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Research Article

Analysis of Sigmoid Functionally Graded Material (S-FGM) Nanoscale Plates Using the Nonlocal Elasticity Theory

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Based on a nonlocal elasticity theory, a model for sigmoid functionally graded material (S-FGM) nanoscale plate with first-order shear deformation is studied. The material properties of S-FGM nanoscale plate are assumed to vary according to sigmoid function (two power law distribution) of the volume fraction of the constituents. Elastic theory of the sigmoid FGM (S-FGM) nanoscale plate is reformulated using the nonlocal differential constitutive relations of Eringen and first-order shear deformation theory. The equations of motion of the nonlocal theories are derived using Hamilton's principle. The nonlocal elasticity of Eringen has the ability to capture the small scale effect. The solutions of S-FGM nanoscale plate are presented to illustrate the effect of nonlocal theory on bending and vibration response of the S-FGM nanoscale plates. The effects of nonlocal parameters, power law index, aspect ratio, elastic modulus ratio, side-to-thickness ratio, and loading type on bending and vibration response are investigated. Results of the present theory show a good agreement with the reference solutions. These results can be used for evaluating the reliability of size-dependent S-FGM nanoscale plate models developed in the future.

1. Introduction

The nanoscale plates have attracted attention of scientific community in solid-state physics, materials science, and nanoelectronics due to their superior mechanical, chemical, and electronic properties. Conducting experiments with nanoscale size specimens is both expensive and difficult. Hence, development of appropriate mathematical models for nanostructures is an important issue concerning the application of nanostructures. The nanostructures are modeled into three main categories using atomistic [1, 2], hybrid atomistic-continuum mechanics [3–5] and continuum mechanics [6, 7]. Continuum mechanics approach is less computationally expensive than the former two approaches. Further, it has been found that continuum mechanics results are in good agreement with those obtained from atomistic and hybrid approaches. Due to the presence of small scale effects at the nanoscale structures, size-dependent continuum mechanics models such as the strain gradient theory (Nix and Gao [8]), couple stress theory (Hadjesfandiari and Dargush [9]), modified couple stress theory (Asghari et al. [10]; Ma et al. [11]; Reddy [12]), and nonlocal elasticity theory (Eringen [13]) are used.

The small size analysis using local theory overpredicts the results. Thus the consideration of small effects is necessary for correct prediction of micro/nano-structures. Peddieson et al. [14] applied nonlocal elasticity to formulate a nonlocal version of the Euler-Bernoulli beam model and concluded that nonlocal continuum mechanics could potentially play a useful role in nanotechnology applications. One of the well-known continuum mechanics theory that includes small scale effects with good accuracy is the nonlocal theory of Eringen [6, 7, 13]. Unlike the local theories which assume that the stress at a point is a function of strain at that point, the nonlocal elasticity theory assumes that the stress at a point is a function of strains at all points in the continuum. Compared to classical continuum mechanics theories, nonlocal theory of Eringen has capability to predict behavior of the large nano-sized structures, while it avoids solving the large number of equations.

Functionally graded material (FGM) is a class of composites in which the material properties vary smoothly and continuously from one surface to the other and thus eliminates the stress concentration found in laminated composites. The other advantage of FGM is that it mitigates singularities at intersections between interfaces usually presented in laminate composites due to their abrupt transitions in material compositions and properties. A generally FGM is made from a mixture of ceramic and metal. The FGM is a composite material whose composition varies according to the required performance. The increase in FGM applications requires accurate models to predict their responses. A critical review of recent works on the bending analysis of functionally graded (FG) plates can be found in Jha et al. [15]. Since the shear deformation has significant effects on the responses of FG plates, shear deformation theories such as first-order shear deformation theory (FSDT) should be used to analyze FG plates. If a high external pressure is applied to the composite plate and shell structures, the high stresses occurred in the structure will affect its integrity, and the structure, as the result, susceptible to failure. For these reasons, understanding the mechanical behavior of FGM plates and shells is very important to assess the safety of the shell and plate structure. Chung and Chi [16] proposed a sigmoid FGM (S-FGM), which is composed of two power-law functions to define a new volume fraction. The effect of loading conditions, the aspect ratio, and the change of elastic modulus on the mechanical behavior of S-FGM plates was investigated in Chi and Chung [17]. Recent work on the vibration, buckling, and geometrically nonlinear analysis of S-FGM plates and shells can be founded in Han et al. [18] and Han et al. [19].

In the literature a great deal of attention has been focused on studying the bending, vibration, and buckling behavior of one-dimensional nanostructures using nonlocal elasticity theory (Aydogdu [20]; Civalek and Demir [21]; Reddy [22]; Reddy and Pang [23]; Reddy [24]; Roque et al. [25]; Wang and Liew [26]; Wang et al. [27]). These nanostructures include nanobeams, nanorods, and carbon nanotubes. In recent years, the application of FGMs has broadly been spread in micro- and nanoscale devices and systems such as thin films [28, 29], atomic force microscopes [30], micro- and nanoelectromechanical systems (MEMS and NEMS) [31, 32]. In such applications, size effects have been experimentally observed [33–36]. On the contrary a few works appear related to the bending analysis of functionally graded material (FGM) nanoscale plate based on first-order shear deformation theory. The present study deals with the use of the nonlocal first-order plate theory in bending response of S-FGM nanoscale plates. Based on the nonlocal constitutive relations of Eringen, equations of motion of nanoscale plates are derived using Hamilton's principle. Closed-form solutions of deflection are obtained for simply supported S-FGM nanoscale plates. The effects of (i) nonlocal parameters, (ii) power law indexes, (iii) E_1/E_2 ratios, (iv) aspect ratios, (v) side-to-thickness ratios, and (vi) loading types on nondimensional bending responses are investigated. To illustrate the accuracy of the present theory, the numerical examples are investigated and compared with those solutions from the previous literatures. The present work would be helpful while designing nano-electro-mechanical system and micro-electro-mechanical systems devices using the S-FGM nanoscale plates.

2. Review of Nonlocal Elasticity

According to Eringen [6, 7, 13], the stress field at a point x in an elastic continuum not only depends on the strain field at the point (hyperelastic case) but also on strains at all other points of the body. Eringen attributed this fact to the atomic theory of lattice dynamics and experimental observations on phonon dispersion. Thus, the nonlocal stress tensor components σ_{ij} at point $\mathbf{x}$ are expressed as

$$\sigma_{ij}(\mathbf{x}) = \int_V K(|\overline{\mathbf{x}} - \mathbf{x}|, \tau)\, t_{ij}(\overline{\mathbf{x}})\, d\overline{\mathbf{x}}, \tag{1}$$

where $t_{ij}(\mathbf{x})$ are the components of the classical macroscopic stress tensor at point $\mathbf{x}$ and the kernel function $K(|\overline{\mathbf{x}} - \mathbf{x}|, \tau)$ represents the nonlocal modulus, $|\overline{\mathbf{x}} - \mathbf{x}|$ being the distance (in Euclidean norm) and τ is a material constant that depends on internal and external characteristic lengths (such as the lattice spacing and wavelength, resp.). The macroscopic stress $\mathbf{t}$ at point $\mathbf{x}$ in a Hookean solid is related to the strain at the point by the generalized Hooke's law

$$t(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : \varepsilon(\mathbf{x}) \quad \text{or} \quad t_{ij} = C_{ijkl}\varepsilon_{kl}, \tag{2}$$

where $\mathbf{C}$ is the fourth-order elasticity tensor and: denotes the "double-dot product" (see Reddy [22]).

