Research Article

Robust Stability and Stabilization for Singular Time-Delay Systems with Linear Fractional Uncertainties: A Strict LMI Approach

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This paper is concerned with the problems of delay-dependent robust stability and stabilization for a class of continuous singular systems with time-varying delay in range and parametric uncertainties. The parametric uncertainties are assumed to be of a linear fractional form, which includes the norm bounded uncertainty as a special case and can describe a class of rational nonlinearities. In terms of strict linear matrix inequalities (LMIs), delay-range-dependent robust stability criteria for the unforced system are presented. Moreover, a strict LMI design approach is developed such that, when the LMI is feasible, a desired state feedback stabilizing controller can be constructed, which guarantees that, for all admissible uncertainties, the closed-loop dynamics will be regular, impulse free, and robustly asymptotically stable. Numerical examples are provided to demonstrate the effectiveness of the proposed methods.

1. Introduction

Singular time-delay systems, which are known as descriptor time-delay systems, implicit time-delay systems, or generalized differential-difference equations, often appear in various engineering systems, for example, aircraft attitude control, flexible arm control of robots, large-scale electric network control, chemical engineering systems, lossless transmission lines, and so forth [1, 2]. Since singular time-delay systems are matrix delay differential equations coupled with matrix difference equations, the study for such systems is much more complicated than that for standard state-space time-delay systems. Recently, a great deal of attention has been devoted to the study of such more general class of delay systems; see [3–27].

The existing stability criteria for singular time-delay systems can be classified into two types: delay independent [3–5] and delay dependent [6–10]. Generally, delay-dependent conditions are less conservative than the delay-independent ones, especially when the time delay is small. To obtain delay-dependent conditions, many efforts have been made in

the literature, among which the model transformation and bounding technology for cross-terms [8–10] are often used. However, it is known that the bounding technology and the model transformation are the main source of conservatism [28]. Recently, some improved stability conditions with less conservatism have been provided by utilizing the free weighting matrix method [11–13], the integral inequality [14], and the delay decomposition approach [15–17], in which neither the bounding technology nor model transformation is involved. However, these conditions in [6–17] were established under the assumption that the delay was time invariant. For the continuous singular systems with time-varying delay, Yue and Han investigated the delay-dependent stability condition by introducing the free weighting matrices [18]. In [19], a delay-dependent stability condition was presented by using the integral inequality method. But the range of the time-varying delay considered in [18, 19] is from 0 to an upper bound. In practice, a time-varying interval delay is often encountered; that is, the range of delay varies in an interval for which the lower bound is not restricted to 0. In this case, the stability criteria in [18, 19] are conservative

because they do not take into account the information of the lower bound of delay. Moreover, when estimating the upper bound of the derivative of Lyapunov functional, some useful terms are ignored in [18, 19]. More recently, continuous singular systems with time-varying delay in a range have been extensively studied; see, for example, [20–27] and references therein.

On the other hand, in recent years, more and more attention has been devoted to derive strict LMI conditions for stability analysis and controller design; see, for example [29, 30] and references therein. The strict LMI conditions, that is, definite LMIs without equality constraints, are highly tractable and reliable when checked by some recently developed algorithms for solving LMIs [31]. However, it should be pointed that the stability conditions derived in [20–27] are formulated in terms of nonstrict LMIs, whose solutions are difficult to calculate since equality constraints are often fragile and usually not met perfectly. Furthermore, up to now, to the best of the authors' knowledge, for a continuous uncertain singular system with a time-varying interval delay, the problems of robust stability, stabilization, and feedback control have not been fully investigated yet [23]. Particularly, strict LMI-based condition has never been reported in the published works.

In this paper, by using a strict LMI approach, we study the robust stability and stabilization problems for a class of singular systems with a time-varying interval delay and uncertainties. Different from the existing results in [13, 19, 21, 23], first, the criteria proposed in our paper do not contain any semidefinite matrix inequality and are expressed as strict LMIs. Second, the new criteria are obtained by only using a well-known integral-inequality and do not employ any free-weighting matrix, which makes our methods more efficient. Third, a new type of uncertainty, namely, linear fractional form, is considered in this paper. Three numerical examples are given to illustrate the effectiveness of the presented method.

