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Research Article

$\mathscr{H}_\infty$ Control for Two-Dimensional Markovian Jump Systems with State-Delays and Defective Mode Information

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This paper investigates the problem of $\mathscr{H}_\infty$ state-feedback control for a class of two-dimensional (2D) discrete-time Markovian jump linear time-delay systems with defective mode information. The mathematical model of the 2D system is established based on the well-known Fornasini-Marchesini local state-space model, and the defective mode information simultaneously consists of the exactly known, partially unknown, and uncertain transition probabilities. By carefully analyzing the features of the transition probability matrices, together with the convexification of uncertain domains, a new $\mathscr{H}_\infty$ performance analysis criterion for the underlying system is firstly derived, and then the $\mathscr{H}_\infty$ state-feedback controller synthesis is developed via a linearisation technique. It is shown that the controller gains can be constructed by solving a set of linear matrix inequalities. Finally, an illustrative example is provided to verify the effectiveness of the proposed design method.

1. Introduction

During the past decades, two-dimensional (2D) systems have drawn considerable attention due to their extensive applications of both theoretical and practical interests in many modern engineering fields, such as process control (thermal processes, gas absorption, water stream heating, etc.) [1], multidimensional digital filtering [2], and image processing (enhancement, deblurring, seismographic data processing, etc.) [3]. In particular, since the well-known Roesser state-space model and the Fornasini-Marchesini local state-space (FMLSS) model were proposed, theory on 2D systems has progressed greatly [4–11].

On the other hand, as a class of stochastic hybrid systems, Markovian jump linear systems (MJLSs) have been extensively investigated [12–26]. The driving force behind this is that MJLSs can model different classes of dynamic systems subject to random abrupt variations in their structures, for example, manufacturing systems, power systems, and networked control systems, where random failure, repairs, and sudden environment changes may occur in Markov chains [27–29]. It is known that MJLSs are described by a set of classical differential (or difference) equations and a Markov stochastic process (or Markov chain) [30]. As a decisive factor, transition probabilities (TPs) in the jumping process determine the system behavior to a large extent and, so, far, many studies on the analysis and synthesis of MJLSs have been carried out in the context of perfect information on TPs [12–15, 25]. In practice, however, defective mode information is often encountered especially when adequate efforts to obtain the accurate TPs are costly or time consuming. Thus, it is more practical and interesting to study more general jump systems with defective mode information. Recently, there have appeared some results on the analysis and synthesis of MJLSs with uncertain TPs or partially unknown TPs [26, 31–38]. To mention a few, the authors in [31] studied the $\mathscr{H}_\infty$ filter synthesis problem for a class of MJLSs with partially unknown TPs; The author in [32] considered the robust stability analysis and stabilization problems for a class of MJLSs with polytopic uncertain TPs; the authors in [36] addressed the robust stability analysis problem for a class of MJLSs with norm-bounded uncertain TPs.

However, the above-mentioned works were only concerned with one-dimensional (1D) systems. Inevitably, when 2D systems are employed to model dynamic systems with random abrupt changes in their structures or parameters

such as chemical process control, the mathematical modeling of such physical systems would be naturally dependent on jumping parameters. For example, information propagation occurs from pass to pass and along a given pass in a gas absorption, water stream heating, and air drying [39]. Therefore, 2D MJLSs emerge as a more reasonable description to account for the parameter jumping phenomenon and have a great potential in engineering applications. Recently, the stability analysis and synthesis of discrete-time delay-free 2D MJLSs described by Roesser model were reported in [39, 40], which are obtained based on the traditional assumption of complete knowledge on TPs. To the authors' best knowledge, the analysis and synthesis for 2D MJLSs with state-delays and defective mode information have not drawn much attention yet, which motivates us for this study.

In this paper, the $\mathscr{H}_\infty$ control problem for a class of 2D discrete-time MJLSs with state-delays and defective mode information will be studied. The mathematical model of the 2D system is established in terms of the FMLSS model subject to state-delays, and the defective mode information simultaneously includes the exactly known, partially unknown and uncertain TPs. By fully considering the properties of the transition probability matrices, together with the convexification of uncertain domains, a new $\mathscr{H}_\infty$ performance analysis criterion for the closed-loop system will be firstly derived. By a linearisation procedure, the corresponding $\mathscr{H}_\infty$ controller synthesis will then be converted into a convex optimization problem in terms of a set of linear matrix inequalities. Finally, an illustrative example will be performed to show the effectiveness of the proposed controller synthesis method.

Notations. The notations used throughout the paper are standard. $\mathbf{R}^n$ and $\mathbf{R}^{m\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $m \times n$ real matrices; $\mathbb{N}^+$ represents the sets of positive integers; the notation $P > 0$ means that P is real symmetric and positive definite; $\mathbf{I}$ and $\mathbf{0}$ represent the identity matrix and a zero matrix, respectively; $(\mathscr{S}, \mathscr{F}, \mathscr{P})$ denotes a complete probability space, in which $\mathscr{S}$ is the sample space, $\mathscr{F}$ is the σ algebra of subsets of the sample space, and $\mathscr{P}$ is the probability measure on $\mathscr{F}$; $\mathbb{E}[\cdot]$ stands for the mathematical expectation; $\|\cdot\|$ refers to the Euclidean norm of a vector or its induced norm of a matrix; $l_2\{[0,\infty),[0,\infty)\}$ denotes the space of square summable sequences on $\{[0,\infty),[0,\infty)\}$. Matrices, if not explicitly stated, are assumed to have appropriate dimensions for algebra operations.

2. Problem Formulation and Preliminaries

Fix a complete probability space $(\mathscr{S}, \mathscr{F}, \mathscr{P})$ and consider the following two-dimensional (2D) discrete-time Markovian jump linear systems (MJLSs), described by the Fornasini-Marchesini local state-space (FMLSS) model with time-delays in the states:

$$\begin{aligned}(\Sigma): x(i+1,j+1) &= A_1(r(i,j+1))\,x(i,j+1)\\ &\quad + A_2(r(i+1,j))\,x(i+1,j)\\ &\quad + A_{d1}(r(i,j+1))\,x(i-d_1,j+1)\\ &\quad + A_{d2}(r(i+1,j))\,x(i+1,j-d_2)\\ &\quad + B_1(r(i,j+1))\,u(i,j+1)\\ &\quad + B_2(r(i+1,j))\,u(i+1,j)\\ &\quad + D_1(r(i,j+1))\,w(i,j+1)\\ &\quad + D_2(r(i+1,j))\,w(i+1,j),\\ z(i,j) &= C(r(i,j))\,x(i,j) + B_3(r(i,j))\,u(i,j)\\ &\quad + D_3(r(i,j))\,w(i,j),\end{aligned} \tag{1}$$

where $x(i,j) \in \mathbf{R}^{n_x}$ is the state vector; $u(i,j) \in \mathbf{R}^{n_u}$ is the control input; $z(i,j) \in \mathbf{R}^{n_z}$ is the controlled output; $w(i,j) \in \mathbf{R}^{n_w}$ denotes the disturbance input vector which belongs to $l_2\{[0,\infty),[0,\infty)\}$; and d_1 and d_2 are two constant positive integers representing delays along vertical and horizontal directions, respectively. $A_1(r(i,j+1))$, $A_2(r(i+1,j))$, $A_{d1}(r(i,j+1))$, $A_{d2}(r(i+1,j))$, $B_1(r(i,j+1))$, $B_2(r(i+1,j))$, $D_1(r(i,j+1))$, $D_2(r(i+1,j))$, $C(r(i,j))$, $B_3(r(i,j))$, and $D_3(r(i,j))$ are real-valued system matrices. These matrices are functions of $r(i,j)$, which is described by a discrete-time, discrete-state homogeneous Markov chain with a finite-state space $\mathscr{I} := \{1,\ldots,N\}$, and a stationary transition probability matrix (TPM) $\Pi = [\pi_{mn}]_{N\times N}$, where

$$\begin{aligned}\pi_{mn} &= \Pr(r(i+1,j+1) = n \mid r(i,j+1) = m)\\ &= \Pr(r(i+1,j+1) = n \mid r(i+1,j) = m), \quad \forall m,n \in \mathscr{I},\end{aligned} \tag{2}$$

with $\pi_{mn} \geq 0$ and $\sum_{n=1}^{N}\pi_{mn} = 1$. For $r(i+1,j) = m \in \mathscr{I}$ or $r(i,j+1) = m \in \mathscr{I}$, the system matrices of the mth mode are denoted by $(A_{1m}, A_{2m}, A_{d1m}, A_{d2m}, B_{1m}, B_{2m}, D_{1m}, D_{2m}, C_m, B_{3m}, D_{3m})$, which are known and with appropriate dimensions. Unless otherwise stated, similar simplification is also applied to other matrices in the following.

