Research Article

Global Exponential Stability of a Unique Almost Periodic Solution for Neutral-Type Cohen-Grossberg Neural Networks with Time Delays

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This paper is concerned with the problem of the existence, uniqueness, and global exponential stability of almost periodic solution for neutral-type Cohen-Grossberg neural networks with time delays. Based on fixed point theory and Lyapunov functional, several sufficient conditions are established for the existence, uniqueness, and global exponential stability of almost periodic solution for the above system. Finally, an example and numerical simulations are given to illustrate the feasibility and effectiveness of our main results.

1. Introduction

In recent years, the Cohen-Grossberg neural networks [1] have been extensively studied because of their immense potentials of application perspective in different areas such as pattern recognition, optimization, and signal and image processing. It is well known that studies on Cohen-Grossberg neural networks not only involve a discussion of stability properties, but also involve many dynamic behaviors such as periodic oscillatory behavior and almost periodic oscillatory properties. In applications, if the various constituent components of the temporally nonuniform environment is with incommensurable (nonintegral multiples) periods, then one has to consider the environment to be almost periodic since there is no a priori reason to expect the existence of periodic solutions. Therefore, if we consider the effects of the environmental factors, almost periodicity is sometimes more realistic and more general than periodicity. In addition, the (almost) periodic oscillatory behavior is of great interest; it has been found that applications in learning theory [2], which is motivated by the fact that learning usually requires repetition. And in pattern recognition or associative memory, the existence of stable almost periodic encoded patterns

(including equilibria or periodic orbits) is an important feature [3, 4]. Hence, it is of prime importance to study the existence and stability of (almost) periodic solutions for Cohen-Grossberg neural networks with delays. We refer the reader to [5–18] and references therein.

On the other hand, owing to the complicated dynamic properties of the neural cells in the real world, the existing neural network models in many cases can not characterize the properties of a neural reaction process precisely. It is natural and important that systems will contain some information about the derivative of the past state to further describe and model the dynamics for such complex neural reactions. This new type of neural networks is called neutral neural networks or neural networks of neutral type. The motivation for us to study neural networks of neutral type comes from three aspects. First, based on biochemistry experiments, neural information may transfer across chemical reactivity, which results in a neutral-type process. Second, in view of electronics, it has been shown that neutral phenomena exist in large-scale integrated (LSI) circuits. Last, the key point is that cerebra can be considered as a super LSI circuit with chemical reactivity, which reasonably implies that the neutral dynamic behaviors should be included in neural dynamic

systems [19]. To the best of our knowledge, the problem of global exponential stability of almost periodic solution for neutral-type Cohen-Grossberg neural networks (see [20–24]) has not been fully investigated in the literature.

In this papers we consider the following neutral-type Cohen-Grossberg neural networks with time delays:

$$\begin{aligned} \dot{u}_i(t) = & -a_i(u_i(t)) \\ & \times \left[b_i(t, u_i(t)) - \sum_{j=1}^n c_{ij}(t) f_j(u_j(t)) \right. \\ & - \sum_{j=1}^n d_{ij}(t) g_j(u_j(t - \mu_j(t))) \\ & \left. - \sum_{j=1}^n v_{ij}(t) k_j(u_j(t - \nu_j(t))) - I_i(t) \right], \end{aligned} \quad (1)$$

where x_i denotes the state variable associated to the i th neuron, a_i represents an amplification function, b_i is an appropriately behaved function; f_j , g_j , and k_j denote the normal and the delayed activation function; c_{ij} , d_{ij} , and v_{ij} denote the connection strengths of the j th neuron on the i th neuron at time t , respectively; $\mu_j \geq 0$ and $\nu_j \geq 0$ are the delays caused during the switching and transmission processes; I_i denotes external input to the i th neuron at time t , $i, j = 1, 2, \dots, n$.

Let $C(\mathbb{X}, \mathbb{Y})$ and $C^1(\mathbb{X}, \mathbb{Y})$ be the space of continuous functions and continuously differential functions which map $\mathbb{X}$ into $\mathbb{Y}$, respectively. In particular, $C(\mathbb{X}) := C(\mathbb{X}, \mathbb{X})$, $C^1(\mathbb{X}) := C^1(\mathbb{X}, \mathbb{X})$. For any bounded function, $f \in C(\mathbb{R})$, $f^+ = \sup_{s \in \mathbb{R}} |f(s)|$, $f^- = \inf_{s \in \mathbb{R}} |f(s)|$.

Throughout this paper, we set

$$x = \{x_i\} = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n. \quad (2)$$

For $x = \{x_i\} \in \mathbb{R}^n$, we define the norm $\|x\| = \max_{1 \leq i \leq n} |x_i|$.

We list some assumptions which will be used in this paper.

(H₁) c_{ij} , d_{ij} , v_{ij} , μ_{ij} , ν_{ij} , and I_{ij} are continuous almost periodic functions, $i, j = 1, 2, \dots, n$.

(H₂) $a_i \in C(\mathbb{R})$ and there exist positive constants $\underline{a}_i$ and $\bar{a}_i$ such that $0 < \underline{a}_i \leq a_i(t) \leq \bar{a}_i$, for all $t \in \mathbb{R}$, $i = 1, 2, \dots, n$.

(H₃) $b_i \in C(\mathbb{R}^2, \mathbb{R})$ is almost periodic about the first argument, and there are positive constants p_i and q_i such that $p_i \leq (\partial b_i(t, u)/\partial u) \leq q_i$ and $b_i(t, 0) = 0$, for all $(t, u) \in (\mathbb{R}, \mathbb{R})$, $i = 1, 2, \dots, n$.