Notations. $\mathbf{R}^n$ denotes the n -dimensional Euclidean space, and $\mathbf{R}^{n \times m}$ denotes the sets of all $n \times m$ matrices. I is the identity matrix with appropriate dimensions. For a real symmetric matrix X , X^T denotes its transpose, the notation $X \geq 0$ ($X > 0$) means that the matrix X is positive semidefinite (positive-definite), and $\lambda_{\min}(X)$ ($\lambda_{\max}(X)$) denotes the minimum (maximum) eigenvalue of X . $C_{n,\tau} := C([- \tau, 0], \mathbf{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[- \tau, 0]$ into $\mathbf{R}^n$, $x_t := x(t + \theta)$, $\theta \in [- \tau, 0]$, and $t \geq 0$ denotes the function family defined on $[- \tau, 0]$ which is generated by n -dimensional real vector valued continuous function $x(t)$, $t \in [- \tau, +\infty)$. Obviously, $x_t \in C_{n,\tau}$. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm, and $\|\varphi\|_c := \sup_{-\tau \leq t \leq 0} \|\varphi(t)\|$ stands for the norm of a function $\varphi \in C_{n,\tau}$. The symmetric terms in a symmetric matrix are denoted by $*$.

2. Problem Formulation and Preliminaries

Consider the following singular time-delay system:

$$\begin{aligned} E\dot{x}(t) = & (A + \Delta A)x(t) + (A_\tau + \Delta A_\tau)x(t - \tau(t)) \\ & + (B + \Delta B)u(t), \end{aligned}$$

$$x(t) = \phi(t), \quad t \in [-h_2, 0], \quad (1)$$

where $x(t) \in \mathbf{R}^n$ is the state vector, and $u(t) \in \mathbf{R}^m$ is the control input. $E, A \in \mathbf{R}^{n \times n}$, $A_\tau \in \mathbf{R}^{n \times n}$, and $B \in \mathbf{R}^{n \times m}$ are real constant matrices with appropriate dimensions, and $0 < \text{rank } E = r \leq n$. $\tau(t)$ denotes the time-varying delay which satisfies $h_1 < \tau(t) \leq h_2$, $\dot{\tau}(t) \leq \mu < 1$. Note that h_1 may not be equal to 0. $\phi(t)$ is a compatible continuous vector-valued initial function on $[-h_2, 0]$. ΔA , ΔA_τ and ΔB are matrices with parametric uncertainties satisfying

$$[\Delta A \quad \Delta A_\tau \quad \Delta B] = D\Delta(t) [E_a \quad E_\tau \quad E_b], \quad (2)$$

$$\Delta(t) = F(t) (I - JF(t))^{-1}, \quad (3)$$

$$I - J^T J > 0, \quad (4)$$

where D, E_a, E_τ, E_b , and J are known real constant matrices of approximate dimensions and $F(t)$ are unknown time-varying matrix function satisfying

$$F^T(t) F(t) \leq I. \quad (5)$$

The parametric uncertainties ΔA , ΔA_τ , and ΔB satisfying (2)–(5) are said to be admissible.

Remark 1. The above-structured linear fractional form includes the norm-bounded uncertainty as a special case when $J = 0$ [3, 8–11, 13, 19, 23] and can describe a class of rational nonlinearities [32]. Note also that conditions (4) and (5) guarantee that $I - JF(t)$ is invertible.

The nominal unforced system of (2) can be written as

$$\begin{aligned} E\dot{x}(t) = & Ax(t) + A_\tau x(t - \tau(t)), \\ x(t) = & \phi(t), \quad t \in [-h_2, 0]. \end{aligned} \quad (6)$$

The following notations are given.

- (i) $S_0 := \{\phi(t) \in C_{n,h_2}, \phi(t) \text{ is the compatible initial function of system (6)}\}$.
- (ii) $S := \{\phi(t) \in C_{n,h_2}, \text{ and there exists a uniquely continuous solution of system (6) on } [0, +\infty) \text{ for } \phi(t)\}$.
- (iii) $B(0, \delta) := \{\phi(t) \mid \phi(t) \in C_{n,h_2}, \|\phi\|_c \leq \delta, \delta > 0\}$.

Definition 2 (see [33]). (1) The pair (E, A) is said to be regular if $\det(sE - A)$ is not identically 0.

(2) The pair (E, A) is said to be impulse-free if $\deg(\det(sE - A)) = \text{rank } E$.

Lemma 3 (see [3]). *If the pair (E, A) is regular and impulse free, then for any compatible initial function $\phi(t) \in C_{n,h_2}$, there exists a uniquely continuous solution of system (6) on $[0, +\infty)$ for $\phi(t)$.*

Definition 4 (see [3]). The singular system (6) is said to be regular and impulse free, if the pair (E, A) is regular and impulse free.

Definition 5 (see [11]). (1) The system (6) is said to be stable, if for any $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$ such that for any compatible initial function $\phi(t) \in B(0, \delta) \cap S$, the solution $x(t)$ of system (6) satisfies $\|x(t)\| \leq \varepsilon$, $t > 0$.

(2) The system (6) is said to be asymptotically stable, if its zero solution is stable, and furthermore, there exists a $b_0 > 0$ such that for any compatible initial function $\phi(t) \in B(0, b_0) \cap S$, the solution $x(t) \rightarrow 0$ as $t \rightarrow \infty$.