In this paper, the transition probabilities (TPs) of the jumping process are assumed to be uncertain and partially accessed; that is, the TPM $\Pi = [\pi_{mn}]_{N\times N}$ is assumed to belong to a given polytope P_Π with vertices Π_s, $s = 1,2,\ldots,M$, $\mathrm{P}_\Pi := \{\Pi \mid \Pi = \sum_{s=1}^{M}\alpha_s\Pi_s;\ \alpha_s \geq 0,\ \sum_{s=1}^{M}\alpha_s = 1\}$, where $\Pi_s = [\pi_{mn}]_{N\times N}$, $m,n \in \mathscr{I}$, are given TPMs containing unknown elements still. For instance, for system (Σ) with four operation modes, the TPM may be as

$$\begin{bmatrix}\pi_{11} & \widetilde{\pi}_{12} & \widehat{\pi}_{13} & \pi_{14}\\ \widehat{\pi}_{21} & \pi_{22} & \widetilde{\pi}_{23} & \pi_{24}\\ \pi_{31} & \widehat{\pi}_{32} & \pi_{33} & \widehat{\pi}_{34}\\ \pi_{41} & \widetilde{\pi}_{42} & \widehat{\pi}_{43} & \widehat{\pi}_{44}\end{bmatrix}, \tag{3}$$

where the elements labeled with "$\widehat{\ }$" and "$\sim$" represent the unknown information and polytopic uncertainties on TPs, respectively, and the others are known TPs. For notational

clarity, for all $m \in \mathcal{I}$, we denote $\mathcal{I} = \mathcal{I}_{\mathcal{K}}^{(m)} \cup \mathcal{I}_{\mathcal{UC}}^{(m)} \cup \mathcal{I}_{\mathcal{UK}}^{(m)}$ as follows:

$$\begin{aligned} \mathcal{I}_{\mathcal{K}}^{(m)} &:= \{n : \pi_{mn} \text{ is known}\}, \\ \mathcal{I}_{\mathcal{UC}}^{(m)} &:= \{n : \widetilde{\pi}_{mn} \text{ is uncertain}\}, \\ \mathcal{I}_{\mathcal{UK}}^{(m)} &:= \{n : \widehat{\pi}_{mn} \text{ is unknown}\}. \end{aligned} \tag{4}$$

Moreover, if $\mathcal{I}_{\mathcal{K}}^{(m)} \neq \emptyset$ and $\mathcal{I}_{\mathcal{UC}}^{(m)} \neq \emptyset$, it is further described as

$$\begin{aligned} \mathcal{I}_{\mathcal{K}}^{(m)} &:= \left\{\mathcal{K}_{1_{(m)}}, \ldots, \mathcal{K}_{t_{(m)}}\right\}, \quad \forall 1 \le t_{(m)} \le N-2, \\ \mathcal{I}_{\mathcal{UC}}^{(m)} &:= \left\{\mathcal{U}_{1_{(m)}}, \ldots, \mathcal{U}_{v_{(m)}}\right\}, \quad \forall 1 \le v_{(m)} \le N, \end{aligned} \tag{5}$$

where $\mathcal{K}_{t_{(m)}} \in \mathbb{N}^+$ represents the $t_{(m)}$th known element with the index $\mathcal{K}_{t_{(m)}}$ in the mth row of the TPM and $\mathcal{U}_{v_{(m)}} \in \mathbb{N}^+$ represents the $v_{(m)}$th uncertain element with the index $\mathcal{U}_{v_{(m)}}$ in the mth row of the TPM. Obviously, $1 \le t_{(m)} + v_{(m)} \le N$. Also, we denote

$$\pi_{\mathcal{UK}}^{(ms)} := \sum_{n \in \mathcal{I}_{\mathcal{UK}}^{(m)}} \widehat{\pi}_{mn} = 1 - \sum_{n \in \mathcal{I}_{\mathcal{K}}^{(m)}} \pi_{mn} - \sum_{n \in \mathcal{I}_{\mathcal{UC}}^{(m)}} \widetilde{\pi}_{mn}^{(s)}, \tag{6}$$

where $\widetilde{\pi}_{mn}^{(s)}$ represents an uncertain TP in the sth polytope, for all $s = 1, \ldots, M$.

The boundary conditions of system (Σ) in (1) are defined by

$$\begin{aligned} &\{x(i,j) = \phi(i,j), \ \forall j \ge 0, \ -d_1 \le i \le 0\}, \\ &\{x(i,j) = \varphi(i,j), \ \forall i \ge 0, \ -d_2 \le j \le 0\}, \\ &\phi(0,0) = \varphi(0,0). \end{aligned} \tag{7}$$

Throughout this paper, the following assumption is made.

Assumption 1. The boundary conditions are assumed to satisfy

$$\begin{aligned} &\lim_{T_1 \to \infty} \mathbb{E}\left\{\sum_{j=0}^{T_1} \sum_{i=-d_1}^{0} \left(\phi^{\mathrm{T}}(i,j)\phi(i,j)\right)\right\} \\ &\quad + \lim_{T_2 \to \infty} \mathbb{E}\left\{\sum_{i=0}^{T_2} \sum_{j=-d_2}^{0} \left(\varphi^{\mathrm{T}}(i,j)\varphi(i,j)\right)\right\} < \infty. \end{aligned} \tag{8}$$

In this paper, we are interested in the $\mathcal{H}_\infty$ controller synthesis for MJLSs (1). The following mode-dependent state-feedback control law is used:

$$u(i,j) = K(r(i,j))\, x(i,j), \tag{9}$$

where $K(r(i,j)) \in \mathbf{R}^{n_u \times n_x}$.

Then the corresponding closed-loop system can be represented as follows:

$$\begin{aligned} (\overline{\Sigma}) : x(i+1, j+1) &= \overline{A}_1(r(i,j+1))\, x(i,j+1) \\ &\quad + \overline{A}_2(r(i+1,j))\, x(i+1,j) \\ &\quad + A_{d1}(r(i,j+1))\, x(i-d_1, j+1) \\ &\quad + A_{d2}(r(i+1,j))\, x(i+1, j-d_2) \\ &\quad + D_1(r(i,j+1))\, w(i,j+1) \\ &\quad + D_2(r(i+1,j))\, w(i+1,j), \\ z(i,j) &= \overline{C}(r(i,j))\, x(i,j) + D_3(r(i,j))\, w(i,j), \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\overline{A}_1(r(i,j+1)) \\ &\quad := A_1(r(i,j+1)) + B_1(r(i,j+1))\, K(r(i,j+1)), \\ &\overline{A}_2(r(i+1,j)) \\ &\quad := A_2(r(i+1,j)) + B_2(r(i+1,j))\, K(r(i+1,j)), \\ &\overline{C}(r(i,j)) := C(r(i,j)) + B_3(r(i,j))\, K(r(i,j)). \end{aligned} \tag{11}$$

Before proceeding further, we introduce the following definitions.

