Research Article

Robust H_∞ Filtering for a Class of Uncertain Markovian Jump Systems with Time Delays

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This paper studies the problem of robust H_∞ filtering for a class of uncertain time-delay systems with Markovian jumping parameters. The system under consideration is subject to norm-bounded time-varying parameter uncertainties. The problem to be addressed is the design of a Markovian jump filter such that the filter error dynamics are stochastically stable and a prescribed bound on the $\mathcal{L}_2$ -induced gain from the noise signals to the filter error is guaranteed for all admissible uncertainties. A sufficient condition for the existence of the desired robust H_∞ filter is given in terms of two sets of coupled algebraic Riccati inequalities. When these algebraic Riccati inequalities are feasible, the expression of a desired H_∞ filter is also presented. Finally, an illustrative numerical example is provided.

1. Introduction

In the past decades, much attention has been focused on the celebrated Kalman filtering which seems to be one of the most popular estimation approaches; see, for example, [1]. Such a kind of filtering approach assumes that the system is subjected to stationary Gaussian noises with known statistics. However, in practical applications, the statistics of the noise sources may not be exactly known. To deal with this problem, an alternative approach named H_∞ filtering was proposed, and a great number of results on this topic have been reported in the literature; see, for example, [2–5]. Note that in the H_∞ filtering setting, the exogenous input signals are assumed to belong to $\mathcal{L}_2[0, \infty)$; furthermore, no exact statistics are required to be known. When parameter uncertainties appear not only in the exogenous input but also in the system model, the problem of robust H_∞ filtering has been investigated and algebraic Riccati equation approach and linear matrix inequality (LMI) approach have been adopted to solve this problem; see [6–9] and the references therein.

Systems with Markovian jumping parameters have been extensively studied due to their both practical and theoretical importance; many issues, such as filtering, stability analysis,

and synthesis, on this kind of systems have been studied [10–13]. The problem of H_∞ filtering for Markovian jumping systems has received much attention. Sufficient conditions for the solvability of this problem were presented in [14], and a design methodology was also proposed based on LMI approach. These results were further extended to uncertain Markovian jump linear systems in [15–18].

In this paper, we consider the problem of robust H_∞ filtering for a class of continuous-time uncertain systems with Markovian jump parameters in all system matrices and time delays in the state variables. We consider uncertain systems with norm-bounded time-varying parameter uncertainties. The objective of this paper is to design a Markovian jump linear filter which guarantees both the stochastic stability and a prescribed H_∞ performance of the filtering error dynamics for all admissible uncertainties. An algebraic Riccati inequalities approach is developed to solve the previous problem and the desired H_∞ filter can be constructed by solving two sets of coupled algebraic Riccati inequalities.

Notation. Throughout this paper, for symmetric matrices X and Y , the notation $X \geq Y$ (resp., $X > Y$) means

that the matrix $X - Y$ is positive semidefinite (resp., positive definite); I is the identity matrix with appropriate dimension. The notation M^T represents the transpose of the matrix M ; $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$; $L_2[0, \infty)$ is the space of square-integrable vector functions over $[0, \infty)$; $\|\cdot\|$ refers to the Euclidean vector norm; $\|\cdot\|_2$ stands for the usual $L_2[0, \infty)$ norm, while $\|\cdot\|_{E_2}$ denotes the norm in $L_2((\Omega, \mathcal{F}, \mathcal{P}), [0, \infty))$; $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space; $\lambda_{\min}(M)$ is used to denote the minimum eigenvalue of the matrix M . Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem Formulation

Fix a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following class of uncertain stochastic linear systems with Markovian jumping parameters and time delay:

$$\Sigma : \begin{cases} \dot{x}(t) = [A(r(t)) + \Delta A(t, r(t))] x(t) \\ \quad + [A_d(r(t)) + \Delta A_d(t, r(t))] \\ \quad \times x(t-d) + B(r(t)) \omega(t), \\ y(t) = [C(r(t)) + \Delta C(t, r(t))] \\ \quad \times x(t) + D(r(t)) \omega(t), \\ z(t) = L(r(t)) x(t), \\ x(t) = \phi(t), \quad \forall t \in [-d, 0], \quad d > 0, \quad r(0) = r_0, \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system state; $y(t) \in \mathbb{R}^r$ is the measurement; $\omega(t) \in \mathbb{R}^m$ is the noise signal which belongs to $L_2[0, \infty)$; $z(t) \in \mathbb{R}^l$ is a linear combination of state variables to be estimated; $d > 0$ is the time delay of the system; and $\phi(t) \in \mathbb{R}^n$ is the continuous initial-value function. The parameter $r(t)$ represents a continuous discrete-state Markov process taking values in a finite set $\mathcal{S} = \{1, 2, \dots, s\}$ with transition probability matrix $\Pi \triangleq \{\pi_{ij}\}$ given by

$$\Pr \{r(t+h) = j \mid r(t) = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j, \\ 1 + \pi_{ii}h + o(h), & i = j, \end{cases} \quad (2)$$

where $h > 0$, $\lim_{h \rightarrow \infty} (o(h)/h) = 0$, and $\pi_{ij} \geq 0$ is the transition rate from mode i at time t to mode j at time $t+h$ and

$$\pi_{ii} = - \sum_{j=1, j \neq i}^s \pi_{ij}. \quad (3)$$

In system Σ , $A(r(t))$, $\Delta A(t, r(t))$, $A_d(r(t))$, $\Delta A_d(t, r(t))$, $B(r(t))$, $C(r(t))$, $\Delta C(t, r(t))$, $D(r(t))$, and $L(r(t))$ are appropriately dimensioned real-valued matrix functions of $r(t)$. For simplicity of notations, in the sequel, for each possible

