Research Article

Stability and Bifurcation in Magnetic Flux Feedback Maglev Control System

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Nonlinear properties of magnetic flux feedback control system have been investigated mainly in this paper. We analyzed the influence of magnetic flux feedback control system on control property by time delay and interfering signal of acceleration. First of all, we have established maglev nonlinear model based on magnetic flux feedback and then discussed hopf bifurcation's condition caused by the acceleration's time delay. The critical value of delayed time is obtained. It is proved that the period solution exists in maglev control system and the stable condition has been got. We obtained the characteristic values by employing center manifold reduction theory and normal form method, which represent separately the direction of hopf bifurcation, the stability of the period solution, and the period of the period motion. Subsequently, we discussed the influence maglev system on stability of by acceleration's interfering signal and obtained the stable domain of interfering signal. Some experiments have been done on CMS04 maglev vehicle of National University of Defense Technology (NUDT) in Tangshan city. The results of experiments demonstrate that viewpoints of this paper are correct and scientific. When time lag reaches the critical value, maglev system will produce a supercritical hopf bifurcation which may cause unstable period motion.

1. Introduction

The maglev vehicle is a new traffic way. It has several advantages, including speeding up rapidly, less energy consumption, no noise, powerful braking effort, high riding quality, and minor radius of bending. So this traffic method will have more developments in short future. The maglev system carries out suspending function and is the core of the maglev train. Figure 1 gives its working principle. The guideway is flexible in engineering environment. Thus, the oscillation induced by Bernoulli-Euler beam equation must be considered. Dynamic response of maglev vehicle/guideway system has great influence on stability of the system. Maglev system is very complicated because it has a lot of uncertain factors and nonlinear coupling components. The resonance may appear when excitation frequency is close to hopf bifurcation frequency. Bifurcation phenomena happen in many industrial control domains. A method to locate bifurcations in time delay systems with a potentially high-dimensional parameter space has been denoted in the literature [1].

It can determine which parameters are relevant to complex dynamical behavior in such networks. Byrnes and Isidori analyze the bifurcation phenomenon [2] of the zero dynamics and the practical stabilization of nonlinear minimum-phase systems. Wang and Hill put forward a “deterministic learning (DL)” theory for identification of nonlinear system dynamics under full-state measurements. A systematic procedure for modeling and robust control of a multivariable magnetic levitation system is described in the literature [3] by scholars Tsujino et al. The discrete-time model of the magnetic levitation system [4] is derived and the stability is guaranteed by the root locus methodology. Banerjee et al. [5] design a control philosophy for simultaneous stabilization and performance improvement of an electromagnetic levitation system. And Shieh et al. have presented a robust optimal sliding-mode control approach [6] for position tracking of a magnetic levitation system. Ji et al. [7] apply an H_∞ control to suppress the spillovers caused by unmodeled dynamics which we estimate using closed loop identification. A real-time operating environment [8] was established for closed-loop

FIGURE 1: National mid-low speed maglev experimental field of Tangshan city.

control over Ethernet. Also, a novel discrete-time repetitive controller [9] design for time delay systems subject to a periodic reference and exogenous periodic disturbances is presented. Ariba et al. designed a new controller for first-order linear time invariant system with time delay based on the Hermite-Biehler theorem [10]. In the paper [11], Ariba et al. propose an original approach: the quadratic separation. At the end of the paper, the delay operator properties are exploited to provide delay range stability conditions. The bifurcation phenomenon also appears in power system [12]. Ben-Kilani and Schlueter [13] denote that the bifurcation subsystem is a singular perturbation problem in fact, and this problem can be analyzed by the center manifold dynamics method. A local bifurcation of a parametrized differential equation occurs at an equilibrium where there is a change in the topological character of the nearby solution curves [14]. The unstable oscillation of autonomous dynamic system [15] of a matrix converter (MC) is studied based on nonlinear dynamic theory. Yang et al. prove that the eigenvalue crossing condition [16] for the hopf curve holds without additional assumption. Zhang and Jiang consider a delayed predator-prey system with Holling II functional response [17]. An effective hopf bifurcation criterion is provided for an induction motor (IM) drive system with indirect field oriented control (IFOC) [18]. The paper [19] deals with the problem of hopf bifurcation stabilization for Rössler system. Rössler system has two cures of equilibria, and hopf bifurcation may occur at some points of the equilibria. It is found that hopf bifurcation [20] occurs when these parameters pass through some critical values where the conditions for local asymptotical stability of the equilibrium are not satisfied. Guan and Chen [21] investigate the local hopf bifurcation in Genesio system with delayed feedback control. The time delay problem is one of the most significant issues [22]. The controller design [23] is cast into a convex multiobjective optimization problem with linear matrix inequality (LMI) constraints by defining a Lyapunov functional and using the delay partitioning method.