The objective of this note is to develop delay-range-dependent robust stability conditions for system (2) with $u(t) = 0$ and to design a state-feedback controller

$$u(t) = Kx(t), \quad K \in \mathbf{R}^{m \times n} \quad (7)$$

so that system (2) is closed-loop *regular, impulse-free, and robustly asymptotically stable* for admissible linear fractional form uncertainties. To this end, the following lemmas are needed.

Lemma 6 (see [34]). For any constant matrix $W \in \mathbf{R}^{n \times n}$, $W = W^T > 0$, scalar $0 \leq r(t) \leq r_M$, and vector function $\dot{x} : [-r_M, 0] \rightarrow \mathbf{R}^n$ such that the following integration is well defined, then

$$\begin{aligned} & -r_M \int_{-r(t)}^0 \dot{x}^T(t + \xi) W \dot{x}(t + \xi) d\xi \\ & \leq \begin{pmatrix} x^T(t) & x^T(t - r(t)) \end{pmatrix} \begin{pmatrix} -W & W \\ W & -W \end{pmatrix} \begin{pmatrix} x(t) \\ x(t - r(t)) \end{pmatrix}. \end{aligned} \quad (8)$$

Lemma 7 (see [35]). Consider the function $\varphi : \mathbf{R}^+ \rightarrow \mathbf{R}$, if φ is bounded on $[0, \infty)$; that is, there exists a scalar $\alpha > 0$ such that $|\varphi(t)| \leq \alpha$ for all $t \in [0, \infty)$, and then φ is uniformly continuous on $[0, \infty)$.

Lemma 8 (see [35]). Consider the function $\varphi : \mathbf{R}^+ \rightarrow \mathbf{R}$, if $\varphi(t)$ is uniformly continuous and $\int_0^\infty \varphi(s) ds < \infty$, then $\lim_{t \rightarrow \infty} \varphi(t) = 0$.

Lemma 9 (see [32]). Given matrices $Q = Q^T$, Γ , and Ξ of approximate dimensions, then

$$Q + \Gamma \Delta(t) \Xi + \Xi^T \Delta^T(t) \Gamma^T < 0, \quad (9)$$

where $\Delta(t)$ is as in (3), if and only if there exists scalar $\varepsilon > 0$ such that

$$Q + \begin{bmatrix} \varepsilon^{-1} \Xi^T & \varepsilon \Gamma \end{bmatrix} \begin{bmatrix} I & -J \\ -J^T & I \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon^{-1} \Xi \\ \varepsilon \Gamma^T \end{bmatrix} < 0. \quad (10)$$

3. Stability Issue

In this section, first of all, we will present new delay-range-dependent stability conditions that guarantee system (6) to be regular, impulse free, and asymptotically stable in terms of LMI, which will play a key role in obtaining the robust stability criterion for the uncertain system (2).

Theorem 10. Given scalars $0 \leq h_1 < h_2$ and μ , the singular system (6) is regular, impulse free, and asymptotically stable if there exist positive-definite matrices P , Q_i , $i = 1, 2, 3$, Z_j , $j = 1, 2$ and matrix S with appropriate dimensions such that

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & 0 & 0 & h_2 A^T Z_1 & h_{12} A^T Z_2 \\ * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & h_2 A_\tau^T Z_1 & h_{12} A_\tau^T Z_2 \\ * & * & \Lambda_{33} & 0 & 0 & 0 \\ * & * & * & \Lambda_{44} & 0 & 0 \\ * & * & * & * & -h_2 Z_1 & 0 \\ * & * & * & * & * & -h_{12} Z_2 \end{bmatrix} < 0, \quad (11)$$

where

$$\Lambda_{11} = E^T P A + A^T P E + S R^T A + A^T R S^T$$

$$+ \sum_{i=1}^3 Q_i - h_2^{-1} E^T Z_1 E,$$

$$\Lambda_{12} = E^T P A_\tau + S R^T A_\tau + h_2^{-1} E^T Z_1 E,$$

$$\begin{aligned} \Lambda_{22} = & -(1 - \mu) Q_3 - h_2^{-1} E^T Z_1 E \\ & - h_{12}^{-1} E^T (Z_1 + Z_2) E - h_{12}^{-1} E^T Z_2 E, \end{aligned} \quad (12)$$

$$\Lambda_{23} = h_{12}^{-1} E^T Z_2 E,$$

$$\Lambda_{24} = h_{12}^{-1} E^T (Z_1 + Z_2) E,$$

$$\Lambda_{33} = -Q_1 - h_{12}^{-1} E^T Z_2 E,$$

$$\Lambda_{44} = -Q_2 - h_{12}^{-1} E^T (Z_1 + Z_2) E,$$

$$h_{12} = h_2 - h_1,$$

and $R \in \mathbf{R}^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E^T R = 0$.