(H₄) There exist positive constants L_j^f , L_j^g , L_j^k , and L_j^a such that

$$\begin{aligned} |f_j(u) - f_j(v)| & \leq L_j^f |u - v|, \\ |g_j(u) - g_j(v)| & \leq L_j^g |u - v|, \\ |k_j(u) - k_j(v)| & \leq L_j^k |u - v|, \\ |a_j(u) - a_j(v)| & \leq L_j^a |u - v|, \end{aligned} \quad (3)$$

for all $u, v \in \mathbb{R}$, $j = 1, 2, \dots, n$.

The organization of this paper is as follows. In Section 2, we give some basic definitions and necessary lemmas which will be used in later sections. In Sections 3 and 4, by using a fixed point theorem and constructing suitable Lyapunov functional, we obtain some sufficient conditions ensuring the existence, uniqueness, and global exponential stability of almost periodic solution of system (1). Finally, an example and numerical simulations are given to illustrate that our results are feasible.

2. Preliminaries

First of all, we shall transform system (1) and state some notations, which will be used in later sections.

From (H₂), the antiderivative of $1/a_i(u_i)$ exists. We choose an antiderivative $h_i(u_i)$ of $1/a_i(u_i)$ that satisfies $h_i(0) = 0$. Obviously, $(d/du_i)h_i(u_i) = 1/a_i(u_i)$. By $a_i(u_i) > 0$, we obtain that $h_i(u_i)$ is strictly monotone increasing about u_i . In view of derivative theorem for inverse function, the inverse function $h_i^{-1}(u_i)$ of $h_i(u_i)$ is differential and $(d/du_i)h_i^{-1}(u_i) = a_i(u_i)$. By (H₃), composition function $b_i(t, h_i^{-1}(z))$ is differentiable. Denote $x_i = h_i(u_i)$. It is easy to see that $\dot{x}_i(t) = \dot{u}_i(t)/a_i(u_i)$, $u_i(t) = h_i^{-1}(x_i(t))$, and $\dot{u}_i(t) = a_i(x_i(t))\dot{x}_i(t)$. Substituting these equalities into system (1), we get

$$\begin{aligned} \dot{x}_i(t) = & -b_i(t, h_i^{-1}(x_i(t))) \\ & + \sum_{j=1}^n c_{ij}(t) f_j(h_j^{-1}(x_j(t))) \\ & + \sum_{j=1}^n d_{ij}(t) g_j(h_j^{-1}(x_j(t - \mu_j(t)))) \\ & + \sum_{j=1}^n v_{ij}(t) k_j(a_j(x_j(t - \nu_j(t))) \\ & \quad \times \dot{x}_j(t - \nu_j(t))) + I_i(t), \end{aligned} \quad (4)$$

which can be rewritten as

$$\begin{aligned} \dot{x}_i(t) = & -w_i(t, x_i(t)) x_i(t) \\ & + \sum_{j=1}^n c_{ij}(t) f_j(h_j^{-1}(x_j(t))) \\ & + \sum_{j=1}^n d_{ij}(t) g_j(h_j^{-1}(x_j(t - \mu_j(t)))) \\ & + \sum_{j=1}^n v_{ij}(t) k_j(a_j(x_j(t - \nu_j(t)))) \\ & \times \dot{x}_j(t - \nu_j(t)) + I_i(t), \end{aligned} \quad (5)$$

where $w_i(t, x_i(t)) = (\partial b_i(t, h_i^{-1}(z))/\partial z)|_{z=\xi_i}$, ξ_i is between 0 and x_i , $i = 1, 2, \dots, n$.

By (H_2) , (H_3) , and the definition of h_i^{-1} , we obtain that $b_i(t, h_i^{-1}(x_i))$ is strictly monotone increasing about x_i . Hence, $w_i(t, x_i)$ is unique for any x_i and continuous about x_i ; moreover,

$$\alpha_i = \underline{a}_i p_i \leq w_i(t, x_i) \leq \bar{a}_i q_i = \beta_i, \quad i = 1, 2, \dots, n. \quad (6)$$

From the definition of h_i^{-1} , using Lagrange mean-value theorem, one gets

$$|h_i^{-1}(u) - h_i^{-1}(v)| \leq \bar{a}_i |u - v|, \quad \forall u, v \in \mathbb{R}, \quad i = 1, 2, \dots, n. \quad (7)$$

The existence and global exponential stability of almost periodic solution for system (1) are equivalent to the existence and global exponential stability of almost periodic solution for system (5), respectively. So, we investigate the existence and global exponential stability of almost periodic solution for system (5).

Let $\tau := \max_{1 \leq i \leq n} \{\mu_i^+, \nu_i^+\}$. The initial conditions associated with system (5) are of the form

$$x_i(s) = \varphi_i(s), \quad \dot{x}_i(s) = \dot{\varphi}_i(s), \quad (8)$$

$$\forall s \in [-\tau, 0], \quad \varphi_i \in C^1([-\tau, 0], \mathbb{R}), \quad i = 1, 2, \dots, n.$$

Now, let us state the following definitions and lemmas, which will be useful in proving our main result.