$r(t) = i$, $i \in \mathcal{S}$, we will denote the matrices associated with the i th mode by

$$\begin{aligned} A_i &\triangleq A(r(t)), & \Delta A_i(t) &\triangleq \Delta A(t, r(t)), \\ A_{di} &\triangleq A_d(r(t)), & \Delta A_{di}(t) &\triangleq \Delta A_d(t, r(t)), \end{aligned} \quad (4)$$

$$B_i \triangleq B(r(t)), \quad C_i \triangleq C(r(t)),$$

$$\Delta C_i(t) \triangleq \Delta C(t, r(t)), \quad D_i \triangleq D(r(t)),$$

$$L_i \triangleq L(r(t)), \quad (5)$$

where A_i , A_{di} , B_i , C_i , D_i , and L_i , are known constant matrices representing the nominal system for each $i \in \mathcal{S}$ and $\Delta A_i(t)$, $\Delta A_{di}(t)$, and $\Delta C_i(t)$ are unknown matrices representing time-varying parameter uncertainties and are assumed to be of the form

$$\begin{aligned} \Delta A_i(t) &= M_{Ai} F_{Ai}(t) N_{Ai}, \\ \Delta A_{di}(t) &= M_{di} F_{di}(t) N_{di}, \end{aligned} \quad (6)$$

$$\Delta C_i(t) = M_{Ci} F_{Ci}(t) N_{Ci},$$

where M_{Ai} , N_{Ai} , M_{di} , N_{di} , M_{Ci} , and N_{Ci} , for each $i \in \mathcal{S}$, are known constant matrices and $F_{Ai}(t)$, $F_{di}(t)$, and $F_{Ci}(t)$, for each $i \in \mathcal{S}$, are the uncertain time-varying matrices satisfying

$$\begin{aligned} F_{Ai}(t)^T F_{Ai}(t) &\leq I, & F_{di}(t)^T F_{di}(t) &\leq I, \\ F_{Ci}(t)^T F_{Ci}(t) &\leq I, & \forall i \in \mathcal{S}. \end{aligned} \quad (7)$$

It is assumed that all the elements of $F_{Ai}(t)$, $F_{di}(t)$, and $F_{Ci}(t)$ are Lebesgue measurable. $\Delta A_i(t)$, $\Delta A_{di}(t)$, and $\Delta C_i(t)$, for each $i \in \mathcal{S}$, are said to be admissible if both (6) and (7) hold.

Throughout the paper we will use the following concept of stochastic stability.

Definition 1. Consider the following stochastic jump system:

$$\begin{aligned} \dot{x}(t) &= A(r(t)) x(t) + A_d(r(t)) x(t-d), \\ x(t) &= \phi(t), \quad \forall t \in [-d, 0], \quad d > 0, \quad r(0) = r_0. \end{aligned} \quad (8)$$

The system (8) is said to be stochastically stable, if, for finite $\phi(t) \in \mathbb{R}^n$ defined on $[-d, 0]$ and $r_0 \in \mathcal{S}$, there exists a scalar $c > 0$ such that

$$\begin{aligned} \lim_{t \rightarrow \infty} \mathcal{E} \left\{ \int_0^t x^T(\tau, \phi, r_0) x(\tau, \phi, r_0) d\tau \mid \phi, r_0 \right\} \\ \leq c \sup_{-d \leq \tau \leq 0} |\phi(\tau)|^2, \end{aligned} \quad (9)$$

where $x(t, \phi, r_0)$ denotes the solution of system (8) at time t under the initial conditions $\phi(t)$ and r_0 .

Now, for each $i \in \mathcal{S}$, consider a Markovian filter of the form

$$\Sigma_F : \begin{cases} \dot{\hat{x}}(t) = \hat{A}_i \hat{x}(t) + \hat{K}_i y(t), \\ \hat{z}(t) = \hat{L}_i \hat{x}(t), \end{cases} \quad (10)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimator state and the matrices $\hat{A}_i$, $\hat{K}_i$, and $\hat{L}_i$ are to be chosen. The filtering error is defined by

$$e(t) \triangleq z(t) - \hat{z}(t). \quad (11)$$

Then the robust H_∞ filtering problem to be dealt with in this paper can be formulated as determining a filter of the form (10) such that, for all admissible uncertainties $\Delta A_i(t)$, $\Delta A_{di}(t)$, and $\Delta C_i(t)$, $i \in \mathcal{S}$, the following requirements are satisfied:

- (R1) the augmented system from system Σ and the filter Σ_F is stochastically stable;
- (R2) with zero initial conditions, the following holds:

$$\|e(t)\|_{E_2} < \gamma \|\omega(t)\|_2, \quad (12)$$

for all nonzero $\omega(t) \in L_2[0, \infty)$, where $\gamma > 0$ is a prescribed scalar.

Before concluding this section, we introduce the following lemma which will be used in the proof of our main results in the next section.

Lemma 2 (see [19, 20]). *Let A, D, E, F , and P be real matrices of appropriate dimensions with $P > 0$ and F satisfying $F^T F \leq I$. Then one has the following:*

- (1) for any scalar $\epsilon > 0$,

$$DFE + (DFE)^T \leq \epsilon^{-1} DD^T + \epsilon E^T E, \quad (13)$$

- (2) for any scalar $\epsilon > 0$ such that $P - \epsilon E^T E > 0$,

$$(A + DFE)P^{-1}(A + DFE)^T \leq A(P - \epsilon E^T E)^{-1}A^T + \epsilon^{-1} DD^T, \quad (14)$$

- (3) $AD + (AD)^T \leq APA^T + D^T P^{-1} D$.

