Research Article

Evaluating the Efficiency of School Preceptors by Fuzzy Risk Analysis

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This paper presents a new method for evaluating the efficiency of school preceptors based on fuzzy number arithmetic operations. It uses fuzzy numbers to represent fuzzy grades. The fuzzy weights of criteria are automatically generated from the opinions of evaluators. The simplified fuzzy number arithmetic operations are used for calculating the average of fuzzy numbers. It can evaluate the efficiency of school preceptors in a more flexible and more intelligent manner.

1. Introduction

The ranking of fuzzy numbers has been a concern in fuzzy Multiple Attribute Decision Making since its inception. More than 20 fuzzy ranking indices have been proposed since 1976. Various techniques are applied to compare the fuzzy numbers. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [1]. Chen and Hwang [2] thoroughly reviewed the existing approaches and pointed out some illogical conditions that arise among them. Among the existing ranking methods, centroid index methods are extensively studied and applied to many decision making problems. Recently, S. J. Chen and S. M. Chen [3] point out the drawback of the existing centroid index ranking methods and proposed a new centroid index method for ranking fuzzy numbers based on the Center of Gravity (COG) point. They applied the COG based ranking method to a human selection problem based on Fuzzy Number Induced Order Weighted Averaging operator (2003). However, the COG based ranking method presented by S. J. Chen and S. M. Chen [3] still has some drawbacks, that is, it cannot correctly calculate the ranking order of the generalized fuzzy numbers in some situations. Also, Chang et al. [4] established the criteria for high school preceptor appraisal. They used fuzzy linguistic questionnaires to evaluate the efficiency of high school preceptors and they used fuzzy linguistic integrating techniques

to calculate preceptor's fuzzy grades, and then used Lee and Li's method to rank preceptors' fuzzy grades. Because Chang et al.'s methods have drawbacks in preceptors' appraisal, it is necessary to develop a new method to overcome these drawbacks.

In this paper, we propose a new centroid index method for ranking fuzzy numbers. First we briefly introduce some existing centroid index ranking methods of fuzzy numbers. Then we propose a new method to evaluate the efficiency of school preceptors based on fuzzy number arithmetic operations. The proposed method can overcome the drawbacks of the other methods.

2. The Centroid Formulae for Fuzzy Numbers

A fuzzy number is a convex fuzzy subset of the real line R and is completely defined by its membership function. Let A be a fuzzy number, whose membership function $f_A(x)$ can generally be defined as [5–9],

$$f_A(x) = \begin{cases} f_A^L(x) & \text{when } a_1 \leq x \leq a_2, \\ \omega & \text{when } a_2 \leq x \leq a_3, \\ f_A^R(x) & \text{when } a_3 \leq x \leq a_4, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where $0 \leq \omega \leq 1$ is a constant, $f_A^L : [a_1, a_2] \rightarrow [0, \omega]$ and $f_A^R : [a_3, a_4] \rightarrow [0, \omega]$ are two strictly monotonically and continuous mappings from R to closed interval $[0, \omega]$. When $\omega = 1$, then A is a normal fuzzy number; otherwise, it is said to be a nonnormal fuzzy number. If the membership function $f_A(x)$ is piecewise linear, then A is referred to as a trapezoidal fuzzy number and is usually denoted by, $A = (a_1, a_2, a_3, a_4; \omega)$. In particular, if $a_2 = a_3$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number. Since $f_A^L(x)$ and $f_A^R(x)$ are both strictly monotonically and continuous functions, their inverse functions exist and should also be continuous and strictly monotonically. Let $g_A^L : [0, \omega] \rightarrow [a_1, a_2]$ and $g_A^R : [0, \omega] \rightarrow [a_3, a_4]$ be the inverse functions of f_A^L and f_A^R , respectively. Then $g_A^L(y)$ and $g_A^R(y)$ should be integrable on the closed interval $[0, \omega]$. In other words, both $\int_0^\omega g_A^L(y) dy$ and $\int_0^\omega g_A^R(y) dy$ should exist. In the case of trapezoidal fuzzy number, the inverse function $g_A^L(y)$ and $g_A^R(y)$ can be analytically expressed as

$$\begin{aligned} g_A^L(y) &= a_1 + \frac{(a_2 - a_1)y}{\omega}, \quad 0 \leq y \leq \omega, \\ g_A^R(y) &= a_4 - \frac{(a_4 - a_3)y}{\omega}, \quad 0 \leq y \leq \omega. \end{aligned} \quad (2)$$

In order to determine the centroid point $(\bar{x}_0, \bar{y}_0)$ of a fuzzy number A , Wang et al. [10] provided the following centroid formulae:

$$\begin{aligned} \bar{x}_0(A) &= \frac{\int_{a_1}^{a_2} x f_A^L(x) dx + \int_{a_2}^{a_3} (x\omega) dx + \int_{a_3}^{a_4} x f_A^R(x) dx}{\int_{a_1}^{a_2} f_A^L(x) dx + \int_{a_2}^{a_3} (\omega) dx + \int_{a_3}^{a_4} f_A^R(x) dx}, \\ \bar{y}_0(A) &= \frac{\int_0^\omega y (g_A^R(y) - g_A^L(y)) dy}{\int_0^\omega (g_A^R(y) - g_A^L(y)) dy}. \end{aligned} \quad (3)$$

The ranking value $R(A)$ of the fuzzy number A is defined as follows [11]:

$$R(A) = \sqrt{\bar{x}_0^2(A) + \bar{y}_0^2(A)}. \quad (4)$$

The larger the value $R(A)$, the better the ranking of A .

In [12], the authors presented a centroid-index ranking method for ordering fuzzy numbers. The centroid point of a fuzzy number A is $(\bar{x}_A, \bar{y}_A)$, where $\bar{x}_A$ and $\bar{y}_A$ are the same as formula (2.2) and (2.3) in [12]. The ranking value $S(A)$ of the fuzzy number A is defined as follows:

$$S(A) = \bar{x}_A \times \bar{y}_A. \quad (5)$$

The larger the value $S(A)$, the better the ranking of A . In [3], S. J. Chen and S. M. Chen proposed a simple method to obtain COG point of fuzzy numbers. If A is a generalized

FIGURE 1

fuzzy number, where $A = (a_1, a_2, a_3, a_4; \omega)$, then the COG point (x_A^*, y_A^*) of A is as follows:

$$\begin{aligned} x_A^* &= \frac{y_A^* (a_2 + a_3) + (a_1 + a_4)(1 - y_A^*)}{2}, \\ y_A^* &= \begin{cases} \frac{\omega((a_3 - a_2)/(a_4 - a_1) + 2)}{6}, & a_1 \neq a_4, \\ \frac{1}{2}, & a_1 = a_4. \end{cases} \end{aligned} \quad (6)$$

After obtaining the COG point of fuzzy number A , where $A = (a_1, a_2, a_3, a_4; \omega_A)$, the ranking value $\text{Rank}(A)$ can be calculated as

$$\text{Rank}(A) = x_A^* + (w_A - y_A^*)^{\hat{s}_A} \times (y_A^* + 0.5)^{1-w_A}, \quad (7)$$

where

$$\begin{aligned} \hat{s}_A &= \sqrt{\frac{\sum_{i=1}^4 (a_i - \bar{a})^2}{3}}, \\ \bar{a} &= \frac{a_1 + a_2 + a_3 + a_4}{4}. \end{aligned} \quad (8)$$

The larger the value $\text{Rank}(A)$, the better the ranking A . However, this method has a drawback in that it cannot correctly rank generalized fuzzy numbers in some situations. The example is used to show the drawback Chen's method.

Example 1. Two generalized fuzzy number A and B are shown as follows (Figure 1):

$$\begin{aligned} A &= (-0.01, -0.01, -0.01, -0.01; 1), \\ B &= (0.01, 0.01, 0.01, 0.01; 0.8). \end{aligned} \quad (9)$$

It can be easily to obtain the COG points of fuzzy numbers A and B , respectively, as follows, $(x_A^*, y_A^*) = (-0.01, 0.5)$ and $(x_B^*, y_B^*) = (0.01, 0.4)$. By applying Chen method, we have $R(A) = 0.99$ and $R(B) = 0.989$. The ranking result shows that ranking order is $A > B$. However, it can be easily seen that the correct order is $A < B$.

To compensate for these shortcomings, a new index of ranking fuzzy numbers is constructed in this article.

3. A New Ranking Method for Generalized Fuzzy Numbers

In this section, the authors propose a novel technique for ranking of fuzzy numbers based on the distance method. The new method not only considers the centroid point of fuzzy numbers, but also considers the min crisp value of a fuzzy number.

Assume that there are n generalized fuzzy numbers $A_1, A_2, \dots, A_n$, where $A_j = (a_{1j}, a_{2j}, a_{3j}, a_{4j}; w_{A_j})$, $1 \leq j \leq n$, $0 < w_{A_j} \leq 1$, $0 \leq a_{1j} \leq a_{2j} \leq a_{3j} \leq a_{4j} \leq k$, k is any real value. The value w_{A_j} denotes the maximum membership value of the generalized trapezoidal fuzzy number A_j , where $1 \leq j \leq n$. The proposed method for ranking fuzzy numbers $A_1, A_2, \dots, A_n$ is now presented as follows.

Step 1. For each generalized trapezoidal fuzzy number A_j , where $1 \leq j \leq n$, if the fuzzy number A_j is not a standardized generalized trapezoidal fuzzy number, where the universe of discourse of the generalized trapezoidal fuzzy number A_j is $[0, k]$, then translate the generalized trapezoidal fuzzy number $A_j = (a_{1j}, a_{2j}, a_{3j}, a_{4j}; w_{A_j})$ into a standardized generalized trapezoidal fuzzy number A_j^* shown as follows:

$$A_j^* = \left(\frac{a_{1j}}{k}, \frac{a_{2j}}{k}, \frac{a_{3j}}{k}, \frac{a_{4j}}{k}; w_{A_j^*} \right) = (a_{1j}^*, a_{2j}^*, a_{3j}^*, a_{4j}^*; w_{A_j^*}), \quad (10)$$

where $w_{A_j} = w_{A_j^*}$, $0 < w_{A_j^*} \leq 1$, $0 \leq a_{1j}^* \leq a_{2j}^* \leq a_{3j}^* \leq a_{4j}^* \leq 1$ or $-1 \leq a_{1j}^* \leq a_{2j}^* \leq a_{3j}^* \leq a_{4j}^* \leq 1$ and $1 \leq j \leq n$.

Step 2. Use formula (6) to calculate the centroid point $(x_{A_j^*}^*, y_{A_j^*}^*)$ of each standardized generalized trapezoidal fuzzy number A_j^* , where $1 \leq j \leq n$.

Step 3. Calculate the standard deviation $s_{A_j^*}$ of each standardized generalized trapezoidal fuzzy number A_j^* from (8). The standard deviation $s_{A_j^*}$ indicates the degree of dispersion of the standardized generalized trapezoidal fuzzy number A_j^* , where $1 \leq j \leq n$.

Step 4. Use the standard deviation $s_{A_j^*}$ and $y_{A_j^*}^*$ of the centroid point $(x_{A_j^*}^*, y_{A_j^*}^*)$ to derive a new value $y_{A_j^*}^{*\hat{s}}$ shown as follows:

$$y_{A_j^*}^{*\hat{s}} = (w_{A_j^*} - y_{A_j^*}^*)^{s_{A_j^*}} \times (y_{A_j^*}^* + 0.5)^{1-w_{A_j^*}}, \quad (11)$$

where $1 \leq j \leq n$, to obtain a new point $(x_{A_j^*}^*, y_{A_j^*}^{*\hat{s}})$.

