Research Article

A Delta Operator Approach for the Discrete-Time Active Disturbance Rejection Control on Induction Motors

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The problem of active disturbance rejection control of induction motors is tackled by means of a generalized PI observer based discrete-time control, using the delta operator approach as the methodology of analyzing the sampled time process. In this scheme, model uncertainties and external disturbances are included in a general additive disturbance input which is to be online estimated and subsequently rejected via the controller actions. The observer carries out the disturbance estimation, thus reducing the complexity of the controller design. The controller efficiency is tested via some experimental results, performing a trajectory tracking task under load variations.

1. Introduction

To obtain high performance control of electric machines there has been a growing interest in the design of controllers based on the discrete-time model of the system. In the case of induction motors, the system is continuous in nature, being necessary to obtain a sampled-time model. Preliminary studies on the sampling of continuous time nonlinear systems can be found in [1]. Many advances have been reported about control of sampled nonlinear systems; see for instance [2, 3] and references therein. Specifically, an analysis about the discretization techniques for the induction motor model can be found in [4].

There exists a variety of control strategies for the induction motor that depend on the difficulty to measure parameters while their closed loop behavior is found to be sensitive to their variations. Generally speaking, the designed feedback control strategies have to exhibit a certain robustness level with respect to unknown bounded additive disturbance, in order to guarantee an acceptable performance. It is possible

to (online or offline) obtain estimates of the motor parameters [5], but some of them can be subject to variations when the system is undergoing actual operation. Frequent misbehavior is also due to external and internal disturbances, such as generated heat, that significantly affect some of the system parameter values. An alternative to overcome this situation is to use robust feedback control techniques which take into account these variations as unknown disturbance inputs that need to be online estimated and rejected. One of the first attempts to solve this problem was proposed by Johnson [6], known as disturbance accommodation control, in which external disturbances are given as “waveform functions”, proposing an unknown input observer to perform the robust controller. On the other hand, the active disturbance rejection scheme [7–9] considers both external disturbances and internal perturbations, as a lumped generalized additive disturbance functions to be canceled out. The main idea of the controller is the fact that the disturbance observer can estimate the lumped disturbance input, which allows to approximately reduce the original nonlinear tracking control

problem to that of a disturbed input tracking problem, suitable for the application of a simple controller.

One variant of this scheme resorts to a local internal model characterization of the lumped disturbance using a representative element of a family of discrete-time polynomial signals of fixed degree. This results in a local self-updating polynomial model of the uncertainty which can be estimated, in an arbitrarily close manner, via a suitable extended linear observer of generalized proportional integral (GPI) nature [10]. The GPI estimation procedure has been extended for fault tolerant control applications as proposed in [11].

For the case of the induction motor control, we consider a robust controller design based upon a simplified discrete model of the system including additive, completely unknown, disturbance inputs lumping nonlinearities and external disturbances whose effect is to be determined in an online fashion by means of a discrete-time linear observer of the GPI. The gathered knowledge will be used in the appropriate canceling of the assumed disturbances themselves while reducing the underlying control problem to a simple linear feedback control task. The control scheme thus requires knowledge of a reduced set of the motor parameters to be implemented.

The controller design for the induction motor is carried out within the philosophy of the classical field oriented controller scheme and implemented through a flux simulator or reconstructor (see Chiasson [12] and Martín and Rouchon [13]). It is considered a two-stage design procedure for the feedback control scheme of an induction motor which allows one to, simultaneously, regulate the motor shaft angular velocity towards a prespecified reference trajectory and to stabilize the flux magnitude to a desired constant level. The first stage designs a controller for the reference trajectory tracking of the rotor shaft angular velocity. The stator currents are used as auxiliary control input variables within a *field oriented* strategy combined with a load torque elimination executed on the basis of an online close estimation of the load disturbance input.

The control configuration for the first stage inherently includes a *flux reconstructor*, and a discrete-time generalized proportional integral observer based control for the efficient and rather accurate online estimation of the unknown but bounded load torque disturbance input function. The second design stage takes the synthesized rotor currents as reference trajectories to be tracked from the rotor input voltages.