Definition 2. System (10) is said to be stochastically stable if, for $w(i,j) = 0$ and the boundary conditions satisfying (8), the following condition holds:

$$\mathbb{E}\left\{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left(\|x(i,j+1)\|^2 + \|x(i+1,j)\|^2\right)\right\} < \infty. \tag{12}$$

Definition 3. Given a scalar $\gamma > 0$, system (10) is said to be stochastically stable with an $\mathcal{H}_\infty$ disturbance attenuation performance index γ if it is stochastically stable with $w(i,j) = 0$, and under zero boundary conditions $\phi(i,j) = \varphi(i,j) = 0$ in (7), for all nonzero, $w \in l_2\{[0,\infty), [0,\infty)\}$ satisfies

$$\|\widetilde{z}\|_{\mathbb{E}_2} < \gamma \|\widetilde{w}\|_2, \tag{13}$$

where

$$\begin{aligned} \|\widetilde{z}\|_{\mathbb{E}_2} &:= \sqrt{\mathbb{E}\left\{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left(\|z(i,j+1)\|^2 + \|z(i+1,j)\|^2\right)\right\}}, \\ \|\widetilde{w}\|_2 &:= \sqrt{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left(\|w(i,j+1)\|^2 + \|w(i+1,j)\|^2\right)}. \end{aligned} \tag{14}$$

Therefore, the purpose of this paper is to design a mode-dependent $\mathscr{H}_\infty$ state-feedback controller in the form of (9), such that the resulting closed-loop system (10) with defective mode information is stochastically stable with a prescribed $\mathscr{H}_\infty$ performance index γ.

3. Main Results

In this section, based on a Markovian Lyapunov-Krasovskii functional (MLKF), a new formulation of bounded real lemma (BRL) for the two-dimensional (2D) Markovian jump linear system (MJLS) (10) with state-delays and defective mode information will be firstly given. Then, via a linearisation procedure, the $\mathscr{H}_\infty$ controller synthesis will be developed.

3.1. $\mathscr{H}_\infty$ Performance Analysis. In this subsection, by invoking the properties of the transition probability matrices (TPMs), together with the convexification of uncertain domains, an $\mathscr{H}_\infty$ performance analysis criterion for the closed-loop system (10) with state-delays and defective mode information is presented, which will play a significant role in solving the $\mathscr{H}_\infty$ controller synthesis problem.

Proposition 4. *The 2D MJLS in* (10) *with state-delays and defective mode information is stochastically stable with a guaranteed $\mathscr{H}_\infty$ performance γ if the matrices $\{P_{1m}, P_{2m}, Q_1, Q_2\} \in \mathbf{R}^{n_x \times n_x}$, with $P_{1m} > 0$, $P_{2m} > 0$, $Q_1 > 0$, and $Q_2 > 0$, $m \in \mathscr{I}$, such that the following matrix inequalities hold:*

$$\begin{gathered} \mathscr{A}_m^T \mathscr{P}_n^{(s)} \mathscr{A}_m + \mathscr{C}_m^T \mathscr{C}_m + \Theta_m < 0, \\ m \in \mathscr{I}, \quad n \in \mathscr{I}_{\mathscr{UK}}^{(m)}, \quad s = 1, \ldots, M, \end{gathered} \tag{15}$$

where

$$\begin{aligned} \Theta_m &:= \operatorname{diag}\left\{-P_{1m} + Q_1, -P_{2m} + Q_2, -Q_1, -Q_2, -\gamma^2 \mathbf{I}, -\gamma^2 \mathbf{I}\right\}, \\ \mathscr{A}_m &:= \left[\overline{A}_{1m} \;\; \overline{A}_{2m} \;\; A_{d1m} \;\; A_{d2m} \;\; D_{1m} \;\; D_{2m}\right], \\ \mathscr{C}_m &:= \begin{bmatrix} \overline{C}_m & \mathbf{0} & \mathbf{0}_{n_z \times 2n_x} & D_{3m} & \mathbf{0} \\ \mathbf{0} & \overline{C}_m & \mathbf{0}_{n_z \times 2n_x} & \mathbf{0} & D_{3m} \end{bmatrix}, \\ \mathscr{P}_n^{(s)} &:= \sum_{n \in \mathscr{I}_{\mathscr{K}}^{(m)}} \pi_{mn} \left(P_{1n} + P_{2n}\right) \\ &\quad + \sum_{n \in \mathscr{I}_{\mathscr{UC}}^{(m)}} \widetilde{\pi}_{mn}^{(s)} \left(P_{1n} + P_{2n}\right) + \pi_{\mathscr{UK}}^{(ms)} \underbrace{\left(P_{1n} + P_{2n}\right)}_{n \in \mathscr{I}_{\mathscr{UK}}^{(m)}}, \\ \pi_{\mathscr{UK}}^{(ms)} &:= 1 - \sum_{n \in \mathscr{I}_{\mathscr{K}}^{(m)}} \pi_{mn} - \sum_{n \in \mathscr{I}_{\mathscr{UC}}^{(m)}} \widetilde{\pi}_{mn}^{(s)}. \end{aligned} \tag{16}$$

Proof. Consider the following MLKF for the 2D MJLS (10):

$$\begin{aligned} V(i,j) &:= \sum_{k=1}^{2} V_k\left(x(i,j+1), r(i,j+1)\right) \\ &\quad + \sum_{k=3}^{4} V_k\left(x(i+1,j), r(i+1,j)\right), \end{aligned} \tag{17}$$

where

$$\begin{aligned} &V_1\left(x(i,j+1), r(i,j+1)\right) \\ &\quad := x^{\mathrm{T}}(i,j+1) P_1\left(r(i,j+1)\right) x(i,j+1), \\ &V_2\left(x(i,j+1), r(i,j+1)\right) \\ &\quad := \sum_{k=i-d_1}^{i-1} x^{\mathrm{T}}(k,j+1) Q_1 x(k,j+1), \\ &V_3\left(x(i+1,j), r(i+1,j)\right) \\ &\quad := x^{\mathrm{T}}(i+1,j) P_2\left(r(i+1,j)\right) x(i+1,j), \\ &V_4\left(x(i+1,j), r(i+1,j)\right) \\ &\quad := \sum_{k=j-d_2}^{j-1} x^{\mathrm{T}}(i+1,k) Q_2 x(i+1,k). \end{aligned} \tag{18}$$

Then, based on the MLKF defined in (17), it is known that the following condition (19) guarantees that the 2D closed-loop system (10) is stochastically stable with an $\mathscr{H}_\infty$ performance γ under zero boundary conditions for any nonzero $w(i,j) \in l_2\{[0,\infty), [0,\infty)\}$:

$$\Omega := \Delta V(i,j) + \|\widetilde{z}\|_{\mathbb{E}_2}^2 - \gamma^2 \|\widetilde{w}\|_2^2 < 0, \tag{19}$$

where

$$\begin{aligned} \Delta V(i,j) := \mathbb{E}\Bigg\{ &\sum_{k=1}^{2} V_k\left(x(i+1,j+1), r(i+1,j+1)\right) \mid \\ &x(i,j+1), r(i,j+1) = m \Bigg\} \\ &+ \mathbb{E}\Bigg\{ \sum_{k=3}^{4} V_k\left(x(i+1,j+1), r(i+1,j+1)\right) \mid \\ &x(i+1,j), r(i+1,j) = m \Bigg\} \end{aligned}$$

$$-\sum_{k=1}^{2} V_k\left(x(i,j+1), r(i,j+1)\right)$$
$$-\sum_{k=3}^{4} V_k\left(x(i+1,j), r(i+1,j)\right), \tag{20}$$

and $\|\widetilde{z}\|_{\mathbb{E}_2}$ and $\|\widetilde{w}\|_2$ are defined in (14).