In the nonlocal linear elasticity, equations of motion can be obtained from nonlocal balance law

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i, \tag{3}$$

where i, j take the symbols x, y, z and f_i, ρ, and u_i are the components of the body force, mass density and displacement vector [13]. By substituting (1) into (3), the integral form of nonlocal constitutive equation is obtained. Because solving an integral equation is more difficult than a differential equation, Eringen [6, 7, 13] proposed a differential form of the nonlocal constitutive equation as

$$t_{ij,j} + \mathscr{L}\left(f_i - \rho \ddot{u}_i\right) = 0, \tag{4}$$

in which the linear differential operator $\mathscr{L}$ was defined by

$$\mathscr{L} = 1 - \mu \nabla^2, \qquad \mu = e_0^2 \overline{a}^2, \tag{5}$$

where μ is the nonlocal parameter, e_0 is material constant which is defined by the experiment, and $\overline{a}$ is the internal characteristic length.

By applying this operator on (1), the constitutive equation can be simplified to

$$\mathscr{L}\left(\sigma_{ij}\right) = C_{ijkl}\varepsilon_{kl}. \tag{6}$$

Equation (6) is simpler and more convenient than the integral relation (1) to apply to various linear elasticity problems.

3. Plate Equations of Nonlocal Elasticity

Using (2) and (6), stress resultants introduced in plate and shell theories can be reformulated in terms of strain for the nonlocal theory. In plate theories based on plane-stress assumption, we take $\sigma_{zz} = 0$ and the resulting theory becomes two-dimensional.

Consider a (x, y, z) coordinate system with the xy-plane coinciding with the mid-plane of the plate. So the stress-strain relations of plane-stress can be expressed as

$$t_{\alpha\beta} = \overline{C}_{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta}, \tag{7}$$

where $\overline{C}_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma\delta} - C_{\alpha\beta zz}C_{zz\gamma\delta}/C_{zzzz}$, and transverse shear stress-strain relation is expressed as

$$t_{\alpha z} = 2\overline{C}_{\alpha z\gamma z}\varepsilon_{\gamma z}, \tag{8}$$

where α, β, γ, s the shear correction factoand δ take the symbols x, y.

The relations between stress resultants in local theory and nonlocal theory are defined by integrating (6) through the plate thickness:

$$\mathscr{L}\left(N_{ij}\right) = N_{ij}^{L}, \qquad \mathscr{L}\left(M_{ij}\right) = M_{ij}^{L}, \tag{9}$$

where

$$\begin{Bmatrix} N_{\alpha\beta}, N_{\alpha\beta}^{L} \\ M_{\alpha\beta}, M_{\alpha\beta}^{L} \end{Bmatrix} = \int_{-h/2}^{h/2} \left\{\sigma_{\alpha\beta}, t_{\alpha\beta}\right\} \begin{Bmatrix} 1 \\ z \end{Bmatrix} dz, \tag{10}$$

$$\left\{N_{\alpha z}, N_{\alpha z}^{L}\right\} = \int_{-h/2}^{h/2} \left\{\sigma_{\alpha z}, t_{\alpha z}\right\} dz. \tag{11}$$

The superscript L denoted the quantities in local first-order shear deformation theory and h is the thickness of the plate. The governing equation of the plate in nonlocal theory can be determined by integrating (3) through the plate thickness and noting (10)

$$N_{\alpha i,\alpha} + F_i = \int_{-h/2}^{h/2} \rho \ddot{u}_i \, dz, \tag{12}$$

where $F_i = \int_{-h/2}^{h/2} f_i dz$. By multiplying (3) by z and then integrating from it through plate thickness and using integration-by-parts, we obtain

$$M_{\alpha\beta,\beta} - N_{\alpha z} = \int_{-h/2}^{h/2} \rho \ddot{u}_\alpha z \, dz. \tag{13}$$

In general, differential operator ∇ in (6) is the 3D Laplace operator. For 2D problems, the operator ∇ may be reduced to 2D one. Thus, the linear differential operator $\mathscr{L}$ becomes

$$\overline{\mathscr{L}} = 1 - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right). \tag{14}$$

It is clear that the operator $\overline{\mathscr{L}}$ is independent of the z direction.

4. Nonlocal First-Order Shear Deformation Theory

The classical plate theory is based on the Kirchhof assumptions, in which transverse normal and shear stresses are neglected. In the first-order shear deformation theory (FSDT), a constant state of transverse shear stresses is accounted for, and often the transverse normal stress is neglected. The displacement field of the first-order theory of plates is given by

$$u_\alpha = u_\alpha^0 + z\phi_\alpha, \qquad u_z = w^0, \tag{15}$$

where u_α are the inplane displacements of point on the mid-plane (i.e., $z = 0$) at $t = 0$, u_z is the transverse displacement of the mid-plane of the plate, and ϕ_α denotes the slope of the transverse normal on mid-plane.

By substituting the displacement field into (12)-(13), we obtain

$$N_{\alpha i,\alpha} + F_i = I_0 \ddot{u}_i^0, \tag{16}$$

$$M_{\alpha\beta,\beta} - N_{\alpha z} = I_2 \ddot{\phi}_\alpha. \tag{17}$$

Then (16) for $i = z$ and (17) can be combined to drive the following governing equations for flexural response of the nonlocal first-order plate theory:

$$N_{\alpha z,\alpha} + q_z = I_0 \ddot{w}^0, \tag{18}$$

$$M_{\alpha\beta,\beta} - N_{\alpha z} = I_2 \ddot{\phi}_\alpha, \tag{19}$$

where $I_k = \int_{-h/2}^{h/2} \rho(z)^k dz$ $(k = 0, 2)$.