In this case, the sampled time system is not defined purely in terms of the time-shift operator but in terms of the unified operator approach proposed by Goodwin et al. [14]. This operator came up with an alternative to obtain better results in high sampling rates, where most traditional discrete-time algorithms may be ill-conditioned when applied to data taken at sampling rates which are high relative to the dynamics of sampled data [15]. The unified approach developed a strategy capable of unifying both continuous and discrete-time formulations [14]. Moreover, this approach overcomes the unstable sampling zero problem as analyzed in [16] and the procedure for the control gains is enhanced since the stability region increases as sampling time decreases,

FIGURE 1: Stability region for delta operator.

avoiding extra reparametrizations as the Tustin approach. In this type of approach, the authors proposed the use of an operator called δ -operator defined as follows: $\delta = (q - 1)/\Delta$, where q is the forward shift operator in the time domain and Δ is the sampling time.

Here, the discrete-time GPI control has been proposed using the delta operator approach taking advantages of the high sampling rates and advantages of working directly in the sampled time system with respect to the continuous scheme, such as the faster implementation in a digital controller.

The remainder of the paper is organized as follows. Section 2 introduces the unified operator framework. In Section 3, the induction motor model is introduced, and some considerations regarding the additive disturbances are reported. Section 4.3 presents the problem of disturbance estimation in the context of the discrete-time GPI observer. Section 5 deals with the field oriented control strategy; the angular velocity and stator current controls are presented as a two-stage design procedure involving inner and outer loop controls. Section 6 provides some experimental results in a test bed to show the behavior of the observer-based control. Finally, some conclusions are reported in Section 7.

2. Brief Remarks about the Delta Operator

In this section, some preliminary concepts regarding the δ operator and its properties are introduced; more details concerning the operator and its applications are found in [14, 17, 18].

Definition 1. The domain of possible nonnegative “times” $\Omega^+(\Delta)$ is defined as follows:

$$\Omega^+(\Delta) = \begin{cases} \mathbb{R}^+ \cup \{0\} & : \Delta = 0, \\ \{0, \Delta, 2\Delta, 3\Delta, \dots\} & : \Delta \neq 0 \end{cases}, \quad (1)$$

where Δ denotes the sampling period in discrete time or $\Delta = 0$ for a continuous time framework.

Definition 2. A time function $x(t)$, $t \in \Omega^+(\Delta)$, is, in general, simply a mapping from times, to either the real or the complex set. That is, $x(t) : \Omega^+ \rightarrow \mathbb{C}$.

Definition 3. The δ operator is defined as follows:

$$\delta x(t) \triangleq \frac{(q - 1)x(t)}{\Delta} = \frac{x(t + \Delta) - x(t)}{\Delta} : \Delta \neq 0, \quad (2)$$

where q is the shift operator and

$$\lim_{\Delta \rightarrow 0^+} \delta(x(t)) = \frac{dx(t)}{dt}. \quad (3)$$

Definition 4. We will consider ρ as a generalized derivative operator, which will denote d/dt in continuous time or δ in discrete time.

Definition 5. The unified integration operation $\mathbf{S}$ is given as follows:

$$\mathbf{S}_{t_1}^{t_2} x(\tau) d\tau = \begin{cases} \int_{t_1}^{t_2} x(\tau) d\tau & : \Delta = 0 \\ \Delta \sum_{l=t_1/\Delta}^{t_2/\Delta-1} x(l\Delta) & : \Delta \neq 0 \end{cases}, \quad (4)$$

$t_1, t_2 \in \Omega^+$.

The integration operator corresponds to the antiderivative operator.

Definition 6 (generalized matrix exponential). In the case of the unified transform theory, the generalized exponential E is defined as follows:

$$E(A, t, \Delta) = \begin{cases} e^{At} & : \Delta = 0 \\ (I + A\Delta)^{t/\Delta} & : \Delta \neq 0 \end{cases}, \quad (5)$$

where $A \in \mathbb{C}^{n \times n}$, I is the identity matrix, and $t \in \Omega^+$. The generalized matrix exponential satisfies to be the fundamental matrix of $\delta x = Ax$, and thus the unique solution to

$$\rho x = Ax, \quad x(0) = x_0 \quad (6)$$

is $x(t) = E(A, t, \Delta)x_0$. The general solution to:

$$\rho x = Ax + Bu, \quad x(0) = x_0 \quad (7)$$

is

$$x(t) = E(A, t, \Delta)x_0 + \mathbf{S}_0^t E(A, t - \tau - \Delta, \Delta) Bu(\tau) d\tau. \quad (8)$$

