Research Article

Travelling Waves Solution of the Unsteady Flow Problem of a Rarefied Nonhomogeneous Charged Gas Bounded by an Oscillating Plate

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The extension of the previous paper (Abdel Wahid and Elagan, 2012) has been made for a nonhomogeneous charged rarefied gas mixture (two-component plasma) instead of a single electron gas. Therefore, the effect of the positive ion collisions with electrons and with each other is taken into consideration, which was ignored, as an approximation, in the earlier work. Thus, we will have four collision terms (electron-electron, electron-ion, ion-ion, and ion-electron) instead of one term, as was studied before. These collision terms are added together with a completely additional system of differential equations for ions. This study is based on the solution of the Bhatnager-Gross-Krook (BGK) model of the Boltzmann kinetic equation coupled with Maxwell’s equations. The initial-boundary value problem of the Rayleigh flow problem applied to the system of the two-component plasma (positive ions + electrons), bounded by an oscillating plate, is solved. This situation, for the best of my knowledge, is presented from the molecular viewpoint for the first time. For this purpose, the traveling wave solution method is used to get the exact solution of the nonlinear partial differential equations. The initial-boundary value problem of the Rayleigh flow problem applied to the system of the two-component plasma (positive ions + electrons), bounded by an oscillating plate, is solved. This situation, for the best of my knowledge, is presented from the molecular viewpoint for the first time. For this purpose, the traveling wave solution method is used to get the exact solution of the nonlinear partial differential equations. In addition, the accurate formula of the whole four-collision frequency terms is presented. The distinction and comparisons between the perturbed and the equilibrium velocity distribution functions are illustrated. Definitely, the equilibrium time for electrons and for ions is calculated. The relation between those times and the relaxation time is deduced for both species of the mixture. The ratios between the different contributions of the internal energy changes are predicted via the extended Gibbs equation for both diamagnetic and paramagnetic plasmas. The results are applied to a typical model of laboratory argon plasma.

1. Introduction

A development of the previous paper [1] is introduced. The nonstationary Krook kinetic model for a rarefied charged binary gas mixture is solved, instead of the single gas. Analytically, the Bhatnager-Gross-Krook (BGK) model of the kinetic equation is applied. The travelling wave solution method is used to get the exact solution of the nonlinear partial differential equations. These equations were produced from applying the moment method to the unsteady BGK equation. Now we should solve eight nonlinear partial differential equations, which represent an arduous task. This situation, for the best of our knowledge, is presented in this paper for the first time. The unsteady solution for a binary charged gas mixture (two-component plasma) gives the problem a great generality and more applications. Taking into consideration the effect of the positive ions on the behavior of electrons, which was not observed in the previous work [1], this effect becomes a pressing matter in some physical situation; see, for example, [2]. The problem is investigated, with the new circumstance, to follow the behavior of the macroscopic properties of the gas such as the mean velocity, the shear stress, and the viscosity coefficient. The important quantities together with both the induced electric and magnetic fields are investigated for the two-component mixture, with respect to both distance and time. This new study is done to examine the behavior of electrons and positive ions in the microscale. This is performed using the kinetic Boltzmann equation coupled with Maxwell’s equation. An important novel thermodynamic treatment of the system is concluded.
The behavior of the nonequilibrium thermodynamic distribution functions for both positive ions and electrons, in different cases, is illustrated. Such treatment holds for the first time for a nonhomogeneous charged gas mixture bounded by oscillating plate. The calculated velocities are substituted into the corresponding two-stream Maxwellian distribution functions permitting us to investigate the nonequilibrium thermodynamic properties of the system. All physical quantities are defined in the Nomenclatures.

2. The Physical Problem and Mathematical Formulation

Let us assume that the upper half of the space \( y \geq 0 \), which is bounded by an infinite flat plate \( y = 0 \), is filled with a nonhomogeneous mixture of charged particles of electrons and positive ions. An infinite flat plate is fixed at \( y = 0 \), and parallel to the \( xz \)-plane. The plate oscillates harmonically in the \( x \) direction with frequency \( \beta \); that is, the velocity of the plate depends on the time \( t \) as follows:

\[
V_w = R_e \left[ U_0 \exp \left( -i\beta t \right) \right] = U_0 \cos (\beta t) ,
\]

where the symbol \( R_e \) denotes the real part of complex expression. The quantity \( U_0 \) is the velocity amplitude, which is assumed to be small when compared with the thermal molecular velocity \( V_{Te} \) of the gas. The charged gas is initially in absolute equilibrium and the wall is at rest. Then the plate starts to oscillate suddenly in its own plane with a velocity \( U_0 \cos(\beta t) \) along the \( x \)-axis (\( U_0 \) and \( \beta \) are constants). Moreover, the plate is considered impermeable, uncharged, and an insulator. The whole system (electrons + ions + plate) is kept at constant temperature. All physical quantities are defined in the Nomenclatures.
Let the forces $\vec{f}_e$ and $\vec{f}_i$ acting on each electron and ion respectively be given by [28–30]
\[
\vec{f}_e = -e\vec{E}_e - \frac{e}{c_0} (\vec{v} \wedge \vec{B}_e),
\]
\[
\vec{f}_i = e\vec{E}_i + \frac{e}{c_0} (\vec{v} \wedge \vec{B}_i).
\]