5. Variational Statements

The variational statements facilitate the direct derivation of the equations of motion in terms of the displacements. Hence, we also present the variational form of governing equations which is useful in integral formulations and displacement finite element formulations. The governing equations of the first-order nonlocal plate theory can be derived using dynamic version of the principle of virtual displacement (Hamilton's principle)

$$0 = \int_0^T (\delta U + \delta V - \delta K) \, dt. \tag{20}$$

By substituting nonlocal stress resultants in terms of the displacements into the principle of virtual displacements and integrate by part, the equations of motion can be obtained as follows:

$$\delta u_x^0 : N_{xx,x} + N_{xy,y} - \overline{\mathscr{L}}\left(I_0 \ddot{u}_x^0\right) = 0, \tag{21}$$

$$\delta u_y^0 : N_{xy,x} + N_{yy,y} - \overline{\mathscr{L}}\left(I_0 \ddot{u}_y^0\right) = 0, \tag{22}$$

$$\delta w^0 : N_{xz,x} + N_{yz,y} - \overline{\mathscr{L}}\left[q_z - I_0 \ddot{w}^0\right] = 0, \tag{23}$$

$$\delta \phi_x : M_{xx,x} + M_{xy,y} - N_{xz} - \overline{\mathscr{L}}\left[I_2 \ddot{\phi}_x\right] = 0, \tag{24}$$

$$\delta \phi_y : M_{xy,x} + M_{yy,y} - N_{yz} - \overline{\mathscr{L}}\left[I_2 \ddot{\phi}_y\right] = 0. \tag{25}$$

6. Constitutive Relations of S-FGM Structures

The functionally graded material (FGM) can be produced by continuously varying the constituents of multiphase materials in a predetermined profile. The most distinct features of an FGM are the nonuniform microstructures with continuously graded properties. An FGM can be defined by the variation in the volume fractions. Most researchers use the power-law function, exponential function, or sigmoid function to describe the volume fractions. This paper uses FGM plates and shells with sigmoid function.

The volume fraction using two power-law functions which ensure smooth distribution of stresses is defined as

$$V_f^1(t) = 1 - \frac{1}{2}\left(\frac{h/2 - t}{h/2}\right)^p \quad \text{for } 0 \le t \le \frac{h}{2}, \tag{26a}$$

$$V_f^2(t) = \frac{1}{2}\left(\frac{h/2 + t}{h/2}\right)^p \quad \text{for } -\frac{h}{2} \le t \le 0. \tag{26b}$$

By using the rule of mixture, the material properties of the S-FGM can be calculated by

$$H(t) = V_f^1(t) H_1 + \left(1 - V_f^1(t)\right) H_2 \quad \text{for } 0 \le t \le \frac{h}{2}, \tag{27a}$$

$$H(t) = V_f^2(t) H_1 + \left(1 - V_f^2(t)\right) H_2 \quad \text{for } -\frac{h}{2} \le t \le 0. \tag{27b}$$

Figure 1 shows that the variation of Young's modulus in (27a) and (27b) represents sigmoid distributions, and this FGM structure is thus called a sigmoid FGM structure (S-FGM structures). In this paper, the volume fraction using two power-law functions by Chung and Chi [16] is used to ensure smooth distribution of stresses among all the interfaces.

Consider an elastic rectangular plate and shell. The local coordinates r and s define the mid-plane of the plate and shell, whereas the t-axis originated at the middle surface of the plate and shell is in the thickness direction. The material properties, both of Young's modulus and the Poisson's ratio, the upper and lower surfaces are different but are pre-assigned according to the performance demands. However, the Young's modulus and Poisson's ratio of the plates and shells vary continuously only in the thickness direction (t-axis); that is, $E = E(t)$, $\nu = \nu(t)$. It is called functionally graded material (FGM) plate and shell.

The constitutive relations of the FGM structures are as follows:

$$\begin{Bmatrix} N_{\alpha\beta} \\ M_{\alpha\beta} \end{Bmatrix} = \begin{bmatrix} A_{\text{FGM}}^{\alpha\beta\gamma\delta} & B_{\text{FGM}}^{\alpha\beta\gamma\delta} \\ B_{\text{FGM}}^{\alpha\beta\gamma\delta} & D_{\text{FGM}}^{\alpha\beta\gamma\delta} \end{bmatrix} \begin{Bmatrix} \varepsilon_{\gamma\delta}^m \\ \varepsilon_{\gamma\delta}^b \end{Bmatrix}, \tag{28}$$

$$\{N_{\alpha z}\} = \left[A_{\text{FGM}}^{\alpha z\beta z}\right] \left\{\varepsilon_{\beta z}^s\right\}. \tag{29}$$

The coefficients of (28) and (29) for FGM structures are defined as follows:

$$\begin{gathered} A_{\text{FGM}}^{\alpha\beta\gamma\delta},\ B_{\text{FGM}}^{\alpha\beta\gamma\delta},\ D_{\text{FGM}}^{\alpha\beta\gamma\delta} = \int_{-h/2}^{h/2} C_{\text{FGM}}^{\alpha\beta\gamma\delta}\left(1, z, z^2\right) dz, \\ A_{\text{FGM}}^{\alpha z\beta z} = k_s \int_{-h/2}^{h/2} C_{\text{FGM}}^{\alpha z\beta z}\, dz, \end{gathered} \tag{30}$$

where k_s is the shear correction factor ($k_s = 5/6$). For details see Han et al. [18].

7. The Navier Solutions of S-FGM Nanoscale Plates

Here, analytical solutions for bending of simply supported S-FGM nanoscale plates are presented using the nonlocal first-order plate theory to illustrate the small scale effects on deflections of the nan-scale plates. For the static case, all time derivative terms are set to zero. For the set of simply supported boundary conditions, the analytical solution can be obtained [37]. According to the Navier solution theory, the generalized displacements at middle of the plane ($z = 0$) are expanded in double Fourier series as follows:

$$\begin{aligned} u_x^0(x, y, t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}\Lambda_1, \\ u_y^0(x, y, t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}\Lambda_2, \\ w^0(x, y, t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\Lambda_3, \\ \phi_x(x, y, t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn}\Lambda_1, \\ \phi_y(x, y, t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn}\Lambda_2, \\ q_z(x, y, t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn}\Lambda_3, \end{aligned} \tag{31}$$

where $\Lambda_1 = \cos\xi x \sin\eta y \cdot e^{i\varpi_{mn}t}$, $\Lambda_2 = \sin\xi x \cos\eta y \cdot e^{i\varpi_{mn}t}$, and $\Lambda_3 = \sin\xi x \sin\eta y \cdot e^{i\varpi_{mn}t}$ in which $\xi = m\pi/a$, $\eta = n\pi/b$, and ϖ_{mn} is the natural frequency.

By substituting (31) into (21)–(25), matrix form is as follows:

$$[\mathbf{K}]\{\Delta\} + [\mathbf{M}]\left\{\ddot{\Delta}\right\} = \{Q\}, \tag{32}$$

where $\{\Delta\} = \{U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}\}$, the superposed dots denote differentiation with respect to time, $[\mathbf{K}]$ is the stiffness matrix, $[\mathbf{M}]$ is the mass matrix, and $\{Q\}$ is the force vector.