3. Main Results

In this section, a Riccati-like inequality approach is proposed to design a robust H_∞ filter for uncertain stochastic linear systems with Markovian jumping parameters and time delay. Our main result is presented in the next theorem.

Theorem 3. *Consider the uncertain jump linear system Σ . If there exist scalars $\epsilon_{1i} > 0$, $\epsilon_{2i} > 0$, $\epsilon_{3i} > 0$, and $\epsilon_{4i} > 0$ and matrices $P_{1i} > 0$, $P_{2i} > 0$, $i = 1, \dots, s$, $Q_1 > 0$, and $Q_2 > 0$, such that $Q_1 - \epsilon_{3i} N_{di}^T N_{di} > 0$, $i = 1, \dots, s$, and the following coupled Riccati matrix inequalities hold:*

$$A_i^T P_{1i} + P_{1i} A_i + \sum_{j=1}^s \pi_{ij} P_{1j} + P_{1i} W_i P_{1i} + \epsilon_{1i} N_{Ai}^T N_{Ai} + \epsilon_{2i} N_{Ci}^T N_{Ci} + Q_1 < 0, \quad (15)$$

$$\hat{A}_i^T P_{2i} + P_{2i} \hat{A}_i + \sum_{j=1}^s \pi_{ij} P_{2j} + P_{2i} U_i P_{2i} + L_i^T L_i + Q_2 < 0,$$

where

$$\begin{aligned} W_i &= \epsilon_{1i}^{-1} M_{Ai} M_{Ai}^T + \epsilon_{3i}^{-1} M_{di} M_{di}^T \\ &\quad + A_{di} (Q_1 - \epsilon_{3i} N_{di}^T N_{di})^{-1} \\ &\quad \times A_{di}^T + \gamma^{-2} B_i B_i^T, \\ U_i &= \epsilon_{1i}^{-1} M_{Ai} M_{Ai}^T + \epsilon_{3i}^{-1} M_{di} M_{di}^T \\ &\quad + A_{di} (Q_1 - \epsilon_{3i} N_{di}^T N_{di})^{-1} A_{di}^T \\ &\quad + \gamma^{-2} B_i (I - D_i^T H_i^{-1} D_i) B_i^T, \\ \hat{A}_i &= A_i + \left[\epsilon_{1i}^{-1} M_{Ai} M_{Ai}^T + \epsilon_{3i}^{-1} M_{di} M_{di}^T \right. \\ &\quad \left. + A_{di} (Q_1 - \epsilon_{3i} N_{di}^T N_{di})^{-1} A_{di}^T \right] P_{1i} \\ &\quad + \gamma^{-2} B_i (I - D_i^T H_i^{-1} D_i) B_i^T P_{1i} - B_i D_i^T H_i^{-1} C_i, \\ H_i &= D_i D_i^T + \gamma^2 \epsilon_{2i}^{-1} M_{Ci} M_{Ci}^T + \epsilon_{4i} I, \end{aligned} \quad (16)$$

for $i = 1, \dots, s$, then the robust H_∞ filtering problem is solvable. In this case, a suitable robust H_∞ filter Σ_F is given in form (10) with parameters as follows:

$$\hat{K}_i = B_i D_i^T H_i^{-1}, \quad \hat{L}_i = L_i, \quad i \in \mathcal{S}. \quad (17)$$

Proof. Let the mode at time t be i , that is; $r(t) = i$, $i \in \mathcal{S}$, and define

$$\bar{x}(t) = x(t) - \hat{x}(t), \quad (18)$$

then from system Σ and filter Σ_F , it is easy to show that

$$\begin{aligned} \dot{\bar{x}}(t) &= \hat{A}_i \bar{x}(t) + [A_i + \Delta A_i(t) - \hat{A}_i - \hat{K}_i (C_i + \Delta C_i(t))] x(t) \\ &\quad + [A_{di} + \Delta A_{di}(t)] x(t-d) + (B_i - \hat{K}_i D_i) \omega(t). \end{aligned} \quad (19)$$

Therefore, by defining $\tilde{x}(t)^T \triangleq [x(t)^T, \bar{x}(t)^T]^T$, we obtain the augmented system from Σ and Σ_F as

$$\Sigma_{\text{aug}} : \begin{cases} \dot{\tilde{x}}(t) = \tilde{A}_i(t) \tilde{x}(t) + \tilde{A}_{di}(t) \tilde{x}(t-d) + \tilde{B}_i \omega(t), \\ e(t) = \tilde{C}_i \tilde{x}(t), \\ \tilde{x}(t) = \varphi(t), \quad \forall t \in [-d, 0], \quad d > 0, \quad r(0) = r_0, \end{cases} \quad (20)$$

where

$$\begin{aligned}\tilde{A}_i(t) &= \tilde{A}_i + \Delta\tilde{A}_i(t), & \tilde{A}_{di}(t) &= \tilde{A}_{di} + \Delta\tilde{A}_{di}(t), \\ \tilde{A}_i &= \begin{bmatrix} A_i - \hat{A}_i - \hat{K}_i C_i & 0 \\ A_i - \hat{A}_i - \hat{K}_i C_i & \hat{A}_i \end{bmatrix}, \\ \Delta\tilde{A}_i(t) &= \begin{bmatrix} \Delta A_i(t) & 0 \\ \Delta A_i(t) - \hat{K}_i \Delta C_i(t) & 0 \end{bmatrix}, \\ \tilde{A}_{di} &= \begin{bmatrix} A_{di} & 0 \\ A_{di} & 0 \end{bmatrix}, & \Delta\tilde{A}_{di}(t) &= \begin{bmatrix} \Delta A_{di}(t) & 0 \\ \Delta A_{di}(t) & 0 \end{bmatrix}, \\ \tilde{B}_i &= \begin{bmatrix} B_i \\ B_i - \hat{K}_i D_i \end{bmatrix}, & \tilde{C}_i &= [0 \quad L_i].\end{aligned}\quad (21)$$