The directions of the considered physical quantities are as follows:
\[
\begin{align*}
\nabla_e &\equiv (V_{eax}, 0, 0), & \nabla_i &\equiv (q_d n V_{eax}, 0, 0), \\
\vec{E}_e &\equiv (E_{eax}, 0, 0), & \vec{B}_i &\equiv (0, 0, B_{eax}),
\end{align*}
\]
where $\alpha = e$ for electrons and $\alpha = i$ for ions.

The $V_{eax}, E_{eax}, B_{eax}$, and $V_{iwx}$ are functions of $y$ and $t$ that satisfy Maxwell's equations. The distribution function $F(y,c,t)$ of the particles for the plasma gas can be obtained from the Bhatnager-Gross-Krook (BGK) model [31] of the kinetic Boltzmann's equation as
\[
\frac{\partial F_\alpha}{\partial t} + \vec{v} \cdot \frac{\partial F_\alpha}{\partial \vec{r}} + \frac{\vec{f}_\alpha}{m_\alpha} \cdot \frac{\partial F_\alpha}{\partial \vec{v}} = \nu_{ce}(F_{0e} - F_e) + \nu_{ei}(F_{0e} - F_e) \text{ for electrons,}
\]
\[
\frac{\partial F_i}{\partial t} + \vec{v} \cdot \frac{\partial F_i}{\partial \vec{r}} + \frac{\vec{f}_i}{m_i} \cdot \frac{\partial F_i}{\partial \vec{v}} = \nu_{ii}(F_{0i} - F_i) + \nu_{ek}(F_{0e} - F_i) \text{ for ions,}
\]
where $F_{0\alpha} = n_\alpha (2\pi RT_\alpha)^{-3/2} \exp\left(-\frac{(\vec{v} - \vec{V}_\alpha)^2}{2RT_\alpha}\right)$ and $F_{0\alpha} = \int (F_{0\alpha} - F_\alpha) \frac{d\vec{v}}{m_\alpha}$

The quantities $n_\alpha, \nu_{ce}, \nu_{ei}, \nu_{ii}, \nu_{ek}$ are the number density, mean drift velocity, and effective temperature obtained by taking moments of $F_\alpha$. Some latitude in the definition of $T_\alpha$ and $\vec{V}_\alpha$ is possible; one choice is in [32]
\[
\vec{T}_e = T_e, \quad \vec{T}_i = T_i, \quad \vec{V}_e = \vec{V}_e, \quad \vec{V}_i = \vec{V}_e.
\]

The particles are reflected from the plate with a full velocity accommodation; that is, the plasma particles are reflected with the plate velocity so that the boundary conditions are $V_{e2ax}(0,t) = U_0 \cos(\beta t)$ for $t > 0$, where $V_{e2ax} = V_{eax}$ as $c_y > 0$ and $V_{eax}$ is finite as $y \to \infty$ for both ions and electrons.