The main work of this paper is investigating the time delay of acceleration signal and the interference of acceleration in magnetic flux feedback maglev control system. We got

FIGURE 2: Simple model of maglev system.

characteristic values by employing the center manifold reduction and normal form theory, which represent separately the direction of hopf bifurcation, the stability of the period solution, and the period of the period motion. Maglev vehicle CMS04 has oscillation phenomenon because of time delay and interference. So the paper has studied the nature of maglev control system's period motion.

2. Foundation of Magnetic Flux Feedback Suspension Control Model

The maglev control model based on magnetic flux feedback is denoted in Figure 2.

Illuminate signs of Figure 2 in Table 1.

Assume the following.

- (1) Magnetic flux formula $\phi_T = \phi_L + \phi_m$, neglect leaking flux, namely, $\phi_L = 0$.
- (2) Neglect magnetic resistance of iron core and guideway, and assume that magnetic field potential is distributed evenly on the suspension gap.
- (3) Neglect deformation and elastic vibration of track, and track has infinite rigidity with regard to electromagnet.

The maglev control model has been founded based on flux feedback in the literature [12] already:

$$\begin{aligned}
 \text{electromagnetics equation: } B &= \frac{\mu_0 N i(t)}{2z(t)}, \\
 \text{mechanics equation: } F &= \frac{B^2 A}{\mu_0}, \\
 \text{kinematics equation: } M \ddot{z}(t) &= -F + Mg, \\
 \text{electric equation: } u(t) &= \frac{2Rz(t)}{\mu_0 N} B(t) + N A \dot{B}(t).
 \end{aligned} \tag{1}$$

The open loop block diagram of maglev control system is given in Figure 3.

Signs	Significations
ϕ_m	Gap flux
ϕ_T	Main flux
ϕ_L	Leaking flux
ψ	Gap flux linkage
$u(t)$	Voltage of magnetic coil
W_m	Magnetic field energy in bulk V
R	Resistance of magnetic coil
B	Gap flux density
μ_0	Magnetic permeability of atmosphere
F_m	Magnetic field potential
R_m	Magnetic resistance
A	Magnetic pole area
$i(t)$	Current of magnetic coil
w_m	Magnetic field energy density
N	Number of magnetic coil
$z(t)$	Gap among electromagnet and track
F	Electromagnetic force
M	Total mass

$$B = \sqrt{\frac{M\mu_0(g-\ddot{z})}{A}},$$

$$= -\frac{M\mu_0}{2A\sqrt{M\mu_0(g-\ddot{z})/A}}\ddot{z}, \quad (4)$$

$$u = \frac{2RBz}{\mu_0 N} + NA\dot{B},$$

$$K_p(z - z_e) + K_d\dot{z} + K_a\ddot{z}_\tau,$$

$$\begin{aligned} \varphi^\tau(z, \dot{z}, \ddot{z}, \ddot{\ddot{z}}, \tau) = & \frac{MN\mu_0}{2\sqrt{M\mu_0(g-\ddot{z})}/A} \ddot{\ddot{z}} \\ & + K_a \ddot{\ddot{z}} + K_d \dot{z} + K_p(z + z_e) \quad (5) \\ & - \frac{2Rz}{\mu_0 N} \sqrt{\frac{M\mu_0(g-\ddot{z})}{A}}. \end{aligned}$$

3.1. Stability and hopf Bifurcation. There are some reasons to cause time delay of acceleration signal, for example, signal transfer delay, the retardation between actuator and sensor, the filtering delay of acceleration signal, and so on. Suppose that delayed time is τ , and acceleration signal after the delayed time is $\ddot{z}_\tau$; in this section, we will investigate stability and hopf bifurcation of the maglev system at the equilibrium. CMS04 generally applies PIDA control algorithm, and this method is represented in (2). When the system reaches stable status, the integral action can be neglected, so we simplify (2) to (3):

$$u(t) = K_p \cdot (z - z_e) + K_i \cdot \int (z - z_e) dt + K_d \cdot \dot{z} + K_a \cdot \ddot{z}_r. \quad (2)$$

$$u(t) = K_p(z - z_e) + K_d\dot{z} + K_a\ddot{z}_\tau. \quad (3)$$

$$\begin{aligned} \varphi^\tau(z, \dot{z}, \ddot{z}, \ddot{z}, \tau) = & \frac{MN\mu_0}{2\sqrt{M\mu_0(g-\ddot{z})/A}}\ddot{z} \\ & + K_a\ddot{z}_\tau + K_d\dot{z} + K_p z \\ & - \frac{2R(z+z_e)}{\mu_0 N} \sqrt{\frac{M\mu_0(g-\ddot{z})}{A}}. \end{aligned} \quad (6)$$

