Robust Adaptive Synchronization of the Energy Resource System with Constraint

Duo Meng

School of Civil and Architectural Engineering, Liaoning University of Technology, Jinzhou 121001, China

Correspondence should be addressed to Duo Meng; mengduo2004@163.com

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Two different chaos synchronization methods are proposed for a class of energy resource demand supply-system with input constraint. Firstly, chaotic synchronization is achieved for a class of energy resource demand supply system with known system parameters based on the Lyapunov theory. Secondly, an adaptive control approach is investigated for a class of energy resource demand supply system with input constraint, and the parameters of the system are unknown based on the Lyapunov stability and robust adaptive theory. To address the input constraint, new auxiliary signals and design systems are employed. Numerical simulations are provided to illustrate the effectiveness of the proposed approach.

1. Introduction

Energy resource system is a kind of complex nonlinear system. Energy resource including coal, petroleum, natural gas, water and electricity, and nuclear power can be classified as renewable energy and nonrenewable energy according to the capability of sustainable utilization. The issue of energy supply and demand has been valued worldwide with increasing development of economy. One of the most noticeable problems in the field of energy resource is how to study energy resource system deeply through nonlinear dynamics, which is currently a rapid developing method [1].

Reference [2] established a three-dimensional energy resource demand-supply system based on the real energy resources demand-supply in the East and the West of China. Furthermore, by adding a new variable to consider the renewable resources, a four-dimensional energy resource system was proposed in [3]. The dynamics behaviors of the four-dimensional energy resource system have been analyzed by means of the Lyapunov exponents and bifurcation diagrams. Also the same as the above-mentioned power systems, this four-dimensional energy resource system is with rich chaos behaviors. The problem of chaotic control for the energy resource system was considered in [1]. Feedback control and adaptive control methods were used to suppress chaos to unstable equilibrium or unstable periodic orbits, where only three of the system's parameters were supposed to be unknown. Reference [4] investigated the robust chaos synchronization problem for the four-dimensional energy resource systems based on the sliding mode control technique. The control of energy resource chaotic system was investigated by time delayed feedback control method in [5]. Based on stability criterion of linear system and Lyapunov stability theory, respectively, the chaos synchronization problems for energy resource demand-supply system were discussed using two novel different control methods in [6].

Although the adaptive synchronization control has achieved a great progress, the aforementioned control approaches assume that all the components of the considered energy resource demand-supply systems are in good operating conditions. As we know, many control systems have constraints on their inputs in the forms of input saturation or dead zone [7–17]. In practice, input saturation constraint is one of the most important input constraints which usually appear in many industries control systems. There are two main motivations for the saturation studies. One is that saturation is a potential problem for actuators of control systems. It often severely degrades the system performance, gives rise to undesirable inaccuracy, or even affects system stability. The other is that the control actions are usually limited in energy or magnitude; the saturation of the control output is necessary in practice.
Motivated by the above observations, two different chaos synchronization control methods are proposed for a class of energy resource demand-supply system with input constraint. Based on Lyapunov stability and robust adaptive theory, on the assumption that all the parameters of the system are known and unknown, nonadaptive and adaptive control approaches are proposed to make the states of two chaotic systems asymptotically synchronization. The main contributions of the proposed algorithm are that (i) the problems of the input constraint are solved by employing a new auxiliary system; (ii) the stability of the energy resource demand-supply system is guaranteed based on the Lyapunov theory.

2. Energy Resource Chaotic System

In the paper, we consider the following energy resource system (see [2, 6]):

\[
\begin{align*}
\dot{x} &= a_1 x \left(1 - \frac{x}{M}\right) - a_2 (y + z), \\
\dot{y} &= -b_1 y - b_2 z + b_3 x [N - (x - z)], \\
\dot{z} &= c_1 z (c_2 x - c_3),
\end{align*}
\]

(1)

where \(x(t)\) is the energy resource shortage in A region, \(y(t)\) is the energy resource supply increment in B region, and \(z(t)\) is the energy resource import in A region; \(M, N, a_i, b_j \ (i = 1, 2, j = 1, 2, 3)\) are parameters that are all positive real.