Substituting from (2), (3), and (6) into (4) and (5) one obtains
\[
\frac{\partial F_e}{\partial t} + c_y \frac{\partial F_e}{\partial y} - \frac{eB_{ex}}{m_e c_0} \left( c_y \frac{\partial F_e}{\partial c_x} - c_x \frac{\partial F_e}{\partial c_y} \right) + \frac{eB_{ex}}{m_e} \frac{\partial F_e}{\partial c_x} = \nu_{ce}(F_{0e} - F_e) + \nu_{ei}(F_{0e} - F_e), \text{ for electrons,}
\]
\[
\frac{\partial F_i}{\partial t} + c_y \frac{\partial F_i}{\partial y} + \frac{eB_{ix}}{m_i c_0} \left( c_y \frac{\partial F_i}{\partial c_x} - c_x \frac{\partial F_i}{\partial c_y} \right) - \frac{eB_{ix}}{m_i} \frac{\partial F_i}{\partial c_x} = \nu_{ii}(F_{0i} - F_i) + \nu_{ek}(F_{0e} - F_i), \text{ for ions,}
\]
where $\nu_{ce}, \nu_{ei}, \nu_{ii}, \nu_{ek}$ are electron-electron, electron-ion, ion-ion, and ion-electron collision frequencies, respectively, which are given by [32–35]
The integrals over the velocity distance are evaluated from the relation
\[ \int Q_j(\vec{c}) F_{\alpha} d\xi = \int_{\epsilon_{\alpha}^0}^{\epsilon_{\alpha}^0} \int_{\epsilon_{\alpha}^0}^{\epsilon_{\alpha}^0} \int_{\epsilon_{\alpha}^0}^{\epsilon_{\alpha}^0} Q_j F_{\alpha} d\xi \]
where \( Q_j = Q_j(\vec{c}) \), \( j = 1, 2, \) and \( d\xi = d\epsilon_{\alpha} d\epsilon_{\beta} d\epsilon_{\gamma} \), where \( \epsilon_x, \epsilon_y, \) and \( \epsilon_z \) are the particles velocities components along \( x-, y-, \) and \( z- \) axes, respectively. Moreover, \( E_x \) and \( B_y \) may be obtained from Maxwell's equation, for electrons as follows:
\[ \frac{\partial E_{xe}}{\partial \epsilon} - \frac{1}{\epsilon_0} \frac{\partial B_{xe}}{\partial \epsilon} = 0, \]
\[ \frac{\partial B_{xe}}{\partial \epsilon} - \frac{1}{\epsilon_0} \frac{\partial E_{xe}}{\partial \epsilon} - 4\pi \epsilon_0 \epsilon V_{xe} = 0. \]
For ions we obtain
\[ \frac{\partial E_{xi}}{\partial \epsilon} - \frac{1}{\epsilon_0} \frac{\partial B_{xi}}{\partial \epsilon} = 0, \]
\[ \frac{\partial B_{xi}}{\partial \epsilon} - \frac{1}{\epsilon_0} \frac{\partial E_{xi}}{\partial \epsilon} + 4\pi \epsilon_0 \epsilon V_{xi} = 0, \]
where \( n_\alpha = \int F_{\alpha} d\xi \), \( n_\alpha V_{xe} = \int c_x F_{\alpha} d\xi \), with the initial and boundary conditions
\[ E_{xe}(y, 0) = B_{xe}(y, 0) = 0; \]
\[ E_{xa}(y, t), B_{xa}(y, t) \] are finite as \( y \to \infty \).

We introduce the dimensionless variables defined by
\[ t = t^* \tau_e, \quad y = y^* \left( \frac{t^* \epsilon V_{Te}}{\sqrt{2\pi}} \right), \]
\[ V_{xa} = V_{xa} V_{Te}, \quad \tau_{xya} = \tau_{xya}^* V_{Te}, \quad \text{Ma} = \frac{U_0}{V_{Te}}, \]
\[ B_{xa} = B_{xa}^* \left( \frac{\sqrt{2\pi} n_\alpha \epsilon_0 \epsilon}{e \tau_e} \right), \quad E_{xa}^* = E_{xa} \left( \frac{m_e V_{Te}}{e \tau_e} \right), \]
\[ \rho_\alpha = n_\alpha m_\alpha, \quad V_{Te} = \frac{2K_B T_e}{m_e}, \]
\[ \epsilon = \frac{m_e}{m_i}, \quad dU_\alpha = dU_\alpha^* \left( K_B T_e \right), \]
\[ F_{ja} = F_{ja}^* n_c (2\pi R T_e)^{-3/2}, \quad j = 0, 1, 2. \]
For \( M^2 \ll 1 \) (low Mach number), we can assume that the density and the temperature variation at each point of the flow and at any time are negligible; that is, \( n_\alpha = 1 + O(M^2) \) and \( T _\alpha = 1 + O(M^2) \). Put
\[ V_{xa} = \frac{1}{2} \left( V_{x1a} + V_{x2a} \right), \]
\[ \tau_{xya} = \frac{P_{xy}}{\rho_\alpha U_0 \sqrt{R T_e}/2\pi} = \left( V_{x2a} - V_{x1a} \right), \]
where \( P_{xy} \) is the shear stress [11] that is defined by \( P_{xy} = m \int (c_x - V_{xc}) c_y F_{xy} d\xi \).

Using the dimensionless variable, if we neglect terms of order \( O(Ma^2) \), (11) for \( Q_1 = c_x \) and \( Q_2 = c_x c_y \) is:
\[ \frac{\partial V'_{xe}}{\partial \epsilon} + \frac{\partial r_{xye}}{\partial \epsilon} - E'_{xe} = 0, \]
\[ \frac{\partial r_{xye}}{\partial \epsilon} + 2\pi \frac{\partial V'_{xye}}{\partial \epsilon} + \left( \frac{\nu_0 e}{\nu_0 e} \frac{\nu_0 e}{\nu_0 e} \right) r'_{xye} = 0. \]

Similarly, (12) becomes:
\[ \frac{\partial V'_{xi}}{\partial \epsilon} + \frac{\partial r'_{yxi}}{\partial \epsilon} + \epsilon E'_{xi} = 0, \]
\[ \frac{\partial r'_{yxi}}{\partial \epsilon} + 2\pi \frac{\partial V'_{yxi}}{\partial \epsilon} + \left( \frac{\nu_0 i e}{\nu_0 i e} \frac{\nu_0 i e}{\nu_0 i e} \right) r'_{yxi} = 0, \]
with the initial and boundary conditions
\[ V'_{xa}(y', 0) = r'_{xya}(y', 0) = 0, \]
\[ 2V'_{xa}(0, t') + r'_{yxa}(0, t') = 2Ma \cos(\beta_1 t'); \]
\[ V'_{xa}, r'_{xya} \] are finite as \( y \to \infty \), \( \beta_1 = \beta_1 \).