Taking the time difference of $V(i,j)$ along the trajectories of the 2D system in (10) yields

$$\begin{aligned}
\Delta V_1 &:= \mathbb{E}\left[V_1\left(x(i+1,j+1), r(i+1,j+1)\right) \mid x(i,j+1), r(i,j+1)=m\right] \\
&\quad - V_1\left(x(i,j+1), r(i,j+1)\right) \\
&= x^{\mathrm{T}}(i+1,j+1)\left(\sum_{n\in\mathcal{J}} \pi_{mn}P_{1n}\right)x(i+1,j+1) \\
&\quad - x^{\mathrm{T}}(i,j+1)P_{1m}x(i,j+1) \\
&= x^{\mathrm{T}}(i+1,j+1) \\
&\quad \times\left(\sum_{n\in\mathcal{J}_{\mathcal{K}}^{(m)}} \pi_{mn}P_{1n} + \sum_{n\in\mathcal{J}_{\mathcal{UC}}^{(m)}}\left(\sum_{s=1}^{M}\alpha_s\widetilde{\pi}_{mn}^{(s)}\right)P_{1n} + \sum_{n\in\mathcal{J}_{\mathcal{UK}}^{(m)}}\widehat{\pi}_{mn}P_{1n}\right) \\
&\quad \times x(i+1,j+1) - x^{\mathrm{T}}(i,j+1)P_{1m}x(i,j+1), \\
\Delta V_2 &:= \mathbb{E}\left[V_2\left(x(i+1,j+1), r(i+1,j+1)\right) \mid x(i,j+1), r(i,j+1)=m\right] \\
&\quad - V_2\left(x(i,j+1), r(i,j+1)\right) \\
&= x^{\mathrm{T}}(i,j+1)Q_1x(i,j+1) \\
&\quad - x^{\mathrm{T}}(i-d_1,j+1)Q_1x(i-d_1,j+1), \\
\Delta V_3 &:= \mathbb{E}\left[V_3\left(x(i+1,j+1), r(i+1,j+1)\right) \mid x(i+1,j), r(i+1,j)=m\right] \\
&\quad - V_3\left(x(i+1,j), r(i+1,j)\right) \\
&= x^{\mathrm{T}}(i+1,j+1)\left(\sum_{n\in\mathcal{J}} \pi_{mn}P_{2n}\right)x(i+1,j+1) \\
&\quad - x^{\mathrm{T}}(i+1,j)P_{2m}x(i+1,j) \\
&= x^{\mathrm{T}}(i+1,j+1) \\
&\quad \times\left(\sum_{n\in\mathcal{J}_{\mathcal{K}}^{(m)}} \pi_{mn}P_{2n} + \sum_{n\in\mathcal{J}_{\mathcal{UC}}^{(m)}}\left(\sum_{s=1}^{M}\alpha_s\widetilde{\pi}_{mn}^{(s)}\right)P_{2n} + \sum_{n\in\mathcal{J}_{\mathcal{UK}}^{(m)}}\widehat{\pi}_{mn}P_{2n}\right) \\
&\quad \times x(i+1,j+1) - x^{\mathrm{T}}(i+1,j)P_{2m}x(i+1,j), \\
\Delta V_4 &:= \mathbb{E}\left[V_4\left(x(i+1,j+1), r(i+1,j+1)\right) \mid x(i+1,j), r(i+1,j)=m\right] \\
&\quad - V_4\left(x(i+1,j), r(i+1,j)\right) \\
&= x^{\mathrm{T}}(i+1,j)Q_2x(i+1,j) \\
&\quad - x^{\mathrm{T}}(i+1,j-d_2)Q_2x(i+1,j-d_2).
\end{aligned} \tag{21}$$

Therefore, based on the MLKF defined in (17), together with consideration of (10) and (21), we have

$$\begin{aligned}
\Omega &= \varsigma^{\mathrm{T}}(i,j)\left[\boldsymbol{\mathcal{A}}_m^{\mathrm{T}}\left(\overline{\boldsymbol{\mathcal{P}}}_{1n} + \overline{\boldsymbol{\mathcal{P}}}_{2n}\right)\boldsymbol{\mathcal{A}}_m + \boldsymbol{\mathcal{C}}_m^{\mathrm{T}}\boldsymbol{\mathcal{C}}_m + \Theta_m\right] \\
&\quad \times \varsigma(i,j), \quad m,n\in\mathcal{J},
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\varsigma(i,j) &:= \left[x^{\mathrm{T}}(i,j+1)\ \ x^{\mathrm{T}}(i+1,j)\ \ x^{\mathrm{T}}(i-d_1,j+1)\ \ x^{\mathrm{T}}(i+1,j-d_2)\ \ w^{\mathrm{T}}(i,j+1)\ \ w^{\mathrm{T}}(i+1,j)\right]^{\mathrm{T}}, \\
\Theta_m &:= \operatorname{diag}\left\{-P_{1m}+Q_1, -P_{2m}+Q_2, -Q_1, -Q_2, -\gamma^2\mathbf{I}, -\gamma^2\mathbf{I}\right\}, \\
\boldsymbol{\mathcal{A}}_m &:= \left[\overline{A}_{1m}\ \ \overline{A}_{2m}\ \ A_{d1m}\ \ A_{d2m}\ \ D_{1m}\ \ D_{2m}\right], \\
\boldsymbol{\mathcal{C}}_m &:= \begin{bmatrix} \overline{C}_m & \mathbf{0} & \mathbf{0}_{n_z\times 2n_x} & D_{3m} & \mathbf{0} \\ \mathbf{0} & \overline{C}_m & \mathbf{0}_{n_z\times 2n_x} & \mathbf{0} & D_{3m} \end{bmatrix}, \\
\overline{\boldsymbol{\mathcal{P}}}_{ln} &:= \sum_{n\in\mathcal{J}_{\mathcal{K}}^{(m)}} \pi_{mn}P_{ln} + \sum_{n\in\mathcal{J}_{\mathcal{UC}}^{(m)}}\left(\sum_{s=1}^{M}\alpha_s\widetilde{\pi}_{mn}^{(s)}\right)P_{ln} + \sum_{n\in\mathcal{J}_{\mathcal{UK}}^{(m)}}\widehat{\pi}_{mn}P_{ln}, \quad l=1,2.
\end{aligned} \tag{23}$$

Considering the fact that $0 \le \alpha_s \le 1$, $\sum_{s=1}^{M} \alpha_s = 1$, and $0 \le \widehat{\pi}_{mn}/\pi_{\mathcal{UK}}^{(ms)} \le 1$, $\sum_{n\in\mathcal{J}_{\mathcal{UK}}^{(m)}} (\widehat{\pi}_{mn}/\pi_{\mathcal{UK}}^{(ms)}) = 1$, (22) can be rewritten as

$$\begin{aligned}\Omega = \sum_{s=1}^{M}\alpha_s \sum_{n\in\mathcal{J}_{\mathcal{UK}}^{(m)}} \frac{\widehat{\pi}_{mn}}{\pi_{\mathcal{UK}}^{(ms)}} \\ \times \left[\varsigma^{\mathrm{T}}(i,j)\left[\boldsymbol{\mathcal{A}}_m^{\mathrm{T}}\boldsymbol{\mathcal{P}}_n^{(s)}\boldsymbol{\mathcal{A}}_m + \boldsymbol{\mathcal{C}}_m^{\mathrm{T}}\boldsymbol{\mathcal{C}}_m + \Theta_m\right]\varsigma(i,j)\right], \\ m\in\mathcal{J},\ n\in\mathcal{J}_{\mathcal{UK}}^{(m)},\ s=1,\dots,M,\end{aligned} \tag{24}$$

where

$$\begin{aligned}\boldsymbol{\mathcal{P}}_n^{(s)} &:= \sum_{n\in\mathcal{J}_{\mathcal{K}}^{(m)}} \pi_{mn}(P_{1n}+P_{2n}) + \sum_{n\in\mathcal{J}_{\mathcal{UC}}^{(m)}} \widetilde{\pi}_{mn}^{(s)}(P_{1n}+P_{2n}) \\ &\quad + \pi_{\mathcal{UK}}^{(ms)}\underbrace{(P_{1n}+P_{2n})}_{n\in\mathcal{J}_{\mathcal{UK}}^{(m)}}, \\ \pi_{\mathcal{UK}}^{(ms)} &:= 1 - \sum_{n\in\mathcal{J}_{\mathcal{K}}^{(m)}} \pi_{mn} - \sum_{n\in\mathcal{J}_{\mathcal{UC}}^{(m)}} \widetilde{\pi}_{mn}^{(s)}.\end{aligned} \tag{25}$$

According to (24), it is easy to see that (19) holds if and only if, for all $s = 1,\dots,M$,

$$\begin{aligned}\varsigma^{\mathrm{T}}(i,j)\left[\boldsymbol{\mathcal{A}}_m^{\mathrm{T}}\boldsymbol{\mathcal{P}}_n^{(s)}\boldsymbol{\mathcal{A}}_m + \boldsymbol{\mathcal{C}}_m^{\mathrm{T}}\boldsymbol{\mathcal{C}}_m + \Theta_m\right]\varsigma(i,j) < 0, \\ m\in\mathcal{J},\ n\in\mathcal{J}_{\mathcal{UK}}^{(m)},\end{aligned} \tag{26}$$

which is implied by condition (15). This completes the proof. □

Remark 5. By fully considering the properties of TPMs, together with the convexification of uncertain domains, a new version of BRL has been derived for the 2D MJLS (10) with state-delays and defective mode information in Proposition 4. It is worth mentioning that the results for Fornasini-Marchesini local state-space (FMLSS) model (10) could be readily applied to Roesser model after the similar transformation as performed in [8]. It is also noted that the condition given in (15) is nonconvex due to the presence of product terms between the Lyapunov matrices and system matrices. For the matrix inequality linearisation purpose, in the following, we shall make a decoupling between the Lyapunov matrices and system matrices, which will be convenient for controller synthesis purpose.