Definition 1 (see [25]). $x \in C(\mathbb{R}, \mathbb{R}^n)$ is called almost periodic, if for any $\epsilon > 0$, it is possible to find a real number $l = l(\epsilon) > 0$, for any interval with length $l(\epsilon)$, there exists a number $\tau = \tau(\epsilon)$ in this interval such that $\|x(t+\tau) - x(t)\| < \epsilon$, for all $t \in \mathbb{R}$. The collection of those functions is denoted by $AP(\mathbb{R}, \mathbb{R}^n)$.

Definition 2 (see [25]). Let $y \in C(\mathbb{R}, \mathbb{R}^n)$ and $P(t)$ be an $n \times n$ continuous matrix defined on $\mathbb{R}$. The linear system

$$\dot{y}(t) = P(t) y(t) \quad (9)$$

is said to be an exponential dichotomy on $\mathbb{R}$ if there exist constants $k, \lambda > 0$, projection S , and the fundamental matrix $Y(t)$ satisfying

$$\begin{aligned} \|Y(t) S Y^{-1}(s)\| &\leq k e^{-\lambda(t-s)}, \quad \forall t \geq s, \\ \|Y(t) (I - S) Y^{-1}(s)\| &\leq k e^{-\lambda(s-t)}, \quad \forall t \leq s. \end{aligned} \quad (10)$$

Lemma 3 (see [25]). If the linear system $\dot{y}(t) = P(t)y(t)$ has an exponential dichotomy, then almost periodic system

$$\dot{y}(t) = P(t) y(t) + g(t) \quad (11)$$

has a unique almost periodic solution $y(t)$ which can be expressed as follows:

$$\begin{aligned} y(t) = & \int_{-\infty}^t Y(t) S Y^{-1}(s) g(s) ds \\ & - \int_t^{\infty} Y(t) (I - S) Y^{-1}(s) g(s) ds. \end{aligned} \quad (12)$$

Lemma 4 (see [25]). Let $a_i(t)$ be an almost periodic function and

$$M[a_i] = \lim_{T \rightarrow \infty} \frac{1}{T} \int_t^{t+T} a_i(s) ds > 0, \quad i = 1, 2, \dots, n. \quad (13)$$

Then the linear system $\dot{y}(t) = -A(t)y(t)$ admits an exponential dichotomy, where $A(t) = \text{diag}\{a_1(t), a_2(t), \dots, a_n(t)\}$.

Definition 5. The almost periodic solution $x = \{x_i\}$ of system (5) with the initial value $\varphi = \{\varphi_i\}$ is said to be globally exponentially stable if there exist constants $\omega > 0$ and $M \geq 1$, for any solution $y = \{y_i\}$ of system (5) with initial value $\phi = \{\phi_i\}$ such that

$$\|x - y\| \leq M \|\varphi - \phi\|_{\tau} e^{-\omega t}, \quad \forall t > 0, \quad (14)$$

where

$$\|\varphi - \phi\|_{\tau} := \sum_{i=1}^n \sup_{s \in [-\tau, 0]} \{|\varphi_i(s) - \phi_i(s)| + |\dot{\varphi}_i(s) - \dot{\phi}_i(s)|\}. \quad (15)$$

Lemma 6 (see [26, 27]). Let $(\mathbb{X}, \|\cdot\|_{\mathbb{X}})$ be a Banach space. Assume that $0 \in \Omega$ is an open bounded subset of $\mathbb{X}$ and $T : \bar{\Omega} \rightarrow \mathbb{X}$ is completely continuous satisfying

$$\|Tx\|_{\mathbb{X}} \leq \|x\|_{\mathbb{X}}, \quad \forall x \in \partial\Omega, \quad (16)$$

then T has a fixed point in $\bar{\Omega}$.

Let

$$\mathbb{X} = \{x \in AP(\mathbb{R}, \mathbb{R}^n) \cap C^1(\mathbb{R}, \mathbb{R}^n) : \dot{x} \in AP(\mathbb{R}, \mathbb{R}^n)\} \quad (17)$$

with the norm $\|x\|_{\mathbb{X}} = \max\{\sup_{s \in \mathbb{R}} \|x(s)\|, \sup_{s \in \mathbb{R}} \|\dot{x}(s)\|\}$. Then $\mathbb{X}$ is a Banach space with the norm $\|\cdot\|_{\mathbb{X}}$.

For given positive constant γ , define an open bounded subset Ω_{γ} in $\mathbb{X}$ by

$$\Omega = \{x \in \mathbb{X} : \|x\|_{\mathbb{X}} < \gamma\}. \quad (18)$$

By Lemmas 3 and 4, system (5) has a unique almost periodic solution

$$x^\varphi = \{x_i^{\varphi_i}\} = (x_1^{\varphi_1}, x_2^{\varphi_2}, \dots, x_n^{\varphi_n})^T \quad (19)$$

which can be expressed as follows:

$$x_i^{\varphi_i} = \int_{-\infty}^t e^{-\int_s^t w_i(r, \varphi_i(r)) dr} F_i(s, \varphi(s)) ds, \quad (20)$$

where

$$\begin{aligned} F_i(t, \varphi(t)) = & \sum_{j=1}^n c_{ij}(t) f_j(h_j^{-1}(\varphi_j(t))) \\ & + \sum_{j=1}^n d_{ij}(t) g_j(h_j^{-1}(\varphi_j(t - \mu_j(t)))) \\ & + \sum_{j=1}^n v_{ij}(t) k_j(a_j(\varphi_j(t - \nu_j(t))) \\ & \quad \times \dot{\varphi}_j(t - \nu_j(t))) + I_i(t), \end{aligned} \quad (21)$$

where $i = 1, 2, \dots, n$, for all $t \in \mathbb{R}$.