Definition 7 (stability boundary). The solution of (6) is said to be asymptotically stable if and only if, for all x_0 , $x(t) \rightarrow 0$ as time elapses. The stability arises if and only if $E(A, t, \Delta) \rightarrow 0$ as $t \rightarrow \infty$ if and only if every eigenvalue of A , denoted as λ_i , $i = 1, \dots, n$, satisfies the following condition:

$$\text{Re}\{\lambda_i\} + \frac{\Delta}{2}|\lambda_i|^2 < 0. \quad (9)$$

Therefore, the stability boundary is the circle with center $(-1/\Delta, 0)$ and radius $1/\Delta$ (see Figure 1). In particular, consider the following n th degree characteristic equation on the complex variable γ (see Definition 8):

$$\prod_{i=1}^n (\gamma - \lambda_i) = \gamma^n + c_{n-1}\gamma^{n-1} + \dots + c_1\gamma + c_0 = 0. \quad (10)$$

If all the roots λ_i , $i = 1, \dots, n$, of the last equation satisfy condition (9), then the solution of the associated system to (10) is asymptotically stable.

Definition 8 (Unified Transform Theory). There is a close connection between the forward shift operator q and the Z -transform variable z . Analogously, consider a new transform variable γ associated with the δ operator as $\gamma = (z - 1)/\Delta$. From the Z -transform, the delta transform is derived as follows:

$$\mathbf{T}(f(k)) = \Delta F(z)|_{z=\Delta\gamma+1} = \Delta \sum_{k=0}^{\infty} f(k) (1 + \Delta\gamma)^{-k}. \quad (11)$$

2.1. Transform Properties. We will just point out to the transform properties to be used throughout the work. A more extensive list of the delta transform properties is found in [17].

(i) **Linearity.** For any scalar α_1, α_2 ,

$$\mathbf{T}\{\alpha_1 f(t) + \alpha_2 g(t)\} = \alpha_1 \mathbf{T}\{f(t)\} + \alpha_2 \mathbf{T}\{g(t)\}. \quad (12)$$

(ii) **Differentiation**

$$\mathbf{T}\{\rho[f(t)]\} = \gamma \mathbf{T}\{f(t)\} - f(0^-)(1 + \Delta\gamma). \quad (13)$$

(iii) **Integration**

$$\mathbf{T}\{\mathbf{S}_0^t f(\tau) d\tau\} = \frac{1}{\gamma} \mathbf{T}\{f(t)\}. \quad (14)$$

(iv) **Frequency Differentiation**

$$\mathbf{T}\{tf(t)\} = -(\Delta\gamma + 1) \frac{d}{d\gamma} [\mathbf{T}\{f(t)\}]. \quad (15)$$

3. System Model and Problem Formulation

Consider the two-phase equivalent mathematical model (a, b) of a three-phase induction motor controlled by the phase voltages represented by the complex input voltage $u_S = u_{Sa} + j u_{Sb} = |u_S|e^{j\theta_u}$. The state variables are given by: θ , which is the rotor angular position, ω denoting the rotor angular velocity, ψ_{R_a} and ψ_{R_b} denote the unmeasured rotor fluxes consolidated by the complex rotor flux: $\psi_R = \psi_{Ra} + j\psi_{Rb} = |\psi_R|e^{j\theta_\psi}$. The variables i_{Sa} and i_{Sb} represent the stator currents, and $i_S = i_{Sa} + j i_{Sb} = |i_S|e^{j\theta_i}$ is their corresponding complex armature current. It is assumed that $j = \sqrt{-1}$ is the imaginary unit, $\bar{x}$ is the conjugate of x , $\mathcal{R}_e(x)$ denote the real part of x and $\mathcal{I}_m(x)$ denote the imaginary part of x . From the abovementioned definitions, we have

$$\begin{aligned} \rho\omega &= \mu \text{Im}(\bar{\psi}_R i_S) - \frac{B}{J}\omega - \frac{\tau_L}{J}, \\ \rho\psi_R &= -(\eta - j n_p \omega) \psi_R + \eta M i_S, \end{aligned} \quad (16)$$

$$\rho i_S = \beta(\eta - j n_p \omega) \psi_R - \gamma i_S + \frac{1}{\sigma L_S} u_S,$$

with ρ the generalized derivative operator, $\eta := R_R/L_R$, $\beta := M/\sigma L_R L_S$, $\mu := n_p M/J L_R$, $\gamma := M^2 R_R/\sigma L_R^2 L_S + R_S/\sigma L_S$, and $\sigma := 1 - M^2/L_R L_S$. R_R and R_S are the rotor and stator resistances. The rotor and stator inductance parameters are

given by L_R and L_S , and M is the mutual inductance constant; the moment of inertia is set by J , the friction coefficient is denoted by B , and n_p is the number of pole pairs. The signal τ_L is the unknown load torque disturbance input.