In the sequel, we focus on the $\mathcal{H}_\infty$ controller design based on the analysis condition given in Proposition 4.

3.2. $\mathcal{H}_\infty$ Controller Synthesis. In this subsection, based on a linearisation procedure, a unified framework for the solvability of the $\mathcal{H}_\infty$ controller synthesis problem will be proposed. It will be shown that the parametrised representations of the controller gains can be constructed in terms of the feasible solutions to a set of strict linear matrix inequalities (LMIs).

Theorem 6. *Consider 2D MJLS* (1) *with state-delays and defective mode information and the state-feedback controller in the form of* (9). *The closed-loop system* (10) *is stochastically stable with an $\mathcal{H}_\infty$ performance γ if there exist matrices $\{X_{1m}, X_{2m}, R_1, R_2\} \in \mathbf{R}^{n_x\times n_x}$, with $X_{1m} > 0$, $X_{2m} > 0$, $R_1 > 0$, and $R_2 > 0$ and matrices $G_m \in \mathbf{R}^{n_x\times n_x}$ and $\overline{K}_m \in \mathbf{R}^{n_u\times n_x}$, $m \in \mathcal{J}$, such that the following LMIs hold:*

$$\begin{bmatrix} -\mathcal{X}_{1n}^{(m)} & \mathbf{0} & \mathbf{0} & \mathcal{F}_m^{(s)}\overline{\boldsymbol{\mathcal{A}}}_m & \mathbf{0} \\ * & -\mathcal{X}_{2n}^{(m)} & \mathbf{0} & \mathcal{F}_m^{(s)}\overline{\boldsymbol{\mathcal{A}}}_m & \mathbf{0} \\ * & * & -\mathbf{I} & \overline{\boldsymbol{\mathcal{C}}}_m & \mathbf{0} \\ * & * & * & -\overline{\Theta}_m & \mathcal{G}_m \\ * & * & * & * & -\mathcal{R} \end{bmatrix} < 0, \tag{27}$$

$$m\in\mathcal{J},\ n\in\mathcal{J}_{\mathcal{UK}}^{(m)},\ s=1,\dots,M,$$

where

$$\begin{aligned}\mathcal{X}_{ln}^{(m)} &:= \operatorname{diag}\left\{X_{l\mathcal{K}_{1(m)}},\dots,X_{l\mathcal{K}_{t(m)}},X_{l\mathcal{U}_{1(m)}},\dots,X_{l\mathcal{U}_{v(m)}},X_{ln}\right\}, \\ &\qquad l=1,2, \\ \mathcal{F}_m^{(s)} &:= \left[\sqrt{\pi_{m\mathcal{K}_{1(m)}}}\mathbf{I},\dots,\sqrt{\pi_{m\mathcal{K}_{t(m)}}}\mathbf{I},\sqrt{\widetilde{\pi}_{m\mathcal{U}_{1(m)}}^{(s)}}\mathbf{I},\dots,\right. \\ &\qquad \left.\sqrt{\widetilde{\pi}_{m\mathcal{U}_{v(m)}}^{(s)}}\mathbf{I},\sqrt{\pi_{\mathcal{UK}}^{(ms)}}\mathbf{I}\right], \\ \overline{\boldsymbol{\mathcal{A}}}_m &:= \left[\overline{\mathcal{A}}_{1m}\ \ \overline{\mathcal{A}}_{2m}\ \ A_{d1m}R_1\ \ A_{d2m}R_2\ \ D_{1m}\ \ D_{2m}\right], \\ \overline{\boldsymbol{\mathcal{C}}}_m &:= \begin{bmatrix}\overline{\mathcal{C}}_m & \mathbf{0} & \mathbf{0}_{n_z\times 2n_x} & D_{3m} & \mathbf{0} \\ \mathbf{0} & \overline{\mathcal{C}}_m & \mathbf{0}_{n_z\times 2n_x} & \mathbf{0} & D_{3m}\end{bmatrix}, \\ \overline{\Theta}_m &:= \operatorname{diag}\left\{G_m + G_m^{\mathrm{T}} - X_{1m}, G_m\right. \\ &\qquad \left. + G_m^{\mathrm{T}} - X_{2m}, \mathcal{R}, \gamma^2\mathbf{I}, \gamma^2\mathbf{I}\right\}, \\ \mathcal{R} &:= \operatorname{diag}\{R_1, R_2\}, \\ \mathcal{G}_m &:= \begin{bmatrix} G_m & \mathbf{0} & \mathbf{0}_{n_x\times 2(n_x+n_w)} \\ \mathbf{0} & G_m & \mathbf{0}_{n_x\times 2(n_x+n_w)}\end{bmatrix}^{\mathrm{T}}, \\ \overline{\mathcal{A}}_{lm} &:= A_{lm} + B_{lm}\overline{K}_m, \\ \overline{\mathcal{C}}_m &:= C_m + B_{3m}\overline{K}_m, \\ \pi_{\mathcal{UK}}^{(ms)} &:= 1 - \sum_{n\in\mathcal{J}_{\mathcal{K}}^{(m)}} \pi_{mn} - \sum_{n\in\mathcal{J}_{\mathcal{UC}}^{(m)}} \widetilde{\pi}_{mn}^{(s)}.\end{aligned} \tag{28}$$

Moreover, if the above conditions have a set of feasible solutions $(X_{1m}, X_{2m}, R_1, R_2, G_m, \overline{K}_m)$, then the state-feedback controller in the form of (9) *can be constructed as*

$$K_m := \overline{K}_m G_m^{-1}. \tag{29}$$

Proof. It follows from Proposition 4 that, if we can show (15), then the claimed results follow. By Schur complement and with $\mathcal{I}_{\mathcal{K}}^{(m)} := \{\mathcal{K}_{1_{(m)}}, \ldots, \mathcal{K}_{t_{(m)}}\}$ and $\mathcal{I}_{\mathcal{UC}}^{(m)} := \{\mathcal{U}_{1_{(m)}}, \ldots, \mathcal{U}_{v_{(m)}}\}$, (15) is equivalent to

$$\begin{bmatrix} -\mathcal{X}_{1n}^{(m)} & \mathbf{0} & \mathbf{0} & F_m^{(s)}\boldsymbol{\mathcal{A}}_m & \mathbf{0} \\ * & -\mathcal{X}_{2n}^{(m)} & \mathbf{0} & F_m^{(s)}\boldsymbol{\mathcal{A}}_m & \mathbf{0} \\ * & * & -\mathbf{I} & \boldsymbol{\mathcal{C}}_m & \mathbf{0} \\ * & * & * & -\widehat{\Theta}_m & \Lambda \\ * & * & * & * & -\mathcal{Q}^{-1} \end{bmatrix} < 0, \tag{30}$$

$$m \in \mathcal{I},\ n \in \mathcal{I}_{\mathcal{UK}}^{(m)},\ s = 1, \ldots, M,$$