Let the map $T : \mathbb{X} \rightarrow \mathbb{X}$ be defined by

$$T(\varphi) = \{T_i(\varphi)\} = \{x_i^{\varphi_i}\}, \quad \forall \varphi = \{\varphi_i\} \in \mathbb{X}. \quad (22)$$

Lemma 7. $T : \overline{\Omega}_\gamma \rightarrow \mathbb{X}$ is completely continuous.

Proof. Obviously, $T : \overline{\Omega} \rightarrow \mathbb{X}$ is continuous. Next, we show that T maps bounded set into itself. Assume c is a positive constant and $\varphi \in \overline{\Omega}_c = \{x \in \Omega : \|x\|_\mathbb{X} \leq c\}$. By the almost periodicity of system (1), there exists a constant $K > 0$ such that

$$\max_{1 \leq i \leq n} \sup_{s \in \mathbb{R}, \varphi \in \overline{\Omega}_c} \{F_i(s, 0), F_i(s, \varphi)\} < K. \quad (23)$$

Assume that $\varphi \in \overline{\Omega}_\gamma$, then

$$\begin{aligned} & |F_i(t, \varphi(t)) - F_i(t, 0)| \\ & \leq \left| \sum_{j=1}^n c_{ij}(t) f_j(h_j^{-1}(\varphi_j(t))) - \sum_{j=1}^n c_{ij}(t) f_j(h_j^{-1}(0)) \right| \\ & \quad + \left| \sum_{j=1}^n d_{ij}(t) g_j(h_j^{-1}(\varphi_j(t - \mu_j(t)))) \right. \\ & \quad \left. - \sum_{j=1}^n d_{ij}(t) g_j(h_j^{-1}(0)) \right| \end{aligned}$$

$$\begin{aligned} & + \left| \sum_{j=1}^n v_{ij}(t) k_j(a_j(\varphi_j(t - \nu_j(t)))) \dot{\varphi}_j(t - \nu_j(t)) \right. \\ & \quad \left. - \sum_{j=1}^n v_{ij}(t) k_j(0) \right| \\ & \leq \sum_{j=1}^n [c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j + v_{ij}^+ M_j^k (\bar{a}_j + L_j^a \gamma)] \|\varphi\|_\mathbb{X}, \end{aligned} \quad (24)$$

which implies that

$$\begin{aligned} |T_i(\varphi)| &= \int_{-\infty}^t e^{-\int_s^t w_i(r, \varphi_i(r)) dr} |F_i(s, \varphi(s)) - F_i(s, 0)| ds \\ & \quad + \int_{-\infty}^t e^{-\int_s^t w_i(r, \varphi_i(r)) dr} |F_i(s, 0)| ds \\ & \leq \frac{1}{\alpha_i} \sum_{j=1}^n [c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j + v_{ij}^+ M_j^k (\bar{a}_j + L_j^a \gamma)] \\ & \quad \times \|\varphi\|_\mathbb{X} + \frac{K}{\alpha_i} \\ & \leq \frac{1}{\alpha_i} \sum_{j=1}^n [c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j + v_{ij}^+ M_j^k (\bar{a}_j + L_j^a \gamma)] \\ & \quad \times c + \frac{K}{\alpha_i}, \\ |\dot{T}_i(\varphi)| &= |-w_i(t, \varphi_i(t)) T_i(\varphi) + F_i(t, \varphi)| \\ & \leq \beta_i \left\{ \frac{1}{\alpha_i} \sum_{j=1}^n [c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j \right. \\ & \quad \left. + v_{ij}^+ M_j^k (\bar{a}_j + L_j^a \gamma)] c + \frac{K}{\alpha_i} \right\} + K, \end{aligned} \quad (25)$$

where $i = 1, 2, \dots, n$. Therefore, $T\overline{\Omega}_c$ is a family of uniformly bounded and equicontinuous subsets. Using Arzela-Ascoli theorem, we obtain that $T\overline{\Omega}_c$ is relatively compact. Hence, T is completely continuous. The proof of this lemma is complete. $\square$

3. Existence of Almost Periodic Solution

In this section, we study the existence of almost periodic solutions of system (1).

Theorem 8. Assume that (H_1) – (H_4) hold, suppose further that

(H_5) $\theta = \max\{\theta_1, \theta_2\} < 1$, where

$$\theta_1 = \max_{1 \leq i \leq n} \left\{ \frac{1}{\alpha_i} \sum_{j=1}^n (c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j + v_{ij}^+ L_j^k \bar{a}_j) \right\},$$

$$\theta_2 = \max_{1 \leq i \leq n} \left\{ \left[1 + \frac{\beta_i}{\alpha_i} \right] \sum_{j=1}^n (c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j + v_{ij}^+ L_j^k \bar{a}_j) \right\}, \quad (26)$$

then system (1) admits at least one almost periodic solution.