Similar to [6], when the system parameters are taken as the following values, this system exhibits chaotic behavior: \(M = 1.8, N = 1, a_1 = 0.09, a_2 = 0.15, b_1 = 0.06, b_2 = 0.082, b_3 = 0.07, c_1 = 0.2, c_2 = 0.5, \) and \(c_3 = 0.4\). Without the particular statement, these values are adopted in this whole paper. Figure 1 shows the phase portrait with initial conditions of \(x(0) = 0.82, y(0) = 0.29, \) and \(z(0) = 0.48\).

3. Synchronization of the Energy Resource System

In this section, we will design a controller in order to make the response system trace the drive system. In order to obtain synchronization of the energy resource system (1), the drive system with subscript 1 is written as

\[
\begin{align*}
\dot{x}_1 &= a_1 x_1 \left(1 - \frac{x_1}{M}\right) - a_2 (y_1 + z_1), \\
\dot{y}_1 &= -b_1 y_1 - b_2 z_1 + b_3 x_1 [N - (x_1 - z_1)], \\
\dot{z}_1 &= c_1 z_1 (c_2 x_1 - c_3).
\end{align*}
\]

(2)

The controlled response system with subscript 2 can be expressed as

\[
\begin{align*}
\dot{x}_2 &= a_1 x_2 \left(1 - \frac{x_2}{M}\right) - a_2 (y_2 + z_2) + u_1 (v_1 (t)), \\
\dot{y}_2 &= -b_1 y_2 - b_2 z_2 + b_3 x_2 [N - (x_2 - z_2)] + u_2 (v_2 (t)), \\
\dot{z}_2 &= c_1 z_2 (c_2 x_2 - c_3) + u_3 (v_3 (t)).
\end{align*}
\]

(3)

where \(v_i \ (i = 1, 2, 3)\) is the controller inputs to be designed, \(u_i (v_i (t)) \ (i = 1, 2, 3)\) denotes the plant input subject to saturation type nonlinearly.

Remark 1. If no input saturation (i.e., \(u_i (v_i (t)) = v_i (t) \ (i = 1, 2, 3)\) is included in (3), then (3) becomes the chaotic systems studied widely; see [2, 6].

\[
u_i (v_i (t)) \text{ is described by}
\begin{align*}
\begin{cases}
\text{sign} (v_i (t)) u_M, & |v_i (t)| \geq u_M, \\
|v_i (t)|, & |v_i (t)| < u_M,
\end{cases}
\end{align*}
\]

(4)

where \(u_M\) is a known bound of \(u_i (v_i (t))\). Clearly, the relationship between the applied control \(u_i (v_i (t))\) and the control input \(v_i (t)\) has a sharp corner when \(|v_i (t)| = u_M\). Similar to [15], define

\[
\begin{align*}
e_1 &= x_2 - x_1 - h_1, \\
e_2 &= y_2 - y_1 - h_2, \\
e_3 &= z_2 - z_1 - h_3,
\end{align*}
\]

(5)

where \(h_i \ (i = 1, 2, 3)\) are filter signals and will be given later.

By using (2), (3), and (5), the error dynamical system can be written as

\[
\begin{align*}
\dot{e}_1 &= -h_1 + a_1 e_1 - a_2 (e_2 + e_3) - \frac{a_1 x_2^2}{M} + \frac{a_1 x_1^2}{M} \\
&\quad + a_1 h_1 + a_2 (h_2 + h_3) + u_1 (v_1 (t)), \\
\dot{e}_2 &= -h_2 - b_1 e_2 - b_2 e_3 + b_3 N e_1 - b_3 x_2^2 \\
&\quad + b_3 x_1^2 + b_3 x_2 z_2 - b_3 x_1 z_1 + b_1 h_2 \\
&\quad + b_2 h_3 - b_3 N h_1 + u_2 (v_2 (t)), \\
\dot{e}_3 &= -h_3 - c_1 c_2 e_3 + c_1 c_2 x_2 z_2 \\
&\quad - c_1 c_2 x_1 z_1 + c_1 c_2 h_3 + u_3 (v_3 (t)).
\end{align*}
\]