where

$$\begin{aligned}
\mathcal{X}_{ln}^{(m)} &:= \operatorname{diag}\left\{X_{l\mathcal{K}_{1_{(m)}}}, \ldots, X_{l\mathcal{K}_{t_{(m)}}},\ X_{l\mathcal{U}_{1_{(m)}}}, \ldots, X_{l\mathcal{U}_{v_{(m)}}}, X_{ln}\right\}, \quad X_{lm} := P_{lm}^{-1},\ l = 1, 2, \\
F_m^{(s)} &:= \left[\sqrt{\pi_{m\mathcal{K}_{1_{(m)}}}}\mathbf{I}, \ldots, \sqrt{\pi_{m\mathcal{K}_{t_{(m)}}}}\mathbf{I}, \sqrt{\widetilde{\pi}_{m\mathcal{U}_{1_{(m)}}}^{(s)}}\mathbf{I}, \ldots, \sqrt{\widetilde{\pi}_{m\mathcal{U}_{v_{(m)}}}^{(s)}}\mathbf{I}, \sqrt{\pi_{\mathcal{UK}}^{(ms)}}\mathbf{I}\right], \\
\boldsymbol{\mathcal{A}}_m &:= \left[\overline{A}_{1m}\ \ \overline{A}_{2m}\ \ A_{d1m}\ \ A_{d2m}\ \ D_{1m}\ \ D_{2m}\right], \\
\boldsymbol{\mathcal{C}}_m &:= \begin{bmatrix} \overline{C}_m & \mathbf{0} & \mathbf{0}_{n_z\times 2n_x} & D_{3m} & \mathbf{0} \\ \mathbf{0} & \overline{C}_m & \mathbf{0}_{n_z\times 2n_x} & \mathbf{0} & D_{3m} \end{bmatrix}, \\
\widehat{\Theta}_m &:= \operatorname{diag}\left\{X_{1m}^{-1}, X_{2m}^{-1}, \mathcal{Q}, \gamma^2\mathbf{I}, \gamma^2\mathbf{I}\right\}, \quad \mathcal{Q} = \operatorname{diag}\{Q_1, Q_2\}, \\
\Lambda &:= \begin{bmatrix} \mathbf{I}_{n_x} & \mathbf{0} & \mathbf{0}_{n_x\times 2(n_x+n_w)} \\ \mathbf{0} & \mathbf{I}_{n_x} & \mathbf{0}_{n_x\times 2(n_x+n_w)} \end{bmatrix}^{\mathrm{T}}.
\end{aligned} \tag{31}$$

Now, by introducing a nonsingular matrix $G_m \in \mathbf{R}^{n_x\times n_x}$ and pre- and postmultiplying (30) by $\operatorname{diag}\{\mathbf{I}_{2((t_{(m)}+v_{(m)}+1)n_x+n_z)}, G_m^{\mathrm{T}}, G_m^{\mathrm{T}}, Q_1^{-1}, Q_2^{-1}, \mathbf{I}_{2(n_w+n_x)}\}$ and its transpose the following yields:

$$\begin{bmatrix} -\mathcal{X}_{1n}^{(m)} & \mathbf{0} & \mathbf{0} & F_m^{(s)}\overline{\boldsymbol{\mathcal{A}}}_m & \mathbf{0} \\ * & -\mathcal{X}_{2n}^{(m)} & \mathbf{0} & F_m^{(s)}\overline{\boldsymbol{\mathcal{A}}}_m & \mathbf{0} \\ * & * & -\mathbf{I} & \overline{\boldsymbol{\mathcal{C}}}_m & \mathbf{0} \\ * & * & * & -\widetilde{\Theta}_m & \mathcal{G}_m \\ * & * & * & * & -\mathcal{R} \end{bmatrix} < 0, \tag{32}$$

$$m \in \mathcal{I},\ n \in \mathcal{I}_{\mathcal{UK}}^{(m)},\ s = 1, \ldots, M,$$

where

$$\begin{aligned}
\overline{\boldsymbol{\mathcal{A}}}_m &:= \left[\overline{\mathcal{A}}_{1m}\ \ \overline{\mathcal{A}}_{2m}\ \ A_{d1m}R_1\ \ A_{d2m}R_2\ \ D_{1m}\ \ D_{2m}\right], \\
\overline{\boldsymbol{\mathcal{C}}}_m &:= \begin{bmatrix} \overline{\mathcal{C}}_m & \mathbf{0} & \mathbf{0}_{n_z\times 2n_x} & D_{3m} & \mathbf{0} \\ \mathbf{0} & \overline{\mathcal{C}}_m & \mathbf{0}_{n_z\times 2n_x} & \mathbf{0} & D_{3m} \end{bmatrix}, \\
\mathcal{G}_m &:= \begin{bmatrix} G_m & \mathbf{0} & \mathbf{0}_{n_x\times 2(n_x+n_w)} \\ \mathbf{0} & G_m & \mathbf{0}_{n_x\times 2(n_x+n_w)} \end{bmatrix}^{\mathrm{T}}, \\
\widetilde{\Theta}_m &:= \operatorname{diag}\left\{G_m^{\mathrm{T}}X_{1m}^{-1}G_m, G_m^{\mathrm{T}}X_{2m}^{-1}G_m, \mathcal{R}, \gamma^2\mathbf{I}, \gamma^2\mathbf{I}\right\}, \\
\mathcal{R} &= \operatorname{diag}\{R_1, R_2\}, \\
\overline{\mathcal{A}}_{lm} &:= A_{lm} + B_{lm}\overline{K}_m, \quad \overline{K}_m := K_mG_m, \\
R_l &:= Q_l^{-1}, \quad l = 1, 2, \\
\overline{\mathcal{C}}_m &:= C_m + B_{3m}\overline{K}_m,
\end{aligned} \tag{33}$$

and $\mathcal{X}_{1n}^{(m)}$, $\mathcal{X}_{2n}^{(m)}$, and $F_m^{(s)}$ are defined in (31).

It follows from

$$\left(X_{lm} - G_m\right)^{\mathrm{T}}X_{lm}^{-1}\left(X_{lm} - G_m\right) \geq 0, \quad l = 1, 2, \tag{34}$$

that

$$-G_m^{\mathrm{T}}X_{lm}^{-1}G_m \leq -G_m - G_m^{\mathrm{T}} + X_{lm}, \quad l = 1, 2. \tag{35}$$

Then, it is easy to see that (27) implies (32).

On the other hand, the condition in (27) implies that $-G_m - G_m^{\mathrm{T}} < 0$, which means that G_m is nonsingular. Thus, the controller gain can be constructed by (29). The proof is thus completed. □

Remark 7. Theorem 6 provides a sufficient condition on the feasibility of $\mathcal{H}_\infty$ state-feedback controller synthesis problem for the 2D MJLSs with state-delays and defective mode information. It is noted that the $\mathcal{H}_\infty$ state-feedback controller synthesis problem for 2D discrete-time MJLSs has also been considered in [40]. However, there still are some remarkable differences between our results and those in [40]. Firstly, in this paper, the state-delays were introduced in the system (1), whereas the 2D delay-free MJLSs are considered in [40]. In addition, in Theorem 6, the exactly known, partially unknown, and uncertain transition probabilities (TPs) have been simultaneously incorporated into the TPM for 2D MJLSs, while in [40], the TPs were assumed to be completely known. It has been recognized that the scenario containing time-delays and such defective TPs is more general and the underlying MJLSs are thereby more practicable for engineering applications.

4. An Illustrative Example

In this section, we use a simulation example to demonstrate the effectiveness of the proposed state-feedback controller design method to two-dimensional (2D) Markovian jump linear systems (MJLSs).

Consider a 2D MJLS with state-delays in the form of (1) with parameters as follows:

$$\left[\begin{array}{c|c|c|c} A_{11} & A_{d11} & B_{11} & D_{11} \\ \hline A_{21} & A_{d21} & B_{21} & D_{21} \\ \hline C_1 & & B_{31} & D_{31} \end{array}\right] = \left[\begin{array}{cc|cc|c|c} -0.5 & 0 & 0 & 0.05 & -0.5 & 0.2 \\ 0 & 0.5 & -0.02 & 0 & 0.5 & 0.6 \\ \hline 0.5 & 1 & 0.04 & 0 & 0.5 & 0.5 \\ 0 & 1 & 0 & 0.02 & 1 & 0.3 \\ \hline 0.2 & 0.5 & & & 0.5 & 0.3 \\ 0.5 & 0.6 & & & 0.2 & -0.5 \end{array}\right],$$