Step 5. Use the new point $(x_{A_j^*}^*, y_{A_j^*}^{*\hat{s}})$ to calculate the ranking value $S(A_j^*)$ of the fuzzy numbers A_j^* as follows:

$$\begin{aligned} S(A_j^*) &= \left[(x_{A_j^*}^* - a_{\min})^2 + (y_{A_j^*}^{*\hat{s}} - 0)^2 \right]^{1/2} \\ &= \left[(x_{A_j^*}^* - a_{\min})^2 + (y_{A_j^*}^{*\hat{s}})^2 \right]^{1/2}, \end{aligned} \quad (12)$$

where $a_{\min}$ denotes the minimum value of the a_{1j}, a_{2j}, a_{3j} and a_{4j} , and $1 \leq j \leq n$. From formula (12), we can see that $S(A_j^*)$ can be considered as the Euclidean distance between the point $(x_{A_j^*}^*, y_{A_j^*}^{*\hat{s}})$ and the point $(a_{\min}, 0)$. We can say that the larger the value of $S(A_j^*)$, the better the ranking of A_j^* , where $1 \leq j \leq n$.

Example 2. As shown above, two generalized fuzzy numbers are shown as follows:

$$\begin{aligned} A &= (-0.01, -0.01, -0.01, -0.01; 1), \\ B &= (0.01, 0.01, 0.01, 0.01; 0.8). \end{aligned} \quad (13)$$

First, the COG points of the fuzzy numbers A and B can be obtained as follows, respectively.

$$\begin{aligned} (x_{A^*}^*, y_{A^*}^*) &= (-0.01, 0.5), \\ (x_{B^*}^*, y_{B^*}^*) &= (0.01, 0.4). \end{aligned} \quad (14)$$

Then, translate the COG points of fuzzy numbers A and B into Index point. The results can be shown as follows:

$$\begin{aligned} x_{A^*}^* &= -0.01, \\ y_{A^*}^{*s} &= (w_{A^*} - y_{A^*}^*)^{s_{A^*}} \times (y_{A^*}^* + 0.5)^{1-w_{A^*}} = 1, \end{aligned} \quad (15)$$

$$\begin{aligned} x_{B^*}^* &= 0.01, \\ y_{B^*}^{*s} &= (w_{B^*} - y_{B^*}^*)^{s_{B^*}} \times (y_{B^*}^* + 0.5)^{1-w_{B^*}} = 1. \end{aligned} \quad (16)$$

The ranking value of fuzzy numbers A and B can be calculated, respectively, as follows:

$$S(A) = 1, \quad S(B) = 1.0002. \quad (17)$$

The ranking order of fuzzy numbers A and B is $B > A$. It can be seen that the proposed method can overcome the drawback of the one presented in [12].

4. A Method for High School Preceptors Evaluation

In this section, we present a new method for high school preceptors appraisal, where the criteria and the fuzzy linguistic questionnaires used for preceptor appraisal in [3] are adopted as shown in Tables 1 and 2, respectively. In the fuzzy linguistic questionnaire shown in Table 3, Saneifard et al. used five linguistic levels to evaluate teachers' performance regarding the subcriterion X_{ij} , that is, very low (VL), low (L), middle (M), high (H) and very high (VH). They used the triangular fuzzy numbers $\tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}$, and $\tilde{5}$ corresponding to the linguistic satisfaction levels VL, L, M, H, and VH, respectively, where $\tilde{1} = (0, 1, 2)$, $\tilde{2} = (1, 2, 3)$, $\tilde{3} = (2, 3, 4)$, $\tilde{4} = (3, 4, 5)$, and $\tilde{5} = (4, 5, 6)$. The fuzzy grade $G(x_{ij})$ of the subcriterion X_{ij} is calculated as follows:

$$G(x_{ij}) = \frac{\sum_{k=1}^5 x_{ijk} f(x_{ijk})}{\sum_{k=1}^5 f(x_{ijk})}, \quad (18)$$

TABLE 1: The criteria for evaluating the efficiency of high school preceptors.

Criteria	Subcriteria	Contents
X_1 (teaching)	X_{11}	According to the standard of the course and the needs of the students to achieve the teaching goals
	X_{12}	To make teaching plans and to prepare teaching materials and tools properly
	X_{13}	To present teaching materials clearly and correctly with proper methods
	X_{14}	To bring the motivation of learning and testing and to increase students' attention on lessons
	X_{15}	To express clearly in speaking (helpful for comprehension and learning)
	X_{16}	To express clearly in writing (helpful for comprehension and learning)
	X_{17}	To make good use of various teaching methods and teaching media
	X_{18}	To make good use of time while teaching
	X_{19}	Giving proper feedback to students while teaching
X_2 (class managing)	X_{21}	Counseling students to obey the rules of life and develop good habits
	X_{22}	To direct students to keep personal and public sanitation
	X_{23}	Management of classroom
	X_{24}	Establishing an appropriate rule of class
	X_{25}	To praise and punish students properly
X_3 (professional attitude)	X_{31}	To take care of students actively
	X_{32}	Working hard
	X_{33}	Researching and pursuing further education
	X_{34}	Being enthusiastic with working
X_4 (cooperation in administration)	X_{41}	To build various kinds of students' files
	X_{42}	To join activities of school actively
	X_{43}	To keep good relationship with coworkers in school
	X_{44}	To keep good relationship and interactions with students' parents
	X_{45}	Treating students fairly
	X_{46}	To keep good relationship with students
X_5 (ability to use computers)	X_{51}	To be able to send and receive email
	X_{52}	To be able to design a web page interacting with students
	X_{53}	To be able to use word edit software to edit handouts and test papers
	X_{54}	To be able to search data on the web to be used in teaching

where i denotes the index of the criterion, j denotes the index of the subcriterion, k denotes the index of linguistic levels, x_{ijk} denotes the k th linguistic satisfaction level of the criterion X_{ij} , $1 \leq k \leq 5$, $x_{ijk} \in \{\tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}, \tilde{5}\}$, $f(x_{ijk})$ denotes the degree that the teacher satisfies the k th satisfaction level of the criterion X_{ij} , and $\sum_{k=1}^5 f(x_{ijk}) = 1$. Saneifard et al. also used fuzzy linguistic questionnaires to determine the weight of a

criterion. A fuzzy linguistic questionnaire for determining the weights of a criterion is shown in Table 3. The fuzzy weight $W(x_i)$ of the criterion X_i is calculated as follows:

$$W(x_j) = \frac{\sum_{k=1}^5 x_{ik} f(x_{ik})}{\sum_{k=1}^5 f(x_{ik})}, \quad (19)$$

TABLE 2: A fuzzy linguistic questionnaire for evaluating the efficiency of a preceptor.

Fuzzy linguistic questionnaire—satisfaction levels of the criterion X_{ij}				
Low				High
←				→
VL	L	M	H	VH
0%	0%	0%	60%	40%

TABLE 3: A fuzzy linguistic questionnaire for determining the weight of the criterion X_i .

Fuzzy linguistic questionnaire—Importance levels of the criterion X_i				
Low				High
←				→
VL	L	M	H	VH
0%	0%	0%	60%	40%

where i denotes the index of the criterion, k denotes the index of linguistic levels, x_{ik} denotes the k th importance level of the criterion X_i , $1 \leq k \leq 5$, $x_{ik} \in \{\bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$, $f(x_{ik})$ denotes the degree of percentage that the criterion X_i satisfies the k th importance level, and $\sum_{k=1}^5 f(x_{ik}) = 1$. The proposed method for high school preceptors appraisal is now presented as follows.

Step 1. Determine the fuzzy weight of each criterion. Let each evaluator use the fuzzy linguistic questionnaire shown in Table 3 to evaluate the importance of each criterion. Then, use (19) to calculate the fuzzy weight of each criterion evaluated by each evaluator, where a fuzzy weight represented by a triangular fuzzy number (k, m, n) should satisfy the rules.

If $k < 1$, then let $k = 1$; if $n > 5$, then let $n = 5$. For example, assume that there are five evaluators E_1, E_2, E_3, E_4, E_5 and nine preceptors $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9$ being evaluated. After the five evaluators fill out the fuzzy linguistic questionnaires, we get the result shown in Table 4. After applying (19) to calculate the fuzzy weight of each criterion evaluated by each evaluator, we get the calculating results as shown in Table 5. Based on (12), we rank the fuzzy weights with respect to each criterion X_i evaluated by the evaluators, where $1 \leq i \leq 5$, we drop the fuzzy weights with the smallest ranking value and the largest ranking value. Then, we calculate the average of the remaining fuzzy weights using the addition operations and the division operations of fuzzy numbers. Finally, we can get the fuzzy weighted vector W , where $W = [w_1 \ w_2 \ w_3 \ w_4 \ w_5]^t$, w_i denotes the average of the remaining fuzzy weights with respect to the i th criterion, and $1 \leq i \leq 5$.

For example, let us consider the first row of Table 6. Based on (12), we can see that $(3.3, 4.3, 5) > (3.2, 4.2, 5) > (3, 4, 5) = (3, 4, 5) > (2.3, 3.3, 4.3)$. Because the fuzzy weights $(3.3, 4.3, 5)$ and $(2.3, 3.3, 4.3)$ have the largest ranking value and smallest ranking value, respectively, we drop the fuzzy weights $(3.3, 4.3, 5)$ and $(2.3, 3.3, 4.3)$, and then we can get

the fuzzy weight $\bar{w}_1$ of criterion X_1 by calculating the average of the remaining fuzzy weights, shown as follows:

$$\begin{aligned} w_1 &= [(3.2, 4.2, 5) \oplus (3, 4, 5) \oplus (3, 4, 5)] \oslash 3 \\ &= [(3.2, 4.2, 5) \oplus (3, 4, 5) \oplus (3, 4, 5)] \oslash (3, 3, 3) \quad (20) \\ &= (3.07, 4.07, 5). \end{aligned}$$

Therefore, the fuzzy weight of criterion X_1 is w_1 , where $w_1 = (3.07, 4.07, 5)$. Now we consider the second row of Table 6. Based on (12), we can see that $(3.75, 4.75, 5) > (3.6, 4.6, 5) > (3.1, 4.1, 5) > (2.9, 3.9, 4.9) > (2.8, 3.8, 4.8)$. Because the fuzzy weights $(3.75, 4.75, 5)$ and $(2.8, 3.8, 4.8)$ have the largest ranking value and smallest ranking value, respectively, we drop the fuzzy weights $(3.75, 4.75, 5)$ and $(2.8, 3.8, 4.8)$, and then we can get the fuzzy weight w_2 of criterion X_2 by calculating the average of the remaining fuzzy weights, shown as follows:

$$\begin{aligned} w_2 &= [(3.6, 4.6, 5) \oplus (3.1, 4.1, 5) \oplus (2.9, 3.9, 4.9)] \\ &\quad \oslash 3 [(3.6, 4.6, 5) \oplus (3.1, 4.1, 5) \oplus (2.9, 3.9, 4.9)] \\ &\quad \oslash (3, 3, 3) \quad (21) \\ &= (3.2, 4.2, 4.97). \end{aligned}$$