Proof. Since $\text{rank } E = r \leq n$, there must exist two invertible matrices $\bar{G}$ and $\bar{H}$ such that

$$\bar{E} = \bar{G} E \bar{H} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \quad (13)$$

Then, R can be parameterized as

$$R = \bar{G}^T \begin{bmatrix} 0 \\ \Phi \end{bmatrix}, \quad (14)$$

where $\bar{\Phi} \in \mathbf{R}^{(n-r) \times (n-r)}$ is any nonsingular matrix.

Similar to (13), we define

$$\begin{aligned} \bar{A} &= \bar{G} A \bar{H} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \\ \bar{P} &= \bar{G}^{-T} P \bar{G}^{-1} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}, \end{aligned} \quad (15)$$

$$\bar{S} = \bar{H}^T S = \begin{bmatrix} \bar{S}_1 \\ \bar{S}_2 \end{bmatrix}, \quad \bar{R} = \bar{G}^{-T} R = \begin{bmatrix} 0 \\ \bar{\Phi} \end{bmatrix},$$

$$\bar{Z}_1 = \bar{G}^{-T} Z_1 \bar{G}^{-1} = \begin{bmatrix} \bar{Z}_{1,11} & \bar{Z}_{1,12} \\ \bar{Z}_{1,21} & \bar{Z}_{1,22} \end{bmatrix}.$$

Since $\Lambda_{11} < 0$ and $Q_i > 0$, $i = 1, 2, 3$, we can formulate the following inequality easily:

$$\Psi = E^T P A + A^T P E + S R^T A + A^T R S^T - h_2^{-1} E^T Z_1 E < 0. \quad (16)$$

Pre- and postmultiplying $\Psi < 0$ by $\bar{H}^T$ and $\bar{H}$, respectively, yields

$$\bar{\Psi} = \bar{H}^T \Psi \bar{H} = \begin{bmatrix} \bar{\Psi}_{11} & \bar{\Psi}_{12} \\ * & \bar{A}_{22}^T \bar{\Phi} \bar{S}_2^T + \bar{S}_2 \bar{\Phi}^T \bar{A}_{22} \end{bmatrix} < 0, \quad (17)$$

where $\bar{\Psi}_{11}$ and $\bar{\Psi}_{12}$ represent the matrices not relevant in the following discussion. From (17), it is easy to see that

$$\bar{A}_{22}^T \bar{\Phi} \bar{S}_2^T + \bar{S}_2 \bar{\Phi}^T \bar{A}_{22} < 0 \quad (18)$$

which gives that $\bar{A}_{22}$ is nonsingular.

Define

$$\bar{G} = \begin{bmatrix} I & -\bar{A}_{12} \bar{A}_{22}^{-1} \\ 0 & I \end{bmatrix} \bar{G}, \quad \bar{H} = \bar{H} \begin{bmatrix} I & 0 \\ -\bar{A}_{22}^{-1} \bar{A}_{21} & \bar{A}_{22}^{-1} \end{bmatrix}. \quad (19)$$

After some algebraic manipulations, we can obtain

$$\tilde{E} = \bar{G} \bar{E} \bar{H} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{A} = \bar{G} \bar{A} \bar{H} = \begin{bmatrix} \tilde{A}_{11} & 0 \\ 0 & I \end{bmatrix}, \quad (20)$$

where $\tilde{A}_{11} = \bar{A}_{11} - \bar{A}_{12} \bar{A}_{22}^{-1} \bar{A}_{21}$. Then, it can be shown that

$$\begin{aligned} \det(sE - A) &= \det(\bar{G}^{-1}) \det(s\bar{E} - \bar{A}) \det(\bar{H}^{-1}) \\ &= \det(\bar{G}^{-1}) (-1)^{(n-r)} \det(sI_r - \tilde{A}_{11}) \det(\bar{H}^{-1}) \end{aligned} \quad (21)$$

which implies that $\det(sE - A)$ is not identically zero and $\deg(\det(sE - A)) = r = \text{rank } E$. Then, the pair of (E, A) is regular and impulse-free, which shows that system (6) is regular and impulse-free. In the following, we will prove that system (6) is also asymptotically stable.