In the expressions for the transport coefficients mentioned previously, the fact that the plasma is neutral was used [33], writing \( n = n_\alpha = Z n_i, \quad T = T_e = T_i, \) and \( Z = 1 \), which exploited the fact that the ratio \( m_i/m_e \) is small. Thus, the ratio between the four distinguished collision frequencies can be rewritten in a form that referred to \( \nu_{ee} \) in formulas (9)
\[ \nu_{ee} : \nu_{ei} : \nu_{ii} : \nu_{ee} = 1 : \sqrt{2} : \sqrt{2} : \sqrt{2} : \sqrt{2} : \epsilon. \]

Therefore, \( \nu_{ee} \approx \nu_{ei} \gg \nu_{ii} \gg \nu_{ee} \); the first of these gross inequalities arises because thermal ion speeds are less than thermal electron speeds by the factor \( \sqrt{m_i/m_e} \) if \( T_e \approx T_i \), and so the ions take longer period of time to meet each other. The second one reflects the fact that the electrons are not very effective in deflection the much heavier ions.

For the sake of simplicity, henceforth, we drop the dash over the dimensionless variables. Therefore, we have the following initial-boundary value problem for electrons (neglecting the displacement current) [30]:
\[ \frac{\partial V'_{xe}}{\partial \epsilon} + \frac{\partial r_{xye}}{\partial \epsilon} - E'_{xe} = 0, \]
\[ \frac{\partial r_{xye}}{\partial \epsilon} + 2\pi \frac{\partial V'_{xye}}{\partial \epsilon} + \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) r'_{xye} = 0, \]
\[ \frac{\partial E'_{xe}}{\partial \epsilon} - \frac{\partial B_{xe}}{\partial \epsilon} = 0, \]
\[ \frac{\partial B_{xe}}{\partial \epsilon} - \alpha_{ee} V_{xe} = 0, \]
with the initial and boundary conditions
\[ V_{xe}(y, 0) = \tau_{xye}(y, 0) = E_{xe}(y, 0) = B_{ze}(y, 0) = 0, \]
\[ 2V_{xe}(0, t) + \tau_{xye}(0, t) = 2Ma \cos(\beta_1 t), \quad \text{for } t > 0; \]  
\[ V_{xe}, \tau_{xye}, E_{xe}, B_{ze} \quad \text{are finite as } y \to \infty. \]

In addition, we have the following initial-boundary problem for ions (neglecting the displacement current):
\[ \frac{\partial V_{xi}(y, t)}{\partial t} + \frac{\partial \tau_{xyi}(y, t)}{\partial y} + eE_{xi}(y, t) = 0, \]  
\[ \frac{\partial \tau_{xyi}(y, t)}{\partial t} + 2\pi \frac{\partial V_{xi}(y, t)}{\partial y} + (\sqrt{\xi} + 1) \tau_{xyi}(y, t) = 0, \]
\[ \frac{\partial E_{xi}(y, t)}{\partial y} - \frac{\partial B_{zi}(y, t)}{\partial t} = 0, \]
\[ \frac{\partial B_{zi}(y, t)}{\partial y} + \alpha_0 V_{xi}(y, t) = 0, \quad \text{where } \alpha_0 = \left( \frac{V_{Te}^2 e^2 n_i}{m_e} \right). \]

Since \( n_i = n_e \); thus, henceforth, we put \( \alpha_0 = \alpha_0 = \alpha_0 \).

We can reduce our basic (23)–(26), for electrons, after some analytical manipulations to a single equation as follows:
\[ \frac{d^2 V_{xe}(y, t)}{dt^2} - 2\pi \frac{d^2 V_{xe}(y, t)}{dt^2} + A_c \frac{d^2 V_{xe}(y, t)}{d\xi d\eta^2} - \alpha_0 \frac{dV_{xe}(y, t)}{d\xi} = 0, \]  
\[ A_c = (1/\sqrt{\xi} + \sqrt{2/\xi}). \]

Similarly, the basic (28)–(31) can be reduced for ions to obtain
\[ \frac{d^2 V_{xi}(y, t)}{dt^2} - 2\pi \frac{d^2 V_{xi}(y, t)}{dt^2} - B_c \frac{d^2 V_{xi}(y, t)}{d\eta^2} - \alpha_0 \frac{dV_{xi}(y, t)}{d\eta} = 0, \]  
where \( B_c = (\sqrt{\xi} + 1) \).