In the same way, we can get the weights w_3, w_4 , and w_5 of the criteria X_3, X_4 , and X_5 , respectively, where $w_3 = (2.87, 3.87, 4.87)$, $w_4 = (2.98, 3.98, 4.97)$, and $w_5 = (2.9, 3.9, 4.9)$. In this situation, the fuzzy weighted vector W is as follows:

$$W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} = \begin{bmatrix} (3.07, 4.07, 5.00) \\ (3.20, 4.20, 4.97) \\ (2.87, 3.87, 4.87) \\ (2.98, 3.98, 4.97) \\ (2.90, 3.90, 4.90) \end{bmatrix}. \quad (22)$$

Step 2. Establish the fuzzy grade matrix. Each evaluator uses the fuzzy linguistic questionnaire shown in Table 3 to evaluate the performance of high school preceptors. Then, use (18) to calculate the fuzzy grade of each subcriterion of each preceptor evaluated by each evaluator, where the fuzzy grade represented by the fuzzy number (k, m, n) should satisfy the rules: if $k < 1$, then let $k = 1$; if $n > 5$, then let $n = 5$. For example, assume that the fuzzy grades evaluated by the five evaluators are as shown in Tables 6, 7, 8, 9, and 10, respectively. Based on (7), we rank the fuzzy grades of a teacher with respect to a sub-criterion evaluated by the evaluators, drop the fuzzy grades with the smallest ranking value and the largest ranking value, and then calculate the average of the remaining fuzzy grades by using the addition operations and the division operations of fuzzy numbers to get the averaged fuzzy grade of the sub-criterion. In the same way we can get the averaged fuzzy grade of each sub-criterion of each teacher. For example, based on Tables 6, 7, 8, 9, and 10 and equation (12), we can rank the fuzzy grades of A_1 with respect to criterion X_{11} evaluated by each evaluator, where the result is as follows: $(3.7, 4.7, 5) > (2.8, 3.8, 4.8) > (2.7, 3.7, 4.7) > (2.5, 3.5, 4.5) = (2.5, 3.5, 4.5)$. Because the fuzzy weights $(3.7, 4.7, 5)$ and $(2.5, 3.5, 4.5)$ have the largest ranking value

TABLE 4: The result after five evaluators till out the fuzzy linguistic questionnaires.

Criterion	Evaluator E_1					Evaluator E_2					Evaluator E_3					Evaluator E_4					Evaluator E_5				
	VL	L	M	H	VH	VL	L	M	H	VH	VL	L	M	H	VH	VL	L	M	H	VH	VL	L	M	H	VH
X_1	0	0	0	80	20	0	0	10	80	10	0	0	20	60	20	0	0	0	70	30	0	0	80	10	10
X_2	0	0	10	70	20	0	0	0	40	60	0	0	30	60	10	0	0	20	70	10	0	0	5	15	80
X_3	0	20	80	0	0	0	0	20	70	10	0	0	0	70	30	0	0	20	70	10	0	0	20	80	0
X_4	0	0	20	60	20	0	0	5	85	10	0	0	20	70	10	0	0	10	75	15	0	0	45	55	0
X_5	0	0	30	50	20	0	0	40	40	20	0	0	20	70	10	0	0	20	30	50	0	0	20	70	10

TABLE 5: The fuzzy weight of each criterion evaluated by each evaluator.

Criterion	Evaluator				
	E_1	E_2	E_3	E_4	E_5
X_1	(3.2, 4.2, 5)	(3, 4, 5)	(3, 4, 5)	(3.3, 4.3, 5)	(2.3, 3.3, 4.3)
X_2	(3.1, 4.1, 5)	(3.6, 4.6, 5)	(2.8, 3.8, 4.8)	(2.9, 3.9, 4.9)	(3.75, 4.75, 5)
X_3	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(3.3, 4.3, 5)	(2.9, 3.9, 9)	(2.8, 3.8, 4.8)
X_4	(3, 4, 5)	(3.05, 4.05, 5)	(2.9, 3.9, 4.9)	(3.05, 4.05, 5)	(2.55, 3.55, 4.55)
X_5	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)	(2.9, 3.9, 4.9)	(3.3, 4.3, 5)	(2.9, 3.9, 4.9)

and smallest ranking value, respectively, we drop the fuzzy weights (3.7, 4.7, 5) and (2.5, 3.5, 4.5), and then we can get the fuzzy grade $g_{11,1}$ of preceptor A_1 with respect to sub-criterion X_{11} by calculating the average of the remaining fuzzy weights, shown as follows:

$$\begin{aligned}
 g_{11,1} &= [(2.8, 3.8, 4.8) \oplus (2.7, 3.7, 4.7) \oplus (2.5, 3.5, 4.5)] \oslash 3 \\
 &= [(2.8, 3.8, 4.8) \oplus (2.7, 3.7, 4.7) \oplus (2.5, 3.5, 4.5)] \\
 &\oslash (3, 3, 3) = (2.67, 3.67, 4.67).
 \end{aligned} \quad (23)$$

Therefore, the fuzzy grade of preceptor A_1 with respect to sub-criterion X_{11} is $g_{11,1}$, where $g_{11,1} = (2.67, 3.67, 4.67)$. Then, based on (7), we can rank the fuzzy grades of preceptor A_2 with respect to criterion X_{12} evaluated by each evaluator. The result is shown as follows: $(3.7, 4.7, 5) > (3.2, 4.2, 5) > (2.6, 3.6, 4.6) = (2.6, 3.6, 4.6) > (2.3, 3.3, 4.3)$. Because the fuzzy weights (3.7, 4.7, 5) and (2.3, 3.3, 4.3) have the largest ranking value and smallest ranking value, respectively, we drop the fuzzy weights (3.7, 4.7, 5) and (2.3, 3.3, 4.3), and then we can get the fuzzy grade $g_{12,1}$ of preceptor A_1 with respect to sub-criterion X_{12} by calculating the average of the remaining fuzzy weights shown as follows:

$$\begin{aligned}
 g_{12,1} &= [(3.2, 4.2, 5) \oplus (2.6, 3.6, 4.6) \oplus (2.6, 3.6, 4.6)] \oslash 3 \\
 &= [(3.2, 4.2, 5) \oplus (2.6, 3.6, 4.6) (2.6, 3.6, 4.6)] \oslash (3, 3, 3) \\
 &= (2.8, 3.8, 4.73).
 \end{aligned} \quad (24)$$

In the same way, we can get the fuzzy grade of each teacher with respect to each sub-criterion to derive the averaged

fuzzygrade table shown in Table 11. Then, we apply the simplified fuzzy numbers addition operations and division operations to the averaged fuzzy grade table shown in Table 11 to get the fuzzy grade of each teacher with respect to each criterion. Finally, we can get the fuzzy grade matrix G defined as follows:

$$G = \begin{bmatrix} \frac{A}{X} & X_1 & X_2 & \cdots & X_k \\ A_1 & g_{11} & g_{12} & \cdots & g_{1k} \\ A_2 & g_{21} & g_{22} & \cdots & g_{2k} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ A_n & g_{n1} & g_{n2} & \cdots & g_{nk} \end{bmatrix}, \quad (25)$$

where g_{ij} denotes the fuzzy grade of the i th preceptor A_i with respect to the j th criterion X_j , where $1 \leq i \leq n$, $1 \leq j \leq k$ and n denotes the number of preceptors, and k denotes the number of criteria. For example, based on Table 11, we can calculate g_{11} as follows:

$$\begin{aligned}
 g_{11} &= \left(\frac{(2.67 + 2.8 + 2.97 + 2.33 + 2.72 + 2.38 + 2.83 + 1.85 + 2.93)}{9}, \right. \\
 &\quad \left. \frac{(3.67 + 3.8 + 3.97 + 3.33 + 3.72 + 3.38 + 3.83 + 2.85 + 3.93)}{9}, \right. \\
 &\quad \left. \frac{(4.67 + 4.73 + 4.92 + 4.33 + 4.5 + 4.38 + 4.75 + 3.85 + 4.83)}{9} \right) \\
 &= (2.61, 3.61, 4.55).
 \end{aligned} \quad (26)$$

In the same way, we can get the fuzzy grade matrix G as follows:

TABLE 6: Fuzzy grades evaluated by the evaluator E_1 .