(6)
Similar to [18, 19], choose Lyapunov function candidate $V$ as
\[ V = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2. \] (7)
The time derivative of $V$ is
\[ \dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3. \] (8)
Define the dynamic system as
\[ \dot{h}_i = -h_i + (u_i - v_i), \quad i = 1, 2, 3. \] (9)
Substituting (6) and (9) into (8) results in
\[ \dot{V} = e_1 \left[ h_1 + v_1 + a_1e_1 - a_2 (e_2 + e_3) - \frac{a_1 x_1^2}{M} + \frac{a_1 x_1^2}{M} + a_1 h_1 + a_2 (h_2 + h_3) \right] \\
+ e_2 \left[ h_2 + v_2 - b_1 e_2 - b_2 e_3 - b_3 N e_1 - b_3 x_1^2 + b_3 x_1 z_2 - b_2 x_1 z_1 + b_1 h_2 + b_2 h_3 - b_3 N h_1 + u_2 \right] \\
+ e_3 \left[ h_3 + v_3 - c_1 c_3 e_3 + c_1 c_2 x_2 z_2 - c_1 c_2 x_1 z_1 - c_1 c_3 h_3 \right]. \] (10)
By using Young’s inequality, we have
\[ -a_2 e_1 (e_2 + e_3) \leq a_2 e_1^2 + \frac{1}{2} e_2^2 + \frac{1}{2} e_3^2, \] (11)
\[ e_2 [-b_2 e_3 + b_3 N e_1] \leq \frac{b_3^2 + b_3^2 N^2}{2} e_1^2 + \frac{1}{2} e_3^2 + \frac{1}{2} e_1^2. \]
Substituting (11) into (10) results in
\[ \dot{V} \leq \left( a_1 + a_2^2 + \frac{1}{2} \right) e_1^2 + \left( h_1 + \frac{b_3^2 + b_3^2 N^2}{2} \right) e_2^2 \\
+ (1 - c_1 c_3) e_3^2 \\
+ e_1 \left[ h_1 + v_1 - \frac{a_1 x_2^2}{M} + \frac{a_1 x_2^2}{M} + a_1 h_1 + a_2 (h_2 + h_3) \right] \\
+ e_2 \left[ h_2 + v_2 - b_1 x_2^2 + b_2 x_2^2 + b_3 x_2 z_2 - b_2 x_1 z_1 + b_1 h_2 + b_2 h_3 - b_3 N h_1 \right] \\
+ e_3 \left[ h_3 + v_3 + c_1 c_2 x_2 z_2 - c_1 c_2 x_1 z_1 + c_1 c_3 h_3 \right]. \] (12)
Choose the actual controllers $v_i$
\[ v_1 = -l_1 e_1 - h_1 + \frac{a_1 x_2^2}{M} - \frac{a_1 x_1^2}{M} - a_1 h_1 - a_2 (h_2 + h_3), \]
\[ v_2 = -l_2 e_2 - h_2 + b_3 x_2^2 + b_3 x_2 z_2 + b_3 x_1 z_1 - b_1 h_2 - b_2 h_3 + b_3 N h_1, \]
\[ v_3 = -l_3 e_3 - h_3 - c_1 c_2 x_2 z_2 + c_1 c_2 x_1 z_1 - c_1 c_3 h_3. \] (13)
where $l_i$ ($i = 1, 2, 3$) are positive design parameters. Substituting (13) into (12) results in
\[ \dot{V} \leq - \left( l_1 - a_1 - a_2^2 - \frac{1}{2} \right) e_1^2 \\
- \left( l_2 - b_1 - \frac{1}{2} b_3^2 + b_3^2 N^2 \right) e_2^2 - (l_3 + c_1 c_3 - 1) e_3^2. \] (14)

Let $l_i$ satisfy $l_1 > a_1 + a_2^2 + 1/2, l_2 > b_1 + 1/2 + (b_3^2 + b_3^2 N^2)/2, \quad \text{and } l_3 + c_1 c_3 > 1.$ We can obtain $\dot{V} \leq 0$, and it is concluded that $e_1, e_2, e_3$ converge to zero as time $t$ tends to infinity. Therefore, the synchronization of response systems (3) and the drive system (2) is finally achieved.