3. Solution of the Initial-Boundary Value Problem

The traveling wave solution method [41, 42] is used, considered
\[ \xi = ly - mt, \]  
to make all the dependent variables as functions of \( \xi \). Here \( l \) and \( m \) are transformation constants, which do not depend on the properties of the fluid but as parameters to be determined by the boundary and initial conditions [41]. Using (34) we obtain the derivatives
\[ \frac{\partial}{\partial t} = -m \frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial y} = l \frac{\partial}{\partial \xi}, \quad \frac{\partial^2}{\partial t^2} = -\alpha_0 m^2 \frac{\partial^2}{\partial \xi^2}, \]
\[ \frac{\partial^4}{\partial t^4} = \alpha_0 l^2 \frac{\partial^4}{\partial \xi^4},. \]

where \( a \) is a positive integer.

Substituting from (34)–(35) into (32), we have
\[ \left( m^2 l^2 - 2m^4 \right) \frac{d^4 V_{xe}(\xi)}{d\xi^4} + A_c ml^2 \frac{d^4 V_{xe}(\xi)}{d\xi^4} - \alpha_0 m^2 \frac{d^2 V_{xe}(\xi)}{d\xi^2} = 0. \]

The boundary and initial conditions became
\[ E_{xe}(\xi = 0) = B_{xe}(\xi = 0) = \tau_{xye}(\xi = 0) = 0, \]
\[ 2V_{xe}(\xi = -m) + \tau_{xye}(\xi = -m) = 2Ma \cos(\beta_1 t), \]
\[ \text{at } y = 0, \quad \text{e.g., } t = 1; \]
\[ V_{xe}, \tau_{xye}, E_{xe}, B_{ze} \quad \text{are finite as } \xi \to -\infty. \]

Now we have an ordinary differential Equation (36) with the boundary and initial conditions (37). Next, let us solve (33) by using the same tackling. Substituting from expressions (34)–(35) into (33), we get
\[ \left( m^2 l^2 - 2\pi m^4 \right) \frac{d^4 V_{xi}(\xi)}{d\xi^4} + B_c ml^2 \frac{d^4 V_{xi}(\xi)}{d\xi^4} - \alpha_0 m^2 \frac{d^2 V_{xi}(\xi)}{d\xi^2} = 0, \]
with the corresponding boundary and initial conditions
\[ E_{xi}(\xi = 0) = B_{zi}(\xi = 0) = \tau_{xyi}(\xi = 0) = 0, \]
\[ 2V_{xi}(\xi = -m) + \tau_{xyi}(\xi = -m) = 2Ma \cos(\beta_1) ; \]
\[ V_{xi}, \tau_{xyi}, E_{xi}, B_{zi} \quad \text{are finite as } \xi \to -\infty. \]

The two ordinary, fourth order homogeneous differential equations (36) and (38) can be solved exactly by the help of symbolic computer software, with their boundary and initial conditions (37) and (39). The sought solutions will be applied to a typical model of laboratory argon plasma.

4. The Investigation of the Behavior of the Internal Energy Change

The studying of the behavior of the internal energy change for the physical systems presents a great importance in science. The extended Gibbs relation for electrons and ions is introduced to study the internal energy change for the system,
based on the solution of the nonstationary Boltzmann equation [43]. It includes the electromagnetic field energy as a part of the complete energy balance. This procedure distinguishes the charged gas into paramagnetic and diamagnetic ones. If there are unpaired electrons in the molecular orbital diagram, the gas is paramagnetic. If all electrons are paired, the gas is considered as a diamagnetic one. We should write the internal energy balance, including the electromagnetic field energy, to get the work term in the first law of thermodynamics as follows.

(a) For paramagnetic plasma, the internal energy change is expressed in terms of the extensive quantities $S$, $P$, and $M$, which are the thermodynamic coordinates corresponding to the conjugate intensive quantities $T$, $E$, and $M$, respectively. The three contributions in the internal energy change in the Gibbs formula we have

$$dU = dU_{\text{sys}} + dU_{\text{pol}} + dU_{\text{para}}$$  \hspace{1cm} (40)

where $dU_{\text{sys}} = TdS_b$ is the internal energy change due to the variation of the entropy, which is written in dimensionless form as

$$S_b = -\pi^{3/2} \left( \left(V_{xla}^2 + V_{x2a}^2\right)^{3/2} - \frac{3}{2} \right).$$  \hspace{1cm} (41)

$$dU_{\text{pol}} = E_d dP_a$$ is the internal energy change due to the variation of polarization, and $dU_{\text{para}} = B_d dM_a$ is the internal energy change due to the variation of magnetization, here $M$ is calculated from the equation [44]

$$\frac{\partial E_a}{\partial M_a} = \frac{T_a}{B_a} \implies M_a = -\left(\frac{T_a}{B_a}\frac{\partial S_a}{\partial y}\right) dy.$$  \hspace{1cm} (42)