Criteria	Subcriteria	Preceptor A_1	Preceptor A_2	Preceptor A_3	Preceptor A_4	Preceptor A_5	Preceptor A_6	Preceptor A_7	Preceptor A_8	Preceptor A_9
X_1	X_{11}	(2.5, 3.5, 4.5)	(2, 3, 4)	(1.6, 2.6, 3.6)	(1.5, 2.5, 3.5)	(2.8, 3.8, 4.8)	(1.6, 2.6, 3.6)	(2.9, 3.9, 4.9)	(2.5, 3.5, 4.5)	(2.3, 3.3, 4.3)
	X_{12}	(3.2, 4.2, 5)	(1.4, 2.4, 3.4)	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)	(1.5, 2.5, 3.5)	(4, 5, 5)	(1.8, 2.8, 3.8)	(1.5, 2.5, 3.5)	(1.8, 2.8, 3.8)
	X_{13}	(3.7, 4.7, 5)	(2.9, 3.9, 4.9)	(3.7, 4.7, 5)	(3.5, 4.5, 5)	(1, 1.2, 2.2)	(3.6, 4.6, 5)	(3.6, 4.6, 5)	(2.3, 3.3, 4.3)	(1.5, 2.5, 3.5)
	X_{14}	(1.8, 2.8, 3.8)	(1.2, 2.2, 3.2)	(2.9, 3.9, 4.9)	(3.3, 4.3, 5)	(1.5, 2.5, 3.5)	(1.6, 2.6, 3.6)	(3.3, 4.3, 5)	(1.7, 2.7, 3.7)	(2.3, 3.3, 4.3)
	X_{15}	(1, 1.6, 2.6)	(3.3, 4.3, 5)	(1.1, 2.2, 2.2)	(2.3, 3.3, 4.3)	(2.5, 3.5, 4.5)	(1, 1.2, 2.2)	(1.8, 2.8, 3.8)	(3.35, 4.35, 5)	(2.9, 3.9, 4.9)
	X_{16}	(1.5, 2.5, 3.5)	(3.7, 4.7, 5)	(2.4, 3.4, 4.4)	(1.7, 2.7, 3.7)	(1.4, 2.4, 3.4)	(2, 3, 4)	(3.5, 4.5, 5)	(1.6, 2.6, 3.6)	(3.3, 4.3, 5)
	X_{17}	(1.4, 2.4, 3.4)	(2.7, 3.7, 4.7)	(2.8, 3.8, 4.8)	(1, 1.2, 2.2)	(3.7, 4.7, 5)	(1.7, 2.7, 3.7)	(2.1, 3.1, 4.1)	(1, 1.2, 2.2)	(2.1, 3.1, 4.1)
	X_{18}	(1, 1.1, 2.1)	(1.6, 2.6, 3.6)	(2.9, 3.9, 4.9)	(4, 5, 5)	(1.2, 2.2, 3.2)	(1.7, 2.7, 3.7)	(1.9, 2.9, 3.9)	(2, 3, 4)	(1, 1.2, 2.2)
	X_{19}	(2.5, 3.5, 4.5)	(3.2, 4.2, 5)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(1.4, 2.4, 3.4)	(1.2, 2.2, 3.2)	(3.7, 4.7, 5)	(1.2, 2.2, 3.2)	(1.7, 2.7, 3.7)
X_2	X_{21}	(2.1, 3.1, 4.1)	(2.1, 3.1, 4.1)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)	(2.9, 3.9, 4.9)	(2.7, 3.7, 4.7)	(2.6, 3.6, 4.6)	(1.5, 2.5, 3.5)	(2.5, 3.5, 4.5)
	X_{22}	(2.7, 3.7, 4.7)	(3.2, 4.2, 5)	(2.6, 3.6, 4.6)	(1.8, 2.8, 3.8)	(2.1, 3.1, 4.1)	(2.1, 3.1, 4.1)	(1.4, 2.4, 3.4)	(2.3, 3.3, 4.3)	(2.9, 3.9, 4.9)
	X_{23}	(2.6, 3.6, 4.6)	(1.5, 2.5, 3.5)	(3.7, 4.7, 5)	(1.9, 2.9, 3.9)	(1.6, 2.6, 3.6)	(3.2, 4.2, 5)	(1.6, 2.6, 3.6)	(1.4, 2.4, 3.4)	(3.3, 4.3, 5)
	X_{24}	(2.5, 3.5, 4.5)	(2.6, 3.6, 4.6)	(2.5, 3.5, 4.5)	(2.6, 3.6, 4.6)	(1, 1.2, 2.2)	(2.6, 3.6, 4.6)	(1.5, 2.5, 3.5)	(1.2, 2.2, 3.2)	(2.9, 3.9, 4.9)
	X_{25}	(3.2, 4.2, 5)	(2.5, 3.5, 4.5)	(1.5, 2.5, 3.5)	(2.5, 3.5, 4.5)	(2.9, 3.9, 4.9)	(2, 3, 4)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(2.1, 3.1, 4.1)
	X_{31}	(2.2, 3.2, 4.2)	(2.6, 3.6, 4.6)	(2.1, 3.1, 4.1)	(2.1, 3.1, 4.1)	(2.6, 3.6, 4.6)	(1.6, 2.6, 3.6)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(1.7, 2.7, 3.7)
X_3	X_{32}	(1.7, 2.7, 3.7)	(1.8, 2.8, 3.8)	(2.6, 3.6, 4.6)	(3.7, 4.7, 5)	(3.7, 4.7, 5)	(1, 1.1, 2.1)	(2.5, 3.5, 4.5)	(2, 3, 4)	(1.4, 2.4, 3.4)
	X_{33}	(2.7, 3.7, 4.7)	(1.2, 2.2, 3.2)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(1, 1.2, 2.2)	(2.5, 3.5, 4.5)	(3.7, 4.7, 5)	(1.7, 2.7, 3.7)	(2.5, 3.5, 4.5)
	X_{34}	(2.6, 3.6, 4.6)	(2.1, 3.1, 4.1)	(1.7, 2.7, 3.7)	(2.8, 3.8, 4.8)	(1.4, 2.4, 3.4)	(3.7, 4.7, 5)	(3.5, 4.5, 5)	(1.6, 2.6, 3.6)	(1.5, 2.5, 3.5)
	X_{41}	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(3.6, 4.6, 5)	(2.1, 3.1, 4.1)	(3.7, 4.7, 5)	(1.7, 2.7, 3.7)
X_4	X_{42}	(3.8, 4.8, 5)	(2.3, 3.3, 4.3)	(2.2, 3.2, 4.2)	(1.7, 2.7, 3.7)	(2.5, 3.5, 4.5)	(2.6, 3.6, 4.6)	(1.9, 2.9, 3.9)	(1.8, 2.8, 3.8)	(3.7, 4.7, 5)
	X_{43}	(2.6, 3.6, 4.6)	(2.5, 3.5, 4.5)	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(1.8, 2.8, 3.8)	(1.2, 2.2, 3.2)	(1.1, 2.2, 2.2)	(2.9, 3.9, 4.9)	(1.6, 2.6, 3.6)
	X_{44}	(1.7, 2.7, 3.7)	(2.5, 3.5, 4.5)	(1.2, 2.2, 3.2)	(2.7, 3.7, 4.7)	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(2.6, 3.6, 4.6)	(1.1, 2.2, 2.2)	(3.3, 4.3, 5)
	X_{45}	(2.6, 3.6, 4.6)	(2.1, 3.1, 4.1)	(2.6, 3.6, 4.6)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)	(2.6, 3.6, 4.6)	(2.9, 3.9, 4.9)	(2.3, 3.3, 4.3)	(1.1, 2.2, 2.2)
	X_{46}	(1.8, 2.8, 3.8)	(2.3, 3.3, 4.3)	(2.8, 3.8, 4.8)	(2.1, 3.1, 4.1)	(1.7, 2.7, 3.7)	(1.7, 2.7, 3.7)	(1.7, 2.7, 3.7)	(2.5, 3.5, 4.5)	(2.9, 3.9, 4.9)
	X_{51}	(2.2, 3.2, 4.2)	(2.5, 3.5, 4.5)	(2.6, 3.6, 4.6)	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)	(2.8, 3.8, 4.8)	(1.6, 2.6, 3.6)	(3.5, 4.5, 5)	(3.3, 4.3, 5)
X_5	X_{52}	(1.4, 2.4, 3.4)	(2.7, 3.7, 4.7)	(2.1, 3.1, 4.1)	(2.3, 3.3, 4.3)	(2.9, 3.9, 4.9)	(2.9, 3.9, 4.9)	(1.1, 2.2, 2.2)	(1.5, 2.5, 3.5)	(1.4, 2.4, 3.4)
	X_{53}	(2.1, 3.1, 4.1)	(2.6, 3.6, 4.6)	(1.5, 2.5, 3.5)	(1.2, 2.2, 3.2)	(1.2, 2.2, 3.2)	(1.2, 2.2, 3.2)	(1.8, 2.8, 3.8)	(3.7, 4.7, 5)	(1.1, 2.2, 2.2)
	X_{54}	(2.3, 3.3, 4.3)	(1.6, 2.6, 3.6)	(2.8, 3.8, 4.8)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(2.9, 3.9, 4.9)	(1, 1.2, 2.2)	(2, 3, 4)	(1.2, 2.2, 3.2)

TABLE 7: Fuzzy grades evaluated by the evaluator E_2 .

Criteria	Subcriteria	Preceptor A_1	Preceptor A_2	Preceptor A_3	Preceptor A_4	Preceptor A_5	Preceptor A_6	Preceptor A_7	Preceptor A_8	Preceptor A_9
X_1	X_{11}	(2.8, 3.8, 4.8)	(3.1, 4.1, 5)	(2.6, 3.6, 4.6)	(2.8, 3.8, 4.8)	(1.4, 2.4, 3.4)	(1.6, 2.6, 3.6)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)
	X_{12}	(2.6, 3.6, 4.6)	(1.8, 2.8, 3.8)	(3.7, 4.7, 5)	(2.8, 3.8, 4.8)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.3, 3.3, 4.3)
	X_{13}	(3.1, 4.1, 5)	(2.9, 3.9, 4.9)	(1.6, 2.6, 3.6)	(1.7, 2.7, 3.7)	(2.6, 3.6, 4.6)	(1, 2, 3)	(3.3, 4.3, 5)	(2.3, 3.3, 4.3)	(1, 2, 3)
	X_{14}	(2.5, 3.5, 4.5)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(3.6, 4.6, 5)	(3.7, 4.7, 5)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(2.7, 3.7, 4.7)
	X_{15}	(3.7, 4.7, 5)	(2.5, 3.5, 4.5)	(2.7, 3.7, 4.7)	(1.2, 2.2, 3.2)	(1.7, 2.7, 3.7)	(2.3, 3.3, 4.3)	(1.2, 2.2, 3.2)	(2.7, 3.7, 4.7)	(2.1, 3.1, 4.1)
	X_{16}	(2.6, 3.6, 4.6)	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(2.6, 3.6, 4.6)	(1.1, 2.2, 2.2)	(3.1, 4.1, 5)	(2.1, 3.1, 4.1)	(1.9, 2.9, 3.9)	(2.9, 3.9, 4.9)
	X_{17}	(3.1, 4.1, 5)	(1.2, 2.2, 3.2)	(1.8, 2.8, 3.8)	(2, 3, 4)	(1.2, 2.2, 3.2)	(1.4, 2.4, 3.4)	(2.3, 3.3, 4.4)	(1.2, 2.2, 3.2)	(2.3, 3.3, 4.3)
	X_{18}	(1.2, 2.2, 3.2)	(3.1, 4.1, 5)	(1.5, 2.5, 3.5)	(2.7, 3.7, 4.7)	(3.7, 4.7, 5)	(2.6, 3.6, 4.6)	(1, 1.1, 2.1)	(2.1, 3.1, 4.1)	(1.4, 2.4, 3.4)
	X_{19}	(1.2, 2.2, 3.2)	(2.1, 3.1, 4.1)	(2.6, 3.6, 4.6)	(2.9, 3.9, 4.9)	(3.2, 4.2, 5)	(3.7, 4.7, 5)	(1.4, 2.4, 3.4)	(2.3, 3.3, 4.3)	(1.7, 2.7, 3.7)
X_2	X_{21}	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(2.6, 3.6, 4.6)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)	(2.1, 3.1, 4.1)	(2.6, 3.6, 4.6)	(2.5, 3.5, 4.5)
	X_{22}	(2.7, 3.7, 4.7)	(2.6, 3.6, 4.6)	(1.5, 2.5, 3.5)	(1.2, 2.2, 3.2)	(3.7, 4.7, 5)	(2, 3, 4)	(3.7, 4.7, 5)	(3.3, 4.3, 5)	(2.9, 3.9, 4.9)
	X_{23}	(3.7, 4.7, 5)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(2.8, 3.8, 4.8)	(1, 1.2, 2.2)	(1.7, 2.7, 3.7)	(4.5, 5)	(3.3, 4.3, 5)
	X_{24}	(1.2, 2.2, 3.2)	(3.1, 4.1, 5)	(1.8, 2.8, 3.8)	(2.1, 3.1, 4.1)	(1.5, 2.5, 3.5)	(3.5, 4.5, 5)	(1, 1.2, 2.2)	(2.3, 3.3, 4.3)	(2.9, 3.9, 4.9)
	X_{25}	(3.1, 4.1, 5)	(2.7, 3.7, 4.7)	(3.7, 4.7, 5)	(3.7, 4.7, 5)	(2.3, 3.3, 4.3)	(3.2, 4.2, 5)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(1.7, 2.7, 3.7)
X_3	X_{31}	(3.7, 4.7, 5)	(2.6, 3.6, 4.6)	(3.1, 4.1, 5)	(2.8, 3.8, 4.8)	(2.6, 3.6, 4.6)	(2.3, 3.3, 4.3)	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.5)
	X_{32}	(2.1, 3.1, 4.1)	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(2.3, 3.3, 4.3)	(1.2, 2.2, 3.2)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(3.3, 4.3, 5)
	X_{33}	(3.7, 4.7, 5)	(3.7, 4.7, 5)	(2.8, 3.8, 4.8)	(2.6, 3.6, 4.6)	(2.1, 3.1, 4.1)	(2.3, 3.3, 4.3)	(2.5, 3.5, 4.5)	(2.3, 3.3, 4.3)	(1.6, 2.6, 3.6)
	X_{34}	(2.3, 3.3, 4.3)	(2.5, 3.5, 4.5)	(2.6, 3.6, 4.6)	(1.8, 2.8, 3.8)	(3.1, 4.1, 5)	(3.6, 4.6, 5)	(3.1, 4.1, 5)	(1.5, 2.5, 3.5)	(2.5, 3.5, 4.5)
X_4	X_{41}	(2.5, 3.5, 4.5)	(1.5, 2.5, 3.5)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(1.4, 2.4, 3.4)
	X_{42}	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(1.5, 2.5, 3.5)	(2.3, 3.3, 4.3)	(1.7, 2.7, 3.7)	(1, 1.2, 2.2)	(2.9, 3.9, 4.9)
	X_{43}	(3.7, 4.7, 5)	(1.2, 2.2, 3.2)	(1.4, 2.4, 3.4)	(2.5, 3.5, 4.5)	(3.7, 4.7, 5)	(1, 1.2, 2.2)	(2.5, 3.5, 4.5)	(2.5, 3.5, 4.5)	(3.3, 4.3, 5)
	X_{44}	(1.5, 2.5, 3.5)	(2.3, 3.3, 4.3)	(3.7, 4.7, 5)	(1.8, 2.8, 3.8)	(3.2, 4.2, 5)	(1.7, 2.7, 3.7)	(2.1, 3.1, 4.1)	(1.6, 2.6, 3.6)	(3.2, 4.2, 5)
	X_{45}	(2.7, 3.7, 4.7)	(1.5, 2.5, 3.5)	(3.2, 4.2, 5)	(2.3, 3.3, 4.3)	(2.9, 3.9, 4.9)	(1.2, 2.2, 3.2)	(1.4, 2.4, 3.4)	(3.7, 4.7, 5)	(1, 1.1, 2.1)
	X_{46}	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(1.8, 2.8, 3.8)	(2.5, 3.5, 4.5)	(1.4, 2.4, 3.4)	(2.5, 3.5, 4.5)	(2.1, 3.1, 4.1)	(1.7, 2.7, 3.7)
X_5	X_{51}	(2.8, 3.8, 4.8)	(2.1, 3.1, 4.1)	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.5)	(2.8, 3.8, 4.8)	(2, 3, 4)	(1, 1.2, 2.2)	(1.9, 2.9, 3.9)	(1.7, 2.7, 3.7)
	X_{52}	(1.8, 2.8, 3.8)	(1.2, 2.2, 3.2)	(2.9, 3.9, 4.9)	(1.4, 2.4, 3.4)	(2, 3, 4)	(1, 1.2, 2.2)	(1.8, 2.8, 3.8)	(2, 3, 4)	(3.3, 4.3, 5)
	X_{53}	(2.1, 3.1, 4.1)	(3.1, 4.1, 5)	(1.2, 2.2, 3.2)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(2.1, 3.1, 4.1)	(1.2, 2.2, 3.3)	(3.2, 4.2, 5)	(2, 3, 4)
	X_{54}	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(2.1, 3.1, 4.1)	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(1.7, 2.7, 3.7)	(1.5, 2.5, 3.5)	(1.4, 2.4, 3.4)	(1.2, 2.2, 3.2)