Expand (6) up to third-order Taylor series about new equilibrium point $(0, 0, 0)$, and we obtain the following:

$$\begin{aligned} \varphi^\tau(z, \dot{z}, \ddot{z}, \ddot{z}, \tau) \\ = M_1 \ddot{z} + \left(\frac{MM_1 R z_e}{\mu_0 AMN^2} + \frac{M_1}{2g\mu_0 MN} \right) \ddot{z} \\ + K_a \ddot{z}_\tau + K_d \dot{z} + \left(K_p - \frac{RM_1}{AN^2 \mu_0} \right) z \\ + f(x, x_\tau) + O^{n, n \geq 3}(z, \dot{z}, \ddot{z}, \ddot{z}), \end{aligned} \quad (7)$$

where,

$$\begin{aligned} f(x, x_\tau) = & \left(\frac{3M_1}{8g^2 MN \mu_0} + \frac{MR z_e M_1}{4g NAMN \mu_0} \right) \ddot{z}^2 \\ & - \frac{2\mu_0 MR z_e}{\mu_0 M_1}, \end{aligned} \quad (8)$$

$$M_1 = \frac{MN \mu_0}{\sqrt{M \mu_0 g / A}}.$$

Delete the nonlinear part, and get

$$\begin{aligned} \varphi_0^\tau(z, \dot{z}, \ddot{z}, \ddot{z}, \tau) \\ = M_1 \ddot{z} + \left(\frac{MM_1 R z_e}{\mu_0 AMN^2} + \frac{M_1}{2g\mu_0 MN} \right) \ddot{z} \\ + K_a \ddot{z}_\tau + K_d \dot{z} + \left(K_p - \frac{RM_1}{AN^2 \mu_0} \right) z. \end{aligned} \quad (9)$$

We transform the time domain (9) into the frequency domain (10):

$$\begin{aligned} \varphi_0^\tau(s, \tau) = M_1 s^3 + \left(\frac{MM_1 R z_e}{\mu_0 AMN^2} + \frac{M_1}{2g\mu_0 MN} \right) s^2 \\ + K_a s^2 e^{-s\tau} + K_d s^1 + \left(K_p - \frac{RM_1}{AN^2 \mu_0} \right) s^0. \end{aligned} \quad (10)$$

Set $x = [z \ \dot{z} \ \ddot{z}]^T$, $x_\tau = [z_\tau \ \dot{z}_\tau \ \ddot{z}_\tau]^T$, and maglev linear model can be got at new equilibrium point:

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + A_2 x_\tau(t) + f(x, x_\tau), \\ y(t) &= C_1 x(t) + C_2 x_\tau(t), \end{aligned} \quad (11)$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -a_{1\tau} \end{bmatrix}, \\ C_1 &= [1 \ 0 \ 0], \quad C_2 = [0 \ 0 \ 0], \\ a_1 &= M_a, \quad a_{1\tau} = \frac{K_a}{M_1}, \quad a_2 = \frac{K_d}{M_1}, \\ a_3 &= \frac{(K_p - (RM_1/AN^2 \mu_0))}{M_1}, \\ M_a &= \frac{MR z_e}{\mu_0 AMN^2} + \frac{1}{2g\mu_0 MN}. \end{aligned} \quad (12)$$

Let $\varphi_0(z, \dot{z}, \ddot{z}, \ddot{z}, \tau) = 0$, and the characteristic equation of maglev system changes as follows:

$$\lambda^3 + a_1 \lambda^2 + a_2 \lambda^1 + a_3 \lambda^0 + a_{1\tau} \lambda^2 \exp(-\lambda \tau) = 0. \quad (13)$$

All properties of the closed loop control system are determined by distributing roots of the previously determined equation. If maglev control system keeps local stability at the equilibrium point $(0, 0, 0)$, it must insure that roots of the characteristic equation all have negative real part:

$$a_1 > 0, \quad (a_1 + a_{1\tau}) a_2 - a_3 > 0, \quad a_3 > 0. \quad (14)$$

When time lag $\tau > 0$, only if it becomes small enough, the above condition can keep the characteristic equation have negative real part, or else the above condition cannot keep that the system's trivial solution has local stability. Because the roots change sequentially, and when $a_3 \neq 0$, zero is not root of the characteristic equation, so if bifurcation occurs, it must be a dynamic hopf bifurcation. When time lag increases close to the critical value, a pair of conjugate pure imaginary roots will appear in characteristic equation, and other roots all have the negative real part. Assume that the pair of conjugate pure imaginary roots are $\lambda = \pm i\omega$ ($\omega > 0$), and substitute them into the characteristic equation:

$$(i\omega)^3 + a_1 (i\omega)^2 + a_2 (i\omega)^1 + a_3 (i\omega)^0 + a_{1\tau} (i\omega)^2 e^{-i\omega\tau} = 0. \quad (15)$$

Separate the real part and the imaginary part, and get

$$\gamma(\omega) \pm i\eta(\omega) = 0, \quad (16)$$

where

$$\begin{aligned} \gamma(\omega) &= a_3 - a_1 \omega^2 - a_{1\tau} \omega^2 \cos(\omega\tau), \\ \eta(\omega) &= a_2 \omega - \omega^3 + a_{1\tau} \omega^2 \sin(\omega\tau). \end{aligned} \quad (17)$$