**Remark 2.** It is noted that the controller in [6] was designed by using the stability criterion of linear system, not based on Lyapunov stability theory. However, this paper designed the control laws based on Lyapunov method. This paper can use a constructive way to obtain the control laws for this class of energy resource demand-supply system. In addition, the control laws’ design parameters of this paper have fewer restrictions compared to [6].

### 4. Simulation Results 1

In this section, the initial values of are chosen as $x_1(0) = 0.82$, $y_1(0) = 0.29$, $z_1(0) = 0.48$, $x_2(0) = 0.69$, $y_2(0) = -0.03$, and $z_2(0) = 1.25$. The saturation values are $u_{1M} = 0.5$, $u_{2M} = 2$, and $u_{3M} = 2$. The design parameters in controllers are chosen as $l_1 = 20, l_2 = 10, \quad \text{and } l_3 = 10.$ The simulation results are shown in Figures 2, 3, 4, and 5.

### 5. Adaptive Synchronization of the Energy Resource System

In this section, we assume that all the parameters of the energy resource system are unknown. For convenience, we define $a_3 = a_1/M, \quad b_3 = b_3 N, \quad d_1 = c_1 c_2, \quad \text{and } d_3 = c_1 c_3.$ The energy resource system (1) can be rewritten as:
\[ \dot{x} = a_1 x - a_2 (y + z) - a_3 x^2, \]
\[ \dot{y} = -b_1 y - b_2 z - b_3 x (x + z) + b_4 x, \]
\[ \dot{z} = d_1 x z - d_2 z \] (15)
and the drive system can be also rewritten as
\[ \dot{x}_1 = a_1 x_1 - a_2 (y_1 + z_1) - a_3 x_1^2, \]
\[ \dot{y}_1 = -b_1 y_1 - b_2 z_1 - b_3 x_1 (x_1 - z_1) + b_4 x_1, \]
\[ \dot{z}_1 = d_1 x_1 z_1 - d_2 z_1, \] (16)
The response system can be expressed as

\[
\begin{align*}
x_2 &= a_1 x_2 - a_2 (y_2 + z_2) - a_3 x_2^2 + u_1 (v_1 (t)), \\
\dot{y}_2 &= -b_1 y_2 - b_2 z_2 - b_3 x_2 (x_2 - z_2) + b_4 x_2 + u_2 (v_2 (t)), \\
\dot{z}_2 &= d_1 x_2 z_2 - d_2 z_2 + u_3 (v_3 (t)).
\end{align*}
\]  

(17)

Choose the same errors \( e_i \) as (5) and the same filter signals \( h_i \) as (9), and we have

\[
\begin{align*}
\dot{e}_1 &= - \hat{h}_1 + a_1 e_1 - a_2 (e_2 + e_3) - a_3 x_2^2 + a_4 x_3^2 \\
&+ a_1 h_1 + a_2 (h_2 + h_3) + u_1 (v_1 (t)), \\
\dot{e}_2 &= - \hat{h}_2 - b_1 e_2 - b_2 e_3 + b_3 e_1 - b_3 x_2^2 + b_4 x_2 z_2 \\
&- b_5 x_1 z_1 - b_6 h_2 + b_7 h_3 - b_8 h_4 + u_2 (v_2 (t)), \\
\dot{e}_3 &= - \hat{h}_3 - d_2 e_3 + d_1 x_2 z_2 - d_2 x_1 z_1 + d_3 h_3 + u_3 (v_3 (t)).
\end{align*}
\]  

(18)

Choose the Lyapunov function candidate \( \bar{V} \) as

\[
\bar{V} = \frac{1}{2} \left( e_1^2 + e_2^2 + e_3^2 + \tilde{a}_1^2 + \tilde{a}_2^2 + \tilde{a}_3^2 + \tilde{b}_1^2 + \tilde{b}_2^2 + \tilde{\alpha}_1^2 + \tilde{\alpha}_2^2 + \tilde{\alpha}_3^2 \right),
\]  

(19)

where \( \tilde{a}_i = a_i - \bar{a}_i \) \((i = 1, 2, 3)\), \( \tilde{b}_j = b_j - \bar{b}_j \) \((j = 1, 2, 3, 4)\), and \( \tilde{\alpha}_k = \bar{\alpha}_k - \hat{\alpha}_k \) \((k = 1, 2)\).