For $r(t) = i, i \in \mathcal{S}$, we define a matrix $P_i > 0$ by

$$P_i = \begin{bmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{bmatrix}. \quad (22)$$

Next, we will show that under the conditions of the theorem, the following matrix inequality

$$\begin{aligned}Z_i(t) &:= \tilde{A}_i(t)^T P_i + P_i \tilde{A}_i(t) + \sum_{j=1}^s \pi_{ij} P_j \\ &+ P_i \tilde{A}_{di}(t) Q^{-1} \tilde{A}_{di}(t)^T P_i + Q + \tilde{C}_i^T \tilde{C}_i \\ &+ \gamma^{-2} P_i \tilde{B}_i \tilde{B}_i^T P_i \leq \begin{bmatrix} V_{1i} & 0 \\ 0 & V_{2i} \end{bmatrix}\end{aligned}\quad (23)$$

holds, when $r(t) = i, i \in \mathcal{S}$, where

$$\begin{aligned}Q &= \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} > 0, \\ V_{1i} &= A_i^T P_{1i} + P_{1i} A_i + \sum_{j=1}^s \pi_{ij} P_{1j} + P_{1i} W_i P_{1i} \\ &+ \varepsilon_{1i} N_{Ai}^T N_{Ai} + \varepsilon_{2i} N_{Ci}^T N_{Ci} + Q_1, \\ V_{2i} &= \tilde{A}_i^T P_{2i} + P_{2i} \tilde{A}_i + \sum_{j=1}^s \pi_{ij} P_{2j} \\ &+ P_{2i} U_i P_{2i} + L_i^T L_i + Q_2.\end{aligned}\quad (24)$$

To this end, we use Lemma 2 to obtain the following matrix inequalities:

$$\begin{aligned}&P_i \Delta\tilde{A}_i(t) + \Delta\tilde{A}_i(t)^T P_i \\ &\leq \varepsilon_{1i}^{-1} P_i \begin{bmatrix} M_{Ai} \\ M_{Ai} \end{bmatrix} \begin{bmatrix} M_{Ai}^T & M_{Ai}^T \end{bmatrix} P_i \\ &+ \varepsilon_{1i} \begin{bmatrix} N_{Ai}^T \\ 0 \end{bmatrix} \begin{bmatrix} N_{Ai} & 0 \end{bmatrix} \\ &+ \varepsilon_{2i}^{-1} P_i \begin{bmatrix} 0 \\ -\hat{K}_i M_{Ci} \end{bmatrix} \begin{bmatrix} 0 & -M_{Ci}^T \hat{K}_i^T \end{bmatrix} P_i \\ &+ \varepsilon_{2i} \begin{bmatrix} N_{Ci}^T \\ 0 \end{bmatrix} \begin{bmatrix} N_{Ci} & 0 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}&= \begin{bmatrix} H_{1i} & \varepsilon_{1i}^{-1} P_{1i} M_{Ai} M_{Ai}^T P_{2i} \\ \varepsilon_{1i}^{-1} P_{2i} M_{Ai} M_{Ai}^T P_{1i} & H_{2i} \end{bmatrix}, \\ &P_i \tilde{A}_{di}(t) Q^{-1} \tilde{A}_{di}(t)^T P_i \\ &\leq P_i \tilde{A}_{di} \left(Q - \varepsilon_{3i} \begin{bmatrix} N_{di}^T \\ 0 \end{bmatrix} \begin{bmatrix} N_{di} & 0 \end{bmatrix} \right)^{-1} \\ &\times \tilde{A}_{di}^T P_i + \varepsilon_{3i}^{-1} P_i \begin{bmatrix} M_{di} \\ M_{di} \end{bmatrix} \begin{bmatrix} M_{di}^T & M_{di}^T \end{bmatrix} P_i \\ &= \begin{bmatrix} G_{1i} & G_{2i}^T \\ G_{2i} & G_{3i} \end{bmatrix},\end{aligned}\quad (25)$$

where (7) and

$$Q_1 - \varepsilon_{3i} N_{di}^T N_{di} > 0, \quad i = 1, \dots, s, \quad (26)$$

are used and

$$\begin{aligned}H_{1i} &= \varepsilon_{1i}^{-1} P_{1i} M_{Ai} M_{Ai}^T P_{1i} + \varepsilon_{1i} N_{Ai}^T N_{Ai} + \varepsilon_{2i} N_{Ci}^T N_{Ci}, \\ H_{2i} &= P_{2i} \left(\varepsilon_{1i}^{-1} M_{Ai} M_{Ai}^T + \varepsilon_{2i}^{-1} \hat{K}_i M_{Ci} M_{Ci}^T \hat{K}_i^T \right) P_{2i}, \\ G_{1i} &= P_{1i} \left[\varepsilon_{3i}^{-1} M_{di} M_{di}^T + A_{di} \left(Q_1 - \varepsilon_{3i} N_{di}^T N_{di} \right)^{-1} A_{di}^T \right] P_{1i}, \\ G_{2i} &= P_{2i} \left[A_{di} \left(Q_1 - \varepsilon_{3i} N_{di}^T N_{di} \right)^{-1} A_{di}^T + \varepsilon_{3i}^{-1} M_{di} M_{di}^T \right] P_{1i}, \\ G_{3i} &= P_{2i} \left[A_{di} \left(Q_1 - \varepsilon_{3i} N_{di}^T N_{di} \right)^{-1} A_{di}^T + \varepsilon_{3i}^{-1} M_{di} M_{di}^T \right] P_{2i}.\end{aligned}\quad (27)$$