Denote

$$\begin{aligned} \bar{A}_\tau &= \bar{G} \bar{A}_\tau \bar{H} = \begin{bmatrix} \bar{A}_{\tau 11} & \bar{A}_{\tau 12} \\ \bar{A}_{\tau 21} & \bar{A}_{\tau 22} \end{bmatrix}, \\ \bar{P} &= \bar{G}^{-T} P \bar{G}^{-1} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}, \\ \bar{R} &= \bar{G}^{-T} R = \begin{bmatrix} 0 \\ \bar{\Phi} \end{bmatrix}, \quad \bar{S} = \bar{H}^T S = \begin{bmatrix} \bar{S}_1 \\ \bar{S}_2 \end{bmatrix}, \\ \bar{Q}_i &= \bar{H}^T Q_i \bar{H} = \begin{bmatrix} \bar{Q}_{i,11} & \bar{Q}_{i,12} \\ \bar{Q}_{i,21} & \bar{Q}_{i,22} \end{bmatrix}, \quad i = 1, 2, 3, \\ \bar{Z}_j &= \bar{G}^{-T} Z_j \bar{G}^{-1} = \begin{bmatrix} \bar{Z}_{j,11} & \bar{Z}_{j,12} \\ \bar{Z}_{j,21} & \bar{Z}_{j,22} \end{bmatrix}, \quad j = 1, 2. \end{aligned} \quad (22)$$

By using Schur complement and noting that $Q_i > 0$, $i = 1, 2$, $\mu < 1$, it follows from (11) that

$$\begin{bmatrix} \bar{\Lambda}_{11} & \bar{\Lambda}_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix} < 0, \quad (23)$$

where

$$\begin{aligned} \bar{\Lambda}_{11} &= E^T P A + A^T P E + S R^T A + A^T R S^T + Q_3 - h_2^{-1} E^T Z_1 E, \\ \bar{\Lambda}_{22} &= -Q_3 - h_2^{-1} E^T Z_1 E - h_{12}^{-1} E^T (Z_1 + Z_2) E - h_{12}^{-1} E^T Z_2 E. \end{aligned} \quad (24)$$

Pre- and post-multiplying (23) by $\text{diag}\{\bar{H}^T, \bar{H}^T\}$ and $\text{diag}\{\bar{H}, \bar{H}\}$, respectively, yields

$$\begin{bmatrix} \tilde{\Lambda}_{11} & \tilde{\Lambda}_{12} \\ * & \tilde{\Lambda}_{22} \end{bmatrix} < 0, \quad (25)$$

where

$$\begin{aligned} \tilde{\Lambda}_{11} &= \tilde{E}^T \tilde{P} \tilde{A} + \tilde{A}^T \tilde{P} \tilde{E} + \tilde{S} \tilde{R}^T \tilde{A} + \tilde{A}^T \tilde{R} \tilde{S}^T + \tilde{Q}_3 - h_2^{-1} \tilde{E}^T \tilde{Z} \tilde{E}, \\ \tilde{\Lambda}_{12} &= \tilde{E}^T \tilde{P} \tilde{A}_\tau + \tilde{S} \tilde{R}^T \tilde{A}_\tau + h_2^{-1} \tilde{E}^T \tilde{Z}_1 \tilde{E}, \\ \tilde{\Lambda}_{22} &= -\tilde{Q}_3 - h_2^{-1} \tilde{E}^T \tilde{Z}_1 \tilde{E} - h_{12}^{-1} \tilde{E}^T (\tilde{Z}_1 + \tilde{Z}_2) \tilde{E} - h_{12}^{-1} \tilde{E}^T \tilde{Z}_2 \tilde{E}. \end{aligned} \quad (26)$$

Substituting (20), (22) into (25), we have

$$\begin{bmatrix} \tilde{S}_2 \bar{\Phi}^T + \bar{\Phi} \tilde{S}_2^T + \tilde{Q}_{3,22} & \tilde{S}_2 \bar{\Phi}^T \bar{A}_{\tau 22}^T \\ * & -\tilde{Q}_{3,22} \end{bmatrix} < 0. \quad (27)$$

Pre- and post-multiplying (27) by $\xi = [\bar{\Phi}^{-T} \tilde{S}_2^{-1} \quad -\bar{A}_{\tau 22} \bar{\Phi}^{-T} \tilde{S}_2^{-1}]$ and ξ^T , respectively, yields

$$\bar{A}_{\tau 22} \bar{\Theta} \bar{A}_{\tau 22}^T - \bar{\Theta} < 0, \quad (28)$$

where

$$\bar{\Theta} = -\bar{\Phi}^{-T} \tilde{S}_2^{-1} (\tilde{S}_2 \bar{\Phi}^T + \bar{\Phi} \tilde{S}_2^T + \tilde{Q}_{3,22}) \tilde{S}_2^{-T} \bar{\Phi}^{-1} > 0. \quad (29)$$