TABLE 8: Fuzzy grades evaluated by the evaluator E_3 .

Criteria	Subcriteria	Preceptor A_1	Preceptor A_2	Preceptor A_3	Preceptor A_4	Preceptor A_5	Preceptor A_6	Preceptor A_7	Preceptor A_8	Preceptor A_9
X_1	X_{11}	(2.8, 3.8, 4.8)	(3.1, 4.1, 5)	(2.6, 3.6, 4.6)	(2.8, 3.8, 4.8)	(1.4, 2.4, 3.4)	(1.6, 2.6, 3.6)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)
	X_{12}	(2.6, 3.6, 4.6)	(1.8, 2.8, 3.8)	(3.7, 4.7, 5)	(2.8, 3.8, 4.8)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.3, 3.3, 4.3)
	X_{13}	(3.1, 4.1, 5)	(2.9, 3.9, 4.9)	(1.6, 2.6, 3.6)	(1.7, 2.7, 3.7)	(2.6, 3.6, 4.6)	(1, 2, 3)	(3.3, 4.3, 5)	(2.3, 3.3, 4.3)	(1, 2, 3)
	X_{14}	(2.5, 3.5, 4.5)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(3.6, 4.6, 5)	(3.7, 4.7, 5)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(2.7, 3.7, 4.7)
	X_{15}	(3.7, 4.7, 5)	(2.5, 3.5, 4.5)	(2.7, 3.7, 4.7)	(1.2, 2.2, 3.2)	(1.7, 2.7, 3.7)	(2.3, 3.3, 4.3)	(1.2, 2.2, 3.2)	(2.7, 3.7, 4.7)	(2.1, 3.1, 4.1)
	X_{16}	(2.6, 3.6, 4.6)	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(2.6, 3.6, 4.6)	(1.1, 2.2, 2.2)	(3.1, 4.1, 5)	(2.1, 3.1, 4.1)	(1.9, 2.9, 3.9)	(2.9, 3.9, 4.9)
	X_{17}	(3.1, 4.1, 5)	(1.2, 2.2, 3.2)	(1.8, 2.8, 3.8)	(2, 3, 4)	(1.2, 2.2, 3.2)	(1.4, 2.4, 3.4)	(2.3, 3.3, 4.4)	(1.2, 2.2, 3.2)	(2.3, 3.3, 4.3)
	X_{18}	(1.2, 2.2, 3.2)	(3.1, 4.1, 5)	(1.5, 2.5, 3.5)	(2.7, 3.7, 4.7)	(3.7, 4.7, 5)	(2.6, 3.6, 4.6)	(1, 1.1, 2.1)	(2.1, 3.1, 4.1)	(1.4, 2.4, 3.4)
	X_{19}	(1.2, 2.2, 3.2)	(2.1, 3.1, 4.1)	(2.6, 3.6, 4.6)	(2.9, 3.9, 4.9)	(3.2, 4.2, 5)	(3.7, 4.7, 5)	(1.4, 2.4, 3.4)	(2.3, 3.3, 4.3)	(1.7, 2.7, 3.7)
	X_{21}	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(2.6, 3.6, 4.6)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)	(2.1, 3.1, 4.1)	(2.6, 3.6, 4.6)	(2.5, 3.5, 4.5)
	X_{22}	(2.7, 3.7, 4.7)	(2.6, 3.6, 4.6)	(1.5, 2.5, 3.5)	(1.2, 2.2, 3.2)	(3.7, 4.7, 5)	(2, 3, 4)	(3.7, 4.7, 5)	(3.3, 4.3, 5)	(2.9, 3.9, 4.9)
	X_{23}	(3.7, 4.7, 5)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(2.8, 3.8, 4.8)	(1, 1.2, 2.2)	(1.7, 2.7, 3.7)	(4, 5, 5)	(3.3, 4.3, 5)
	X_{24}	(1.2, 2.2, 3.2)	(3.1, 4.1, 5)	(1.8, 2.8, 3.8)	(2.1, 3.1, 4.1)	(1.5, 2.5, 3.5)	(3.5, 4.5, 5)	(1, 1.2, 2.2)	(2.3, 3.3, 4.3)	(2.9, 3.9, 4.9)
	X_{25}	(3.1, 4.1, 5)	(2.7, 3.7, 4.7)	(3.7, 4.7, 5)	(3.7, 4.7, 5)	(2.3, 3.3, 4.3)	(3.2, 4.2, 5)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(1.7, 2.7, 3.7)
	X_{31}	(3.7, 4.7, 5)	(2.6, 3.6, 4.6)	(3.1, 4.1, 5)	(2.8, 3.8, 4.8)	(2.6, 3.6, 4.6)	(2.3, 3.3, 4.3)	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.5)
	X_{32}	(2.1, 3.1, 4.1)	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(2.3, 3.3, 4.3)	(1.2, 2.2, 3.2)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(3.3, 4.3, 5)
	X_{33}	(3.7, 4.7, 5)	(3.7, 4.7, 5)	(2.8, 3.8, 4.8)	(2.6, 3.6, 4.6)	(2.1, 3.1, 4.1)	(2.3, 3.3, 4.3)	(2.5, 3.5, 4.5)	(2.3, 3.3, 4.3)	(1.6, 2.6, 3.6)
	X_{34}	(2.3, 3.3, 4.3)	(2.5, 3.5, 4.5)	(2.6, 3.6, 4.6)	(1.8, 2.8, 3.8)	(3.1, 4.1, 5)	(3.6, 4.6, 5)	(3.1, 4.1, 5)	(1.5, 2.5, 3.5)	(2.5, 3.5, 4.5)
X_4	X_{41}	(2.5, 3.5, 4.5)	(1.5, 2.5, 3.5)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(1.4, 2.4, 3.4)
	X_{42}	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(1.5, 2.5, 3.5)	(2.3, 3.3, 4.3)	(1.7, 2.7, 3.7)	(1, 1.2, 2.2)	(2.9, 3.9, 4.9)
	X_{43}	(3.7, 4.7, 5)	(1.2, 2.2, 3.2)	(1.4, 2.4, 3.4)	(2.5, 3.5, 4.5)	(3.7, 4.7, 5)	(1, 1.2, 2.2)	(2.5, 3.5, 4.5)	(2.5, 3.5, 4.5)	(3.3, 4.3, 5)
	X_{44}	(1.5, 2.5, 3.5)	(2.3, 3.3, 4.3)	(3.7, 4.7, 5)	(1.8, 2.8, 3.8)	(3.2, 4.2, 5)	(1.7, 2.7, 3.7)	(2.1, 3.1, 4.1)	(1.6, 2.6, 3.6)	(3.2, 4.2, 5)
	X_{45}	(2.7, 3.7, 4.7)	(1.5, 2.5, 3.5)	(3.2, 4.2, 5)	(2.3, 3.3, 4.3)	(2.9, 3.9, 4.9)	(1.2, 2.2, 3.2)	(1.4, 2.4, 3.4)	(3.7, 4.7, 5)	(1, 1.1, 2.1)
	X_{46}	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(1.8, 2.8, 3.8)	(2.5, 3.5, 4.5)	(1.4, 2.4, 3.4)	(2.5, 3.5, 4.5)	(2.1, 3.1, 4.1)	(1.7, 2.7, 3.7)
X_5	X_{51}	(2.8, 3.8, 4.8)	(2.1, 3.1, 4.1)	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.5)	(2.8, 3.8, 4.8)	(2, 3, 4)	(1, 1.2, 2.2)	(1.9, 2.9, 3.9)	(1.7, 2.7, 3.7)
	X_{52}	(1.8, 2.8, 3.8)	(1.2, 2.2, 3.2)	(2.9, 3.9, 4.9)	(1.4, 2.4, 3.4)	(2, 3, 4)	(1, 1.2, 2.2)	(1.8, 2.8, 3.8)	(2, 3, 4)	(3.3, 4.3, 5)
	X_{53}	(2.1, 3.1, 4.1)	(3.1, 4.1, 5)	(1.2, 2.2, 3.2)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(2.1, 3.1, 4.1)	(1.2, 2.2, 3.3)	(3.2, 4.2, 5)	(2, 3, 4)
	X_{54}	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(2.1, 3.1, 4.1)	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(1.7, 2.7, 3.7)	(1.5, 2.5, 3.5)	(1.4, 2.4, 3.4)	(1.2, 2.2, 3.2)

TABLE 9: Fuzzy grades evaluated by the evaluator E_4 .