8. Numerical Results and Discussion

In order to validate, several numerical examples are solved to test the performance in bending analysis. Examples include

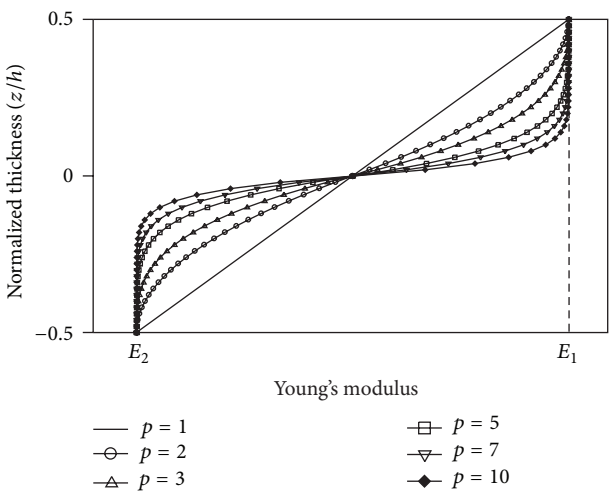

Figure 1: The variation of Young's modulus of S-FGM plate and shell.

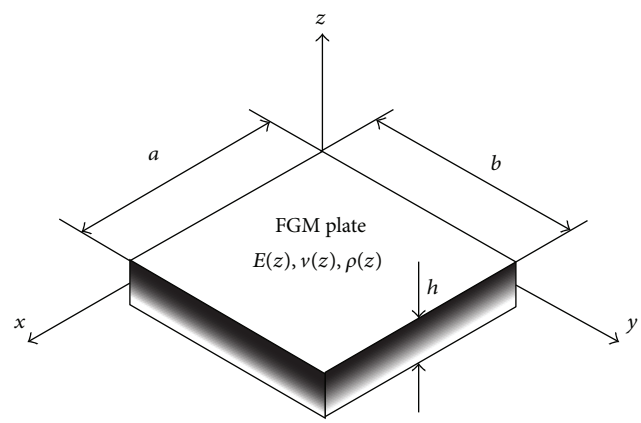

Figure 2: Geometry of S-FGM plate.

P-FGM to check some crucial features and to make comparison with previous published analysis results.

8.1. Validation. Firstly, since the results of nanoscale plate made of S-FGM are not available in the open literature, homogeneous and P-FGM ($p = 1$) plates are used herein for the verification.

Table 1 shows the nondimensional displacements of simply supported plates with various values of side-to-thickness ratio a/h in homogeneous and functionally graded ($p = 1$) plates. The nanoscale plate is made of epoxy with the following material properties:

$$E_1 = 14.4 \text{ GPa}, \qquad E_2 = 1.44 \text{ GPa}, \qquad \nu = 0.38,$$
$$h = 17.6 \times 10^{-6} \text{ m}, \qquad q_0 = 1.0 \text{ N/m}, \tag{33}$$
$$\rho_1 = 12.2 \times 10^3 \text{ kg/m}^3, \qquad \rho_2 = 1.22 \times 10^3 \text{ kg/m}^3.$$

The nondimensional displacement and frequency are defined as

$$\overline{w} = w\frac{E_2 h^3}{q_0 a^4} \times 10^2, \qquad \overline{\omega} = \omega\frac{a^2}{h}\sqrt{\frac{\rho_2}{E_2}}. \tag{34}$$

A shear correction factor of 5/6 is used for FSDT plate theory. The calculated displacements based on FSDT plate theory with S-FGM power law index ($p = 1$) are compared with those reported by Thai and Choi [38] based on Mindlin plate theory (MPT) with P-FGM index (Table 2). It can be observed that the present results are identical with those given by Thai and Choi [38] based on MPT. There is no difference between the present S-FGM results and those P-FGM results given by Thai and Choi [38]. This is due to the fact that the S-FGM material properties are identical with P-FGM, when the power law index is 1.

Secondly, the analytical bending solutions are numerically evaluated here for an isotropic plate to discuss the effects of nonlocal parameter μ on the plate bending response.

Table 3 shows the non-dimensional displacements of simply supported plates with various values of nonlocal parameter μ in homogeneous plates. The nanoscale plate is made of the following material properties:

$$E = 30 \times 10^6, \qquad \nu = 0.3, \qquad q_0 = 1.0. \tag{35}$$

The results based on FSDT plate theory with various values of nonlocal parameter μ are compared with those reported by Lee et al. [39] based on HSDT. It can be observed that the present results are identical with those given by Lee et al. [39] when the side-to-thickness ratio is 100. For the case of the $a/h = 10$, there is small difference between the present results and those given by Lee et al. [39]. This is due to the fact that Lee et al. [39] used HSDT to calculate the displacement, whereas the present results are based on the FSDT.

Thirdly, in Tables 4 and 5, the calculated frequency based on FSDT plate theory with S-FGM power law index ($p = 1$) is compared with those reported by Thai and Choi [38] with P-FGM index. The present results are identical with those given by Thai and Choi [38]. As we expected, there is no difference between the present S-FGM results and those P-FGM results given by Thai and Choi [38].

Fourthly, the results of S-FGM plates (see Figure 2) using the classical plate theory [17] are compared with present solutions using the FSDT for validation. The material properties are

$$E_1 = 2.1 \times 10^6 \text{ kg/cm}^2, \qquad E_2 = \text{varied}, \qquad \nu = 0.3,$$
$$a = 100 \text{ cm}, \qquad h = 2 \text{ cm}, \qquad q_0 = 1.0 \text{ kg/cm}^2. \tag{36}$$

The results of classical plate theory by Chi and Chung [17] and the results of first-order shear deformation theory are plotted in Figure 3. It shows that the more of E_1/E_2 brings the larger deflection, because lager E_1/E_2 decreases the stiffness of the FGM plate. The present and reference results agree very well.

In order to investigate the effects of the aspect ratio a/b, the center deflection of the FGM plate is shown in Figure 4. The center deflection increases upon raising the aspect ratio

Table 1: Nondimensional displacement of simply supported FGM plate.

a/h	A homogeneous plate		$p = 1$	
	P-FGM (Thai and Choi [38])	Present (S-FGM)	P-FGM (Thai and Choi [38])	Present (S-FGM)
5	0.5147	0.5147	1.1536	1.1536
10	0.4415	0.4415	1.0205	1.0205
20	0.4232	0.4232	0.9873	0.9873
100	—	0.4173	—	0.9766

Table 2: Nondimensional displacement of simply supported nano-scale FGM plate.

