{\theta}(t)] \\ &\quad - R_2\omega_2(t)\dot{T}_1(t) + (T_0 - AE)\omega_1(t)\dot{R}_1(t)) \\ &\quad \times (L_1 - 2L_{D2}\theta(t))^{-1} \\ &\quad + \frac{R_1(t)(T_0 - AE)[R_1(t)(T_1(t) - T_0) - f\omega_1(t)]}{J_1(t)(L_1 - 2L_{D2}\theta(t))} \end{aligned}$$

$$\begin{aligned} &+ \frac{C_1 R_1(t)(T_0 - AE)}{J_1(t)(L_1 - 2L_{D2}\theta(t))} u_1(t) \\ &+ \frac{R_2(AE - T_1(t))[R_2(T_2(t) - T_1(t)) - f\omega_2(t)]}{J_2(L_1 - 2L_{D2}\theta(t))} \\ &+ \frac{C_2 R_2(AE - T_1(t))}{J_2(L_1 - 2L_{D2}\theta(t))} u_2(t), \\ \ddot{T}_2(t) &= \frac{R_2\omega_2(t)\dot{T}_1(t) - R_3\omega_3\dot{T}_2(t)}{L_2} \\ &+ \frac{R_2(T_1(t) - AE)[R_2(T_2(t) - T_1(t)) - f\omega_2(t)]}{J_2 L_2} \\ &+ \frac{C_2 R_2(T_1(t) - AE)}{J_2 L_2} u_2(t). \end{aligned} \quad (10)$$

FIGURE 7: Decoupling response curves for $\omega_3 = 100 \text{ r/min}$.

We define the dynamic coupling vector $\mathbf{f}(t)$ and input matrix $\mathbf{B}(t)$ as follows:

$$\mathbf{f}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix},$$

$$\begin{aligned} f_1(t) &= (2L_{D2} [(T_1(t) - AE) \ddot{\theta}(t) + 2\dot{T}_1(t) \dot{\theta}(t)] \\ &\quad - R_2 \omega_2(t) \dot{T}_1(t) + (T_0 - AE) \omega_1(t) \dot{R}_1(t)) \\ &\quad \times (L_1 - 2L_{D2}\theta(t))^{-1} \\ &\quad + \frac{R_1(t) (T_0 - AE) [R_1(t) (T_1(t) - T_0) - f\omega_1(t)]}{J_1(t) (L_1 - 2L_{D2}\theta(t))} \\ &\quad + \frac{R_2 (AE - T_1(t)) [R_2 (T_2(t) - T_1(t)) - f\omega_2(t)]}{J_2 (L_1 - 2L_{D2}\theta(t))}, \end{aligned}$$

$$\begin{aligned} f_2(t) &= \frac{R_2 \omega_2(t) \dot{T}_1(t) - R_3 \omega_3 \dot{T}_2(t)}{L_2} \\ &\quad + \frac{R_2 (T_1(t) - AE) [R_2 (T_2(t) - T_1(t)) - f\omega_2(t)]}{J_2 L_2}, \\ \mathbf{B}(t) &= \begin{bmatrix} \frac{C_1 R_1(t) (T_0 - AE)}{J_1(t) (L_1 - 2L_{D2}\theta(t))} & \frac{C_2 R_2 (AE - T_1(t))}{J_2 (L_1 - 2L_{D2}\theta(t))} \\ \frac{C_2 R_2 (T_1(t) - AE)}{J_2 L_2} \end{bmatrix}. \end{aligned} \quad (11)$$

Substituting (11) into (10), (10) can be rewritten as follows:

$$\begin{bmatrix} \ddot{T}_1(t) \\ \ddot{T}_2(t) \end{bmatrix} = \mathbf{f}(t) + \mathbf{B}(t) \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \quad (12)$$

FIGURE 8: Decoupling response curves for $R_1 = 0.2 \text{ m}$.