Eliminate $\sin(\omega\tau)$ and $\cos(\omega\tau)$ from the above equation, and we can obtain

$$\omega^6 + (a_1^2 - 2a_2) \omega^4 + (a_2^2 + 2a_1 a_3) \omega^2 + a_3^2 - a_{1\tau}^2 \omega^4 = 0. \quad (18)$$

If (18) has no positive real root, the stability of the system will have no change with the time delay varying. If it has a positive real root, then put it into the following equation:

$$\begin{aligned} \sin(\omega\tau) &= \frac{\omega^3 - a_2 \omega}{a_{1\tau} \omega^2}, \\ \cos(\omega\tau) &= \frac{a_3 - a_1 \omega^2}{a_{1\tau} \omega^2}. \end{aligned} \quad (19)$$

The minimal time lag obtained is critical value τ_0 , and the corresponding value of ω is ω_0 . hopf bifurcation happens at

the moment. The differential equation of formula (13) is as follows:

$$\begin{aligned} & \left[a_2 - 3w_0^2 + 2a_{1\tau}w_0 \sin(w_0\tau_0) + a_{1\tau}\tau_0w_0^2 \cos(w_0\tau_0) \right] d\lambda \\ & + \left[2a_1w_0 + 2a_{1\tau}w_0 \cos(w_0\tau_0) \right. \\ & \quad \left. - a_{1\tau}\tau_0w_0^2 \sin(w_0\tau_0) \right] i d\lambda \\ & = a_{1\tau}w_0^3 \sin(w_0\tau_0) d\tau - a_{1\tau}w_0^3 \cos(w_0\tau_0) i d\tau, \end{aligned} \quad (20)$$

$$\operatorname{Re} \left(\frac{d\lambda}{d\tau} \right)_{\tau=\tau_0}^{\lambda=iw_0} = \frac{a_{1\tau}w_0^3 \sin(w_0\tau_0)}{q_{11}^2 + q_{12}^2}, \quad (21)$$

where the terms

$$\begin{aligned} q_{11} &= a_2 - 3w_0^2 + 2a_{1\tau}w_0 \sin(w_0\tau_0) + a_{1\tau}\tau_0w_0^2 \cos(w_0\tau_0), \\ q_{12} &= 2a_1w_0 + 2a_{1\tau}w_0 \cos(w_0\tau_0) - a_{1\tau}\tau_0w_0^2 \sin(w_0\tau_0). \end{aligned} \quad (22)$$

If $\operatorname{Re}(d\lambda/d\tau)_{\tau=\tau_0}^{\lambda=iw_0} \neq 0$, then a pair of complex eigenvalues crosses the imaginary axis with the time lag changing; at the end the hopf bifurcation will occur.

3.2. Orientation of hopf Bifurcation and Stability of Periodic Solution. The critical point of hopf bifurcation has been discussed in front section. We will investigate the stability of periodic solution of hopf bifurcation by employing center manifold reduction and normal form theory in this section.

Set $\tau = \tau_0 + \mu$, and the phase space of the maglev closed loop control system is $C = C([- \tau_0, 0], R^3)$, so the characteristic equation can be transformed into a function differential equation as follows:

$$\dot{x}_t = D_\mu x_t + f(x_t, \mu), \quad x_t \in R^3, \quad t > 0, \quad (23)$$

where $x_t(\theta) = x(t + \theta)$, $\theta \in [- \tau_0, 0]$, and $D_\mu x_t$, $f(x_t, \mu)$ separately are linear part and nonlinear part. The terms $D_\mu : C \rightarrow C$ and $f : C \rightarrow C$ are defined as

$$\begin{aligned} D_\mu \Phi &= A_1 \Phi(0) + (\tau_0 + \mu) A_2 \Phi(-\tau_0), \\ f(\mu, \Phi) &= (\tau_0 + \mu) \begin{pmatrix} 0 \\ 0 \\ f(x, x_\tau) \end{pmatrix}. \end{aligned} \quad (24)$$

There is a bounded function $\gamma(\mu, \theta)$, based on Riesz representation theorem. We obtain $\theta \in [- \tau_0, 0]$:

$$D_\mu \Phi = \int_{-\tau_0}^0 d\gamma(\mu, \theta) \Phi(\theta), \quad \Phi \in C, \quad (25)$$

where

$$\gamma(\mu, \theta) = A_1 \delta(\theta) - (\tau_0 + \mu) A_2 \delta(\theta + \tau_0), \quad (26)$$

and $\delta(\theta)$ is an impulse function.