Introduce the dimensionless variables $U' = U_a/K T_a$, $M_a' = M_a(1/e_a T_a V_T a)$, $P_a' = P_a(1/e_a T_e V_T a)$ in the Gibbs formula to get (after dropping the primes)

$$dU = dS_a + f_{\alpha 1} E_a dP_a + f_{\alpha 1} B_a dM_a.$$  \hspace{1cm} (43)

(b) On the other hand, if the plasma is diamagnetic, the internal energy change due to the extensive variables $S$, $P$, and $B$ represents the thermodynamic coordinates conjugate to the intensive quantities $T$, $E$, and $M$, respectively; therefore, we have three contributions in the internal energy change in the Gibbs formula given by

$$dU = dU_{\text{sys}} + dU_{\text{pol}} + dU_{\text{dia}},$$  \hspace{1cm} (44)

where $dU_{\text{dia}} = -M_a dB_a$ is the internal energy change due to the variation of the induced magnetic induction, where $M_a = T_a (\partial S_a/\partial B_a)$. Hence, the dimensionless form for $dU$ in this case takes the following form:

$$dU = dS_a + f_{\alpha 1} E_a dP_a - f_{\alpha 1} B_a dM_a.$$  \hspace{1cm} (45)

where

$$f_1 = \left(\frac{m_e V_{T_e}^2}{KT_0}\right), \quad dS_a = \left(\frac{\partial S_a}{\partial r}\right) dy + \left(\frac{\partial S_a}{\partial t}\right) dt;$$  \hspace{1cm} (46)

$$\delta y = 1, \quad \delta t = 4.35 \approx t_{\text{cge}}.$$

5. Discussion

In this problem, the unsteady behavior of an inhomogeneous mixture of charged gas, bounded by an oscillating plate, is investigated. This study is based on the kinetic theory via the BGK model of the Boltzmann equation. Our computations are performed according to typical data for argon plasma as a paramagnetic medium in the case of the argon gas losing single electrons subjected to the following conditions and parameters: $K_B = 1.3807 \times 10^{-16}$ erg/K, $T_0 = 600$ K, $n_e = 7 \times 10^{16}$ cm$^{-3}$, $d = 3.84 \times 10^{-8}$ cm (diameter of the argon atom); the electron rest mass, mass of ions, and charge are $m_e = 9.093 \times 10^{-28}$ gm, $m_i = 6.633 \times 10^{-21}$ gm, $e = 4.8 \times 10^{-10}$ esu, which are used to calculate electron-ion relaxation time $\tau_{\text{e}} = 1.4859 \times 10^{-10}$ sec. The dimensionless parameter is $\delta_t = 7.9 \times 10^{-2}$ and the mean free path of the electrons $\lambda_{\text{e}} = 2.110^{-3}$ cm compared to the electron Debye length.

Consider $\lambda_{\text{De}} = (K_B T_0/4 m_e e^2)^{1/2} = 6.39 \times 10^{-7}$ cm, $f_1 = 2$. Using the idea of the shooting numerical calculation method, we find the transformation constants are evaluated to obtain $m = 2.8, l = -1.2, \text{ and the plate Mach number } M_a = 6.5 \times 10^{-2}$.

The fundamental and the essential inequalities, which we must bear in mind when analyzing the results, are

$$m_i \gg m_e \quad \text{makes } \varepsilon^2 = \frac{m_e}{m_i} = 1.37 \times 10^{-5},$$  \hspace{1cm} (I-a)

$$\tau_{\text{e}} : \tau_{\text{el}} : \tau_{\text{ii}} : \tau_{\text{e}} = 1 : 7.07 \times 10^{-1} : 2.69 \times 10^{2} : 5.15 \times 10^{4}$$

$$\implies \tau_{\text{e}} \sim \tau_{\text{el}} \ll \tau_{\text{ii}} \ll \tau_{\text{e}}.$$  \hspace{1cm} (I-b)

These inequalities will control the major behavior of both electrons and positive ions in the rest of the discussion. Figure I(a) clarifies that in the course of time the perturbed velocity distribution functions $F_1$ and $F_2$ approach the equilibrium velocity distribution function $F_0$ at $y = 0.003$, for example. The ions are still departing from equilibrium very slowly; see Figure I(b).