Equation (12) illustrates that the unwinding system of the gravure printing machines is a second-order coupling system with two inputs and two outputs. In this study, we define the virtual control quantities $\tau_1(t)$ and $\tau_2(t)$ as follows:

$$\begin{bmatrix} \tau_1(t) \\ \tau_2(t) \end{bmatrix} = \mathbf{B}(t) \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \quad (13)$$

Combining (12) and (13), the nonlinear mathematical model for the unwinding system can also be rewritten as follows:

$$\begin{bmatrix} \ddot{T}_1(t) \\ \ddot{T}_2(t) \end{bmatrix} = \mathbf{f}(t) + \begin{bmatrix} \tau_1(t) \\ \tau_2(t) \end{bmatrix}. \quad (14)$$

Because the dynamic coupling vector $\mathbf{f}(t)$ in the ADRC framework can be actively estimated and compensated in real time, the relationships between the virtual control quantities $\tau_1(t)$ and $\tau_2(t)$ and the controlled output variables $T_1(t)$ and $T_2(t)$ are single-input, single-output, and also decoupled, as shown in (14).

In this work, because $T_0 \ll AE$, $T_1(t) \ll AE$, and $2L_{D2}\theta(t) \ll L_1$, so (15) can be got as follows:

$$|\mathbf{B}(t)| = \frac{C_1 C_2 R_2 R_1(t) (T_0 - AE) (T_1(t) - AE)}{J_2 L_2 J_1(t) (L_1 - 2L_{D2}\theta(t))} \neq 0. \quad (15)$$

So $\mathbf{B}(t)$ is an invertible matrix.

Therefore, the relationships between the virtual control quantities $\tau_1(t)$ and $\tau_2(t)$ and the actual control variables $u_1(t)$ and $u_2(t)$ can be expressed as

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \mathbf{B}^{-1}(t) \begin{bmatrix} \tau_1(t) \\ \tau_2(t) \end{bmatrix}, \quad (16)$$

where matrix $\mathbf{B}^{-1}(t)$ is the decoupling plant of the unwinding system as follows:

$$\mathbf{B}^{-1}(t) = \begin{bmatrix} \frac{J_1(t) (L_1 - 2L_{D2}\theta(t))}{C_1 R_1(t) (T_0 - AE)} & \frac{L_2 J_1(t)}{C_1 R_1(t) (T_0 - AE)} \\ \frac{J_2 L_2}{C_2 R_2 (T_1(t) - AE)} & \frac{J_2 L_2}{C_2 R_2 (T_1(t) - AE)} \end{bmatrix}. \quad (17)$$

FIGURE 9: Antidisturbance curves for $\omega_3 = 100$ r/min.

3.2. ADRC Decoupling Control Methodology. Based on the order and decoupling plant of the unwinding system, an ADRC decoupling control methodology is designed to enhance tension stability, as shown in Figure 3. ADRC1 and ADRC2 are relevant to $T_1(t)$ and $T_2(t)$, respectively, for constant tension control. Because the unwinding system is a second-order system, two second-order ADRC controllers are needed. Each ADRC controller consists of a second-order tracking differentiator (TD), a third-order extended state observer (ESO), and a nonlinear state error feedback (NLSEF).

The second-order TD is a nonlinear component; given an input signal, we can acquire a corresponding tracking input signal and an approximately differentiated input signal, even for a nondifferentiable or noncontinuous input signal. As shown in Figure 3, v_{11} and v_{21} are tracking signals of $T_{1\text{ref}}$ and $T_{2\text{ref}}$, respectively, and v_{12} and v_{22} are approximately differentiated signals of $T_{1\text{ref}}$ and $T_{2\text{ref}}$, respectively. According to [22], the discrete forms of second-order TD i ($i = 1, 2$) are expressed as

$$\begin{aligned} fh_i &= \text{fhan}(v_{i1}(k) - T_{i\text{ref}}(k), v_{i2}(k), r_i, h), \\ v_{i1}(k+1) &= v_{i1}(k) + hv_{i2}(k), \end{aligned}$$

$$v_{i2}(k+1) = v_{i2}(k) + hf h_i, \quad (18)$$

where r_i is the velocity factor; h is the sampling step; and $\text{fhan}(x_1, x_2, r, h)$ is the time-optimal control synthesis function defined as follows:

$$\begin{aligned} d &= rh; \quad d_0 = hd; \quad y = x_1 + hx_2, \\ a_0 &= \sqrt{d^2 + 8r|y|}, \\ a &= \begin{cases} x_2 + \frac{a_0 - d}{2} \text{sign}(y), & |y| > d_0, \\ x_2 + \frac{y}{h}, & |y| \leq d_0, \end{cases} \quad (19) \\ \text{fhan}(x_1, x_2, r, h) &= - \begin{cases} r \text{sign}(a), & |a| > d, \\ r \frac{a}{d}, & |a| \leq d. \end{cases} \end{aligned}$$