Proof. Define

$$\Omega_{r_0} = \left\{ x \in \mathbb{X} : \|x\|_{\mathbb{X}} < r_0 = \frac{\delta}{1-\theta} \right\}, \quad (27)$$

where $\delta = \max\{\delta_1, \delta_2\}$,

$$\delta_1 = \max_{1 \leq i \leq n} \left\{ \frac{1}{\alpha_i} \sum_{j=1}^n (c_{ij}^+ f_j(0) + d_{ij}^+ g_j(0) + v_{ij}^+ k_j(0)) + \frac{I_i^+}{\alpha_i} \right\},$$

$$\delta_2 = \max_{1 \leq i \leq n} \left\{ \left[1 + \frac{\beta_i}{\alpha_i} \right] \left[\sum_{j=1}^n (c_{ij}^+ f_j(0) + d_{ij}^+ g_j(0) + v_{ij}^+ k_j(0)) + I_i^+ \right] \right\}. \quad (28)$$

Consider the following nonlinear operator:

$$T(\phi) = \{T_i(\phi)\} = \{x_i^{\phi_i}\}, \quad \forall \phi = \{\phi_i\} \in \bar{\Omega}_{r_0}. \quad (29)$$

For $\forall \phi = \{\phi_i\} \in \partial\Omega_{r_0}$, we have $\|\phi\|_{\mathbb{X}} = r_0$. So, we obtain

$$\begin{aligned} \|T(\phi(t))\| &= \max_{1 \leq i \leq n} |T_i(\phi(t))| \\ &= \max_{1 \leq i \leq n} \left| \int_{-\infty}^t e^{-\int_s^t w_i(r, \phi_i(r)) dr} \right. \\ &\quad \times \left[\sum_{j=1}^n c_{ij}(s) f_j(h_j^{-1}(\phi_j(s))) \right. \\ &\quad + \sum_{j=1}^n d_{ij}(s) g_j(h_j^{-1}(\phi_j(s - \mu_j(s)))) \\ &\quad + \sum_{j=1}^n v_{ij}(s) k_j(a_j(\phi_j(s - \nu_j(s))) \\ &\quad \left. \left. \times \phi_j(s - \nu_j(s))) + I_i(s) \right] ds \right| \end{aligned}$$

$$\begin{aligned} &\leq \max_{1 \leq i \leq n} \left\{ \int_{-\infty}^t e^{-\int_s^t w_i(r, \phi_i(r)) dr} \right. \\ &\quad \times \left[\sum_{j=1}^n (c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j + v_{ij}^+ L_j^k \bar{a}_j) \|\phi\|_{\mathbb{X}} \right. \\ &\quad + \sum_{j=1}^n (c_{ij}^+ f_j(0) + d_{ij}^+ g_j(0) \\ &\quad \left. \left. + v_{ij}^+ k_j(0)) + I_i^+ \right] ds \right\} \\ &\leq \max_{1 \leq i \leq n} \left\{ \frac{1}{\alpha_i} \sum_{j=1}^n (c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j + v_{ij}^+ L_j^k \bar{a}_j) \right\} r_0 \\ &\quad + \max_{1 \leq i \leq n} \left\{ \frac{1}{\alpha_i} \sum_{j=1}^n (c_{ij}^+ f_j(0) + d_{ij}^+ g_j(0) \right. \\ &\quad \left. + v_{ij}^+ k_j(0)) + \frac{I_i^+}{\alpha_i} \right\} \\ &:= \theta_1 r_0 + \delta_1, \\ \|\dot{T}(\phi(t))\| &= \max_{1 \leq i \leq n} |\dot{T}_i(\phi(t))| \\ &= \max_{1 \leq i \leq n} |-w_i(t, \phi) T_i(\phi) + F_i(t, \phi(t))| \\ &\leq \max_{1 \leq i \leq n} \left\{ \left[1 + \frac{\beta_i}{\alpha_i} \right] \right. \\ &\quad \times \sum_{j=1}^n \left(c_{ij}^+ L_j^f \bar{a}_j + d_{ij}^+ L_j^g \bar{a}_j \right. \\ &\quad \left. \left. + v_{ij}^+ L_j^k \bar{a}_j \right) \right\} r_0 \\ &\quad + \max_{1 \leq i \leq n} \left\{ \left[1 + \frac{\beta_i}{\alpha_i} \right] \right. \\ &\quad \times \left[\sum_{j=1}^n (c_{ij}^+ f_j(0) + d_{ij}^+ g_j(0) \right. \\ &\quad \left. \left. + v_{ij}^+ k_j(0)) + I_i^+ \right] \right\} \\ &:= \theta_2 r_0 + \delta_2, \end{aligned} \quad (30)$$

which yield

$$\begin{aligned}
 \|T(\varphi)\|_{\infty} &= \max \left\{ \sup_{s \in \mathbb{R}} \|T(\varphi(s))\|, \sup_{s \in \mathbb{R}} \|\dot{T}(\varphi(s))\| \right\} \\
 &\leq \max \{\theta_1, \theta_2\} r_0 + \max \{\delta_1, \delta_2\} \\
 &= \theta r_0 + \delta \\
 &\leq \frac{\delta}{1 - \theta} \\
 &= r_0 \\
 &= \|\varphi\|_{\infty}, \quad \forall \varphi \in \partial \Omega_{r_0}.
 \end{aligned} \tag{31}$$

By Lemma 6, there exists at least one fixed point $\varphi_0 \in \bar{\Omega}$ satisfying $T(\varphi_0) = \varphi_0$, which implies that system (5) has at least one almost periodic solution φ_0 with $\|\varphi_0\|_{\infty} \leq r_0$. That is, system (1) has at least one almost periodic solution. This completes the proof. $\square$