3.1. Flux Observer. A simple way to obtain a discretization of the flux observer is using δ -operator approximation for the derivatives (which is equivalent to Euler's approximation); that is,

$$\dot{\psi}_R(t) \approx \delta \psi_R(t) = \frac{[\psi_R(t + \Delta) - \psi_R(t)]}{\Delta}. \quad (17)$$

A discretized version of flux dynamics is given by

$$\psi_R(t + \Delta) = [1 - \eta\Delta + jn_p\Delta\omega(t)] \psi_R(t) + \eta M \Delta i_S(t). \quad (18)$$

An observer for this discretized system is given by

$$\hat{\psi}_R(t + \Delta) = [1 - \eta\Delta + jn_p\Delta\omega(t)] \hat{\psi}_R(t) + \eta M \Delta i_S(t). \quad (19)$$

In order to analyze the stability of the discrete-time flux estimator, the estimation error is defined as: $e_\psi(t + \Delta) = \psi_R(t + \Delta) - \hat{\psi}_R(t + \Delta)$; these errors satisfy $e_\psi(t + \Delta) = [1 - \eta\Delta + jn_p\Delta\omega(t)]e_\psi(t)$. Consider the Lyapunov function candidate $V(k) = |e_\psi(t)|^2$ under simple algebraic manipulations yields

$$V(t + \Delta) = \alpha(t) V(t) \quad (20)$$

with $0 \leq \alpha(t) = (1 - \eta\Delta)^2 + (n_p\Delta\omega(t))^2$. The stability is guaranteed when $0 \leq \alpha(t) < 1$. It is possible to find a condition on the sample period, Δ , and speed, ω , such that the origin of the complex error space, $e_\psi(t) = 0$, is a globally asymptotic equilibrium point for (18):

$$\Delta < \frac{2\eta}{\eta^2 + n_p^2 \omega_{\max}^2} \implies 0 \leq \alpha(t) < 1, \quad (21)$$

where $\omega_{\max}$ is the maximum angular velocity of the motor.

The flux simulator variable, $\hat{\psi}_R$, will be used, henceforth, in place of the actual flux without further considerations.

3.2. Assumptions

- (i) It is assumed that only the shaft's angular position, θ , and the stator currents, i_{Sa} , i_{Sb} , are measured.
- (ii) The motor parameters are assumed to be known.
- (iii) The load torque $\tau_L(t)$ is assumed to be time-varying but unknown.
- (iv) Let us assume that the sampling period Δ is sufficiently small to achieve accurate results when using, as a discretization methodology, the unified operator ρ (the use of Euler methods is in [4] in a different context). In particular, Δ is small enough to satisfy (21).

3.3. Problem Formulation. Under the above assumptions, consider the induction motor dynamic model (16). Given a reference trajectory for the motor angular velocity, $\omega^*(t)$, and a reference for the magnitude of the complex flux, $|\psi_R^*|$, the main objective of this paper is to devise multivariable discrete-time feedback control laws for the stator voltages, $u_{Sa}(t)$, $u_{Sb}(t)$, such that they force, in an arbitrary fashion, $\omega(t)$ to track $\omega^*(t)$ and $|\psi_R|$ to track $|\psi_R^*|$ regardless of the values adopted by the time-varying torque, $\tau_L(t)$, the discretization errors resulting from the δ -operator discretization procedure, and eventually parameter uncertainty.

4. Control Strategy

4.1. Simplified Model. The proposed control strategy is based on a simplified vision of the system model (16), which is systematically advocated in the ADRC approach. One adopts the simplified models, defined in terms of complex variables notation:

$$\rho\omega = \mu \operatorname{Im}(\bar{\psi}_R i_S) + \xi_1(t), \quad (22)$$

$$\rho|\psi_R|^2 = -2\eta|\psi_R|^2 + 2\eta M \operatorname{Re}(\bar{\psi}_R i_S), \quad (23)$$

$$\rho\theta_\psi = n_p\omega + \frac{R_r M}{L_r |\psi_R|^2} \operatorname{Im}(\bar{\psi}_R i_S), \quad (24)$$

$$\rho i_S = \frac{1}{\sigma L_S} u_S + \xi_2(t), \quad (25)$$

where $\tilde{\xi}_1(t)$ is the exogenous disturbance function that takes into account the load torque disturbance term, $-\tau_L(t)/J$, the viscous friction term, $-(B/J)\omega$, and discretization errors due to δ -operator approximation; $\tilde{\xi}_2(t)$ is the endogenous state dependent disturbance function that represents nonlinear and linear additive dissipation terms, depending on the stator currents, i_{Sa} , i_{Sb} , and the angular velocity, ω , and it also includes discretization errors.