The time derivative of \( \bar{V} \) along with the solution of (18) is

\[
\begin{align*}
\dot{\bar{V}} &= e_1 \left[ h_1 + v_1 + \tilde{a}_1 e_1 - \tilde{a}_2 (e_2 + e_3) - \tilde{a}_3 x_2^2 \\
&+ \tilde{a}_1 h_1 + \tilde{a}_2 (h_2 + h_3) \right] \\
&+ e_2 \left[ h_2 + v_2 - \tilde{b}_1 e_2 - \tilde{b}_2 e_3 + \tilde{b}_4 e_1 - \tilde{b}_3 x_2^2 \\
&+ \tilde{b}_3 x_1 z_1 + \tilde{b}_4 h_2 + \tilde{b}_5 h_3 - \tilde{b}_6 h_4 \right] \\
&+ e_3 \left[ h_3 + v_3 - \tilde{d}_2 e_3 + \tilde{d}_1 x_2 z_2 - \tilde{d}_2 x_1 z_1 + \tilde{d}_3 h_3 \right]
\end{align*}
\]
Choose the actual controllers $v_i$ and update the laws of $\hat{a}_i$ ($i = 1, 2, 3$), $\hat{b}_j$ ($j = 1, 2, 3, 4$), and $\hat{d}_k$ ($k = 1, 2$) as follows:

$$v_1 = -\tilde{l}_1 e_1 - h_1 - \hat{a}_1 e_1 + \hat{a}_2 (e_2 + e_3) + \hat{a}_3 (e_3^2 + e_3 x_2^2 + e_1 h_1 + e_1 (h_2 + h_3) - \hat{a}_2)$$

$$v_2 = -\tilde{l}_2 e_2 - h_2 + \hat{b}_1 e_2 + \hat{b}_2 e_3 + \hat{b}_3 e_1 + \hat{b}_4 x_3^2 - \hat{b}_5 x_3^2$$

$$v_3 = -\tilde{l}_3 e_3 - h_3 + \hat{d}_1 e_3 - \hat{d}_2 x_2 z_2 + \hat{d}_3 x_2 z_1 - \hat{d}_4 h_3,$$

where $\tilde{l}_i$ ($i = 1, 2, 3$) are positive design parameters:

$$\hat{a}_1 = e_1 h_1 + e_1^2,$$

$$\hat{a}_2 = -(e_2 + e_3) e_1 + (h_2 + h_3) e_1,$$

$$\hat{a}_3 = -e_1 x_2^2 + e_1 x_1^2 + e_1 h_1 + e_1 (h_2 + h_3),$$

$$\hat{b}_1 = -e_2^2 + e_2 h_2,$$

$$\hat{b}_2 = -e_3 e_3 + e_3 h_3,$$

$$\hat{b}_3 = -e_2 x_2^2 + e_2 x_1^2 + e_2 x_2 z_2 - e_2 x_1 z_1,$$

$$\hat{b}_4 = e_1 e_2 - e_3 h_1,$$

$$\hat{d}_1 = e_3 x_2 z_2 + e_3 h_3,$$

$$\hat{d}_2 = -e_3^2 - e_3 x_1 z_1.$$  

Substituting (21) and (22) into (20) results in

$$\mathbf{V} = -\tilde{l}_i e_i^2 - \tilde{l}_j e_j^2 - \tilde{l}_k e_k^2.$$  

From (23), we can conclude that the states $x_2$, $y_2$, and $z_2$ of response system (17) and the states $x_1$, $y_1$, and $z_1$ of drive system (16) are ultimately synchronized asymptotically.
of the proposed algorithm are that (i) the problems of the input constraint have been solved by employing a new auxiliary system; (ii) the stability of the energy resource demand-supply system has been guaranteed based on Lyapunov theory. The results all have demonstrated the validity and feasibility of the proposed approaches.

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References