Criteria	Subcriteria	Preceptor A_1	Preceptor A_2	Preceptor A_3	Preceptor A_4	Preceptor A_5	Preceptor A_6	Preceptor A_7	Preceptor A_8	Preceptor A_9
X_1	X_{11}	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(2.3, 3.3, 4.3)	(3.1, 4.1, 5)	(3.1, 4.1, 5)	(1.4, 2.4, 3.4)	(2.1, 3.1, 4.1)	(1.4, 2.4, 3.3)	(1.8, 2.8, 3.8)
	X_{12}	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(2.8, 3.8, 4.8)	(1.4, 2.4, 3.4)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(1.9, 2.9, 3.9)	(2.8, 3.8, 4.8)
	X_{13}	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(2.6, 3.6, 4.6)	(2.2, 3.2, 4.2)	(3.1, 4.1, 5)	(2.2, 3.2, 4.2)	(1, 1.2, 2.2)	(1.8, 2.8, 3.8)	(1.7, 2.7, 3.7)
	X_{14}	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(2.1, 3.1, 4.1)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(1.7, 2.7, 3.7)
	X_{15}	(2.3, 3.3, 4.3)	(2.2, 3.2, 4.2)	(1, 1.2, 2.2)	(3.6, 4.6, 5)	(1.5, 2.5, 3.5)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(1.4, 2.4, 3.4)	(1.2, 2.2, 3.2)
	X_{16}	(3.6, 4.6, 5)	(3.2, 4.2, 5)	(2.7, 3.7, 4.7)	(1, 1.2, 2.2)	(2.5, 3.5, 4.5)	(1.4, 2.4, 3.4)	(3.1, 4.1, 5)	(1.8, 2.8, 3.8)	(2.8, 3.8, 4.8)
	X_{17}	(3.1, 4.1, 5)	(3.7, 4.7, 5)	(2.6, 3.6, 4.6)	(4, 5, 6)	(1.4, 2.4, 3.4)	(2.3, 3.3, 4.3)	(2.5, 3.5, 4.5)	(3.3, 4.3, 5)	(2.2, 3.2, 4.2)
	X_{18}	(3.7, 4.7, 5)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(2.2, 3.2, 4.2)	(1.8, 2.8, 3.8)	(1.6, 2.6, 3.6)	(2.3, 3.3, 4.3)	(1.4, 2.4, 3.4)	(3.3, 4.3, 5)
	X_{19}	(3.2, 4.2, 5)	(2.8, 3.8, 4.8)	(2.6, 3.6, 4.6)	(2.9, 3.9, 4.9)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(2.9, 3.9, 4.9)
X_2	X_{21}	(3.6, 4.6, 5)	(3.1, 4.1, 5)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(2, 3, 4)	(1.4, 2.4, 3.4)	(1.8, 2.8, 3.8)	(1, 2, 3)	(1.8, 2.8, 3.8)
	X_{22}	(2.6, 3.6, 4.6)	(2.6, 3.6, 4.6)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.2, 3.2, 4.2)	(2.3, 3.3, 4.3)	(1.8, 2.8, 3.8)	(2.2, 3.2, 4.2)
	X_{23}	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(2.7, 3.7, 4.7)	(1, 2, 3)	(3.7, 4.7, 5)	(2.3, 3.3, 4.3)	(2.3, 3.3, 4.3)	(1.2, 2.2, 3.2)	(2.7, 3.7, 4.7)
	X_{24}	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(3.2, 4.2, 5)	(3.5, 4.5, 5)	(1.8, 2.8, 3.8)	(2.2, 3.2, 4.2)	(2.2, 3.2, 4.2)	(2.3, 3.3, 4.3)	(1.4, 2.4, 3.4)
	X_{25}	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.1, 3.1, 4.1)	(2.8, 3.8, 4.8)	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(1.4, 2.4, 3.4)	(2.9, 3.9, 4.9)	(2.3, 3.3, 4.3)
X_3	X_{31}	(2.7, 3.7, 4.7)	(2.8, 3.8, 4.8)	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)
	X_{32}	(3.7, 4.7, 5)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(1, 1.2, 2.2)	(2.7, 3.7, 4.7)	(1.7, 2.7, 3.7)	(2.2, 3.2, 4.2)	(1.8, 2.8, 3.8)	(1.6, 2.6, 3.8)
	X_{33}	(2.6, 3.6, 4.6)	(2.3, 3.3, 4.3)	(2.5, 3.5, 4.5)	(2.7, 3.7, 4.7)	(2.8, 3.8, 4.8)	(1.5, 2.5, 3.5)	(2.7, 3.7, 4.7)	(1.4, 2.4, 3.4)	(2.8, 3.8, 4.8)
	X_{34}	(1.4, 2.4, 3.4)	(1.8, 2.8, 3.8)	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(1.2, 2.2, 3.2)	(1.2, 2.2, 3.2)	(1.5, 2.5, 3.5)	(2, 3, 4)	(2.8, 3.8, 4.8)
X_4	X_{41}	(2.2, 3.2, 4.2)	(3.1, 4.1, 5)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(3.1, 4.1, 5)	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)	(1.4, 2.4, 3.4)
	X_{42}	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(2.2, 3.2, 4.2)	(1.4, 2.4, 3.4)
	X_{43}	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(3.1, 4.1, 5)	(1.7, 2.7, 3.7)	(1, 1.1, 2.1)	(2.5, 3.5, 4.5)	(1.6, 2.6, 3.6)	(3.3, 4.3, 5)
	X_{44}	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.5)	(2.7, 3.7, 4.7)	(2.2, 3.2, 4.2)	(2.2, 3.2, 4.2)	(3.6, 4.6, 5)	(3.1, 4.1, 5)	(1.4, 2.4, 3.4)	(1.8, 2.8, 3.8)
	X_{45}	(2.5, 3.5, 4.5)	(2.1, 3.1, 4.1)	(1.4, 2.4, 3.4)	(3.1, 4.1, 5)	(3.7, 4.7, 5)	(3.1, 4.1, 5)	(1.7, 2.7, 3.7)	(2, 3, 4)	(2.1, 3.1, 4.1)
	X_{46}	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(1.4, 2.4, 3.4)	(2.2, 3.2, 4.2)	(1.4, 2.4, 3.4)
X_5	X_{51}	(1.8, 2.8, 3.8)	(2.3, 3.3, 4.3)	(1, 1.2, 2.2)	(1.8, 2.8, 3.8)	(3.1, 4.1, 5)	(1, 2, 3)	(1, 1.2, 2.2)	(1.8, 2.8, 3.8)	(1.7, 2.7, 3.7)
	X_{52}	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(1.5, 2.5, 3.5)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(1.8, 2.8, 3.8)	(1.4, 2.4, 3.4)	(2.9, 3.9, 4.9)	(1.5, 2.5, 3.5)
	X_{53}	(2.3, 3.3, 4.3)	(2.8, 3.8, 4.8)	(2.8, 3.8, 4.8)	(3.2, 4.2, 5)	(1.5, 2.5, 3.5)	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(2, 3, 4)
	X_{54}	(3.6, 4.6, 5)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(2.2, 3.2, 4.2)	(2.1, 3.1, 4.1)	(1, 1.2, 2.2)	(3.7, 4.7, 5)	(2, 3, 4)	(3.3, 4.3, 5)

TABLE 10: Fuzzy grades evaluated by the evaluator E_5 .

Criteria	Subcriteria	Preceptor A_1	Preceptor A_2	Preceptor A_3	Preceptor A_4	Preceptor A_5	Preceptor A_6	Preceptor A_7	Preceptor A_8	Preceptor A_9
X_1	X_{11}	(2.5, 3.5, 4.5)	(2.2, 3.2, 4.2)	(1.6, 2.6, 3.6)	(3.2, 4.2, 5)	(2.9, 3.9, 4.9)	(3.2, 4.2, 5)	(2.7, 3.7, 4.7)	(1.6, 2.6, 3.6)	(2.9, 3.9, 4.9)
	X_{12}	(3.7, 4.7, 5)	(1.4, 2.4, 3.4)	(3.1, 4.1, 5)	(3.6, 4.6, 5)	(2.7, 3.7, 4.7)	(1.4, 2.4, 3.4)	(1.6, 2.6, 3.6)	(3.9, 4.9, 5)	(2.2, 3.2, 4.2)
	X_{13}	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(2.7, 3.7, 4.7)	(1.4, 2.4, 3.4)	(3.1, 4.1, 5)	(2.9, 3.9, 4.9)
	X_{14}	(3.7, 4.7, 5)	(2.2, 3.2, 4.2)	(1.4, 2.4, 3.4)	(2.2, 3.2, 4.2)	(2, 3, 4)	(1.5, 2.5, 3.5)	(2.5, 3.5, 4.5)	(1.4, 2.4, 3.4)	(1.6, 2.6, 3.6)
	X_{15}	(3.6, 4.6, 5)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(1.8, 2.8, 3.8)	(1.4, 2.4, 3.4)	(1.9, 2.9, 3.9)	(2.9, 3.9, 4.9)	(1.5, 2.5, 3.5)	(2.8, 3.8, 4.8)
	X_{16}	(2.2, 3.2, 4.2)	(2.9, 3.9, 4.9)	(2.7, 3.7, 4.7)	(3.2, 4.2, 5)	(2, 3, 4)	(3.3, 4.3, 5)	(2.8, 3.8, 4.8)	(1.9, 2.9, 3.9)	(2.9, 3.9, 4.9)
	X_{17}	(2.2, 3.2, 4.2)	(2.9, 3.9, 4.9)	(2.5, 3.5, 4.5)	(2.7, 3.7, 4.7)	(1.6, 2.6, 3.6)	(2.9, 3.9, 4.9)	(1, 1.2, 2.2)	(1, 1.2, 2.2)	(1, 1.2, 2.2)
	X_{18}	(1.4, 2.4, 3.4)	(1.7, 2.7, 3.7)	(3.1, 4.1, 5)	(1.9, 2.9, 3.9)	(2.7, 3.7, 4.7)	(3.2, 4.2, 5)	(1.8, 2.8, 3.8)	(1.4, 2.4, 3.4)	(3.3, 4.3, 5)
	X_{19}	(3.1, 4.1, 5)	(2.8, 3.8, 4.8)	(3.4, 4.4, 5)	(1.8, 2.8, 3.8)	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(2.1, 3.1, 4.1)	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)
X_2	X_{21}	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(3.7, 4.7, 5)	(2.7, 3.7, 4.7)	(2.2, 3.2, 4.2)	(0.2, 1.2, 2.2)	(2.7, 3.7, 4.7)	(1.6, 2.6, 3.6)	(1.4, 2.4, 3.4)
	X_{22}	(1.5, 2.5, 3.5)	(1.6, 2.6, 3.6)	(3.1, 4.1, 5)	(3.6, 4.6, 5)	(2.5, 3.5, 4.5)	(1.4, 2.4, 3.4)	(1.8, 2.8, 3.8)	(2.8, 3.8, 4.8)	(1.4, 2.4, 3.4)
	X_{23}	(1.4, 2.4, 3.4)	(2.8, 3.8, 4.8)	(3.6, 4.6, 5)	(3.3, 4.3, 5)	(2.7, 3.7, 4.7)	(2.2, 3.2, 4.2)	(1.5, 2.5, 3.5)	(2.1, 3.1, 4.1)	(1, 1.2, 2.2)
	X_{24}	(1.8, 2.8, 3.8)	(1.4, 2.4, 3.4)	(3.2, 4.2, 5)	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(2.8, 3.8, 4.8)	(1.2, 2.2, 3.2)	(1.5, 2.5, 3.5)	(1.7, 2.7, 3.7)
	X_{25}	(1.4, 2.4, 3.4)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(2.8, 3.8, 4.8)	(3.7, 4.7, 5)	(1.2, 2.2, 3.2)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(2.1, 3.1, 4.1)
	X_{31}	(2.7, 3.7, 4.7)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(1.4, 2.4, 3.4)	(3.1, 4.1, 5)	(1.8, 2.8, 3.8)	(2.2, 3.2, 4.2)	(1.6, 2.6, 3.6)
X_3	X_{32}	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(2, 3, 4)	(4, 5, 5)	(3.1, 4.1, 5)	(2.8, 3.8, 4.8)	(1.7, 2.7, 3.7)	(2.2, 3.2, 4.2)
	X_{33}	(2.7, 3.7, 4.7)	(2.8, 3.8, 4.8)	(1.8, 2.8, 3.8)	(1.6, 2.6, 3.6)	(3.7, 4.7, 5)	(1.9, 2.9, 3.9)	(3.8, 4.8, 5)	(2.5, 3.5, 4.5)	(1.2, 2.2, 3.2)
	X_{34}	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(3.7, 4.7, 5)	(2.7, 3.7, 4.7)	(3.3, 4.3, 5)	(1.8, 2.8, 3.8)	(3.7, 4.7, 5)	(2.8, 3.8, 4.8)	(2.1, 3.1, 4.1)
	X_{41}	(2.7, 3.7, 4.7)	(3.1, 4.1, 5)	(1.4, 2.4, 3.4)	(3.1, 4.1, 5)	(1.5, 2.5, 3.5)	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(1.4, 2.4, 3.4)	(2.8, 3.8, 4.8)
X_4	X_{42}	(3.7, 4.7, 5)	(1.4, 2.4, 3.4)	(2.9, 3.9, 4.9)	(1.4, 2.4, 3.4)	(2.2, 3.2, 4.2)	(3.3, 4.3, 5)	(2.2, 3.2, 4.2)	(2.2, 3.2, 4.2)	(1.7, 2.7, 3.7)
	X_{43}	(3.6, 4.6, 5)	(3, 4, 5)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(3.2, 4.2, 5)	(1.6, 2.6, 3.6)	(3.3, 4.3, 5)	(1.4, 2.4, 3.4)	(2.8, 3.8, 4.8)
	X_{44}	(1.4, 2.4, 3.4)	(3.1, 4.1, 5)	(2.5, 3.5, 4.5)	(2.9, 3.9, 4.9)	(1.8, 2.8, 3.8)	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(3.2, 4.2, 5)	(1.6, 2.6, 3.6)
	X_{45}	(1.4, 2.4, 3.4)	(2.7, 3.7, 4.7)	(1.6, 2.6, 3.6)	(1.9, 2.9, 3.9)	(3.7, 4.7, 5)	(1.4, 2.4, 3.4)	(3.6, 4.6, 5)	(2.8, 3.8, 4.8)	(2.1, 3.1, 4.1)
	X_{46}	(2.1, 3.1, 4.1)	(1.8, 2.8, 3.8)	(2.7, 3.7, 4.7)	(2.1, 3.1, 4.1)	(2.8, 3.8, 4.8)	(3.1, 4.1, 5)	(3.5, 4.5, 5)	(3.7, 4.7, 5)	(3.2, 4.2, 5)
	X_{51}	(1.5, 2.5, 3.5)	(2.7, 3.7, 4.7)	(3.7, 4.7, 5)	(3.7, 4.7, 5)	(3.3, 4.3, 5)	(3.1, 4.1, 5)	(1, 1.2, 2.2)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)
X_5	X_{52}	(2.8, 3.8, 4.8)	(3.1, 4.1, 5)	(1.4, 2.4, 3.4)	(2.5, 3.5, 4.5)	(3.9, 4.9, 5)	(1.6, 2.6, 3.6)	(2, 3, 4)	(1.9, 2.9, 3.9)	(1, 1.2, 2.2)
	X_{53}	(1.9, 2.9, 3.9)	(3.7, 4.7, 5)	(1.4, 2.4, 3.4)	(2.8, 3.8, 4.8)	(3.7, 4.7, 5)	(1, 1.2, 2.2)	(1, 1.2, 2.2)	(3, 4, 5)	(1.7, 2.7, 3.7)
	X_{54}	(1.4, 2.4, 3.4)	(1.6, 2.6, 3.6)	(2.1, 3.1, 4.1)	(1, 1.1, 2.1)	(3.3, 4.3, 5)	(2.7, 3.7, 4.7)	(2, 3, 4)	(1, 2, 3)	(1, 1.2, 2.2)