The third-order ESO is the core of the ADRC, and it not only tracks system output variables and their differentiated signals but also actively estimates dynamic coupling and

FIGURE 10: Antidisturbance curves for $R_1 = 0.2$ m.

external disturbances in real time. As shown in Figure 3, z_{11} and z_{21} track the output variables $T_1(t)$ and $T_2(t)$, respectively; z_{12} and z_{22} estimate the differentiated signals of $T_1(t)$ and $T_2(t)$, respectively; and z_{13} and z_{23} actively estimate $f_1(t)$ and $w_1(t)$ and $f_2(t)$ and $w_2(t)$, respectively. The z_{13} and z_{23} variables are considered to be extended states of the system. According to [22], the discrete forms of third-order ESO $_i$ ($i = 1, 2$) are expressed as

$$\begin{aligned}
 e_i(k) &= z_{i1}(k) - T_i(k), \\
 z_{i1}(k+1) &= z_{i1}(k) + h(z_{i2}(k) - \beta_{i1}e_i(k)), \\
 z_{i2}(k+1) &= z_{i2}(k) \\
 &\quad + h(z_{i3}(k) - \beta_{i2}\text{fal}(e_i(k), 0.5, \delta) + \tau_i(k)), \\
 z_{i3}(k+1) &= z_{i3}(k) - h\beta_{i3}\text{fal}(e_i(k), 0.25, \delta),
 \end{aligned} \tag{20}$$

where β_{i1} , β_{i2} , and β_{i3} are the ESO gains; δ is the interval length of the linear segment; and $\text{fal}(e, \alpha, \delta)$ is a nonlinear

function defined as follows:

$$\text{fal}(e, \alpha, \delta) = \begin{cases} \frac{e}{\delta^{1-\alpha}}, & |e| \leq \delta, \\ |e|^\alpha \text{sign}(e), & |e| > \delta. \end{cases} \tag{21}$$

The NLSEF is a nonlinear combination of the resulting differences of the state variables and estimations generated by the TD and ESO, respectively, as shown in Figure 3. The control law actively compensates for dynamic coupling and external disturbances that are estimated by the ESO in real time. According to [22], the discrete forms of NLSEF $_i$ ($i = 1, 2$) are defined as

$$\begin{aligned}
 e_{i1}(k) &= v_{i1}(k) - z_{i1}(k), \\
 e_{i2}(k) &= v_{i2}(k) - z_{i2}(k), \\
 \tau_{0i}(k) &= k_{ip}\text{fal}(e_{i1}(k), 0.5, \delta) + k_{id}\text{fal}(e_{i2}(k), 1.5, \delta), \\
 \tau_i(k) &= \tau_{0i} - \frac{z_{i3}(k)}{b_{0i}},
 \end{aligned} \tag{22}$$

FIGURE 11: Step response curves for $\omega_3 = 100 \text{ r/min}$.

where k_{ip} is the proportionality coefficient; k_{id} is the differential coefficient; and b_{0i} is the compensating factor.

Combining (3.2), (20), and (22), the discrete algorithm of the ADRCi ($i = 1, 2$) is shown in

$$\begin{aligned}
 fh_i(k) &= fhan(v_{i1}(k) - T_{iref}(k), v_{i2}(k), r_i, h), \\
 v_{i1}(k+1) &= v_{i1}(k) + hv_{i2}(k), \\
 v_{i2}(k+1) &= v_{i2}(k) + hf h_i(k), \\
 e_i(k) &= z_{i1}(k) - T_i(k), \\
 z_{i1}(k+1) &= z_{i1}(k) + h(z_{i2}(k) - \beta_{i1}e_i(k)), \\
 z_{i2}(k+1) &= z_{i2}(k) \\
 &\quad + h(z_{i3}(k) - \beta_{i2}fal(e_i(k), 0.5, h) + \tau_i(k)), \\
 z_{i3}(k+1) &= z_{i3}(k) - h\beta_{i3}fal(e_i(k), 0.25, h), \\
 e_{i1}(k+1) &= v_{i1}(k+1) - z_{i1}(k+1),
 \end{aligned}$$

$$e_{i2}(k+1) = v_{i2}(k+1) - z_{i2}(k+1),$$

$$\tau_i(k+1) = k_{ip}fal(e_{i1}(k+1), 0.5, \delta)$$

$$\begin{aligned}
 &+ k_{id}fal(e_{i2}(k+1), 1.5, \delta) - \frac{z_{i3}(k+1)}{b_{0i}}. \\
 &\hspace{15em} (23)
 \end{aligned}$$