Therefore,

$$\rho(\bar{A}_{\tau 22}) < 1. \quad (30)$$

Now, let

$$\tilde{x}(t) = \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix} = \bar{H}^{-1} x(t), \quad (31)$$

where $\tilde{x}_1(t) \in \mathbf{R}^r$ and $\tilde{x}_2(t) \in \mathbf{R}^{n-r}$. Using the expressions in (20), (22), and (31), system (6) can be decomposed as

$$\dot{\tilde{x}}_1(t) = \tilde{A}_{11} \tilde{x}_1(t) + \tilde{A}_{\tau 11} \tilde{x}_1(t - \tau(t)) + \tilde{A}_{\tau 12} \tilde{x}_2(t - \tau(t)), \quad (32)$$

$$0 = \tilde{x}_2(t) + \tilde{A}_{\tau 21} \tilde{x}_1(t - \tau(t)) + \tilde{A}_{\tau 22} \tilde{x}_2(t - \tau(t)), \quad (33)$$

or equivalently rewritten as

$$\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{A}_\tau\tilde{x}(t - \tau(t)). \quad (34)$$

It is easy to see that the stability of system (6) is equivalent to that of system (34).

Construct the Lyapunov-Krasovskii functional for system (34) as

$$\begin{aligned} \bar{V}(\tilde{x}_t) = & \tilde{x}^T(t) \tilde{E}^T \tilde{P} \tilde{E} \tilde{x}(t) \\ & + \sum_{i=1}^2 \int_{t-h_i}^t \tilde{x}^T(s) \tilde{Q}_i \tilde{x}(s) ds \\ & + \int_{t-\tau(t)}^t \tilde{x}^T(s) \tilde{Q}_3 \tilde{x}(s) ds \\ & + \int_{-h_2}^0 \int_{t+\theta}^t \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_1 \tilde{E} \dot{\tilde{x}}(s) ds d\theta \\ & + \int_{-h_2}^{-h_1} \int_{t+\theta}^t \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_2 \tilde{E} \dot{\tilde{x}}(s) ds d\theta. \end{aligned} \quad (35)$$

By Lemma 6, the following inequalities are true

$$\begin{aligned} & -h_2 \int_{t-h_2}^t \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_1 \tilde{E} \dot{\tilde{x}}(s) ds \\ & = -h_2 \int_{t-\tau(t)}^t \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_1 \tilde{E} \dot{\tilde{x}}(s) ds \\ & \quad - h_2 \int_{t-h_2}^{t-\tau(t)} \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_1 \tilde{E} \dot{\tilde{x}}(s) ds \\ & \leq (\tilde{x}^T(t) \tilde{x}^T(t - \tau(t))) \\ & \quad \times \begin{pmatrix} -\tilde{E}^T \tilde{Z}_1 \tilde{E} & \tilde{E}^T \tilde{Z}_1 \tilde{E} \\ \tilde{E}^T \tilde{Z}_1 \tilde{E} & -\tilde{E}^T \tilde{Z}_1 \tilde{E} \end{pmatrix} \begin{pmatrix} \tilde{x}(t) \\ \tilde{x}(t - \tau(t)) \end{pmatrix} \\ & \quad + h_{12}^{-1} (\tilde{x}^T(t - \tau(t)) \tilde{x}^T(t - h_2)) \\ & \quad \times \begin{pmatrix} -\tilde{E}^T \tilde{Z}_1 \tilde{E} & \tilde{E}^T \tilde{Z}_1 \tilde{E} \\ \tilde{E}^T \tilde{Z}_1 \tilde{E} & -\tilde{E}^T \tilde{Z}_1 \tilde{E} \end{pmatrix} \begin{pmatrix} \tilde{x}(t - \tau(t)) \\ \tilde{x}(t - h_2) \end{pmatrix}, \\ & -h_{12} \int_{t-h_2}^{t-h_1} \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_2 \tilde{E} \dot{\tilde{x}}(s) ds \\ & = -h_{12} \int_{t-h_2}^{t-\tau(t)} \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_2 \tilde{E} \dot{\tilde{x}}(s) ds \\ & \quad - h_{12} \int_{t-\tau(t)}^{t-h_1} \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_2 \tilde{E} \dot{\tilde{x}}(s) ds \end{aligned}$$

$$\begin{aligned} & \leq (\tilde{x}^T(t - \tau(t)) \tilde{x}^T(t - h_2)) \\ & \quad \times \begin{pmatrix} -\tilde{E}^T \tilde{Z}_2 \tilde{E} & \tilde{E}^T \tilde{Z}_2 \tilde{E} \\ \tilde{E}^T \tilde{Z}_2 \tilde{E} & -\tilde{E}^T \tilde{Z}_2 \tilde{E} \end{pmatrix} \begin{pmatrix} \tilde{x}(t - \tau(t)) \\ \tilde{x}(t - h_2) \end{pmatrix} \\ & \quad + (\tilde{x}^T(t - h_1) \tilde{x}^T(t - \tau(t))) \\ & \quad \times \begin{pmatrix} -\tilde{E}^T \tilde{Z}_2 \tilde{E} & \tilde{E}^T \tilde{Z}_2 \tilde{E} \\ \tilde{E}^T \tilde{Z}_2 \tilde{E} & -\tilde{E}^T \tilde{Z}_2 \tilde{E} \end{pmatrix} \begin{pmatrix} \tilde{x}(t - h_1) \\ \tilde{x}(t - \tau(t)) \end{pmatrix}. \end{aligned} \quad (36)$$