TABLE II: The averaged fuzzy grades table.

Criteria	Subcriteria	Preceptor A_1	Preceptor A_2	Preceptor A_3	Preceptor A_4	Preceptor A_5	Preceptor A_6	Preceptor A_7	Preceptor A_8	Preceptor A_9
X_1	X_{11}	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(2, 3, 4)	(3, 4, 4.9)	(2.9, 3.9, 4.9)	(2, 3, 4)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.6)
	X_{12}	(2.8, 3.8, 4.7)	(1.8, 2.8, 3.8)	(3, 4, 5)	(3, 4, 4.9)	(2, 3, 4)	(2.7, 3.7, 4.7)	(2, 3, 4)	(2.3, 3.3, 4.2)	(2.4, 3.4, 4.4)
	X_{13}	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.5)	(2.8, 3.8, 4.7)	(2.1, 3.1, 4.1)	(2.4, 3.4, 4.3)	(2.1, 3.1, 4.1)	(1.9, 2.9, 3.9)
	X_{14}	(2.3, 3.3, 4.3)	(2.1, 3.1, 4.1)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.6)	(2.8, 3.8, 4.6)	(1.6, 2.6, 3.6)	(2.5, 3.5, 4.5)	(2.2, 3.2, 4.2)	(2, 3, 4)
	X_{15}	(2.7, 3.7, 4.5)	(2.6, 3.6, 4.6)	(2.2, 3.2, 4.2)	(2.3, 3.3, 4.3)	(1.8, 2.8, 3.8)	(2.1, 3.1, 4.1)	(2.4, 3.4, 4.4)	(2.1, 3.1, 4.1)	(2.2, 3.2, 4.2)
	X_{16}	(2.3, 3.3, 4.3)	(3.2, 4.2, 4.9)	(2.7, 3.7, 4.7)	(2.4, 3.4, 4.4)	(1.9, 2.9, 3.9)	(2.8, 3.8, 4.6)	(2.9, 3.9, 4.9)	(1.9, 2.9, 3.9)	(2.9, 3.9, 4.9)
	X_{17}	(2.8, 3.8, 4.7)	(2.8, 3.8, 4.8)	(2.3, 3.3, 4.3)	(2.6, 3.6, 4.5)	(1.8, 2.8, 3.8)	(2.2, 3.2, 4.2)	(1.8, 2.8, 3.8)	(1.2, 1.9, 2.9)	(1.9, 2.9, 3.9)
	X_{18}	(1.8, 2.8, 3.8)	(2, 3, 4)	(3, 4, 4.9)	(2.6, 3.6, 4.6)	(2.5, 3.5, 4.5)	(2, 3, 4)	(2, 3, 4)	(1.6, 2.6, 3.6)	(2.5, 3.5, 4.4)
	X_{19}	(2.9, 3.9, 4.8)	(2.9, 3.9, 4.9)	(2.7, 3.7, 4.7)	(2.9, 3.9, 4.9)	(2.3, 3.3, 4.3)	(2.1, 3.1, 4.1)	(2.5, 3.5, 4.5)	(2, 3, 4)	(2.5, 3.5, 4.5)
X_2	X_{21}	(2.6, 3.6, 4.5)	(2.8, 3.8, 4.8)	(2.6, 3.6, 4.6)	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.5)	(1.9, 2.9, 3.9)	(2.3, 3.3, 4.3)	(1.9, 2.9, 3.9)	(1.5, 2.5, 3.5)
	X_{22}	(2.2, 3.2, 4.2)	(2.6, 3.6, 4.6)	(2.4, 3.4, 4.4)	(1.6, 2.6, 3.6)	(2.4, 3.4, 4.3)	(2.1, 3.1, 4.1)	(1.9, 2.9, 3.9)	(2.3, 3.3, 4.3)	(2.3, 3.3, 4.3)
	X_{23}	(2.2, 3.2, 4.2)	(1.9, 2.9, 3.9)	(3.1, 4.1, 4.9)	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(2, 3, 4)	(1.8, 2.8, 3.8)	(2.3, 3.3, 4.1)	(2.9, 3.9, 4.8)
	X_{24}	(2.2, 3.2, 4.2)	(2.5, 3.5, 4.4)	(3, 4, 4.8)	(2.6, 3.6, 4.6)	(2, 3, 4)	(2.5, 3.5, 4.5)	(1.6, 2.6, 3.6)	(1.7, 2.7, 3.7)	(2.5, 3.5, 4.5)
	X_{25}	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.4)	(2.9, 3.9, 4.8)	(2.7, 3.7, 4.7)	(2.6, 3.6, 4.5)	(1.9, 2.9, 3.9)	(2.8, 3.8, 4.8)	(1.9, 2.9, 3.9)
	X_{31}	(3.0, 4.0, 4.8)	(2.4, 3.4, 4.4)	(2.5, 3.5, 4.5)	(2.8, 3.8, 4.8)	(2.5, 3.5, 4.5)	(2.4, 3.4, 4.4)	(1.8, 2.8, 3.8)	(2.5, 3.5, 4.5)	(2.1, 3.1, 4.1)
X_3	X_{32}	(1.7, 2.7, 3.7)	(2.2, 3.2, 4.2)	(2.7, 3.7, 4.7)	(1.7, 2.7, 3.7)	(3.7, 4.7, 5.7)	(2.1, 3.1, 4.1)	(2.6, 3.6, 4.6)	(1.8, 2.8, 3.8)	(1.8, 2.8, 3.8)
	X_{33}	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(2.4, 3.4, 4.4)	(2.4, 3.4, 4.4)	(2.5, 3.5, 4.5)	(2.2, 3.2, 4.2)	(2.9, 3.9, 4.7)	(1.8, 2.8, 3.8)	(2.3, 3.3, 4.3)
	X_{34}	(2.2, 3.2, 4.2)	(2.4, 3.4, 4.4)	(2.5, 3.5, 4.5)	(2.7, 3.7, 4.7)	(2.3, 3.3, 4.2)	(2.3, 3.3, 4)	(2.8, 3.8, 4.6)	(2.1, 3.1, 4.1)	(2, 3, 4)
	X_{41}	(2.6, 3.6, 4.6)	(2.9, 3.9, 4.9)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(1.9, 2.9, 3.9)	(2.9, 3.9, 4.9)	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(1.9, 2.9, 3.9)
X_4	X_{42}	(3.0, 4.0, 4.7)	(2.5, 3.5, 4.5)	(1.9, 2.9, 3.9)	(2.4, 3.4, 4.4)	(2.1, 3.1, 4.1)	(2.7, 3.7, 4.7)	(1.9, 2.9, 3.9)	(1.9, 2.9, 3.9)	(2.2, 3.2, 4.2)
	X_{43}	(3.3, 4.3, 4.8)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.7)	(2.7, 3.7, 4.6)	(1.3, 2, 3)	(2.6, 3.6, 4.6)	(1.9, 2.9, 3.9)	(2.5, 3.5, 4.4)
	X_{44}	(1.9, 2.9, 3.9)	(2.4, 3.4, 4.4)	(2.8, 3.8, 4.8)	(2.4, 3.4, 4.4)	(1.9, 2.9, 3.9)	(2.4, 3.4, 4.4)	(2.6, 3.6, 4.6)	(1.5, 2.5, 3.5)	(2.7, 3.7, 4.6)
	X_{45}	(2.6, 3.6, 4.6)	(2, 3, 4)	(1.9, 2.9, 3.9)	(2.6, 3.6, 4.5)	(3, 4, 4.8)	(2.1, 3.1, 4.1)	(2.3, 3.3, 4.3)	(2.9, 3.9, 4.7)	(1.6, 2.3, 3.3)
	X_{46}	(2.2, 3.2, 4.2)	(2.3, 3.3, 4.3)	(2.1, 3.1, 4.1)	(2, 3, 4)	(2.6, 3.6, 4.6)	(1.5, 2.5, 3.5)	(1.9, 2.9, 3.9)	(2.6, 3.6, 4.5)	(2.5, 3.5, 4.5)
	X_{51}	(2.1, 3.1, 4.1)	(2.3, 3.3, 4.3)	(2.7, 3.7, 4.7)	(2.5, 3.5, 4.5)	(3.1, 4.1, 4.9)	(2.4, 3.4, 4.4)	(1.2, 1.6, 2.6)	(1.8, 2.8, 3.8)	(2.1, 3.1, 4.1)
X_5	X_{52}	(2.1, 3.1, 4.1)	(2.5, 3.5, 4.5)	(1.7, 2.7, 3.7)	(2.4, 3.4, 4.4)	(2.5, 3.5, 4.5)	(1.7, 2.7, 3.7)	(1.5, 2.5, 3.5)	(1.8, 2.8, 3.8)	(1.3, 2, 3)
	X_{53}	(2.2, 3.2, 4.2)	(3.2, 4.2, 4.9)	(1.6, 2.6, 3.6)	(2, 3, 4)	(2.1, 3.1, 4.1)	(1.7, 2.7, 3.7)	(1.5, 2.5, 3.5)	(2.6, 3.6, 4.5)	(1.7, 2.7, 3.7)
	X_{54}	(2.6, 3.6, 4.6)	(1.8, 2.8, 3.8)	(2.3, 3.3, 4.3)	(1.9, 2.9, 3.9)	(2.8, 3.8, 4.8)	(1.7, 2.7, 3.7)	(1.8, 2.8, 3.8)	(1.6, 2.6, 3.6)	(1.9, 2.9, 3.9)