For any $\Phi \in C^1([- \tau_0, 0], R^3)$, define the adjoint operators E and E^* :

$$E(\mu) \Phi = \begin{cases} \frac{d\Phi(\theta)}{d\theta}, & \theta \in [- \tau_0, 0], \\ \int_{-\tau_0}^0 d\gamma(\mu, \xi) \Phi(\xi), & \theta = 0, \end{cases} \quad (27)$$

$$F(\mu) \Phi = \begin{cases} 0, & \theta \in [- \tau_0, 0], \\ f(\mu, \Phi), & \theta = 0. \end{cases}$$

Equation (23) can be written as $\dot{x}_t = E(\mu)x_t + F(\mu)x_t$, according to $dx_t/d\theta = dx_t/dt$ and the definition before.

The conjugate operator can be defined as follows for any $\psi \in C^1([- \tau_0, 0], (R^3)^*)$.

$$E(0) \Phi(\theta) = \begin{cases} \frac{-d\Phi(\theta)}{d\theta} & \theta \in (0, \tau_0], \\ \int_{-\tau_0}^0 d\gamma(\theta, 0) \Phi(-\theta), & \theta = 0. \end{cases} \quad (28)$$

The inner product of the vector ψ and Φ is

$$\begin{aligned} \langle \psi(\theta), \Phi(\theta) \rangle &= \bar{\psi}(0) \Phi(0) \\ &\quad - \int_{-\tau_0}^0 \int_{\xi=0}^0 \bar{\psi}(\xi - \theta) d\gamma(\theta) \Phi(\xi) d\xi, \end{aligned} \quad (29)$$

where $\bar{\psi}$ denotes the complex conjugation operation for transposition of the vector ψ .

Proof of the Periodic Solution and hopf Bifurcation's Stability

Step 1. Suppose that the operators E and E^* are the eigenvectors corresponding to the eigenvalues $\pm iw_0\tau_0$ and satisfy

$$\begin{aligned} p(\theta) &= [1 \quad \alpha \quad \beta]^T e^{iw_0\tau_0\theta}, \\ p^*(\theta) &= H[1 \quad \alpha \quad \beta]^T e^{iw_0\tau_0\theta}. \end{aligned} \quad (30)$$

According to (24), yield

$$\begin{aligned} Ep(\theta) &= iw_0\tau_0\theta p(\theta), \\ E^*p^*(\theta) &= -iw_0\tau_0\theta p^*(\theta). \end{aligned} \quad (31)$$

The solutions can be obtained

$$\begin{aligned} \alpha &= \frac{a_{1\tau}w_0 \cos(w_0\tau_0) - (a_1 + 1)w_0 - a_3}{(a_2 + 1)a_{1\tau} \sin(w_0\tau_0)}, \\ \beta &= \frac{w_0}{a_{1\tau} \sin(w_0\tau_0)}, \\ \alpha_1 &= \frac{(a_1 + iw_0 - a_{1\tau}e^{iw_0\tau_0})(1 - iw_0)}{(1 - iw_0)(a_2 + iw_0)}, \\ \beta_1 &= \frac{1 - iw_0}{a_2 + iw_0}. \end{aligned} \quad (32)$$

$q(\theta)$ and $q^*(\theta)$ can be solved according to orthonormality conditions $\langle q^*(\theta), q(\theta) \rangle = 1$, $\langle q^*(\theta), \bar{q}(\theta) \rangle = 0$, and we will get

$$H = \frac{1}{1 + \bar{\alpha}\alpha_1 + \bar{\beta}\beta_1}. \quad (33)$$

Step 2. Introduce new variables $z(t) = \langle q^*, x_t \rangle$, $W(t, \theta) = x_t(\theta) - 2 \operatorname{Re}\{z(t)q(\theta)\}$, and the *Poincaré* normal form of (20) can be written as

$$W(t, \theta) = W(z(t), \bar{z}(t), \theta),$$

$$\begin{aligned} W(z(t), \bar{z}(t), \theta) = & W_{20}(\theta) \frac{z^2}{2} + W_{11}(\theta) z\bar{z} \\ & + W_{02}(\theta) \frac{\bar{z}^2}{2} + W_{30}(\theta) \frac{z^3}{2} + \dots \end{aligned} \quad (34)$$

The plane spanned by eigenvectors z and $\bar{z}$ is tangent to center manifold at the origin. This means that center manifold can be locally approximated as a truncated power series of W depending on the second order of the coordinates z and $\bar{z}$.