4. Global Exponential Stability of Almost Periodic Solution

Theorem 9. Assume that (H_1) – (H_5) hold, suppose further that

(H_6) there exists a positive constant $\lambda < 1$ such that

$$\begin{aligned}
 \Theta_1 &= \max_{1 \leq i \leq n} \sup_{s \in \mathbb{R}} \left\{ -\alpha_i + \lambda \beta_i + (1 + \lambda) \right. \\
 &\quad \times \sum_{j=1}^n |c_{ij}(s)| L_j^f \bar{a}_j \\
 &\quad + \sum_{j=1}^n (1 + \lambda) |d_{ij}(s)| L_j^g \bar{a}_j \\
 &\quad \left. + \sum_{j=1}^n (1 + \lambda) |v_{ij}(s)| L_j^k L_j^a r_0 \right\} < 0, \\
 \Theta_2 &= \max_{1 \leq i, j \leq n} \sup_{s \in \mathbb{R}} \left\{ \sum_{j=1}^n \lambda |v_{ij}(s)| L_j^k \bar{a}_j \right. \\
 &\quad \left. + \sum_{i=1}^n |v_{ij}(s)| L_j^k \bar{a}_j - \lambda (1 - \dot{v}_j^+) \right\} < 0,
 \end{aligned} \tag{32}$$

where r_0 is defined as that in Theorem 8.

Then system (1) has a unique almost periodic solution, which is globally exponentially stable.

Proof. It follows from Theorem 8 that system (5) has at least one almost periodic solution $y = \{y_i\}$ with initial value $\phi = \{\phi_i\}$. Next we show that the almost periodic solution y is globally exponentially stable.

Make a transformation for system (5): $x_i = z_i - y_i$, $i = 1, 2, \dots, n$, where $z = \{z_i\}$ is arbitrary solution of system (5) with initial value $\psi = \{\psi_i\}$.

By (H_6) , there exists a small enough positive constant ω such that

$$\begin{aligned}
 \Theta_{1\omega} &= \max_{1 \leq i \leq n} \sup_{s \in \mathbb{R}} \left\{ \omega - \alpha_i + \lambda \beta_i + (1 + \lambda) \right. \\
 &\quad \times \sum_{j=1}^n |c_{ij}(s)| L_j^f \bar{a}_j \\
 &\quad + \sum_{j=1}^n (1 + \lambda) |d_{ij}(s)| L_j^g \bar{a}_j e^{\omega \mu_j^+} \\
 &\quad \left. + \sum_{j=1}^n (1 + \lambda) |v_{ij}(s)| L_j^k L_j^a r_0 e^{\omega \mu_j^+} \right\} < 0, \\
 \Theta_{2\omega} &= \max_{1 \leq i, j \leq n} \sup_{s \in \mathbb{R}} \left\{ \sum_{j=1}^n \lambda |v_{ij}(s)| L_j^k \bar{a}_j \right. \\
 &\quad \left. + \sum_{i=1}^n |v_{ij}(s)| L_j^k \bar{a}_j - \lambda e^{-\omega \nu_j^+} (1 - \dot{v}_j^+) \right\} < 0.
 \end{aligned} \tag{33}$$

Define

$$V(t) = V_1(t) + V_2(t), \tag{34}$$

where

$$V_1(t) = \sum_{i=1}^n e^{\omega t} |x_i(t)|, \tag{35}$$

$$V_2(t) = \lambda \sum_{i=1}^n \int_{t-\nu_i(t)}^t e^{\omega s} |\dot{x}_i(s)| ds.$$

In view of system (5), we have

$$\begin{aligned}
 D^+ V_1(t) &\leq \omega \sum_{i=1}^n e^{\omega t} |x_i(t)| \\
 &\quad + \sum_{i=1}^n e^{\omega t} \operatorname{sgn}(x_i(t)) \dot{x}_i(t) \\
 &\leq \omega \sum_{i=1}^n e^{\omega t} |x_i(t)| \\
 &\quad + \sum_{i=1}^n e^{\omega t} \left\{ -[b_i(t, h_i^{-1}(z_i(t))) \right. \\
 &\quad \left. - b_i(t, h_i^{-1}(y_i(t)))] \right. \\
 &\quad \left. \times \operatorname{sgn}(z_i(t) - y_i(t)) \right\}
 \end{aligned}$$

$$\begin{aligned}
& + \left| \sum_{j=1}^n c_{ij}(t) f_j(h_j^{-1}(z_j(t))) \right. \\
& \quad \left. - \sum_{j=1}^n c_{ij}(t) f_j(h_j^{-1}(y_j(t))) \right| \\
& + \left| \sum_{j=1}^n d_{ij}(t) g_j(h_j^{-1}(z_j(t - \mu_j(t)))) \right. \\
& \quad \left. - \sum_{j=1}^n d_{ij}(t) g_j(h_j^{-1}(y_j(t - \mu_j(t)))) \right| \\
& + \left| \sum_{j=1}^n v_{ij}(t) k_j \right. \\
& \quad \times (a_j(z_j(t - \nu_j(t))) \dot{z}_j(t - \nu_j(t))) \\
& \quad - \sum_{j=1}^n v_{ij}(t) k_j \\
& \quad \times (a_j(y_j(t - \nu_j(t))) \\
& \quad \times \dot{y}_j(t - \nu_j(t))) \left. \right| \Bigg\} \\
& \leq \omega \sum_{i=1}^n e^{\omega t} |x_i(t)| \\
& + \sum_{i=1}^n e^{\omega t} \left\{ -\alpha_i |x_i(t)| \right. \\
& + \sum_{j=1}^n |c_{ij}(t)| L_j^f \bar{a}_j |x_j(t)| \\
& + \sum_{j=1}^n |d_{ij}(t)| L_j^g \bar{a}_j |x_j(t - \mu_j(t))| \\
& + \sum_{j=1}^n |v_{ij}(t)| L_j^k L_j^a r_0 |x_j(t - \nu_j(t))| \\
& + \sum_{j=1}^n |v_{ij}(t)| L_j^k \bar{a}_j |\dot{x}_j(t - \nu_j(t))| \left. \right\}. \tag{36}
\end{aligned}$$