$$G = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \end{matrix} \begin{bmatrix} (2.61, 3.61, 4.55) & (2.41, 3.41, 4.40) & (2.44, 3.44, 4.38) & (2.63, 3.63, 4.52) & (2.28, 3.28, 4.28) \\ (2.58, 3.58, 4.54) & (2.54, 3.54, 4.53) & (2.49, 3.49, 4.47) & (2.45, 3.45, 4.45) & (2.49, 3.49, 4.42) \\ (2.57, 3.54, 4.52) & (2.77, 3.77, 4.65) & (2.58, 3.58, 4.58) & (2.32, 3.32, 4.32) & (2.10, 3.10, 4.10) \\ (2.70, 3.70, 4.65) & (2.56, 3.56, 4.54) & (2.46, 3.46, 4.46) & (2.49, 3.49, 4.48) & (2.25, 3.25, 4.25) \\ (2.36, 3.36, 4.33) & (2.44, 3.44, 4.43) & (2.79, 3.79, 4.78) & (2.42, 3.42, 4.36) & (2.67, 3.67, 4.64) \\ (2.33, 3.33, 4.20) & (2.25, 3.25, 4.23) & (2.29, 3.29, 4.23) & (2.20, 3.20, 4.15) & (1.91, 2.91, 3.91) \\ (2.40, 3.40, 4.38) & (1.96, 2.96, 3.96) & (2.57, 3.57, 4.46) & (2.22, 3.22, 4.22) & (1.55, 2.41, 3.41) \\ (2.01, 2.98, 3.97) & (2.24, 3.24, 4.24) & (2.09, 3.09, 4.09) & (2.31, 3.31, 4.27) & (2.00, 3.00, 3.96) \\ (2.61, 3.61, 4.55) & (2.41, 3.41, 4.40) & (2.44, 3.44, 4.38) & (2.63, 3.63, 4.52) & (2.28, 3.28, 4.28) \end{bmatrix}. \quad (27)$$

Step 3. Calculate the total fuzzy grade vector R , where

$$R = G \otimes W = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1k} \\ g_{21} & g_{22} & \cdots & g_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nk} \end{bmatrix} \otimes \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_k \end{bmatrix} = \begin{bmatrix} g_{11} \otimes w_1 \oplus g_{12} \otimes w_2 \oplus \cdots \oplus g_{1k} \otimes w_k \\ g_{21} \otimes w_1 \oplus g_{22} \otimes w_2 \oplus \cdots \oplus g_{2k} \otimes w_k \\ \vdots \\ g_{n1} \otimes w_1 \oplus g_{n2} \otimes w_2 \oplus \cdots \oplus g_{nk} \otimes w_k \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}, \quad (28)$$

where R_i denotes the total fuzzy grade of the i th preceptor A_i and $1 \leq i \leq n$. For example, based on the fuzzy weighted vector W obtained in Step 1, the fuzzy grade matrix G obtained in Step 2 and (28), we can calculate the total fuzzy grade R_1 of teacher A_1 as follows:

$$\begin{aligned} R_1 &= (2.61, 3.61, 4.55) \otimes (3.07, 4.07, 5) \oplus (2.61, 3.61, 4.55) \\ &\quad \otimes (3.2, 4.2, 4.97) \oplus (2.61, 3.61, 4.55) \otimes (2.87, 3.87, 4.87) \\ &\quad \oplus (2.98, 3.98, 4.97) \otimes (2.98, 3.98, 4.97) \\ &\quad \oplus (2.61, 3.61, 4.55) \otimes (2.9, 3.9, 4.9) \\ &= (37.18, 69.57, 109.39). \end{aligned} \quad (29)$$

In the same way, we can get the total fuzzy grade vector R shown as follows:

$$R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \\ R_9 \end{bmatrix} = \begin{bmatrix} (37.18, 69.57, 109.39) \\ (37.72, 70.29, 110.76) \\ (37.16, 69.4, 109.58) \\ (37.49, 69.57, 110.62) \\ (38.02, 70.72, 111.35) \\ (33.02, 64.02, 102.41) \\ (32.13, 62.30, 100.98) \\ (32.02, 62.57, 101.47) \\ (32.58, 62.84, 101.55) \end{bmatrix},$$

$$R_1^* = \left(\frac{37.18}{120}, \frac{69.57}{120}, \frac{109.39}{120} \right),$$

$$R_2^* = \left(\frac{37.72}{120}, \frac{70.29}{120}, \frac{110.76}{120} \right),$$

$$R_3^* = \left(\frac{37.16}{120}, \frac{69.40}{120}, \frac{109.58}{120} \right),$$

$$R_4^* = \left(\frac{37.49}{120}, \frac{69.57}{120}, \frac{110.62}{120} \right),$$

$$R_5^* = \left(\frac{38.02}{120}, \frac{70.72}{120}, \frac{111.35}{120} \right),$$

$$R_6^* = \left(\frac{33.02}{120}, \frac{64.02}{120}, \frac{102.41}{120} \right),$$

$$R_7^* = \left(\frac{32.13}{120}, \frac{62.30}{120}, \frac{100.98}{120} \right),$$

$$R_8^* = \left(\frac{32.02}{120}, \frac{62.57}{120}, \frac{101.47}{120} \right),$$

$$R_9^* = \left(\frac{32.58}{120}, \frac{62.84}{120}, \frac{101.55}{120} \right). \quad (30)$$

Step 4. Based on (12), calculate the ranking value $S(R_i)$ of the fuzzy grade R_i , where $1 \leq i \leq n$. The larger the value of $S(R_i)$, the better the ranking of R_i and the better the choice of preceptor A_i , where $1 \leq i \leq n$ and n denotes the number of teachers. For example, after applying (12) to rank the fuzzy numbers $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8$, and R_9 of the fuzzy grade vector R obtained in Step 3, we can get the ranking results as shown in Table 12. We can see that $R_5 > R_2 > R_4 > R_1 > R_3 > R_6 > R_9 > R_7 > R_8$. Because R_5 has the largest ranking value, preceptor A_5 is the best choice.

5. Conclusions

In this paper, we proposed a new centroid index method for ranking fuzzy numbers. First we briefly introduce some existing centroid index ranking methods of fuzzy numbers. Then we propose a new method to evaluate the efficiency of

TABLE 12: Ranking values of the total fuzzy grades.

	$x_{R_i}^*$	$y_{R_i}^{*s}$	$s_{R_i}^*$	Ranking value $s(R_i^*)$
R_1^*	0.6003	0.9049	0.2462	0.9644
R_2^*	0.6076	0.9038	0.2492	0.9660
R_3^*	0.6003	0.9046	0.2471	0.9641
R_4^*	0.6046	0.9037	0.2497	0.9647
R_5^*	0.6113	0.9035	0.2502	0.9669
R_6^*	0.5540	0.9084	0.2367	0.9527
R_7^*	0.5427	0.9090	0.2351	0.9500
R_8^*	0.5445	0.9083	0.2371	0.9498
R_9^*	0.5471	0.9089	0.2355	0.9511

school preceptors based on fuzzy number arithmetic operations. The proposed method can overcome the drawbacks of the other methods.

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References

- [1] G. Bortolan and R. Degani, "A review of some methods for ranking fuzzy subsets," *Fuzzy Sets and Systems*, vol. 15, no. 1, pp. 1–19, 1985.
- [2] S. J. Chen and C. L. Hwang, *Fuzzy Multiple Attribute Decision Making*, Springer, Berlin, Germany, 1993.
- [3] S. J. Chen and S. M. Chen, "A new method for handling multicriteria fuzzy decision-making problems using FN-IOWA operators," *Cybernetics and Systems*, vol. 34, no. 2, pp. 109–137, 2003.
- [4] J. R. Chang, C. H. Cheng, and C. Y. Kuo, "Conceptual procedure for ranking fuzzy numbers based on adaptive two dimensions dominance," *Soft Computing*, vol. 10, no. 3, pp. 74–103, 2006.
- [5] D. Dubois and H. Prade, "Operations on fuzzy numbers," *International Journal of Systems Science*, vol. 9, no. 6, pp. 613–626, 1978.
- [6] R. Saneifard, T. Allahviranloo, F. Hosseinzadeh, and N. Mikaeilvand, "Euclidean ranking DMUs with fuzzy data in DEA," *Applied Mathematical Sciences*, vol. 60, pp. 2989–2998, 2007.
- [7] R. Saneifard and R. Ezzati, "Defuzzification through a bi-symmetrical weighted function," *Australian Journal of Basic and Applied Sciences*, vol. 4, no. 10, pp. 4976–4984, 2010.
- [8] R. Saneifard, "Ranking L-R fuzzy numbers with weighted averaging based on levels," *International Journal of Industrial Mathematics*, vol. 2, pp. 163–173, 2009.
- [9] R. Ezzati and R. Saneifard, "A new approach for ranking of fuzzy numbers with continuous weighted quasi-arithmetic means," *Mathematical Sciences*, vol. 2, pp. 143–158, 2010.
- [10] Y.-M. Wang, J.-B. Yang, D.-L. Xu, and K.-S. Chin, "On the centroids of fuzzy numbers," *Fuzzy Sets and Systems*, vol. 157, no. 7, pp. 919–926, 2006.
- [11] C.-H. Cheng, "A new approach for ranking fuzzy numbers by distance method," *Fuzzy Sets and Systems*, vol. 95, no. 3, pp. 307–317, 1998.
- [12] T.-C. Chu and C.-T. Tsao, "Ranking fuzzy numbers with an area between the centroid point and original point," *Computers & Mathematics with Applications*, vol. 43, no. 1-2, pp. 111–117, 2002.