Step 3. Only consider real solution. When $\mu = 0$, the solution of (20) is $x_t \in C_0$. According to the above equations, get

$$\begin{aligned} \dot{z}(t) = \langle q^*, \dot{x}_t \rangle &= \langle q^*, E\dot{x}_t + Fx \rangle \\ &= iw_0\tau_0 z(t) + \bar{q}^*(0) f_0(z, \bar{z}) \\ &= iw_0\tau_0 z(t) + k(z, \bar{z}), \end{aligned} \quad (35)$$

$$\begin{aligned} f_0(z, \bar{z}) &= f(0, x_t(\theta)) \\ &= f(0, W(z, \bar{z}, \theta)) + 2 \operatorname{Re}\{zq(\theta)\}. \end{aligned}$$

Define

$$\begin{aligned} g(z, \bar{z}) &= \bar{q}^*(0) f_0(z, \bar{z}) \\ &= k_{20}(\theta) \frac{z^2}{2} + k_{11}(\theta) z\bar{z} + k_{02}(\theta) \frac{\bar{z}^2}{2} \\ &\quad + k_{30}(\theta) \frac{z^2\bar{z}}{2} + \dots, \\ x_t(\theta) &= W(t, \theta) + 2 \operatorname{Re}\{zq(\theta)\} \\ &= k_{20}(\theta) \frac{z^2}{2} + k_{11}(\theta) z\bar{z} + k_{02}(\theta) \frac{\bar{z}^2}{2} \\ &\quad + (1 \ \alpha \ \beta)^T e^{iw_0\tau_0\theta} z + (1 \ \bar{\alpha} \ \bar{\beta})^T e^{-iw_0\tau_0\theta} \bar{z} + \dots \end{aligned} \quad (36)$$

The values of k_{20} , k_{02} , k_{11} , and k_{21} can be calculated by (35) and (36).

Step 4. Calculate $W_{20}(\theta)$ and $W_{11}(\theta)$

$$\begin{aligned} \dot{W} &= \dot{x}_t - \dot{z}p - \dot{\bar{z}}\bar{p} \\ &= \begin{cases} EW - 2 \operatorname{Re}\{\bar{p}^*(0) f_0(z, \bar{z}) p(\theta)\}, & \theta \in [-\tau_0, 0), \\ EW - 2 \operatorname{Re}\{\bar{p}^*(0) f_0(z, \bar{z}) p(0)\}, & \theta = 0, \end{cases} \end{aligned} \quad (37)$$

where

$$\begin{aligned} M(z, \bar{z}, \theta) &= M_{20}(\theta) \frac{z^2}{2} + M_{11}(\theta) z\bar{z} \\ &\quad + M_{02}(\theta) \frac{\bar{z}^2}{2} + M_{30}(\theta) \frac{z^3}{6} + \dots \end{aligned} \quad (38)$$

Differentiating M from t , yield

$$\dot{M} = M_z \dot{z} + M_{\bar{z}} \dot{\bar{z}}. \quad (39)$$

Further, yield

$$\begin{aligned} H_{20}(\theta) &= W_{20}(\theta) (2iw_0 - A_0), \\ H_{11}(\theta) &= -A_0 W_{11}(\theta), \\ H_{02}(\theta) &= -W_{02}(\theta) (A_0 + 2iw_0). \end{aligned} \quad (40)$$

According to the above equations and $\theta \in [-\tau_0^*)$, we can obtain

$$\begin{aligned} M(z, \bar{z}, \theta) &= -2 \operatorname{Re}\{\bar{q}^*(0) f_0(z, \bar{z}) p(\theta)\} \\ &= -k(z, \bar{z}) p(\theta) - \bar{k}(z, \bar{z}) \bar{p}(\theta) \\ &= -\left(\frac{1}{2} k_{20} z^2 + k_{11} z\bar{z} + \frac{1}{2} k_{02} \bar{z}^2 + \dots\right) p(\theta) \\ &\quad - \left(\frac{1}{2} k_{20} \bar{z}^2 + k_{11} z\bar{z} + \frac{1}{2} k_{02} z^2 + \dots\right) \bar{p}(\theta). \end{aligned} \quad (41)$$

Compare coefficients of (36) and (39), and obtain

$$\begin{aligned} M_{20}(\theta) &= -k_{20} p(\theta) - \bar{k}_{02} \bar{p}(\theta), \\ M_{11}(\theta) &= -k_{11} p(\theta) - \bar{k}_{11} \bar{p}(\theta). \end{aligned} \quad (42)$$

Further

$$\begin{aligned} \dot{W}_{20}(\theta) &= 2iw_0 W_{20}(\theta) + k_{20} p(\theta) + \bar{k}_{02} \bar{p}(\theta), \\ \dot{W}_{11}(\theta) &= k_{11} p(\theta) + \bar{k}_{11} \bar{p}(\theta). \end{aligned} \quad (43)$$

The solution of (43) is

$$\begin{aligned} W_{20}(\theta) &= -\frac{k_{20} p(0)}{iw_0} e^{iw_0\theta} - \frac{\bar{k}_{02} \bar{p}(0)}{3iw_0} e^{-iw_0\theta} + R_1 e^{2iw_0\theta}, \\ W_{20}(\theta) &= -\frac{k_{11} p(0)}{iw_0} e^{iw_0\theta} - \frac{\bar{k}_{02} \bar{p}(0)}{iw_0} e^{-iw_0\theta} + R_2, \end{aligned} \quad (44)$$

where R_1 and R_2 are constant vectors which can be determined by setting $\theta = 0$ in $H(z, \bar{z}, \theta)$.