μ	A homogeneous plate		$p = 1$	
	P-FGM (Thai and Choi [38])	Present (S-FGM)	P-FGM (Thai and Choi [38])	Present (S-FGM)
0	0.4415	0.4415	1.0205	1.0205
1.0	—	0.6969	—	1.6123
2.25	—	1.0160	—	2.3521
4.0	—	1.4629	—	3.3878

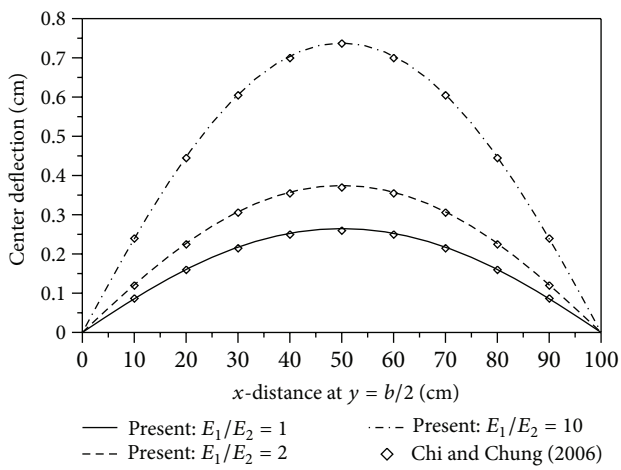

Figure 3: The deflection of S-FGM plate along the x direction for different E_1/E_2.

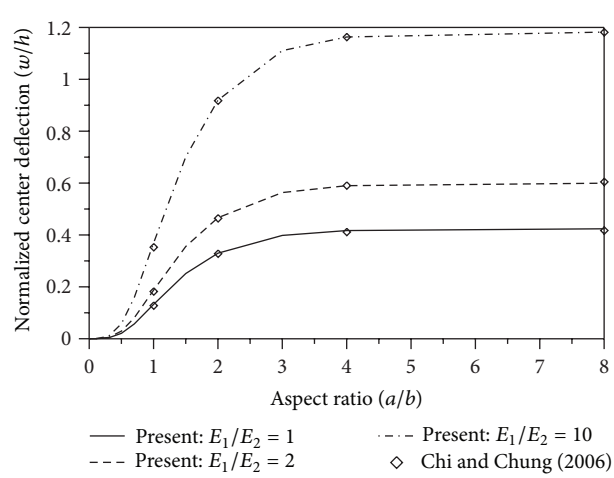

Figure 4: Normalized center deflection of S-FGM plate versus the aspect ratio for different E_1/E_2.

Table 3: Nondimensional displacement of simply supported nano-scale plate (100 term series).

μ	$a/h = 10$		$a/h = 100$	
	Lee et al. [39]	Present	Lee et al. [39]	Present
0	4.6658	4.6658	4.4384	4.4384
0.5	5.0836	5.0836	4.8408	4.8408
1	5.5014	5.5012	5.2432	5.2432
1.5	5.9192	5.9189	5.6456	5.6456
2	6.3370	6.3365	6.0480	6.0480
2.5	6.7548	6.7542	6.4504	6.4504
3	7.1726	7.1718	6.8528	6.8528
3.5	—	7.5895	—	7.2552
4	—	8.0071	—	7.6576

for a/b is less than 3. In Figures 3 and 4, it is clear that the results show very good agreement, because of the large side-to-thickness ratio. As expected, the less side-to-thickness ratio is, the error is larger. In this study, all discussions are based on the first-order shear deformation theory.

8.2. Parameter Studies. Consider a simply supported square plate with the material properties of (33). Parameter studies are presented to investigate the influences of transverse shear deformation, nonlocal parameter ($NT = \mu$), and power law index p on bending responses of S-FGM nanoscale plate.

To illustrate the effect of nonlocal parameter on responses of S-FGM nanoscale plate, Figure 5 plots the deflection with respect to dimensionless nonlocal parameter μ for a simply supported S-FGM plate with $p = 1.0$ and $a/h = 10$. The nonlocal parameters are taken as $\mu = 0, 1.0, 2.25$, and 4. These values are taken because $e_0\bar{a}$ in (5) should be smaller than 2.0 nm for carbon nanotubes as described by Q. Wang and C. M. Wang [40]. The inclusion of the nonlocal scale effect

Table 4: Nondimensional frequency of simply supported FGM plate.

a/h	A homogeneous plate		$p = 1$	
	P-FGM (Thai and Choi [38])	Present (S-FGM)	P-FGM (Thai and Choi [38])	Present (S-FGM)
5	5.3871	5.3871	4.8744	4.8744
10	5.9301	5.9301	5.2697	5.2697
20	6.0997	6.0997	5.3880	5.3880
100	—	6.1579	—	5.4280

Table 5: Nondimensional frequency of simply supported nano-scale FGM plate.

μ	A homogeneous plate		$p = 1$	
	P-FGM (Thai and Choi [38])	Present (S-FGM)	P-FGM (Thai and Choi [38])	Present (S-FGM)
0	5.9301	5.9301	5.2697	5.2697
1.0	—	4.6345	—	4.1184
2.25	—	3.8012	—	3.3779
4.0	—	3.1478	—	2.7973

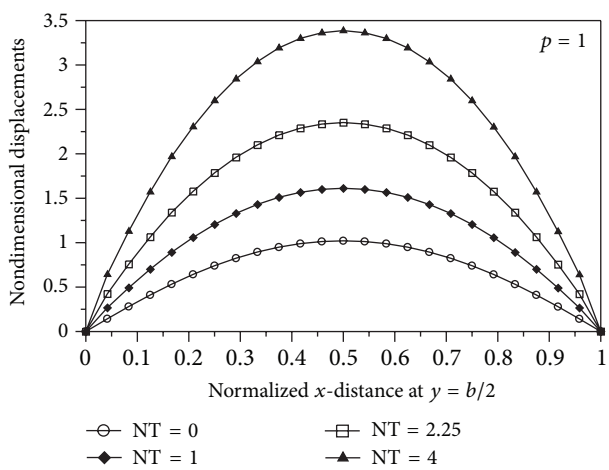

Figure 5: Effect of the nonlocal parameter on the nondimensional displacement of a simply supported S-FGM plate with $a/h = 10$.

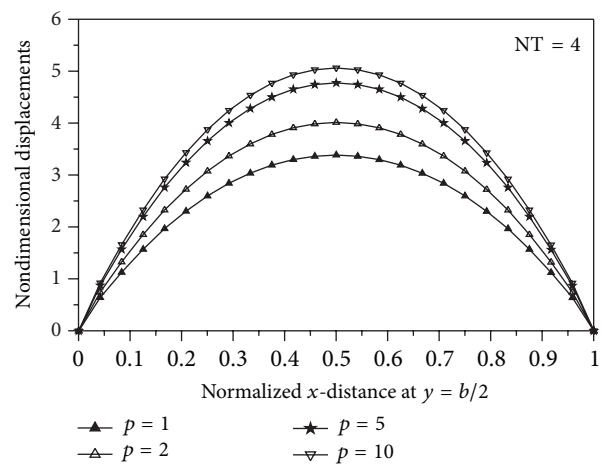

Figure 6: Effect of the power law index on the nondimensional displacement of a simply supported S-FGM plate with $a/h = 10$.

will decrease the stiffness of the S-FGM nanoscale plate, and consequently, leads to an enlargement of deflection.