Further, it follows from (36) that

$$\begin{aligned}
D^+ V_2(t) & \leq \lambda \sum_{i=1}^n e^{\omega t} |\dot{x}_i(t)| - \lambda \\
& \times \sum_{i=1}^n e^{\omega(t-\nu_i(t))} (1 - \dot{\nu}_i(t)) |\dot{x}_i(t - \nu_i(t))|
\end{aligned}$$

$$\begin{aligned}
& \leq \lambda \sum_{i=1}^n e^{\omega t} \left[\beta_i |x_i(t)| \right. \\
& + \sum_{j=1}^n |c_{ij}(t)| L_j^f \bar{a}_j |x_j(t)| \\
& + \sum_{j=1}^n |d_{ij}(t)| L_j^g \bar{a}_j |x_j(t - \mu_j(t))| \\
& + \sum_{j=1}^n |v_{ij}(t)| L_j^k L_j^a r_0 |x_j(t - \nu_j(t))| \left. \right] \\
& - \lambda \sum_{j=1}^n e^{\omega t} \left[e^{-\omega \nu_j^u} (1 - \dot{\nu}_j^+) - \sum_{j=1}^n |v_{ij}(t)| L_j^k \bar{a}_j \right] \\
& \times |\dot{x}_j(t - \nu_j(t))|. \tag{37}
\end{aligned}$$

By (36) and (37), we have

$$\begin{aligned}
D^+ V(t) & \leq \sum_{i=1}^n \left[\omega - \alpha_i + \lambda \beta_i + (1 + \lambda) \right. \\
& \quad \times \sum_{j=1}^n |c_{ij}(t)| L_j^f \bar{a}_j \left. \right] e^{\omega t} |x_i(t)| \\
& + \sum_{i=1}^n \sum_{j=1}^n (1 + \lambda) |d_{ij}(t)| L_j^g \bar{a}_j e^{\omega \mu_j^+} e^{\omega(t-\mu_j(t))} \\
& \times |x_j(t - \mu_j(t))| \\
& + \sum_{i=1}^n \sum_{j=1}^n (1 + \lambda) |v_{ij}(t)| L_j^k L_j^a r_0 e^{\omega \nu_j^+} e^{\omega(t-\nu_j(t))} \\
& \times |x_j(t - \nu_j(t))| \\
& - \sum_{j=1}^n e^{\omega t} \left[\lambda e^{-\omega \nu_j^u} (1 - \dot{\nu}_j^+) \right. \\
& \quad - \sum_{j=1}^n \lambda |v_{ij}(t)| L_j^k \bar{a}_j \\
& \quad \left. - \sum_{i=1}^n |v_{ij}(t)| L_j^k \bar{a}_j \right] |\dot{x}_j(t - \nu_j(t))|. \tag{38}
\end{aligned}$$

For $t \leq 0$, note that

$$\begin{aligned}
 \sum_{i=1}^n e^{\omega t} |x_i(t)| &= V_1(t) \\
 &\leq V(t) \\
 &= \sum_{i=1}^n e^{\omega t} |x_i(t)| \\
 &\quad + \lambda \sum_{i=1}^n \int_{t-\nu_i(t)}^t e^{\omega s} |\dot{x}_i(s)| ds \\
 &\leq (1 + e^{\omega \tau}) \|\psi - \phi\|_{\tau} \\
 &:= M \|\psi - \phi\|_{\tau},
 \end{aligned} \tag{39}$$

where $M = 1 + e^{\omega \tau}$. Next, we claim that

$$V(t) \leq M \|\psi - \phi\|_{\tau}, \quad \forall t \in [0, +\infty). \tag{40}$$

Contrarily, there is $t_0 \in [0, +\infty)$ such that

$$\begin{aligned}
 V(t_0) &= M \|\psi - \phi\|_{\tau}, \quad D^+ V(t_0) > 0, \\
 V(t) &\leq M \|\psi - \phi\|_{\tau}, \quad \forall t \in [-\tau, t_0].
 \end{aligned} \tag{41}$$

By (38), that we have from (33)

$$\begin{aligned}
 0 &< D^+ V(t_0) \\
 &\leq \sum_{i=1}^n \left[\omega - \alpha_i + \lambda \beta_i + (1 + \lambda) \right. \\
 &\quad \times \sum_{j=1}^n |c_{ij}(t_0)| L_j^f \bar{a}_j \\
 &\quad + \sum_{j=1}^n (1 + \lambda) |d_{ij}(t_0)| L_j^g \bar{a}_j e^{\omega \mu_j^+} \\
 &\quad \left. + \sum_{j=1}^n (1 + \lambda) |\nu_{ij}(t_0)| L_j^k L_j^a r_0 e^{\omega \nu_j^+} \right] M \|\psi - \phi\|_{\tau} \\
 &\quad - \sum_{j=1}^n e^{\omega t_0} \left[\lambda e^{-\omega \nu_j^+} (1 - \nu_j^+) \right. \\
 &\quad - \sum_{j=1}^n \lambda |\nu_{ij}(t_0)| L_j^k \bar{a}_j \\
 &\quad \left. - \sum_{i=1}^n |\nu_{ij}(t_0)| L_j^k \bar{a}_j \right] |\dot{x}_j(t_0 - \nu_j(t_0))| \\
 &< 0.
 \end{aligned} \tag{42}$$

This is a contradiction. So, our claim is valid. Therefore,

$$\sum_{i=1}^n |x_i(t)| \leq M \|\psi - \phi\|_{\tau} e^{-\omega t}, \quad \forall t > 0. \tag{43}$$

Thus, the almost periodic solution of system (5) is globally exponentially stable. That is, the corresponding almost periodic solution of system (1) is globally exponentially stable.