A comparison between Figures I(a) and I(b) shows that the collisions of ions with electrons have a very little effect on the form of the ions nonequilibrium distribution function. On the other hand, collisions of electrons with ions have an important effect on the form of the electrons nonequilibrium distribution function, which is in a good agreement with the well-known study of Braginskii [33]. This is because the transfer of momentum from ions to electrons occurs in about the same time $\tau_{\text{el}}$ as the transfer of the energy, hence, ion-electron momentum transfer is small compared with ion-ion momentum transfer. The transfer of momentum from electrons to ions occurs in a time of the same order $\tau_{\text{e}}$ as electron-electron transfer time, so that collisions of electrons...
Figure 1: (a) The comparison between the combined perturbed dimensionless velocity distribution functions for electrons $F_e (F_{1e} (green), F_{2e} (red))$ and electrons equilibrium velocity distribution function $F_{0e} (grid)$ at $t = 10^{-3}, 2.175,$ and $4.35$ for a fixed $y$ value ($0.003$) with the Mach number of the plate $Ma = 0.065$. (b) The comparison between the combined perturbed dimensionless velocity distribution functions for ions $F_i (F_{1i} (green), F_{2i} (red))$ and ions equilibrium velocity distribution function $F_{0i} (grid)$ at $t = 10^{-3}, 2.175,$ and $4.35$ for a fixed $y$ value ($0.003$) with the Mach number of the plate $Ma = 0.065$. (c) The comparison between the combined perturbed dimensionless velocity distribution functions for ions $F_i (F_{1i} (gray), F_{2i} (black))$ and ions equilibrium dimensionless velocity distribution function $F_{0i} (grid)$ at $t = 690, 1380$ for a fixed $y$ value ($0.003$) with the Mach number of the plate $Ma = 0.065$. 

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with the ions affect the electrons nonequilibrium distribution function.

Figures 1(a) and 1(b) indicate that the lighter species (electrons) in the mixture of gases reach equilibrium before the heavier one (positive ions) that is in a qualitative agreement with the investigation made by Galkin [45]. Figure 1(a) makes it clear that the time needed for the electrons to reach equilibrium is \( t_{eq} \sim 4.35 \) which will be taken into consideration, henceforth.

In a relative long period, for example, \( t = 690 \), the departure of positive ions from equilibrium becomes obvious; the distinction between ions equilibrium distribution function and their nonequilibrium distribution function becomes substantial. After a relatively long time the heavy species (positive ions) reach equilibrium at \( t \equiv 1380 \); see Figure 1(c).

All Figures 1(a) and 1(b) to 8(a) and 8(b) shed light upon the boundary and initial conditions for both electrons and ions hold. As revealed in Figures 2(a) and 2(b) at the vicinity of the suddenly oscillating plate the mean velocities for both electrons and positive ions have a value \( = Ma \) of the oscillating plate which satisfies the conditions of the problem. The \( V_{xe} \) gets a severe decrement with time while \( V_{xi} \) shows a gradually linear decrement; this is due to two reasons, firstly, because thermal ion speeds are less than the thermal electron speeds, by the amount \( \sqrt{m_e/m_i} \) as \( T_e \equiv T_i \), and so ions take longer time to encounter each other while electrons do not. Secondly, this behavior reflects the fact that the electrons are not very effective in deflecting the much heavier ions, while ions have a very successful effect on electrons motion, referring to inequalities (I-a) and (I-b). Therefore, the change of all kinetic and thermodynamic variables belongs to the positive ions that has unnoticed nonlinear gradually slow changes; see Figures 1(b) to 8(b). This is in a qualitative agreement with the previous paper [15].

The shear stress \( \tau_{xye} \) is beginning from zero value. It is increasing nonlinearly towards its maximum value at \( t \sim 0.5t_{eq} \). After that, it decreases with the same behavior until it vanishes; see Figure 3(a). This is according to the behavior of the electrons velocity itself. Since the deviation from equilibrium is small, the electron gas is rarefied and the flow is slow. Thus, the gas is Newtonian [46]. It follows that the viscosity \( \mu_e = -\tau_{xye}/(\partial V_{xe}/\partial y) \) represents the resistance to the motion. It gradually increases nonlinearly as the gas particles move away from the plate for ions \( \mu_i \). The \( \mu_e \) approaches zero for electrons except at a small time interval around \( t \sim 0.5t_{eq} \); this is due to the corresponding maximum value of \( \tau_{xye} \) in the same time interval.
The electrons induced electric field has a sudden increase in the beginning until it reaches its maximum value \( E_{xe\,\text{Max}} \approx 0.1 \) due to the sudden oscillation of the plate itself. It decreases nonlinearly until it vanishes at \( t \approx 0.5 t_{eq} \). After that, it changes its direction until the same maximum value \( E_{xe\,\text{Max}} \approx -0.1 \) in the opposite direction. It is pushing electrons towards equilibrium. This is because of the famous le Chatelier principle that states, "If the system going away from equilibrium, its particles take the behavior that decreases the departure from equilibrium" that is pushing the system towards equilibrium again; see Figure 4(a). The same behavior holds for electrons induced magnetic field; see Figure 5(a).

Upon passing through a plasma, a charged particle (electron) losses (or gains) part of its energy because of the interaction with the surroundings (positive ions) due to plasma polarization and collisions. The energy loss (or gain) of an electron is determined by the work of the forces acting on the electrons in the plasma by the electromagnetic field generated by the moving particles themselves [47], since the suddenly oscillating plate causes work to be done on the gas, changing the internal energy of the gas \( U \). As shown in Figures 6–8, the change in the internal energy due to the variation of entropy and polarization varies smoothly with time by the energy lost to and gained from the ions and plate, respectively. The change in internal energy varies chaotically because of the intensive variables, corresponding to paramagnetic plasma; at the end each tends to zero.