After calculating the virtual control variables $\tau_1(t)$ and $\tau_2(t)$ with the ADRC controller, the actual control variables $u_1(t)$ and $u_2(t)$ can be obtained by (16).

Two diagrams are provided in Figures 3 and 4 that show the differences between PID and ADRC control synthesis methodology for tension decoupling control of the unwinding system. First, the total disturbance in the ADRC methodology, which includes the external disturbances ($w_1(t)$ and $w_2(t)$), dynamic coupling components of the system ($f(t)$), and unknown internal dynamics, is defined as an extended state (z_{i3}) of the system. The total disturbance can be actively estimated by ESO in real time and compensated in the

FIGURE 12: Step response curves for $R_1 = 0.2$ m.

corresponding control signal in each sampling period. However, in PID methodology, the total disturbance is managed passively by parameter tuning. Second, because the differentiated signals v_{i2} and z_{i2} are obtained by the TD and ESO, respectively, in ADRC methodology, the errors of the differentiation are not produced by direct differentiation of the inputs and the outputs. This attribute makes the ADRC algorithm less sensitive to discontinuities in the inputs and noise in the outputs. However, due to noise sensitivity, PID methodology is often implemented without the derivative component.

4. Simulation and Experiments

To investigate the performance of the proposed ADRC decoupling control methodology, a comparison of the unwinding systems of PID and ADRC control strategies was performed through simulation and experiments. The parameters of the unwinding system used in the simulation and experiments are summarised in Table 1. All parameters of ADRC and PID were determined at $\omega_3 = 100$ r/min and $R_1 = 0.2$ m.

TABLE 1: Mechanical parameters of the unwinding system.

Parameters	Value	Units
L_1	1	m
L_2	0.35	m
A	5×10^{-6}	m ²
L_{D1}	0.064	m
L_{D2}	0.128	m
R_2	0.03	m
R_3	0.03	m
E	1.6×10^8	Pa
K	1×10^4	N/m

Although the tests were conducted under different operating conditions, particularly for different unwinding diameters, the parameters remain constant.

4.1. Simulation Results. MATLAB simulation models were constructed to test the performance of the ADRC and PID

FIGURE 13: Decoupling response curves for $\omega_3 = 100$ r/min.

TABLE 2: Controller parameters in simulation.

Controller	Controller parameters
ADRC1	$\beta_{11} = 100, \beta_{12} = 5560, \beta_{13} = 81200,$ $k_{1p} = 920, k_{1d} = 760, r_1 = 60$
ADRC2	$\beta_{21} = 100, \beta_{22} = 2870, \beta_{23} = 51600,$ $k_{2p} = 3460, k_{2d} = 750, r_2 = 60$
PID1	$K_{1p} = 30, K_{1i} = 2.5, K_{1d} = 12$
PID2	$K_{2p} = 65, K_{2i} = 32, K_{2d} = 5$

control strategies. The simulation adopted a fixed-step size mode; the fixed-step size is 10 ms. According to [22, 30, 32], the adjusted parameters of the ADRC are listed in Table 2. The adjusted parameters of the PID are also listed in Table 2.

4.1.1. Case A: Robustness against Dynamic Changes. To verify the robustness against dynamic changes of the developed

ADRC control methodology, the $T_1(t)$ and $T_2(t)$ step change is from 50 N to 80 N when ω_3 is equal to 100 r/min and R_1 is equal to 0.2 m. The simulation behaviours of the PID and ADRC controllers are shown in Figures 5 and 6.