4.2. Field Oriented Control. From (22) and (23), we can obtain an interesting control decoupling property: the angular velocity is governed by $\operatorname{Im}(\bar{\psi}_R i_S)$, while the squared flux magnitude is commanded by $\operatorname{Re}(\bar{\psi}_R i_S)$. Consequently, taking the current, i_S , as auxiliary control input, both constitutive parts of the system can be controlled independently of each other. This classical indirect control decoupling property is equivalent to the field oriented control approach. We use this property to set, with the help of auxiliary input variable, $v = v_a + jv_b$, the following input current field oriented controller:

$$i_S = \left(\frac{\psi_R}{|\psi_R|^2} \right) v, \quad (26)$$

yielding the following set of control decoupled linear disturbed systems:

$$\rho\omega = \mu v_b + \xi_1(t), \quad (27)$$

$$\rho|\psi_R|^2 = -2\eta|\psi_R|^2 + 2\eta M v_a. \quad (28)$$

The control law v should accomplish the simultaneous tracking tasks for $|\psi_R|$ and ω (see (28), (27)) and the control law u_s for the tracking of i_s (see (25)) in a two-stage feedback observer based control configuration. The key observation of this observer based control approach is that the disturbance inputs, ξ_1, ξ_2 , involved in (27), (25), can be approximately estimated and then canceled at the controller stage. This procedure goes with the total active disturbance rejection paradigm (see [8]).

4.3. Disturbance Estimation. In this subsection, a methodology of disturbance estimation by means of a delta operator discrete-time observer, which can be associated to an extended Luenberger like linear observer, is developed.

The ideal performance of control systems and its dual estimation is to achieve zero steady-state errors in an asymptotic fashion. Given the uncertainty of unified disturbance signals (regarding external disturbances and the dynamics of the system) involved in the dynamics of the inner and outer loops of the proposed control scheme, it is necessary to make an approach to a generic model for signals. The approximation used and which is simpler to determine the internal model is given by the approximation of the truncated Taylor series. These families of functions with respect to disturbance signals are in agreement with the model $\rho^{m_i} \xi_i = ((q-1)^{m_i} / \Delta^{m_i}) \xi_i \approx 0$ with $i = 1, 2$. The approach uses the fact that the disturbance inputs, $\xi_i, i = 1, 2$, can be approximately modeled by

$$\rho^{m_i} \xi_i \approx 0, \quad (29)$$

where m_1, m_2 are integers large enough. So, taking $\xi_i, i = 1, 2$, as augmented variables it is possible to establish generalized state observers (see [19]).

The disturbance inputs ξ_i can be expressed as functions of the output, the input and a finite application of the delta operator on them; therefore, the algebraic observability property is achieved, and a delta operator based observer can be proposed in each equation.

The construction of the delta generalized proportional integral disturbance observer for ξ_1 is described in the following proposition.

Proposition 9. Define the observation error as $\tilde{e}_\theta(t) = \theta(t) - \hat{\theta}(t)$. The following system

$$\begin{aligned} \widehat{\rho\theta}(t) &= \widehat{\omega}(t) + l_{m_1+1} \tilde{e}_\theta(t), \\ \widehat{\rho\omega}(t) &= \mu v_b(t) + l_{m_1} \tilde{e}_\theta(t) + \widehat{z}_{1,1}(t), \\ \widehat{\rho z}_{1,1}(t) &= l_{m_1-1} \tilde{e}_\theta(t) + \widehat{z}_{1,2}(t), \\ \widehat{\rho z}_{1,2}(t) &= l_{m_1-2} \tilde{e}_\theta(t) + \widehat{z}_{1,3}(t), \\ &\vdots \\ \widehat{\rho z}_{1,m_1-1}(t) &= l_1 \tilde{e}_\theta(t) + \widehat{z}_{1,m_1}(t), \\ \widehat{\rho z}_{1,m_1}(t) &= l_0 \tilde{e}_\theta(t), \end{aligned}$$

$$\widehat{\theta}(0) = \widehat{\omega}(0) = \widehat{z}_{1,1}(0) = \cdots = \widehat{z}_{1,m_1}(0) = 0,$$

$$\widehat{\xi}_1(t) = \widehat{z}_{1,1}(t) \quad (30)$$

constitutes an asymptotic unified discrete generalized proportional integral observer of order m for the disturbance ξ_1 , where $l_0, \dots, l_{m_1+1}$ are the design constants which regulate the convergence rate of the observation error.