The effect of the power law index p on the dimensionless deflection is presented in Figure 6 for a simply supported square plate with $\mu = 4.0$ and $a/h = 10$. The increasing value of the power law index decreases the stiffness of the S-FGM nanoscale plates. It can be seen that increasing value of the power law index leads to an increase in the magnitude of deflection. When the nonlocal parameter (NT) is 0, a nonlocal S-FGM nanoscale plate is treated as a local nanoplate.

To show the effect of E_1/E_2 of S-FGM nanoscale plate, Figure 7 plots the nondimensional displacement with $\mu = 4.0$, $p = 1.0$, and $a/h = 10$. In this case, E_1 is varied and E_2 is fixed. It shows that the more of E_1/E_2 brings the smaller deflection, because lager E_1/E_2 increases the stiffness of the S-FGM nanoscale plate.

It is shown that the effects of side-to-thickness ratio on the dimensionless deflection is presented in Figure 8 for a simply supported square plate with $\mu = 4.0$ and $a/h = 10$. Figure 8 clearly shows the diminishing effect of transverse shear deformation on deflections, the effect being negligible for side-to-thickness ratios larger than 30. The increasing value of the power law index leads to an increase in the magnitude of deflection and the increasing value of side-to-thickness ratio leads to a decrease in the deflection. As expected, the effect of transverse shear deformation is to increase deflection. The differences in deflection values predicted by the present model and the classical model are significant when the side-to-thickness ration is small, but they are negligible when the side-to-thickness ratio becomes larger.

Figure 9 shows that the effect of power law index on the dimensionless deflection is presented for a simply supported square plate with various nonlocal parameters. The increasing

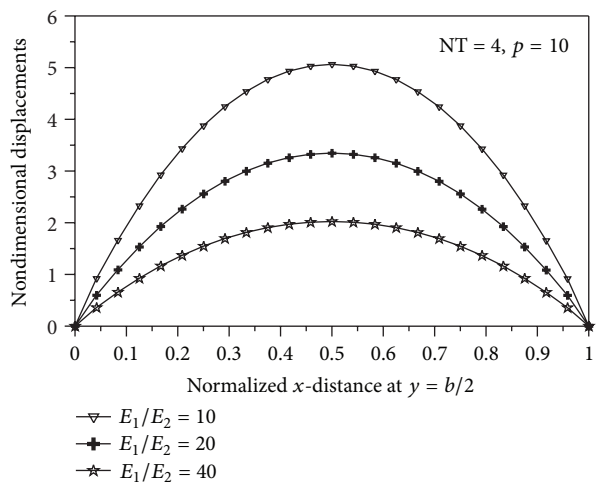

Figure 7: Effect of the E_1/E_2 ratio on the nondimensional displacement of a simply supported S-FGM plate with $a/h = 10$.

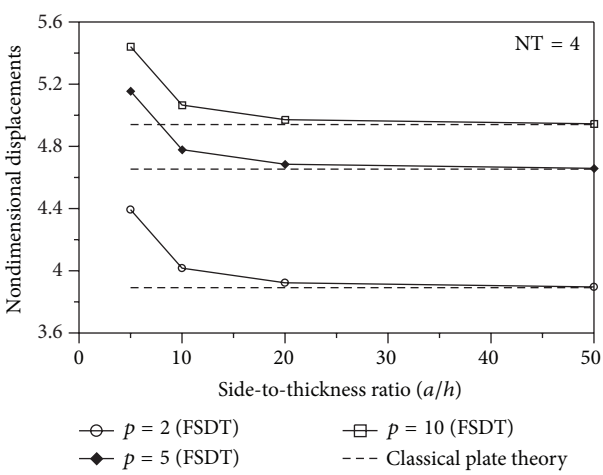

Figure 8: Effect of the side-to-thickness ratio on the nondimensional displacement of a simply supported S-FGM plate with $\mu = 4.0$.

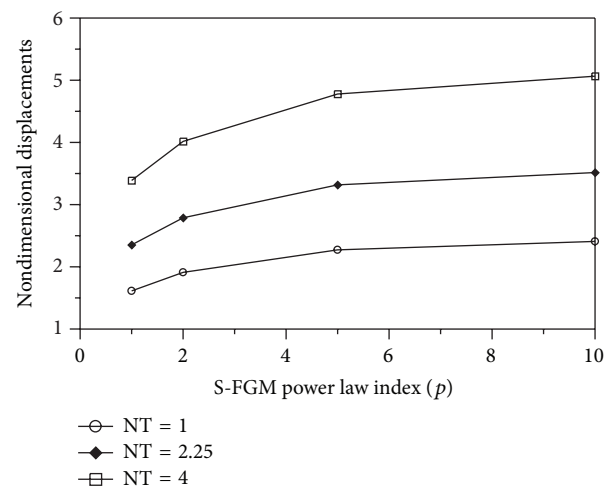

Figure 9: Effect of the power law index on the nondimensional displacement of a simply supported S-FGM plate with various nonlocal parameters.

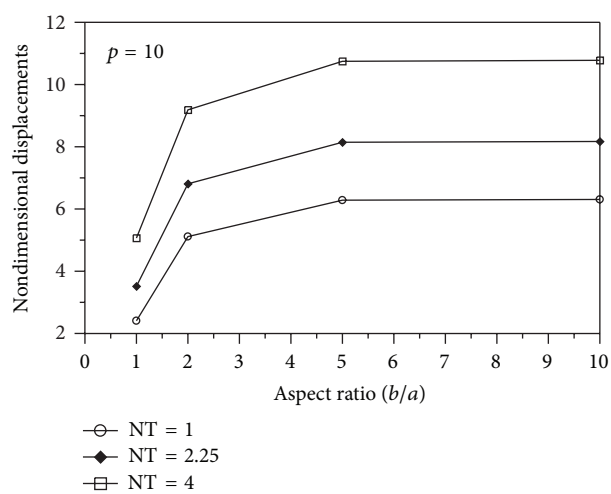

Figure 10: Effect of the aspect ratio on the nondimensional displacement of a simply supported S-FGM plate with various nonlocal parameters.

value of the power law index decreases the stiffness and the increasing value of the nonlocal parameter increases the load vector of the S-FGM nanoscale plates. The increasing value of the power law index leads to an increase in the magnitude of deflection and the increasing value of the nonlocal parameter leads to an increase in the deflection.

It is shown that the effect of the aspect ratio on the dimensionless deflection is presented in Figure 10 for a simply supported square plate with various nonlocal parameters and $p = 10.0$. The increasing values of the aspect ratio (b/a) and nonlocal parameter decrease the stiffness of the S-FGM nanoscale plates. When the width b of a nanoscale plate is very small compared to the length a, it is treated as a nanoscale beam. The increasing value of the aspect ratio leads to an increase in the magnitude of deflection and the increasing value of the nonlocal parameter leads to an increase in the deflection. The nondimensional center deflection increases upon raising the aspect ratio for b/a less than 5.