As shown in Figure 5, when R_1 is equal to 0.2 m, 0.1 m, or 0.05 m, both $T_1(t)$ and $T_2(t)$ quickly reach 80 N without overshoot or oscillation in the ADRC controller. However, in the PID controller, a decrease in R_1 leads to deterioration in the control abilities of $T_1(t)$ and $T_2(t)$. For example, when R_1 is equal to 0.05 m, $T_1(t)$ demonstrates an overshoot of approximately 6.38% and arrives at 80 N after 4 s. Figure 6 illustrates that when ω_3 is equal to 100 r/min, 300 r/min, or 500 r/min, both $T_1(t)$ and $T_2(t)$ quickly reach 80 N without overshoot or oscillation in the ADRC controller. In contrast, an increase in ω_3 contributes to deterioration in the control abilities of $T_1(t)$ and $T_2(t)$ in the PID controller.

The simulation results show that once the ADRC controller is set up properly, it can handle a wide range of dynamic changes. Compared with the PID controller, Figures 5 and 6

FIGURE 14: Decoupling response curves for $R_1 = 0.2$ m.

show that the ADRC controller can obtain better robustness against dynamic changes in the unwinding system. The primary reason is that uncertainties and variations in the internal model parameters can be actively estimated by the ESO and compensated by the NLSEF in the ADRC controller; the PID controller does not have these capabilities.

4.1.2. Case B: Decoupling Performance. To determine the decoupling performance of the proposed controller, the $T_1(t)$ step change is from 50 N to 80 N at 5 s and from 80 N to 50 N at 10 s. Figures 7 and 8 illustrate the simulation performance of the PID and ADRC controllers when ω_3 is equal to 100 r/min and R_1 is equal to 0.2 m, respectively.

Figure 7 indicates that when R_1 is equal to 0.2 m, 0.1 m, or 0.05 m, variations in $T_1(t)$ have no influence on $T_2(t)$ in the ADRC controller. In contrast, in the PID controller, $T_2(t)$ is continuously influenced by variations in $T_1(t)$, and variations in $T_2(t)$ increase with a decrease in R_1 . As shown in Figure 8,

when ω_3 is equal to 100 r/min, 300 r/min, or 500 r/min, variations in $T_1(t)$ have no influence on $T_2(t)$ in the ADRC controller. However, in the PID controller, $T_2(t)$ is continuously influenced by variations in $T_1(t)$, and variations in $T_2(t)$ increase with an increase in ω_3 . For example, when ω_3 is equal to 500 r/min, variations in $T_1(t)$ at 5 s and 10 s cause $T_2(t)$ to vary by 6.5 N.

The simulation results illustrate that the designed ADRC controller has better decoupling capability than the PID controller. Because the dynamic coupling components and the unknown coupled dynamics can be combined as the total disturbance estimated and compensated in real time, the decoupling control of the unwinding system in the ADRC controller can be realised, depending on the system order which can be easily obtained and the decoupling plant which may be inaccurate. However, the decoupling control cannot be realised, only depending on the imprecise of decoupling plant in the PID controller.

FIGURE 15: Tension experimental curves for $\omega_3 = 100$ r/min.

4.1.3. Case C: External Antidisturbance Ability. To demonstrate the external antidisturbance ability of the proposed controller, a step disturbance of T_0 is set at 5 s with a value of 10 N, and a step disturbance of $T_1(t)$ is set at 10 s with a value of 2 N when ω_3 is equal to 100 r/min and R_1 is equal to 0.2 m. The simulation behaviours of the PID and ADRC controllers are shown in Figures 9 and 10.

Figures 9 and 10 illustrate that with a decrease in R_1 and an increase in ω_3 , the web tension in the ADRC controller exhibits a trend similar to that of the PID controller, with a gradual increase in tension fluctuation. The range and duration of the tension fluctuation in the ADRC controller are much smaller than the range and duration of the tension fluctuation in the PID controller under similar simulation conditions. For example, when R_1 is equal to 0.05 m, the range and duration of the fluctuation in $T_1(t)$ at 5 s are 0.5 N and 1.5 s, respectively, in the ADRC controller. However, in the PID controller, the range and duration of the fluctuation in $T_1(t)$ at 5 s are 0.9 N and 4 s, respectively. In the ADRC controller, when ω_3 is equal to 500 r/min, the range and duration of the fluctuation in $T_1(t)$ at 5 s are 0.8 N and 0.8 s, respectively. However, the

range and duration of the fluctuation in $T_1(t)$ under similar simulation conditions in the PID controller are 1.2 N and 2 s, respectively.