Proof. The disturbance estimation procedure is the dual counterpart of disturbance rejection mechanism which resides in the application of m_1 discrete-time successive differences; that is, $\rho^{m_1} = \delta^{m_1} = (q-1)^{m_1} / \Delta^{m_1}$ (see (29)).

Let us use (30) in (27) and the internal model of the disturbance input ξ_1 . Then, applying the unified operator and some algebraic manipulations in the resulting expression, the observation error satisfies the following disturbed linear dynamics:

$$\begin{aligned} \rho^{m_1+2} \tilde{e}_\theta + l_{m_1+1} \rho^{m_1+1} \tilde{e}_\theta + l_{m_1} \rho^{m_1} \tilde{e}_\theta + \cdots + l_1 \rho \tilde{e}_\theta + l_0 \tilde{e}_\theta \\ = \rho^{m_1} \xi_1(t). \end{aligned} \quad (31)$$

According to the assumptions, ξ_1 is uniformly bounded, therefore, successive differences of ξ_1 , namely, $\rho^{m_1} \xi_1$, remain also uniformly bounded. If $\rho^{m_1} \xi_1(t)$ is uniformly absolutely bounded, by selecting the gain parameters $l_j, j = 0, 1, \dots, m_1+1$, such that the characteristic polynomial in the variable γ , associated to the linear undisturbed part of (31)

$$p_{o,\theta}(\gamma) = \gamma^{m_1+2} + l_{m_1+1} \gamma^{m_1+1} + \cdots + l_1 \gamma + l_0, \quad (32)$$

satisfies (9), the estimation error $\tilde{e}_\theta$ is restricted to a vicinity of zero as time elapses. Thus, $\tilde{\xi}_1(t)$ tends to be located in the neighborhood of $\xi_1(t)$. The size of the vicinity is related to the achieved rank of attenuation in the term $\rho^{m_1} \xi_1(t)$. The parameter m_1 is related to the complexity of the signal to estimate, as in the case of Taylor polynomial approximation [10]. $\square$

Remark 10. ADRC-GPI observer-based controllers use an internal model approximation of the perturbation functions to reconstruct and reject the perturbations. Under this disturbance model approximation setting, several authors have applied it to different areas. Parker and Johnson used a first-order perturbation approximation to model wind speed perturbations in a wind turbine operating in region 3 [20]. Freidovich and Khalil [21] used a first-order perturbation model approximation to estimate the model uncertainty and disturbance on a nonlinear system. Zhao and Gao also used a first-order internal model disturbance approximation to estimate the resonance in two-inertia systems [22] and a first- and second-order approximation to estimate the nonlinearities of an actuator [23]. Zheng et al. also used disturbance model approximation applied to disturbance decoupling control [24].

Remark 11. The parameter m is related to the complexity of the signal to estimate, as in the case of Taylor polynomial

approximation. A first-order perturbation model approximation means that the internal model naturally converges towards a constant disturbance. Equation (30) is a more generalized extension of the internal model perturbation function which provides extra information and increases the ability to track different types of disturbances. For example, $m = 2$ allows convergence to a disturbance with a constant derivative, $m = 3$ allows convergence to a disturbance with a constant acceleration, and so forth.

Remark 12. The ultimate boundedness of the estimation errors, produced by the GPI observer, is strongly dependent on the product of poles magnitudes of the dominant characteristic polynomial for the estimation error. Given a desired ultimate value the use of a larger m in the internal model approximation can alleviate the need for high gain related to the observer parameters. In practice, however, m can be small and chosen within the range of 2 to 5. We recall here a quote by J. von Neumann: “With four parameters I can fit an elephant, and with five I can make him wiggle his trunk!”

Remark 13. GPI observers are bandwidth limited by the roots location of the estimation error characteristic polynomial. Generally, the larger the observer bandwidth is, the more accurate the estimation will be. However, a large observer bandwidth will increase noise sensitivity. Then, the selection of the roots of the estimation error characteristic polynomial affects the bandwidth of the GPI observer and also the influence of measurement noises on the estimations. Therefore, GPI observers are usually tuned in a compromise between disturbance estimation performance (set by the internal model approximation degree) and noise sensitivity.