$$\left[\begin{array}{c|c|c|c} A_{12} & A_{d12} & B_{12} & D_{12} \\ \hline A_{22} & A_{d22} & B_{22} & D_{22} \\ \hline C_2 & & B_{32} & D_{32} \end{array}\right] = \left[\begin{array}{cc|cc|c|c} 0.6 & 0 & 0 & 0.05 & 0.5 & 0.2 \\ 0.3 & 0.5 & -0.02 & 0.04 & 0.5 & 0.6 \\ \hline 0.5 & 0.5 & 0.04 & 0 & 0.5 & 0.5 \\ 0 & 0.8 & 0.04 & 0.02 & 1 & 0.3 \\ \hline 0.2 & 0.5 & & & 0.5 & 0.3 \\ 0.5 & 0.6 & & & 0.2 & -0.5 \end{array}\right],$$

$$\left[\begin{array}{c|c|c|c} A_{13} & A_{d13} & B_{13} & D_{13} \\ \hline A_{23} & A_{d23} & B_{23} & D_{23} \\ \hline C_3 & & B_{33} & D_{33} \end{array}\right] = \left[\begin{array}{cc|cc|c|c} 0.5 & 0 & 0 & 0.05 & 0.5 & 0.2 \\ 0.3 & 0.3 & -0.02 & 0.1 & 0.5 & 0.6 \\ \hline 0.2 & 0.5 & 0.04 & 0 & 0.5 & 0.5 \\ 0 & 0.4 & 0.1 & 0.02 & 0.2 & 0.3 \\ \hline 0.2 & 0.5 & & & 0.5 & 0.3 \\ 0.5 & 0.6 & & & 0.2 & -0.5 \end{array}\right],$$

$$\left[\begin{array}{c|c|c|c} A_{14} & A_{d14} & B_{14} & D_{14} \\ \hline A_{24} & A_{d24} & B_{24} & D_{24} \\ \hline C_4 & & B_{34} & D_{34} \end{array}\right] = \left[\begin{array}{cc|cc|c|c} 0.4 & 0 & 0 & 0.05 & 0.3 & 0.2 \\ 0.5 & 0.6 & -0.02 & 0.1 & 0.5 & 0.6 \\ \hline 0.6 & 0.5 & 0.04 & 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.1 & 0.02 & 0.4 & 0.3 \\ \hline 0.2 & 0.5 & & & 0.5 & 0.3 \\ 0.5 & 0.6 & & & 0.2 & -0.5 \end{array}\right]. \tag{36}$$

Four different cases for the transition probability matrix (TPM) are given in Table 1, where the transition probabilities (TPs) labeled with "^" and "~" represent the unknown and uncertain elements, respectively. Specifically, Case 1, Case 2, Case 3, and Case 4 stand for the completely known TPs, defective mode information (including known, partially unknown, and uncertain TPs), partially unknown TPs, and completely unknown TPs, respectively.

Table 1: Four different TPMs.

Case 1: completely known TPM	Case 2: defective TPM1
$\begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ 0.3 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.3 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ \widehat{\pi}_{21} & \widehat{\pi}_{22} & 0.3 & 0.2 \\ \widehat{\pi}_{31} & \widetilde{\pi}_{32} & \widehat{\pi}_{33} & \widetilde{\pi}_{34} \\ 0.2 & \widehat{\pi}_{42} & \widehat{\pi}_{43} & \widehat{\pi}_{44} \end{bmatrix}$
Case 3: defective TPM2	**Case 4: completely unknown TPM**
$\begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ \widehat{\pi}_{21} & \widehat{\pi}_{22} & 0.3 & 0.2 \\ \widehat{\pi}_{31} & \widehat{\pi}_{32} & \widehat{\pi}_{33} & \widehat{\pi}_{34} \\ 0.2 & \widehat{\pi}_{42} & \widehat{\pi}_{43} & \widehat{\pi}_{44} \end{bmatrix}$	$\begin{bmatrix} \widehat{\pi}_{11} & \widehat{\pi}_{12} & \widehat{\pi}_{13} & \widehat{\pi}_{14} \\ \widehat{\pi}_{21} & \widehat{\pi}_{22} & \widehat{\pi}_{23} & \widehat{\pi}_{24} \\ \widehat{\pi}_{31} & \widehat{\pi}_{32} & \widehat{\pi}_{33} & \widehat{\pi}_{34} \\ \widehat{\pi}_{41} & \widehat{\pi}_{42} & \widehat{\pi}_{43} & \widehat{\pi}_{44} \end{bmatrix}$

Table 2: Comparison of minimum $\mathscr{H}_\infty$ performance for different TPMs.

TPMs	Case 1	Case 2	Case 3	Case 4
$\gamma_{\min}$	0.9884	1.0415	1.0906	2.2398

For Case 2, it is assumed that the uncertain TPs comprise four vertices Π_s, $s = 1, 2, 3, 4$, where the third rows $\Pi_{s(3)}$, $s = 1, 2, 3, 4$, are given by

$$\begin{aligned} \Pi_{1(3)} &= \left[\widehat{\pi}_{31} \;\; 0.2 \;\; \widehat{\pi}_{33} \;\; 0.4\right], \\ \Pi_{2(3)} &= \left[\widehat{\pi}_{31} \;\; 0.5 \;\; \widehat{\pi}_{33} \;\; 0.3\right], \\ \Pi_{3(3)} &= \left[\widehat{\pi}_{31} \;\; 0.3 \;\; \widehat{\pi}_{33} \;\; 0.1\right], \\ \Pi_{4(3)} &= \left[\widehat{\pi}_{31} \;\; 0.1 \;\; \widehat{\pi}_{33} \;\; 0.45\right], \end{aligned} \tag{37}$$

and the other rows in the four vertices are given with the same elements; that is,

$$\begin{aligned} \Pi_{s(1)} &= \left[0.3 \;\; 0.2 \;\; 0.1 \;\; 0.4\right], \\ \Pi_{s(2)} &= \left[\widehat{\pi}_{21} \;\; \widehat{\pi}_{22} \;\; 0.3 \;\; 0.2\right], \\ \Pi_{s(4)} &= \left[0.2 \;\; \widehat{\pi}_{42} \;\; \widehat{\pi}_{43} \;\; \widehat{\pi}_{44}\right], \quad s = 1, 2, 3, 4. \end{aligned} \tag{38}$$

The objective is to design a state-feedback controller of the form (9) for the above system such that the 2D closed-loop system is stochastically stable with an $\mathscr{H}_\infty$ performance γ. By applying Theorem 6, a detailed comparison of the obtained minimum $\mathscr{H}_\infty$ performance indices $\gamma_{\min}$ by the state-feedback controller (9) with four TPM cases being shown in Table 2. It is shown in Tables 1 and 2 that the lower the level of defectiveness of the TPM is, the better the $\mathscr{H}_\infty$ performance can be obtained, which is effective to reduce the design conservatism. Therefore, the introduction of the uncertain TPs is meaningful.

Specifically, in the following, considering the four TPM cases shown in Table 1 and by applying Theorem 6, the feasible

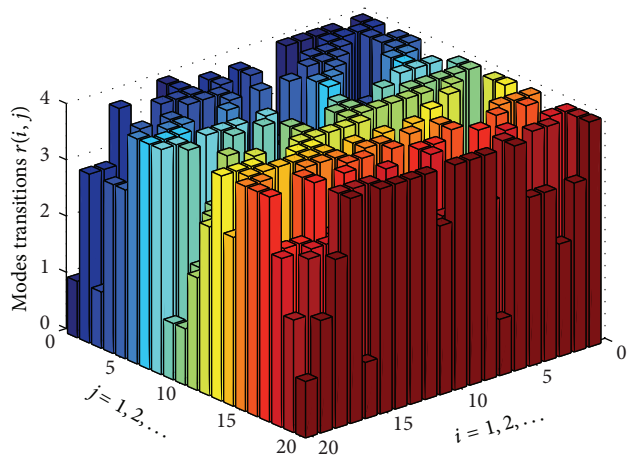

FIGURE 1: One possible system mode evolution.