Hence

$$\begin{aligned}Z_i(t) &\leq \tilde{A}_i^T P_i + P_i \tilde{A}_i \\ &+ \sum_{j=1}^s \pi_{ij} P_j + Q + \tilde{C}_i^T \tilde{C}_i + \gamma^{-2} P_i \tilde{B}_i \tilde{B}_i^T P_i \\ &+ \begin{bmatrix} H_{1i} + G_{1i} & \varepsilon_{1i}^{-1} P_{1i} M_{Ai} M_{Ai}^T P_{2i} + G_{2i}^T \\ \varepsilon_{1i}^{-1} P_{2i} M_{Ai} M_{Ai}^T P_{1i} + G_{2i} & H_{2i} + G_{3i} \end{bmatrix}.\end{aligned}\quad (28)$$

Substituting (16) and (17) into the right-hand side of the previous inequality and using algebraic manipulations we have that (23) holds.

Now, we will show the stochastic stability of the augmented system Σ_{aug} when $\omega(t) = 0$. In the following we will simply use $\tilde{x}(t)$ to stand for the solution of the system Σ_{aug} at time t with the initial conditions $\varphi(t)$ and r_0 .

Introduce the following stochastic Lyapunov functional candidate for system Σ_{aug} :

$$V(\tilde{x}(t), r(t)) = \tilde{x}(t)^T P(r(t)) \tilde{x}(t) + \int_{t-d}^t \tilde{x}(\tau)^T Q \tilde{x}(\tau) d\tau. \quad (29)$$

It can then be shown that the weak infinitesimal generator $\mathcal{A}$ of the random process $\{\tilde{x}(t), r(t)\}$ is given by

$$\begin{aligned} \mathcal{A}V(\tilde{x}(t), i) &= \lim_{h \rightarrow \infty} \frac{1}{h} [\mathcal{E}\{V(\tilde{x}(t+h), r(t+h)) \mid \tilde{x}(t), \\ &\quad r(t) = i\} - V(\tilde{x}(t), r(t) = i)] \\ &= \tilde{x}(t)^T \left[\tilde{A}_i(t)^T P_i + P_i \tilde{A}_i(t) + \sum_{j=1}^s \pi_{ij} P_j + Q \right] \\ &\quad \times \tilde{x}(t) + \tilde{x}(t)^T P_i \tilde{A}_{di}(t) \tilde{x}(t-d) \\ &\quad + \tilde{x}(t-d)^T \tilde{A}_{di}(t)^T P_i \tilde{x}(t) \\ &\quad - \tilde{x}(t-d)^T Q \tilde{x}(t-d), \end{aligned} \quad (30)$$

for any $r(t) = i, i \in \mathcal{S}$. Then, from Lemma 2, it follows that

$$\begin{aligned} \mathcal{A}V(\tilde{x}(t), i) &\leq \tilde{x}(t)^T \left[\tilde{A}_i(t)^T P_i + P_i \tilde{A}_i(t) \right. \\ &\quad \left. + \sum_{j=1}^s \pi_{ij} P_j + P_i \tilde{A}_{di}(t) Q^{-1} \tilde{A}_{di}(t)^T P_i + Q \right] \tilde{x}(t). \end{aligned} \quad (31)$$

Using (23), we have

$$\mathcal{A}V(\tilde{x}(t), i) \leq \tilde{x}(t)^T \begin{bmatrix} V_{1i} & 0 \\ 0 & V_{2i} \end{bmatrix} \tilde{x}(t) \leq -\beta_1 \tilde{x}(t)^T \tilde{x}(t), \quad (32)$$

where $\beta_1 = \min_{i \in \mathcal{S}} \{\lambda_{\min}(-V_{1i}), \lambda_{\min}(-V_{2i})\}$. From (15), it is easy to see that $\beta_1 > 0$.

Now using Dynkin's formula, we have, for each $r(t) = i, i \in \mathcal{S}, t > 0$,

$$\begin{aligned} &\mathcal{E}\{V(\tilde{x}(t), i)\} - V(\tilde{x}_0, r_0) \\ &= \mathcal{E}\left\{\int_0^t \mathcal{A}V(\tilde{x}(\tau), r(\tau)) d\tau\right\} \\ &\leq -\beta_1 \int_0^t \mathcal{E}\{\tilde{x}(\tau)^T \tilde{x}(\tau)\} d\tau. \end{aligned} \quad (33)$$

On the other hand, for each $r(t) = i, i \in \mathcal{S}$, we can show that

$$\begin{aligned} \mathcal{E}\{V(\tilde{x}(t), i)\} &= \mathcal{E}\{\tilde{x}(t)^T P_i \tilde{x}(t)\} \\ &\quad + \mathcal{E}\left\{\int_{t-d}^t \tilde{x}(\tau)^T Q \tilde{x}(\tau) d\tau\right\} \\ &\geq \beta_2 \mathcal{E}\{\tilde{x}(t)^T \tilde{x}(t)\}, \end{aligned} \quad (34)$$

where $\beta_2 = \min_{i \in \mathcal{S}} \{\lambda_{\min}(P_i)\}$ and $\beta_2 > 0$. The previous inequality and (33) imply that

$$\begin{aligned} \mathcal{E}\{\tilde{x}(t)^T \tilde{x}(t)\} &\leq -\lambda_1 \int_0^t \mathcal{E}\{\tilde{x}(\tau)^T \tilde{x}(\tau)\} d\tau \\ &\quad + \lambda_2 V(\tilde{x}_0, r_0), \end{aligned} \quad (35)$$

where $\lambda_1 = \beta_1 \beta_2^{-1} > 0$ and $\lambda_2 = \beta_2^{-1} > 0$. Then, it can be verified that

$$\mathcal{E}\{\tilde{x}(t)^T \tilde{x}(t)\} \leq \lambda_2 \exp(-\lambda_1 t) V(\tilde{x}_0, r_0). \quad (36)$$