6. Conclusions

The solution of the unsteady BGK equation in the case of an inhomogeneous rarefied charged gas, bounded by an oscillating plate, is investigated. We use the method of moments of the two-sided distribution function together with Maxwell’s equations. This is developed within the restrictions of small deviation from equilibrium, rarified gas mixture, and slow flow. This solution allows for the calculation of the components of the velocity of the flow for both electrons and positive ions. Inserting them into the suggested two-sided distribution functions and analyzing the results, it is found that:

(a) the lighter species (electrons) of the gas mixture reaches equilibrium before the heave one (positive ions), which is in a qualitative agreement with Galkin [37];
(b) definitely, the equilibrium times for electrons and for ions are calculated. The relation between those times and the relaxation time for both species of the mixture is deduced. We proved that the collision of ions with electrons has very little effect on the form of the ions nonequilibrium distribution function. On the other hand, we found that the collisions of electrons with ions have an important effect on the form of the electrons nonequilibrium distribution function, which is in a qualitative agreement with the study done by Park et al. [21];

(c) the ratio between the time that electrons \( t_{eq_e} = 4.35 \) and ions \( t_{eq_i} = 1380 \) need to reach equilibrium is approximately equal to the same order of the reverse ratio of the ion-ion collision frequency to the mean value of the electron-electron and electron-ion collision frequencies; that is, \( t_{eq_i}/t_{eq_e} = ((\nu_{ee} + \nu_{ei})/2)/\nu_{ii} \).

This conclusion is different from the situation when the plate is moving with damping velocity and not oscillating; see the previous paper [15].

The predictions, estimated using Gibbs’ equation, reveal that the following order of maximum magnitude ratios between the different contributions to the internal energy change based on the total derivatives of the extensive parameters is, for ions

\[
dU_{Si} : dU_{pi} : dU_{pari} = 1 : 0.6 \times 10^{-2} : 0.8 \times 10^{-4}. \quad (47)
\]

It is concluded that the effect of the changes of the internal energies for positive ions \( dU_{pi} \) and \( dU_{pari} \) due to electric and magnetic fields are small in comparison with \( dU_{Se} \). This happens with the recognition of the fact that these fields are self-induced by the sudden motion of the oscillating plate.

The same conclusion is applied in the case of electrons such that

\[
dU_{Se} : dU_{pe} : dU_{pare} = 1 : 10^{-6} : 10^{-2}. \quad (48)
\]

**Nomenclature**

- \( \vec{B} \): The induced magnetic field vector
- \( B \): The induced magnetic field
- \( \vec{E} \): The induced electric vector
- \( E \): The induced electric field
- \( F \): The velocity distribution function
$F_0$: The local Maxwellian distribution function

$F_1$: Distribution function for going downward particles $c_y < 0$

$F_2$: Distribution function for going upward particles $c_y > 0$

$J$: The current density

$K_B$: Boltzmann’s constant (Erg/K°) $1.38 \times 10^{-16}$

$Ma$: The plate Mach number

$M$: Specific magnetization

$P$: Polarization

$R$: The gas constant

$S$: Entropy per unit mass

$T$: The temperature

$U$: The internal energy of the gas

$U_0$: Plate initial velocity

$V_c$: The mean velocity

$V_{a1}$: The mean velocity related to $F_1$

$V_{a2}$: The mean velocity related to $F_2$

$V$: Gas volume

$V_e$: Thermal velocity of electrons

$V_i$: Thermal velocity of ions

$a_o$: The speed of light

$c$: The velocity of the particles

$d$: Particle diameter

$e$: The electron charge

$f$: Lorentz’s force vector

$m_e$: Electron mass

$m_i$: Ion mass

$n$: The mean density

$n_e$: Electrons concentration

$n_i$: Ions concentration

$p$: Pressure

$r$: The position vector of the particle

$t$: Time variable

$u$: The mean velocity of the particle

$dU_S$: The internal energy change due to the variation of entropy

$dU_P$: The internal energy change due to the variation of polarization

$dU_{par}$: The internal energy change due to the variation of magnetization

$dU_{dia}$: The internal energy change due to the variation of the induced magnetic field

$y$: Displacement variable

$Z$: Ionization.

Superscripts

$*$: Dimensionless variable.

Subscripts

$e$: Related to electrons

$i$: Related to ions

$q$: Equilibrium

$\alpha$: = $e$ for electrons or = $i$ for ions.

Greek Letters

$\tau$: The relaxation time

$\tau_{xy}$: The shear stress

$\mu$: Viscosity coefficient

$\lambda$: The mean free path

$\alpha_0$: Dimensionless parameter

$\nu$: Frequency

$\nu$: Collision frequency

$\epsilon$: Mass ratio

$\lambda$: Mean free path

$\lambda_D$: Debye radius.

References


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