Without loss of generality, we assume that $y = \{x_i\}$ and $z = \{z_i\}$ are two almost periodic solutions of system (5). Then $x = \{x_i\} = z - y = \{z_i - y_i\} \in AP(\mathbb{R}, \mathbb{R}^n)$. Similar to the above argument, (43) is valid. Hence,

$$\lim_{t \rightarrow \infty} \sum_{i=1}^n |x_i(t)| = 0, \tag{44}$$

which implies from $x \in AP(\mathbb{R}, \mathbb{R}^n)$ that $x \equiv 0$. That is, $y = z$. So the almost periodic solution of system (5) is unique. This completes the proof. $\square$

5. An Example and Numerical Simulations

Example 1. Consider the following neutral-type Cohen-Grossberg neural networks:

$$\begin{aligned}
 \dot{u}_i(t) &= -a_i(u_i(t)) \left[u_i(t) - \sum_{j=1}^2 c_{ij}(t) f(u_j(t)) \right. \\
 &\quad - \sum_{j=1}^2 d_{ij}(t) g(u_j(t - 0.01 \sin^2(\sqrt{2}t))) \\
 &\quad \left. - \sum_{j=1}^2 0.1k(u_j(t-1)) - I_i(t) \right],
 \end{aligned} \tag{45}$$

where $i = 1, 2$, $f(s) = g(s) = k(s) = 0.1s$,

$$\begin{aligned}
 \begin{pmatrix} a_1(s) \\ a_2(s) \end{pmatrix} &= \begin{pmatrix} 0.11 + 0.01 \sin(\sqrt{2}s) \\ 0.11 + 0.01 \cos(\sqrt{13}s) \end{pmatrix}, \\
 \begin{pmatrix} I_1(s) \\ I_2(s) \end{pmatrix} &= \begin{pmatrix} 0.01 \sin(\sqrt{7}s) \\ 0.01 \sin(s) \end{pmatrix}, \\
 \begin{pmatrix} c_{11}(s) & c_{12}(s) \\ c_{21}(s) & c_{22}(s) \end{pmatrix} &= \begin{pmatrix} d_{11}(s) & d_{12}(s) \\ d_{21}(s) & d_{22}(s) \end{pmatrix} \\
 &= \begin{pmatrix} 0.1 \sin(\sqrt{2}s) & 0 \\ 0.1 \cos(\sqrt{3}s) & 0.1 \end{pmatrix}, \quad \forall s \in \mathbb{R}.
 \end{aligned} \tag{46}$$

Then system (45) has a unique almost periodic solution, which is globally exponentially stable.

Proof. Corresponding to system (1), $a_i = \alpha_i = 0.1$, $\bar{a}_i = \beta_i = 0.12$, $c_{ij}^+ = d_{ij}^+ = \nu_{ij}^+ = 0.1$, $L_j^f = L_j^g = L_j^k = 0.1$, $L_j^a = 0.05$, $I_i^+ = 0.01$, and $i, j = 1, 2$. Taking $\lambda = 0.5$, by an easy calculation, we obtain

$$\theta = 0.072, \quad \delta = 0.1, \quad r_0 = 0.1. \tag{47}$$

Further, we also have

$$\Theta_1 \leq -0.03 < 0, \quad \Theta_2 \leq -0.49 < 0. \tag{48}$$

FIGURE 1: State variables u_1 and u_2 of Example 1. FIGURE 2: Exponential stability of state variable u_1 of Example 1.

It is easy to verify that all the conditions of Theorem 9 are satisfied and that the result follows from Theorem 9. Numerical simulations show that system (45) has a unique almost periodic solution, which is globally exponentially stable (see Figures 1, 2, and 3). This completes the proof. $\square$

In [28–30], the authors studied the existence and global exponential stability of almost periodic solution for a kind of Cohen-Grossberg neural networks without neutral delays based on the exponential dichotomy with fixed point theorem. Therefore, in some sense, we generalize the results in [28–30].

6. Conclusions

Compared with periodic effects, almost periodic effects are more frequent. In recent years, the study on the almost periodic behavior of neural networks has been the object of intensive analysis by numerous authors, and some of these results can be found in the literature. However, the neural networks of neutral type have not been fully investigated in the literature. In this paper, we give some sufficient conditions to ensure the existence, uniqueness, and global exponential stability of almost periodic solution for neutral-type Cohen-Grossberg neural networks. Numerical simulation is given to illustrate the feasibility of our main result. By using

FIGURE 3: Exponential stability of state variable u_2 of Example 1.

the method in this paper, we could study the existence, uniqueness, and global exponential stability of almost periodic solution of other (impulsive) neural networks (on time scales). We leave these for our future work.

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