Therefore,

$$\begin{aligned} &\mathcal{E}\left\{\int_0^t \tilde{x}(\tau)^T \tilde{x}(\tau) d\tau \mid \varphi, r_0\right\} \\ &\leq \lambda_1^{-1} \lambda_2 [1 - \exp(-\lambda_1 t)] V(\tilde{x}_0, r_0). \end{aligned} \quad (37)$$

Taking limit as $t \rightarrow \infty$, we have

$$\lim_{t \rightarrow \infty} \mathcal{E}\left\{\int_0^t \tilde{x}(\tau)^T \tilde{x}(\tau) d\tau \mid \varphi, r_0\right\} \leq \lambda_1^{-1} \lambda_2 V(\tilde{x}_0, r_0). \quad (38)$$

Note that there always exists a scalar $c > 0$ such that

$$\lambda_1^{-1} \lambda_2 V(\tilde{x}_0, r_0) \leq c \sup_{-d \leq \tau \leq 0} |\varphi(\tau)|^2. \quad (39)$$

Considering (38), we have that the augmented system Σ_{aug} is stochastically stable. Next we will show that

$$\|e(t)\|_{E_2} < \gamma \|\omega(t)\|_2, \quad (40)$$

for all nonzero $\omega(t) \in L_2[0, \infty)$. To this end, we introduce

$$J = \mathcal{E}\left\{\int_0^\infty [e(\tau)^T e(\tau) - \gamma^2 \omega(\tau)^T \omega(\tau)] d\tau\right\}. \quad (41)$$

For any $r(t) = i, i \in \mathcal{S}, \omega(t) \neq 0$, we have

$$\begin{aligned} \mathcal{A}V(\tilde{x}(t), i) &= \tilde{x}(t)^T \left[\tilde{A}_i(t)^T P_i + P_i \tilde{A}_i(t) + \sum_{j=1}^s \pi_{ij} P_j + Q \right] \\ &\quad \times \tilde{x}(t) + \tilde{x}(t)^T P_i \tilde{A}_{di}(t) \tilde{x}(t-d) \\ &\quad + \tilde{x}(t-d)^T \tilde{A}_{di}(t)^T P_i \tilde{x}(t) \\ &\quad - \tilde{x}(t-d)^T Q \tilde{x}(t-d) \\ &\quad + \tilde{x}(t)^T P_i \tilde{B}_i \omega(t) + \omega(t)^T \tilde{B}_i^T P_i \tilde{x}(t). \end{aligned} \quad (42)$$

Thus, under zero initial condition, for any $t > 0$, $r(t) = i$, $i \in \mathcal{S}$, $\omega(t) \neq 0$,

$$\begin{aligned}
J_1 &= \mathcal{E} \left\{ \int_0^t [e(\tau)^T e(\tau) - \gamma^2 \omega(\tau)^T \omega(\tau)] d\tau \right\} \\
&= \mathcal{E} \left\{ \int_0^t [e(\tau)^T e(\tau) - \gamma^2 \omega(\tau)^T \omega(\tau) \right. \\
&\quad \left. + \mathcal{A}V(\tilde{x}(\tau), i)] d\tau \right\} \\
&\quad - \mathcal{E} \left\{ \int_0^t \mathcal{A}V(\tilde{x}(\tau), i) d\tau \right\} \\
&= \mathcal{E} \left\{ \int_0^t [\tilde{x}(\tau)^T Z_i(\tau) \tilde{x}(\tau) \right. \\
&\quad - (\tilde{x}(t-d)^T - \tilde{x}(t)^T P_i \tilde{A}_{di}(t) Q^{-1}) \\
&\quad \times Q (\tilde{x}(t-d) - Q^{-1} \tilde{A}_{di}(t)^T P_i \tilde{x}(t)) \\
&\quad - \gamma^2 (\omega(\tau)^T - \gamma^{-2} \tilde{x}(t)^T P_i \tilde{B}_i) \\
&\quad \times (\omega(\tau) - \gamma^{-2} \tilde{B}_i^T P_i \tilde{x}(t))] d\tau \Big\} \\
&\quad - \mathcal{E} \{V(\tilde{x}(t), i)\}.
\end{aligned} \tag{43}$$

Hence

$$J \leq \mathcal{E} \left\{ \int_0^\infty \tilde{x}(\tau)^T Z_i(\tau) \tilde{x}(\tau) d\tau \right\}. \tag{44}$$

Finally, from (23), (40) follows immediately. This completes the proof. $\square$

Remark 4. Theorem 3 provides a sufficient condition for the solvability of robust H_∞ filtering problem for uncertain stochastic time-delay systems with Markovian jumping parameters, and the desired filter can be constructed by solving two sets of coupled algebraic Riccati inequalities.