Step 5. Calculate the characteristic quantity β_2 of the stability of hopf bifurcation, the characteristic quantity μ_2 of the direction of bifurcation, and the characteristic quantity T_2 of the periodic variation. Before the calculation, we need to obtain canonical form coefficient $c_1(0)$:

$$\begin{aligned} c_1(0) &= \frac{i}{2w_0\tau_0} \left(k_{20}k_{11} - 2|k_{11}|^2 - \frac{1}{3}|k_{02}|^2 \right) + \frac{1}{2}k_{21}, \\ \mu_2 &= \frac{\operatorname{Re}\{c_1(0)\}}{\operatorname{Re}\{\lambda'(\tau_0)\}}, \\ \nu_2 &= 2 \operatorname{Re}\{c_1(0)\}, \end{aligned} \quad (45)$$

$$T_2 = -\frac{1}{w_0\tau_0} \left[\operatorname{Im}\{c_1(0)\} + \mu_2 \operatorname{Im}\{\lambda'(\tau_0)\} \right].$$

Step 6. To sum up, we get the theorem and it is represented as follows.

Theorem 1. Suppose $f_i, k_i \in C^3, i = 1, 2, 3$; then the direction of hopf bifurcation can be determined by μ_2 . If $\mu_2 > 0$ ($\mu_2 < 0$), hopf bifurcation is supercritical (subcritical). When time lag $\tau > \tau_0$ ($\tau < \tau_0$), the periodic solution of bifurcation is stable (unstable). T_2 can determine the periodic law of bifurcation: if $T_2 > 0$ ($T_2 < 0$), the periodic solution will crease (decrease). Otherwise, the period of periodic motion can be evaluated by

$$T(\mu) = \frac{2\pi}{\omega_0} (1 + T_2\mu^2 + \dots). \quad (46)$$

4. Stability and Interference

We find disturbing signal of acceleration can be defined as $\ddot{z}_\sigma = (\sigma^n + 1) \cdot \ddot{z}$, through a number of maglev experiments on the vehicle CMS04. The term n is the power of disturbing signal. The positive direction is equal to the direction of gravity acceleration. So suppose disturbing model of acceleration is $p(x) = [0 \ 0 \ \sigma^n] x$, and then the characteristic equation of maglev system about σ is

$$\begin{aligned} \varphi_0^\sigma(z, \dot{z}, \ddot{z}, \ddot{\ddot{z}}, \sigma) \\ = M_1 \ddot{\ddot{z}} + \left(\frac{MM_1 R z_e}{\mu_0 AMN^2} + \frac{M_1}{2g\mu_0 MN} + K_a (1 + \sigma^n) \right) \ddot{\ddot{z}} \\ + K_d \dot{z} + \left(K_p - \frac{RM_1}{AN^2 \mu_0} \right) z. \end{aligned} \quad (47)$$

Simplify the characteristic equation, and get

$$\lambda^3 + a_1^\sigma \lambda^2 + a_2^\sigma \lambda^1 + a_3^\sigma = 0, \quad (48)$$

where

$$\begin{aligned} a_1^\sigma &= \frac{MRz_e}{\mu_0 AMN^2} + \frac{1}{2g\mu_0 MN} + \frac{K_a (1 + \sigma^n)}{M_1}, \\ a_2^\sigma &= \frac{K_d}{M_1}, \quad a_3^\sigma = \frac{K_p}{M_1} - \frac{R}{AN^2 \mu_0}. \end{aligned} \quad (49)$$

Let $\varphi_0^\sigma(z, \dot{z}, \ddot{z}, \ddot{\ddot{z}}, \sigma) = 0$; then

$$\lambda^3 + a_1^\sigma \lambda^2 + a_2^\sigma \lambda^1 + a_3^\sigma = 0. \quad (50)$$

Namely,

$$\lambda^3 + \left(a_1 + \frac{K_a}{M_1} \sigma^n \right) \lambda^2 + a_2 \lambda^1 + a_3 = 0. \quad (51)$$

The stability of system about σ should be discussed at once.

According to Routh criterion, the condition of maglev system stability must be satisfied: all of eigenvalues must be positive and satisfy

$$\begin{aligned} D_1 &= a_1 + \frac{K_a}{M_1} \sigma^n > 0, \\ D_2 &= \begin{vmatrix} a_1 + \frac{K_a}{M_1} \sigma^n & a_3 \\ 1 & a_2 \end{vmatrix} > 0, \\ D_3 &= a_3 D_2 > 0. \end{aligned} \quad (52)$$

The above equation can be simplified to

$$\sigma^n > \left(\frac{a_3}{a_2} - a_1 \right) \frac{K_a}{M_1}, \quad (53)$$

where

$$\left(\frac{a_3}{a_2} - a_1 \right) K_a < 0 \quad (54)$$

according to the stable condition of the original system: $(a_3/a_2) - a_1 < 0, K_a > 0$.