Figure 11 shows that the aspect ratio on the dimensionless deflection is presented for a simply supported square plate with various power law indexes and $\mu = 4.0$. As we expected, the increasing value of power law index decreases the stiffness of the S-FGM nanoscale plates. For this reason, it increases the deflection. The nondimensional center deflection increases upon raising the aspect ratio for b/a less than 5. When the aspect ratio (b/a) is greater than 5, a S-FGM nanoscale plate is treated as a nanobeam.

To illustrate the effect of the loading type on responses of S-FGM nanoscale plate, Figure 12 plots the deflection with respect to uniform and sinusoidal load for a simply supported S-FGM plate with $\mu = 4.0$, $p = 10.0$, and $a/h = 10$. The sinusoidal load on the S-FGM nanoscale plate is small in

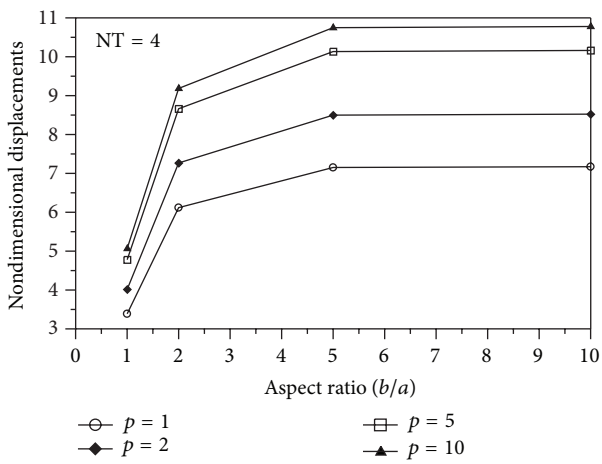

Figure 11: Effect of the aspect ratio on the nondimensional displacement of a simply supported S-FGM plate with various power law indexes.

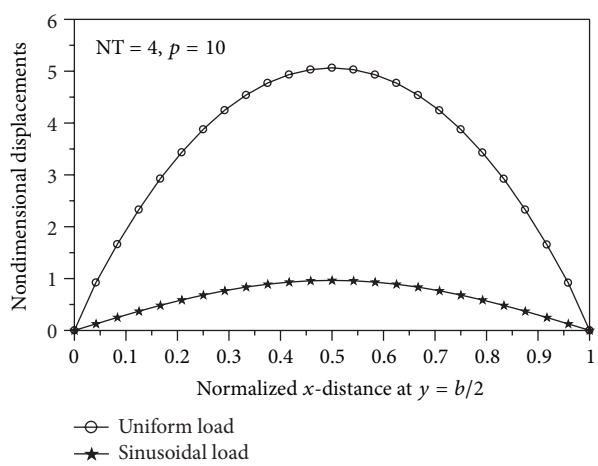

Figure 12: Effect of the loading type on the nondimensional displacement of a simply supported S-FGM plate with $a/h = 10$.

magnitude compared to the uniform load. As we expected, it is observed that the deflections investigated under the uniform load are larger than those investigated under the sinusoidal load. It may be noticed that the load-displacement curve under the sinusoidal load exhibits the value of 20% of the uniform load.

9. Conclusions

Nonlocal elasticity model for bending analysis of sigmoid functionally graded materials (S-FGM) nanoscale plates is presented using a first-order shear deformation theory and Hamilton's principle. The present models contain one nonlocal parameter and can capture the size effect, and two-constituent material variation through the plate thickness. Also, the present model can be reduced to the homogeneous nanoscale plates by setting $E_1 = E_2$ and S-FGM plates of local elasticity theory by setting the nonlocal parameter equal to zero. Analytical solutions for deflection of a simply supported rectangular S-FGM nanoscale plate are presented. The numerical results reveal that the inclusion of the small scale effect and power law index leads to an enlargement of the magnitude of deflection. The differences in deflection values predicted by the present model and the classical model are significant when the side-to-thickness ratio is small, but they are negligible when the side-to-thickness ratio becomes larger.

From the present work the following conclusions are drawn.

(1) The inclusion of the nonlocal scale effect and increasing value of the power law index will decrease stiffness of the S-FGM nanoscale plate, and consequently, leads to an enlargement of deflection.

(2) The increasing value of side-to-thickness ratio leads to a decrease in the deflection. As expected, the effect of transverse shear deformation is to increase deflection.

(3) The increasing value of the aspect ratio leads to an increase in the magnitude of deflection. The nondimensional center deflection increases upon raising the aspect ratio for b/a is less than 5.

(4) As expected, it is observed that the deflections investigated under the uniform load are larger than those investigated under the sinusoidal load.

These predicted trends agree with the size effect at the micron scale observed in experiments. These results can be used for evaluating the reliability of size-dependent plate models developed in the future. Further, in the analysis of S-FGM structures it is necessary to include the nonlocal elasticity theory for nanoscale shell and other boundary conditions.

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References

[1] P. Ball, "Roll up for the revolution," *Nature*, vol. 414, pp. 142–144, 2001.

[2] R. H. Baughman, A. A. Zakhidov, and W. A. de Heer, "Carbon nanotubes: the route toward applications," *Science*, vol. 297, no. 5582, pp. 787–792, 2002.

[3] B. H. Bodily and C. T. Sun, "Structural and equivalent continuum properties of single walled carbon nanotubes," *International Journal of Materials and Product Technology*, vol. 18, pp. 381–397, 2003.

[4] C. Li and T. W. Chou, "A structural mechanics approach for the analysis of carbon nanotubes," *International Journal of Solids and Structures*, vol. 40, pp. 2487–2499, 2003.

[5] C. Li and T. Chou, "Single-walled carbon nanotubes as ultrahigh frequency nanomechanical resonators," *Physical Review B*, vol. 68, no. 7, Article ID 073405, pp. 734051–734053, 2003.

[6] A. C. Eringen, "Nonlocal polar elastic continua," *International Journal of Engineering Science*, vol. 10, no. 1, pp. 1–16, 1972.

[7] A. C. Eringen and D. G. B. Edelen, "On nonlocal elasticity," *International Journal of Engineering Science*, vol. 10, no. 3, pp. 233–248, 1972.

[8] W. D. Nix and H. Gao, "Indentation size effects in crystalline materials: a law for strain gradient plasticity," *Journal of the Mechanics and Physics of Solids*, vol. 46, no. 3, pp. 411–425, 1998.

[9] A. R. Hadjesfandiari and G. F. Dargush, "Couple stress theory for solids," *International Journal of Solids and Structures*, vol. 48, no. 18, pp. 2496–2510, 2011.

[10] M. Asghari, M. H. Kahrobaiyan, and M. T. Ahmadian, "A nonlinear Timoshenko beam formulation based on the modified couple stress theory," *International Journal of Engineering Science*, vol. 48, no. 12, pp. 1749–1761, 2010.