In the case when $\mathcal{S} = \{1\}$, that is, there is only one mode operation, we have $\pi_{ii} = 0$, $i \in \mathcal{S} = \{1\}$, and system Σ reduces to the following uncertain time-delay system with no jumping parameters:

$$\Sigma_1 : \begin{cases} \dot{x}(t) = [A + \Delta A(t)] x(t) \\ \quad + [A_d + \Delta A_d(t)] x(t-d) + B\omega(t), \\ y(t) = [C + \Delta C(t)] x(t) + D\omega(t), \\ z(t) = Lx(t), \\ x(t) = \phi(t), \quad \forall t \in [-d, 0], \quad d > 0, \quad r(0) = r_0, \end{cases} \tag{45}$$

where $\Delta A(t)$, $\Delta A_d(t)$, and $\Delta C(t)$ are unknown matrices and are assumed to be of the form

$$\begin{aligned}
\Delta A(t) &= M_A F_A(t) N_A, & \Delta A_d(t) &= M_d F_d(t) N_d, \\
\Delta C(t) &= M_C F_C(t) N_C, \end{aligned} \tag{46}$$

where M_A , N_{Ai} , M_d , N_d , M_C , and N_C are known constant matrices and $F_A(t)$, $F_d(t)$, and $F_C(t)$ are the uncertain time-varying matrices satisfying

$$\begin{aligned} F_A(t)^T F_A(t) &\leq I, & F_d(t)^T F_d(t) &\leq I, \\ F_C(t)^T F_C(t) &\leq I. \end{aligned} \tag{47}$$

Then, from Theorem 3, we have the following robust H_∞ filtering result for the previous system.

Corollary 5. Consider the uncertain time-delay system (Σ_1) . If there exist scalars $\varepsilon_i > 0$, $i = 1, \dots, 4$, and matrices $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, and $Q_2 > 0$, such that $Q_1 - \varepsilon_3 N_d^T N_d > 0$, and the following coupled Riccati matrix inequalities hold:

$$\begin{aligned} A^T P_1 + P_1 A + P_1 W P_1 + \varepsilon_1 N_A^T N_A + \varepsilon_2 N_C^T N_C + Q_1 &< 0 \\ \widehat{A}^T P_2 + P_2 \widehat{A} + P_2 U P_2 + L^T L + Q_2 &< 0, \end{aligned} \tag{48}$$

where

$$\begin{aligned} W &= \varepsilon_1^{-1} M_A M_A^T + \varepsilon_3^{-1} M_d M_d^T + A_d (Q_1 - \varepsilon_3 N_d^T N_d)^{-1} A_d^T \\ &\quad + \gamma^{-2} B B^T, \\ U &= \varepsilon_1^{-1} M_A M_A^T + \varepsilon_3^{-1} M_d M_d^T + A_d (Q_1 - \varepsilon_3 N_d^T N_d)^{-1} A_d^T \\ &\quad + \gamma^{-2} B (I - D^T H^{-1} D) B^T, \\ \widehat{A} &= A + [\varepsilon_1^{-1} M_A M_A^T + \varepsilon_3^{-1} M_d M_d^T \\ &\quad + A_d (Q_1 - \varepsilon_3 N_d^T N_d)^{-1} A_d^T] P_1 \\ &\quad + \gamma^{-2} B (I - D^T H^{-1} D) B^T P_1 - B D^T H^{-1} C, \\ H &= D D^T + \gamma^2 \varepsilon_2^{-1} M_C M_C^T + \varepsilon_4 I, \end{aligned} \tag{49}$$

then the robust H_∞ filtering problem is solvable. In this case, a suitable robust H_∞ filter is given by

$$\begin{aligned} \hat{\dot{x}}(t) &= \widehat{A} \hat{x}(t) + \widehat{K} y(t), \\ \widehat{z}(t) &= \widehat{L} \hat{x}(t), \end{aligned} \tag{50}$$

where

$$\widehat{K} = B D^T H^{-1}, \quad \widehat{L} = L. \tag{51}$$

Remark 6. The solvability for robust H_∞ filtering problem for uncertain continuous delay-free systems can be easily derived from Corollary 5, in this case. It can be shown that the result coincides with that proposed in [8].

4. Numerical Example

In this section, we will give a numerical example to demonstrate the applicability of the proposed approach.

Consider the uncertain time-delay stochastic linear systems with Markovian jumping parameters in form (1) with two modes. For mode 1, the dynamics of the system are described as follows:

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -5 & 2 & 0 \\ 0 & -4 & 1 \\ 0 & 0 & -5 \end{bmatrix}, & A_{d1} &= \begin{bmatrix} -1 & 0 & 0.2 \\ 0 & 0.5 & 1 \\ 0.5 & 0 & 0 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} 1 & -2.3 \\ 0 & 1 \\ 0.5 & 0 \end{bmatrix}, & C_1 &= \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \\
 D_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & M_{C1} &= \begin{bmatrix} 0.5 & 1 \\ 0 & 0.5 \end{bmatrix}, \\
 N_{C1} &= \begin{bmatrix} 0 & 0.5 & 1 \\ 0 & -0.5 & 1 \end{bmatrix}, & M_{A1} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.5 & 1 \end{bmatrix}, \\
 M_{d1} &= \begin{bmatrix} 0 & 0.5 \\ 0.5 & 1 \\ 0.5 & 0 \end{bmatrix}, & N_{A1}^T &= \begin{bmatrix} -0.2 \\ 0.1 \\ 0 \end{bmatrix}, \\
 N_{d1}^T &= \begin{bmatrix} -0.1 \\ 0 \\ 0.2 \end{bmatrix}, & L_1 &= \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.3 & -0.1 \end{bmatrix}.
 \end{aligned} \tag{52}$$