In summary, if the additive interfering signal of acceleration is positive, it cannot influence stability of maglev system. Through the experiments, this view has been proved and the additive positive interference can improve stability of system improve, on the contrary, because the positive interference changes the poles of maglev system far from the origin. Similarly, the positive multiplicative interference cannot influence stability of the system.

Discuss how the positive additive disturbs stability of maglev system

$$\lambda^3 + \left(a_1 - \frac{K_a}{M_1} \sigma^n \right) \lambda^2 + a_2 \lambda^1 + a_3 = 0. \quad (55)$$

According to Routh criterion, the system must satisfy (55) for keeping itself asymptotically stable

$$\sigma^n < \min \left(\frac{M_1 a_1}{K_a}, \frac{M_1 (a_1 - (a_3/a_2))}{K_a} \right) = \frac{(a_1 a_2 - a_3) M_1}{a_2 K_a}. \quad (56)$$

If the negative interfering signal of acceleration reaches the critical condition, the maglev control system will change from stability to divergence. If the gain of interfering signal belongs to the set $U_\sigma^s = \{\sigma \mid \sigma^n > (a_3 - a_1 a_2) M_1 / a_2 K_a\}$, maglev system will be asymptotic stability. If the gain of interfering signal belongs to the set $U_\sigma^u = \{\sigma \mid \sigma^n < (a_3 - a_1 a_2) M_1 / a_2 K_a\}$, maglev system will be divergence, and the term $\sigma = ((a_3 - a_1 a_2) M_1 / a_2 K_a)^{1/n}$.

5. Experiments

Some experiments have been implemented on CMS04 maglev control experiment platform designed by NUDT. Maglev control system applies position and flux double cascade control algorithm, the outer loop adopting PID control

FIGURE 4: Block diagram of maglev closed control system.

FIGURE 5: Position and current curves when $\tau = 0.62$.

method, and the inner loop adopting P control method. There is a detailed presentation about this control algorithm in the literature [12], so it is not stated in this paper. Some experiments have been done in terms of the theoretical result in this paper. The control parameters are shown in Table 2.

The maglev closed control system's block diagram is shown in Figure 4.

The control parameters in experiments are

$$\begin{aligned} K_P &= 4500, & K_I &= 4000, & K_D &= 800, \\ K_A &= 2, & K_B &= 60. \end{aligned} \quad (57)$$

Experiment 1 (time delay of acceleration). The control parameters designed in experiment satisfy (11); namely, when

$\tau = 0$, the system is asymptotic stability. We obtain the values $\omega_0 = 2.1465$ and $\tau_0 = 0.6208$ from (45). Put the critical value into (21), and yield $\text{Re} [d\lambda/d\tau]_{\tau=\tau_0}^{\lambda=i\omega_0} > 0$; namely, it satisfies the crossing condition. When time lag gets close to τ_0 , hopf bifurcation will appear. Set delayed time $\tau = 0.62$ s, and the solutions $\beta_2 \approx 0.0432 > 0$, $\mu_2 \approx -0.0371 < 0$, $T_2 \approx 3.897 > 0$, which are calculated according to the theory of this paper. So we can diagnose that hopf bifurcation of maglev system is supercritical; namely, when delayed time is greater than τ_0 , hopf bifurcation happens, and the periodic solution of bifurcation is unstable, and the period increases gradually. When $\tau = 0.62$, the response of maglev system is shown in Figure 5. The response proves that period motion

TABLE 2: Standard maglev bogie's parameters of CMS04 maglev vehicle.

Parameters	Values	Parameters	Values
M/kg	653	z_0/m	0.009
N	324	B_0/T	0.6
A/m^2	0.0235	g	9.8
R/Ω	0.5	μ_0	$4\pi \times 10^{-7}$

appears in maglev control system, and the system's manifold will be divergent generally, and the phenomenon is that the suspension system has been vibrating consistently until divergence.

6. Conclusions

This paper has discussed hopf bifurcation caused by time delay of acceleration and denoted that when delayed time is close to the critical value τ_0 , hopf bifurcation will appear. This paper also obtains characteristic values by employing the center manifold reduction theory and the normal form method, which represent separately the direction of hopf bifurcation, the stability of period solution, and the period of period motion. Subsequently, we discuss the influence on the stability of maglev system by acceleration's interfering signal and obtain the stable domain of the interfering signal. Some experiments have been done on CMSO4 maglev vehicle by NUDT of Tangshan city. The results of experiments demonstrate that viewpoints in this paper are correct and scientific. When time lag reaches the critical value, maglev system will produce a supercritical hopf bifurcation which may cause unstable period motion. Therefore, the control engineers should decrease the retarded time between actuator and sensor and augment the accuracy of the suspending guideway with a view to make delayed time less than the critical value and make the control system escape from unstable period motion. The entire analysis illustrates that maglev system has complicated dynamic attribute, so this paper put more important references for investigating the dynamic property of maglev system further.

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