The simulation results indicate that the proposed ADRC control methodology has a better external antidisturbance ability than the PID controller because the ADRC algorithm is less sensitive to disturbance.

4.2. Experimental Results. To establish practical application results of the ADRC control methodology, comparative experiments between the ADRC and PID control methodologies are performed in the experimental setup, as shown in Figure 1, with the servo cycle and sampling period set to 10 ms. According to the parameters used in the simulation, the adjusted parameters of the PID and ADRC in experiments are presented in Table 3.

4.2.1. Case A: Robustness against Dynamic Changes. To determine the practical robustness against dynamic changes of the ADRC controller, the $T_1(t)$ and $T_2(t)$ step change is from 50 N

FIGURE 16: Tension experimental curves for $\omega_3 = 500$ r/min.

TABLE 3: Controller parameters in experiments.

Controller	Controller parameters
ADRC1	$\beta_{11} = 100, \beta_{12} = 6560, \beta_{13} = 84700,$ $k_{1p} = 830, k_{1d} = 520, r_1 = 68$
ADRC2	$\beta_{21} = 100, \beta_{22} = 2950, \beta_{23} = 54600,$ $k_{2p} = 2760, k_{2d} = 670, r_2 = 57$
PID1	$K_{1p} = 43, K_{1i} = 3.2, K_{1d} = 7$
PID2	$K_{2p} = 81, K_{2i} = 46, K_{2d} = 8$

to 80 N at 5 s when ω_3 is equal to 100 r/min and R_1 is equal to 0.2 m. The performance of the PID and ADRC controllers in the experiments is shown in Figures 11 and 12.

Figures 11 and 12 show that with a decrease in R_1 and an increase in ω_3 , the control abilities of $T_1(t)$ and $T_2(t)$ deteriorate in the PID controller. However, when ω_3 is equal to 100 r/min or when R_1 is equal to 0.2 m, $T_1(t)$ and $T_2(t)$ quickly reach 80 N without overshoot in the ADRC controller. The experimental results are consistent with the simulation results

and indicate that the proposed ADRC controller is superior to the PID controller in practical robustness against dynamic changes in the unwinding system. This finding is mainly due to the fact that uncertainties and variations in the internal model parameters can be actively estimated and compensated in the ADRC controller but are passively managed by parameters tuning in the PID controller.

4.2.2. Case B: Decoupling Performance. To establish the practical decoupling performance of the proposed controller, the $T_1(t)$ step change is from 50 N to 80 N at 5 s and from 80 N to 50 N at 10 s when ω_3 is equal to 100 r/min and R_1 is equal to 0.2 m. Figures 13 and 14 show the performance of the PID and ADRC controllers.

Figures 13 and 14 illustrate that $T_2(t)$ is always influenced by variation in $T_1(t)$ and the variation in $T_2(t)$ increases with a decrease in R_1 and an increase in ω_3 in the PID controller. Unfortunately, when ω_3 is equal to 100 r/min or R_1 is equal to 0.2 m, the variation in $T_1(t)$ has no influence on $T_2(t)$ in the ADRC controller. The experimental results agree with

TABLE 4: S_{T_1} and S_{T_2} in various test conditions.

Conditions	Controller type			
	PID controller		ADRC controller	
	S_{T_1} (N)	S_{T_2} (N)	S_{T_1} (N)	S_{T_2} (N)
$\omega_3 = 100$ r/min, $R_1 = 0.2$ m	0.3683	0.2949	0.2400	0.2163
$\omega_3 = 100$ r/min, $R_1 = 0.05$ m	0.5540	0.4271	0.2686	0.2350
$\omega_3 = 500$ r/min, $R_1 = 0.2$ m	0.9326	0.6572	0.4880	0.4064
$\omega_3 = 500$ r/min, $R_1 = 0.05$ m	1.3349	0.8514	0.5831	0.4658

the simulation results and illustrate that the designed ADRC controller has better decoupling capability than the PID controller. Because the dynamic coupling components and the unknown coupled dynamics can be combined as the total disturbance, which is estimated and compensated in real time, the decoupling control of the unwinding system can be realised, depending on the system order and the decoupling plant in ADRC controller. However, only depending on the decoupling plant, the decoupling control cannot be realised in the PID controller.