Remark 14. The trajectory tracking problem is formulated in terms of the angular velocity. The disturbance observer, however, is treated in terms of the angular position second-order dynamics. This allows an alternative estimation of the angular velocity, $\hat{\omega}$.

For the estimation of ξ_2 a similar procedure can be proposed which is synthesized in the following proposition.

Proposition 15. Consider the observation error $\tilde{e}_{iS} = i_S - \hat{i}_S$, and consider the following characteristic polynomial: $p_{o,iS} = \gamma^{m_2+1} + \alpha_{m_2}\gamma^{m_2} + \dots + \alpha_1\gamma + \alpha_0$, with all the roots into the stable region related to δ operator (see Figure 1); then the system

$$\begin{aligned}\hat{\rho}\tilde{i}_S(t) &= \frac{1}{\sigma L_S}u_S(t) + \alpha_{m_2}\tilde{e}_\theta(t) + \hat{z}_{2,1}(t), \\ \hat{\rho}\tilde{z}_{2,1}(t) &= \alpha_{m_2-1}\tilde{e}_\theta(t) + \hat{z}_{2,2}(t), \\ \hat{\rho}\tilde{z}_{2,2}(t) &= \alpha_{m_2-2}\tilde{e}_\theta(t) + \hat{z}_{2,3}(t), \\ &\vdots \\ \hat{\rho}\tilde{z}_{2,m_2-1}(t) &= \alpha_1\tilde{e}_\theta(t) + \hat{z}_{2,m_2}(t), \\ \hat{\rho}\tilde{z}_{2,m_2}(t) &= \alpha_0\tilde{e}_\theta(t),\end{aligned}$$

$$\hat{i}_S(0) = \hat{z}_{2,1}(0) = \dots = \hat{z}_{2,m_2}(0) = 0,$$

$$\hat{\xi}_2(t) = \hat{z}_{2,1}(t) \quad (33)$$

constitutes an asymptotic unified discrete generalized proportional integral observer of order m for the disturbance ξ_2 , where $\alpha_0, \dots, \alpha_{m_2}$ are the design constants which regulate the convergence rate of the observation error.

Proof. The proof is similar to that of the previous proposition. $\square$

5. Controller Design

A two-stage feedback control law is considered for this system. In the first stage (outer loop), the angular position of the motor shaft is forced to track a reference signal $\omega^*(t)$, while regulating the flux magnitude towards a given constant value $|\psi_R^*|$. This stage devises a set of desirable current trajectories, which are taken as output references for the second stage. The second stage (inner loop) designs a feedback controller to track the current trajectories from the first stage; in this case, the stator voltages are the control inputs. For both stages, observer based controls will be implemented.

5.1. Outer Loop Design

5.1.1. Flux Magnitude Regulation. Consider again the linear system (27) and (28). According to the problem formulation, for the case of rotor flux magnitude regulation, a simple control law can be proposed:

$$v_a = \frac{|\psi_R^*|^2}{M}. \quad (34)$$

From (34), it is guaranteed that the tracking of the rotor flux modulus approaches to the given reference modulus flux. Indeed, in closed loop, the square modulus of the rotor flux satisfies $\rho(|\psi_R|^2) = -2\eta[|\psi_R|^2 - |\psi_R^*|^2]$, and then $|\psi_R|$ tends to $|\psi_R^*|$ in an exponential asymptotic manner for a constant reference flux modulus. Notice that the partial feedback (28) requires no cancellations of exogenous or endogenous disturbances. In the case of a time variant reference flux modulus, the decoupling property allows one to propose an independent flux magnitude control law. This fact is properly used in [25].

5.1.2. Stator Current Control. Assuming a proper observer behavior related to system (30), accurate estimations for the disturbance input ξ_1 and angular velocity ω are provided. The following observer based control is proposed:

$$v_b = \frac{1}{\mu} [\rho\omega^*(t) - k_{0\omega}(\hat{\omega} - \omega^*) - \hat{\xi}_1(t)]. \quad (35)$$

The characteristic polynomial of the tracking error, $e_\omega = \omega - \omega^*$, is given by $p_e(\gamma) = \gamma + k_{0\omega} = \gamma + p_k$, where $k_{0\omega} = p_k$ and $\text{Re}\{-p_k\} + (\Delta/2)|p_k|^2 < 0$, to ensure the closed loop stability property.

FIGURE 2: Velocity tracking results.