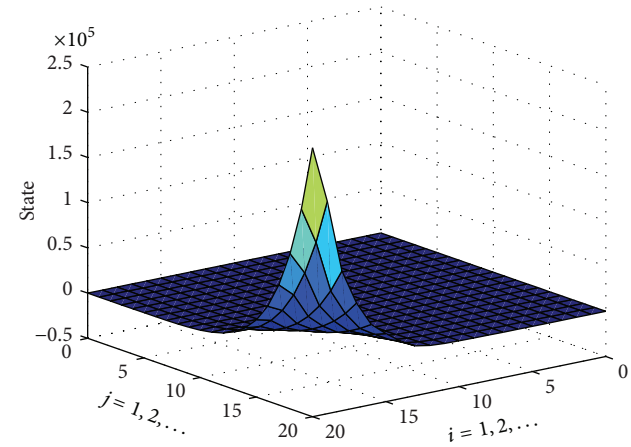

FIGURE 2: State responses of the open-loop system: the 1st component.

solution of $\gamma_{\min} = 1.0415$ for the state-feedback controller is obtained under Case 2 with controller gains given by

$$\begin{aligned} K_1 &= \left[-0.2467 \quad -0.7888\right], \\ K_2 &= \left[-0.2125 \quad -0.7064\right], \\ K_3 &= \left[-0.4563 \quad -0.5998\right], \\ K_4 &= \left[-0.2015 \quad -0.7041\right]. \end{aligned} \tag{39}$$

The feasible solutions for the other three TPM cases in Table 1 are omitted for brevity.

In order to further illustrate the effectiveness of the designed $\mathscr{H}_\infty$ state-feedback controllers, we present some simulation results. Let the boundary conditions be

$$x(t,i) = x(i,t) = \begin{cases} \left[-1 \quad 1.4\right]^{\mathrm{T}}, & 0 \le i \le 10, \\ \left[0 \quad 0\right]^{\mathrm{T}}, & i > 10, \end{cases} \tag{40}$$

where $-4 \le t \le 0$, and choose the delays $d_1 = 4$ (vertical direction), $d_2 = 4$ (horizontal direction), and disturbance input $w(i, j)$ as

$$w(i,j) = \begin{cases} 0.2, & 0 \le i, j \le 10, \\ 0, & \text{otherwise}. \end{cases} \tag{41}$$

With the previous obtained controllers under Case 2 in Table 1, one possible realization of the Markovian jumping mode is plotted in Figure 1. The state responses for the open-loop and closed-loop systems are shown in Figures 2 and 3 and Figures 4 and 5, respectively, and Figures 6 and 7 are the controlled output trajectories of the closed-loop system. It can be clearly observed from the simulation curves that, despite the defective TPs, the performance of the designed controller is satisfactory.

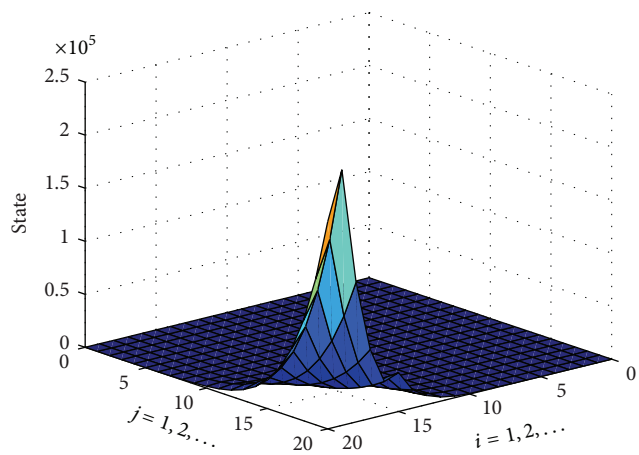

FIGURE 3: State responses of the open-loop system: the 2nd component.

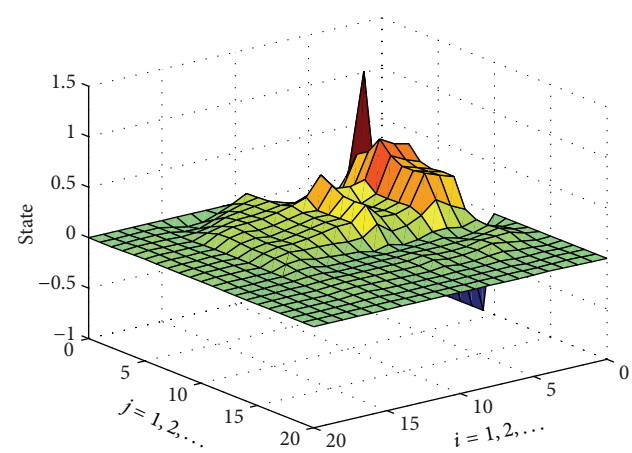

FIGURE 4: State responses of the closed-loop system: the 1st component.

5. Conclusions

This paper has addressed the problem of $\mathscr{H}_\infty$ control for a class of two-dimensional (2D) Markovian jump linear

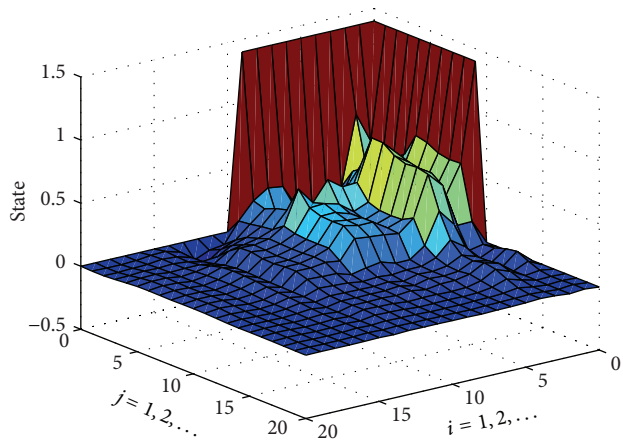

Figure 5: State responses of the closed-loop system: the 2nd component.

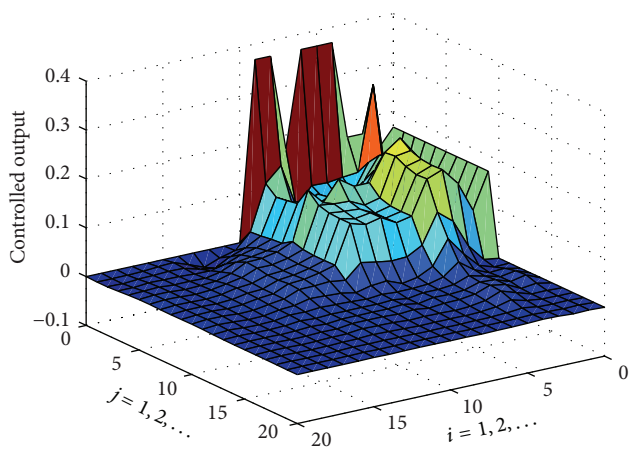

Figure 6: Responses of the controlled output $z(i, j)$ for the closed-loop system: the 1st component.

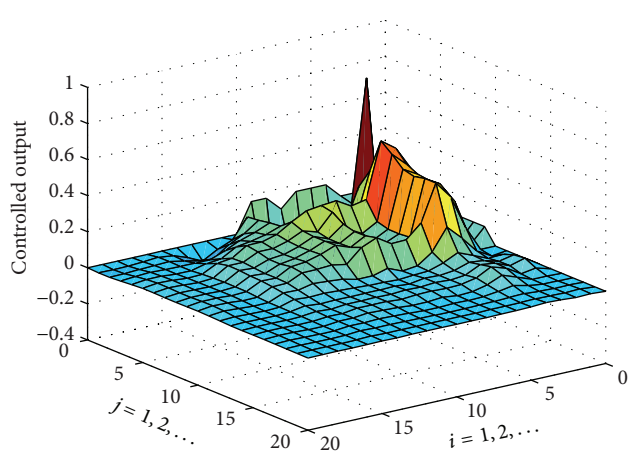

Figure 7: Responses of the controlled output $z(i, j)$ for the closed-loop system: the 2nd component.

systems (MJLSs) with state-delays and defective mode information. Such defective mode information simultaneously includes the exactly known, partially unknown, and uncertain transition probabilities, which contributes to the practicability of 2D MJLSs. By fully considering the properties of the transition probability matrices, together with the convexification of uncertain domains, an $\mathscr{H}_\infty$ performance analysis criterion for the 2D Fornasini-Marchesini local state-space model has been firstly developed, and then via a linearisation procedure, a unified framework has been proposed for the state-feedback controller synthesis that guarantees the stochastic stability of the closed-loop system with an $\mathscr{H}_\infty$ disturbance attenuation level. A simulation example has been given to illustrate the effectiveness of the proposed method.

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