On the other hand, noticing that $E^T R = 0$, we can deduce that

$$0 = 2\tilde{x}^T(t) \tilde{S} \tilde{R}^T \tilde{E} \dot{\tilde{x}}(t), \quad (37)$$

where $\tilde{S}$ is any matrix with appropriate dimensions.

Taking the derivative of $\bar{V}(\tilde{x}_t)$ with respect to t along the trajectory of system (34) and using (36) and (37), we have

$$\begin{aligned} \dot{\bar{V}}(\tilde{x}_t) & = \dot{\tilde{x}}^T(t) \tilde{E}^T \tilde{P} \tilde{E} \tilde{x}(t) + \tilde{x}^T(t) \tilde{E}^T \tilde{P} \tilde{E} \dot{\tilde{x}}(t) \\ & \quad + \sum_{i=1}^2 \{ \tilde{x}^T(t) \tilde{Q}_i \tilde{x}(t) - \tilde{x}^T(t - h_i) \tilde{Q}_i \tilde{x}(t - h_i) \} \\ & \quad + \tilde{x}^T(t) \tilde{Q}_3 \tilde{x}(t) - (1 - \dot{\tau}(t)) \tilde{x}^T(t - \tau(t)) \tilde{Q}_3 \tilde{x}(t - \tau(t)) \\ & \quad + h_2 \dot{\tilde{x}}^T(t) \tilde{E}^T \tilde{Z}_1 \tilde{E} \dot{\tilde{x}}(t) - \int_{t-h_2}^t \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_1 \tilde{E} \dot{\tilde{x}}(s) ds \\ & \quad + (h_2 - h_1) \dot{\tilde{x}}^T(t) \tilde{E}^T \tilde{Z}_2 \tilde{E} \dot{\tilde{x}}(t) - \int_{t-h_2}^{t-h_1} \dot{\tilde{x}}^T(s) \tilde{E}^T \tilde{Z}_2 \tilde{E} \dot{\tilde{x}}(s) ds \\ & \leq \tilde{\psi}^T(t) \tilde{\Lambda} \tilde{\psi}(t), \end{aligned} \quad (38)$$

where

$$\begin{aligned} \tilde{\psi}(t) & = [\tilde{x}^T(t) \quad \tilde{x}^T(t - \tau(t)) \quad \tilde{x}^T(t - h_1) \quad \tilde{x}^T(t - h_2)]^T, \\ \tilde{\Lambda} & = \begin{bmatrix} \tilde{\Lambda}_{11} & \tilde{\Lambda}_{12} & 0 & 0 \\ * & \tilde{\Lambda}_{22} & \tilde{\Lambda}_{23} & \tilde{\Lambda}_{24} \\ * & * & \tilde{\Lambda}_{33} & 0 \\ * & * & * & \tilde{\Lambda}_{44} \end{bmatrix}, \end{aligned} \quad (39)$$

with

$$\begin{aligned} \tilde{\Lambda}_{11} & = \tilde{E}^T \tilde{P} \tilde{A} + \tilde{A}^T \tilde{P} \tilde{E} + \tilde{S} \tilde{R}^T \tilde{A} + \tilde{A}^T \tilde{R} \tilde{S}^T \\ & \quad + \sum_{i=1}^3 \tilde{Q}_i + h_2 \tilde{A}^T \tilde{Z}_1 \tilde{A} + h_{12} \tilde{A}^T \tilde{Z}_2 \tilde{A} - h_2^{-1} \tilde{E}^T \tilde{Z}_1 \tilde{E}, \\ \tilde{\Lambda}_{12} & = \tilde{E}^T \tilde{P} \tilde{A}_\tau + \tilde{S} \tilde{R}^T \tilde{A}_\tau + h_2 \tilde{A}^T \tilde{Z}_1 \tilde{A}_\tau + h_{12} \tilde{A}^T \tilde{Z}_2 \tilde{A}_\tau \\ & \quad + h_2^{-1} \tilde{E}^T \tilde{Z}_1 \tilde{E}, \\ \tilde{\Lambda}_{22} & = -(1 - \mu) \tilde{Q}_3 + h_2 \tilde{A}_\tau^T \tilde{Z}_1 \tilde{A}_\tau + h_{12} \tilde{A}_\tau^T \tilde{Z}_2 \tilde{A}_\tau \\ & \quad - h_2^{-1} \tilde{E}^T \tilde{Z}_1 \tilde{E} - h_{12}^{-1} \tilde{E}^T (\tilde{Z}_1 + \tilde{Z}_2) \tilde{E} - h_{12}^{-1} \tilde{E}^T \tilde{Z}_2 \tilde{E}, \end{aligned}$$