5.2. Inner Loop Design. Let $i_s^*(t)$ be the desired stator current vector reference trajectory as represented by (26). At this stage, the given structure for the outer loop control is also proposed for the current regulation scheme. We have

$$u_s(t) = \sigma L_s [\rho i_s^*(t) - k_{0i} e_{i_s}(t) - \hat{\xi}_2(t)], \quad (36)$$

where $e_{i_s}(t) = i_s(t) - i_s^*(t)$ and the estimation $\hat{\xi}_2$ is provided by the observer in (33). Finally, the closed loop tracking error for the stator currents is given by $p_{i_s}(\gamma) = \gamma + k_{0i}$.

6. Experimental Results

To assess the control approach, some experiments were carried out in a test bed including a controlled load, by means of a controlled coupled DC motor. The experimental induction motor has the following parameters: $J = 2 \times 10^{-3}$ [Kg·m²], $n_p = 1$, $M = 0.2374$ [H], $L_R = 0.2505$ [H], $L_S = 0.2505$ [H], $R_S = 4.32$ [Ω], and $R_R = 2.8807$ [Ω]. The flux absolute desired value was selected to maximize the induced torque subject to the nominal current constraints. That is, $\psi_R^* = M i_{\text{nom}} / \sqrt{2} = 0.5036$ [Wb], for $i_{\text{nom}} = 3$ [A].

The controller was devised in a MATLAB-xPC Target environment using a sampling period of 0.125 [ms]. The communication between the plant and the controller was performed by two data acquisition devices: a National Instruments PCI-6025E data acquisition card for the analog data, and the digital I/O implementation was performed in a National Instruments PCI-6602 data acquisition card. The voltage and current signals are conditioned for acquisition system by means of low pass filters with cut frequency of 1 [kHz].

The reference trajectory of the velocity consisted in a series of rest to rest transitions with values 0, 15, 13, and

FIGURE 3: Regulation of the flux magnitude and control input.

21 [rad/s]. The gain parameter associated to the velocity control was $k_{0\omega} = -70.3071$, and the gain constant of the current control was set to be $k_{0i} = -357.7691$. The characteristic polynomial of the disturbance GPI observer in the velocity loop was set to be $(\gamma^2 + 35.96\gamma + 339.23)^2$, and the characteristic polynomial for the disturbance GPI observer of the current control loop was $(\gamma^2 + 98.9\gamma + 2500)^2$. The characteristic polynomial selection was based on the transformation of continuous time transfer functions (s -domain) to the unified operator domain (γ) (further details concerning this procedure are found in [17]). The responses were given in terms of two nested second-order damped responses with damping coefficients 4 and 6 for the velocity and current loops and natural frequencies of 4 and 50, respectively.

Figure 2 shows the behavior of the tracking velocity with respect to the reference value, achieving accurate results. Figure 3 illustrates that the rotor flux magnitude is regulated with an approximate error of about 15×10^{-4} [Wb]. In Figure 4, a precise tracking of the stator currents is depicted. Additionally, to illustrate the robustness of the scheme, a time varying load torque was applied through the manipulation of the DC motor armature current, such that the generated external torque load described a trajectory of a chaotic type, corresponding to the output of a Chua's circuit, respectively. The peak value of the applied torque was 1.7 [N·m]. The

FIGURE 4: Trajectory tracking of the stator currents and associated disturbance estimations.

estimation of the disturbance, as well as the applied torque, are shown in Figure 5.

The main advantage of the control algorithm, using the xPC target environment, in a single tasking execution mode was the minimization of the execution time; in the case of the discrete-time control scheme, this time was 4.810^{-5} [s], in contrast with a similar control scheme in a continuous time design, which has an execution time of 6.510^{-5} [s].

7. Concluding Remarks

In this work, a discrete-time disturbance observer based control was proposed to solve the problem of controlling an induction motor. The discrete-time process based on the delta operator allows a faster digital control implementation scheme as well as some easy tuning strategies for both control and observer processes in relation to the pole placement for the closed loop tracking (and injection) errors. The presence of the observer in the control loop makes the proposed scheme quite simple and easy to implement. Besides, it is accurate in presence of different nature disturbance inputs.

The degree of polynomial approximation of the disturbance input, denoted by m , depends on the sampling frequency parameter; for high sampling frequencies the approximation needs a smaller degree of polynomial approximation;

FIGURE 5: Mechanical lumped disturbance estimation.

in particular, the treated case study was satisfied with $m = 2$, which reduces considerably the implementation complexity.

Even though the control loops were proposed for first-order plants, the proposed observer based control can be extended without loss of generality to higher order systems.

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