For mode 2, the dynamics of the system are described as follows:

$$\begin{aligned}
 A_2 &= \begin{bmatrix} -6 & 1 & 0.5 \\ -1 & -7 & 0 \\ 0 & 0.5 & -6 \end{bmatrix}, & A_{d2} &= \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & -1 & -0.1 \\ 0 & 0 & 1 \end{bmatrix}, \\
 B_2 &= \begin{bmatrix} -1 & 0.5 \\ -3 & 1 \\ 0 & 1 \end{bmatrix}, & C_2 &= \begin{bmatrix} 0.5 & 1 & 2 \\ -1 & 0 & 2 \end{bmatrix}, \\
 D_2 &= \begin{bmatrix} -1 & 0.5 \\ 0.2 & 1 \end{bmatrix}, & M_{C2} &= \begin{bmatrix} 0.1 & -0.3 \\ 0.5 & 0 \end{bmatrix}, \\
 N_{C2} &= \begin{bmatrix} -1 & 0.2 & 0.3 \\ 0.5 & 0.5 & 0 \end{bmatrix}, & M_{A2} &= \begin{bmatrix} 0.5 & 0.3 \\ 0 & 0.5 \\ 0.2 & 1 \end{bmatrix}, \\
 M_{d2} &= \begin{bmatrix} -0.6 & 0.5 \\ 0 & 1 \\ 0.5 & 1 \end{bmatrix}, & N_{A2}^T &= \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix}, \\
 N_{d2}^T &= \begin{bmatrix} 0.2 \\ 0 \\ 0 \end{bmatrix}, & L_2 &= \begin{bmatrix} -0.2 & 0.1 & 0 \\ 0.3 & 0 & 0.5 \end{bmatrix}.
 \end{aligned} \tag{53}$$

Let the transition probability matrix be given by

$$\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 1 & -1 \end{bmatrix}. \tag{54}$$

In this example, we set $\gamma = 2.8$. By solving (15), we obtain

$$\begin{aligned}
 P_{11} &= \begin{bmatrix} 2.1592 & 0.3579 & -0.0589 \\ 0.3579 & 3.2018 & 0.1337 \\ -0.0589 & 0.1337 & 4.0867 \end{bmatrix}, \\
 P_{12} &= \begin{bmatrix} 2.9083 & -0.0024 & -0.2047 \\ -0.0024 & 1.6855 & 0.0031 \\ -0.2047 & 0.0031 & 2.0020 \end{bmatrix}, \\
 Q_1 &= \begin{bmatrix} 11.1343 & -0.4670 & -0.3949 \\ -0.4670 & 10.5734 & -0.6361 \\ -0.3949 & -0.6361 & 11.8166 \end{bmatrix}, \\
 P_{21} &= \begin{bmatrix} 0.2157 & 0.1094 & 0.0221 \\ 0.1094 & 0.3324 & 0.0524 \\ 0.0221 & 0.0524 & 0.1848 \end{bmatrix}, \\
 P_{22} &= \begin{bmatrix} 0.1572 & -0.0161 & 0.0122 \\ -0.0161 & 0.0989 & -0.0176 \\ 0.0122 & -0.0176 & 0.1414 \end{bmatrix}, \\
 Q_2 &= \begin{bmatrix} 0.7630 & -0.0624 & -0.0257 \\ -0.0624 & 0.7896 & -0.0205 \\ -0.0257 & -0.0205 & 0.8370 \end{bmatrix}, \\
 \varepsilon_{11} &= 12.1023, & \varepsilon_{21} &= 9.2498, \\
 \varepsilon_{31} &= 12.1293, & \varepsilon_{41} &= 0.0032, \\
 \varepsilon_{12} &= 12.1346, & \varepsilon_{22} &= 10.1634, \\
 \varepsilon_{32} &= 12.1219, & \varepsilon_{42} &= 0.0012.
 \end{aligned} \tag{55}$$

Therefore, using Theorem 3, a suitable Markovian robust H_∞ filter can be constructed with parameters given as follows:

$$\begin{aligned}
 \hat{A}_1 &= \begin{bmatrix} -3.5504 & 4.4496 & 0.3304 \\ 0.0239 & -3.8366 & 0.9941 \\ 0.2927 & 0.4860 & -4.7540 \end{bmatrix}, \\
 \hat{K}_1 &= \begin{bmatrix} 0.9411 & -2.2211 \\ -0.1821 & 0.8865 \\ 0.2611 & -0.0911 \end{bmatrix}, \\
 \hat{L}_1 &= \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.3 & -0.1 \end{bmatrix}, \\
 \hat{A}_2 &= \begin{bmatrix} -5.9695 & 0.0301 & -1.2747 \\ -2.8680 & -9.2528 & -4.3890 \\ 0.7840 & 0.5184 & -7.3116 \end{bmatrix}, \\
 \hat{K}_2 &= \begin{bmatrix} 0.9452 & -0.0162 \\ 2.7441 & -0.4287 \\ 0.1826 & 0.7602 \end{bmatrix}, \\
 \hat{L}_2 &= \begin{bmatrix} -0.2 & 0.1 & 0 \\ 0.3 & 0 & 0.5 \end{bmatrix}.
 \end{aligned} \tag{56}$$

5. Conclusions

In this paper, we have studied the problem of robust H_∞ filtering for a class of uncertain Markovian jump systems

with time-delay and norm-bounded time-varying parameter uncertainties. A Markovian jump filter is designed which guarantees the stochastic stability of the filter error dynamics and a prescribed bound on the $\mathcal{L}_2$ -induced gain from the noise signals to the filter error irrespective of the parameter uncertainties. It has been shown that the desired robust H_∞ filter can be constructed by solving two sets of coupled algebraic Riccati inequalities.

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