[11] H. M. Ma, X. Gao, and J. N. Reddy, "A microstructure-dependent Timoshenko beam model based on a modified couple stress theory," *Journal of the Mechanics and Physics of Solids*, vol. 56, no. 12, pp. 3379–3391, 2008.

[12] J. N. Reddy, "Microstructure-dependent couple stress theories of functionally graded beams," *Journal of the Mechanics and Physics of Solids*, vol. 59, no. 11, pp. 2382–2399, 2011.

[13] A. C. Eringen, "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves," *Journal of Applied Physics*, vol. 54, no. 9, pp. 4703–4710, 1983.

[14] J. Peddieson, G. R. Buchanan, and R. P. McNitt, "Application of nonlocal continuum models to nanotechnology," *International Journal of Engineering Science*, vol. 41, no. 3-5, pp. 305–312, 2003.

[15] D. K. Jha, T. Kant, and R. K. Singh, "A critical review of recent research on functionally graded plates," *Composite Structures*, vol. 96, pp. 833–849, 2013.

[16] Y. L. Chung and S. H. Chi, "The residual stress of functionally graded materials," *Journal of the Chinese Institute of Civil and Hydraulic Engineering*, vol. 13, pp. 1–9, 2001.

[17] S. H. Chi and Y. L. Chung, "Mechanical behavior of functionally graded material plates under transverse load-Part II: numerical results," *International Journal of Solids and Structures*, vol. 43, pp. 3675–3691, 2006.

[18] S. Han, G. R. Lomboy, and K. Kim, "Mechanical vibration and buckling analysis of FGM plates and shells using a four-node quasi-conforming shell element," *International Journal of Structural Stability and Dynamics*, vol. 8, no. 2, pp. 203–229, 2008.

[19] S. Han, W. Lee, and W. Park, "Non-linear analysis of laminated composite and sigmoid functionally graded anisotropic structures using a higher-order shear deformable natural Lagrangian shell element," *Composite Structures*, vol. 89, no. 1, pp. 8–19, 2009.

[20] M. Aydogdu, "A general nonlocal beam theory: its application to nanobeam bending, buckling and vibration," *Physica E*, vol. 41, no. 9, pp. 1651–1655, 2009.

[21] Ö. Civalek and Ç. Demir, "Bending analysis of microtubules using nonlocal Euler-Bernoulli beam theory," *Applied Mathematical Modelling*, vol. 35, no. 5, pp. 2053–2067, 2011.

[22] J. N. Reddy, "Nonlocal theories for bending, buckling and vibration of beams," *International Journal of Engineering Science*, vol. 45, no. 2-8, pp. 288–307, 2007.

[23] J. N. Reddy and S. D. Pang, "Nonlocal continuum theories of beams for the analysis of carbon nanotubes," *Journal of Applied Physics*, vol. 103, no. 2, Article ID 023511, 2008.

[24] J. N. Reddy, "Nonlocal nonlinear formulations for bending of classical and shear deformation theories of beams and plates," *International Journal of Engineering Science*, vol. 48, no. 11, pp. 1507–1518, 2010.

[25] C. M. C. Roque, A. J. M. Ferreira, and J. N. Reddy, "Analysis of Timoshenko nanobeams with a nonlocal formulation and meshless method," *International Journal of Engineering Science*, vol. 49, no. 9, pp. 976–984, 2011.

[26] Q. Wang and K. M. Liew, "Application of nonlocal continuum mechanics to static analysis of micro- and nano-structures," *Physics Letters*, vol. 363, no. 3, pp. 236–242, 2007.

[27] C. M. Wang, S. Kitipornchai, C. W. Lim, and M. Eisenberger, "Beam bending solutions based on nonlocal Timoshenko beam theory," *Journal of Engineering Mechanics*, vol. 134, no. 6, pp. 475–481, 2008.

[28] Y. Fu, H. Du, and S. Zhang, "Functionally graded TiN/TiNi shape memory alloy films," *Materials Letters*, vol. 57, no. 20, pp. 2995–2999, 2003.

[29] C. Lu, D. Wu, and W. Chen, "Non-linear responses of nano-scale FGM films including the effects of surface energies," *IEEE Transactions on Nanotechnology*, vol. 10, no. 6, pp. 1321–1327, 2011.

[30] M. Rahaeifard, M. H. Kahrobaiyan, and M. T. Ahmadian, "Sensitivity analysis of atomic force microscope cantilever made of functionally graded materials," in *Proceedings of the 3rd International conference on micro- and nanosystems (DETC '09)*, pp. 539–544, September 2009.

[31] A. Witvrouw and A. Mehta, "The use of functionally graded poly-SiGe layers for MEMS applications," *Materials Science Forum*, vol. 492-493, pp. 255–260, 2005.

[32] Z. Lee, C. Ophus, L. M. Fischer et al., "Metallic NEMS components fabricated from nanocomposite Al-Mo films," *Nanotechnology*, vol. 17, no. 12, article 042, pp. 3063–3070, 2006.

[33] N. A. Fleck, G. M. Muller, M. F. Ashby, and J. W. Hutchinson, "Strain gradient plasticity: theory and experiment," *Acta Metallurgica et Materialia*, vol. 42, no. 2, pp. 475–487, 1994.

[34] J. S. Stolken and A. G. Evans, "A microbend test method for measuring the plasticity length scale," *Acta Materialia*, vol. 46, no. 14, pp. 5109–5115, 1998.

[35] A. C. M. Chong, F. Yang, D. C. C. Lam, and P. Tong, "Torsion and bending of micron-scaled structures," *Journal of Materials Research*, vol. 16, no. 04, pp. 1052–1058, 2001.

[36] D. C. C. Lam, F. Yang, A. C. M. Chong, J. Wang, and P. Tong, "Experiments and theory in strain gradient elasticity," *Journal of the Mechanics and Physics of Solids*, vol. 51, no. 8, pp. 1477–1508, 2003.

[37] J. N. Reddy, *Mechanics of Composite Plates and Shells: Theory and Analysis*, CRC Press, Boca Raton, Fla, USA, 2nd edition, 2004.

[38] H. T. Thai and D. H. Choi, "Size-dependent functionally graded Kirchhoff and Mindlin plate models based on a modified couple stress theory," *Composite Structures*, vol. 96, pp. 376–383, 2013.

[39] W. H. Lee, S. C. Han, and W. T. Park, "Nonlocal elasticity theory for bending and free vibration analysis of nano plates," *Journal of the Korea Academia-Industrial Cooperation Society*, vol. 13, no. 7, pp. 3207–3215, 2012 (Korean).

[40] Q. Wang and C. M. Wang, "The constitutive relation and small scale parameter of nonlocal continuum mechanics for modelling carbon nanotubes," *Nanotechnology*, vol. 18, no. 7, Article ID 075702, 2007.