4.2.3. Case C: Tension Control Ability of Steady Speeds. Because tension control at steady speeds is essential for the performance of the gravure machine, further attention should be paid to tension fluctuation at steady speeds. To investigate the steady performance of the ADRC controller, tension control experiments were performed at motor steady speeds of $\omega_3 = 100$ r/min and $\omega_3 = 500$ r/min. In these experiments, the steady tension was 50 N and the test time was 20 s. The performance of the PID and ADRC controllers in the experiments is shown in Figures 15 and 16. S_{T_1} and S_{T_2} are standard deviations of $T_1(t)$ and $T_2(t)$, respectively, and were obtained under different test conditions, just as shown in Table 4.

Figures 15 and 16 and Table 4 demonstrate that as the disturbance due to web speed and system dynamics increases, the web tension in the ADRC controller exhibits a trend similar to that of the PID controller, with a gradual increase in tension fluctuation. However, compared with the PID controller under similar experimental conditions, the resulting tension fluctuation in the ADRC controller is considerably smaller. For example, when R_1 is equal to 0.05 m and ω_3 is equal to 500 r/min, the standard deviations of $T_1(t)$ and $T_2(t)$ in the PID controller are 1.3349 N and 0.8514 N, respectively. Under the ADRC controller, when R_1 is equal to 0.05 m and ω_3 is equal to 500 r/min, the standard deviations of $T_1(t)$ and $T_2(t)$ are 0.5831 N and 0.4658 N, respectively. The experimental results indicate that the proposed ADRC controller has better tension control at steady speeds than the PID controller. The main reason is that the ADRC algorithm is much less sensitive to internal and external disturbances than the PID controller.

5. Conclusions

Based on ADRC, an innovative control synthesis methodology is proposed for the design of a tension decoupling controller for the unwinding system of the gravure printing

machine. First, according to the working principle of the unwinding system, a nonlinear mathematical model is established. Second, a decoupling model is constructed to obtain the order and decoupling plant of the unwinding system, which establishes a basis for the ADRC decoupling control methodology. Last, the results of the simulation and experiments demonstrate that the proposed control methodology can achieve not only good decoupling control performance but also better stability and robustness than the traditional PID controllers. From this study, it can be found that in the ADRC framework, all of the unknown coupling terms that may contribute to the total disturbance can be actively estimated and cancelled out in the corresponding control signal in real time; this greatly simplifies the design of the decoupling control methodology.

The main contributions of this paper are as follows. (1) We have addressed the design of a tension decoupling controller based on ADRC for the unwinding system of gravure printing machines. (2) Simulation and experiments of the proposed decoupling controller for the unwinding system were completed. (3) The study demonstrates that the proposed method is a promising solution for the problem of tension control in the unwinding systems of gravure printing machines because it is inherently robust against internal and external disturbances and decoupling control over the unwinding system of the gravure printing machine can be achieved in a simple manner.

Nomenclature

- T_0 : Constant tension of the unwinding roll (N)
- T_i : Web tension in the i th span (N)
- w_i : External disturbances that influence T_i
- V_i : Tangential velocity of the i th roll (m/s)
- ω_i : Rotational speed of the i th roll (r/min)
- u_i : Torque control signal of the i th servo motor (Nm)
- L_i : Nominal span length of the web in the i th span (m)
- L_{Di} : The i th arm length of the dancer roll (m)
- R_i : Radius of the i th roll (m)
- J_i : Inertia of the i th roll (kgm^2)
- J_{Da} : Inertia of the dancer roll (kgm^2)
- C_i : Constant of the i th servo motor
- T_{iref} : Reference inputs of $T_i(t)$ (N)
- V_{ref} : Reference inputs of V_3 (m/s)

- K : Young's modulus of the spring (N/m)
 A : Cross-sectional area of the web (m^2)
 E : Modulus of elasticity of the web material (Pa)
 f : Viscous friction coefficient (Nm/(rad/s))
 f_D : Viscous friction coefficient of the dancer roll (Nm/(rad/s))
 θ : Angle of the dancer roll arm from a neutral position (rad)
 M_i : Servo motors driving the i th roll.

Conflict of Interests

The authors declare that they have no financial or personal relationships with other people or organizations that can inappropriately influence their work; there is no professional or other personal interest of any nature or kind in any product or company that could be construed as influencing the position presented in, or the review of, the paper.

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