$$\begin{aligned}
\bar{\Lambda}_{23} &= h_{12}^{-1} \bar{E}^T \bar{Z}_2 \bar{E}, \quad \bar{\Lambda}_{24} = h_{12}^{-1} \bar{E}^T (\bar{Z}_1 + \bar{Z}_2) \bar{E}, \\
\bar{\Lambda}_{33} &= -\bar{Q}_1 - h_{12}^{-1} \bar{E}^T \bar{Z}_2 \bar{E}, \\
\bar{\Lambda}_{44} &= -\bar{Q}_2 - h_{12}^{-1} \bar{E}^T (\bar{Z}_1 + \bar{Z}_2) \bar{E}.
\end{aligned} \tag{40}$$

It is easy to see that (17) guarantees $\dot{\bar{V}}(\bar{x}_t) < 0$ and

$$\begin{aligned}
\lambda_1 \|\bar{x}_1(t)\|^2 - \bar{V}(\bar{x}(0)) &\leq -\bar{V}(\bar{x}(t)) - \bar{V}(\bar{x}(0)) \\
&= \int_0^t \dot{\bar{V}}(\bar{x}(s)) ds \leq -\lambda_2 \int_0^t \|\bar{x}(s)\|^2 ds \\
&\leq -\lambda_2 \int_0^t \|\bar{x}_1(s)\|^2 ds < 0,
\end{aligned} \tag{41}$$

where $\lambda_1 = \lambda_{\min}(\bar{P}_{11}) > 0$, $\lambda_1 = -\lambda_{\max}(\bar{\Lambda}) > 0$.

Taking into account (41), we can deduce that

$$\lambda_1 \|\bar{x}_1(t)\|^2 + \lambda_2 \int_0^t \|\bar{x}_1(s)\|^2 ds \leq \bar{V}(\bar{x}(0)). \tag{42}$$

Therefore,

$$\|\bar{x}_1(t)\|^2 \leq m_1, \tag{43}$$

$$\int_0^t \|\bar{x}_1(s)\|^2 ds \leq m_2, \tag{44}$$

where $m_1 = (1/\lambda_1)\bar{V}(\bar{x}(0)) > 0$, $m_2 = (1/\lambda_2)\bar{V}(\bar{x}(0)) > 0$. Thus, $\|\bar{x}_1(t)\|$ is bounded. Considering this and (30), it can be deduced from (33) that $\|\bar{x}_2(t)\|$ is bounded; hence, it follows that from (32) that $\|\dot{\bar{x}}_1(t)\|$ is bounded. Therefore, $(d/dt)\|\bar{x}_1(t)\|^2$ is bounded too. By Lemma 7, we obtain that $\|\dot{\bar{x}}_1(t)\|^2$ is uniformly continuous. Therefore, noting (44) and using Lemma 8, we get

$$\lim_{t \rightarrow \infty} \|\bar{x}_1(t)\| = 0. \tag{45}$$

This, together with (33), implies that

$$\lim_{t \rightarrow \infty} \|\bar{x}_2(t)\| = 0. \tag{46}$$

Thus, according to Definition 5, system (34) is stable. This completes the proof. $\square$

Remark 11. From the proof of Theorem 10, it is clear to see that neither model transformation nor bounding technique for cross-terms is involved. Hence, the conservatism inherited from these ideas will no longer exist in Theorem 10.

Remark 12. Free-weighting matrices in [11–13, 22, 23, 25] plays an important role to reducing the conservatism of delay-dependent stability conditions. However, too many free-weighting matrices will complicate the system analysis and increase the computational demand. It is worth pointing out that no free-weighting matrix is involved in Theorem 10.

Remark 13. Recently, the delay-partitioning or delay-fractioning method [36] was widely used to reduce the conservatism of the delay-dependent results of standard time-delay systems. This method can be extended to study the stability of singular time-delay system (2). Suppose we decompose the delay interval $[h_1, h_2]$ into N equidistant subintervals. Defining $h^i = h_1 + i(h_2 - h_1)/N$, $i = 1, 2, \dots, N-1$, and constructing the following Lyapunov-Krasovskii functional:

$$\begin{aligned}
\bar{V}(\bar{x}_t) &= \bar{x}^T(t) \bar{E}^T \bar{P} \bar{E} \bar{x}(t) + \sum_{i=1}^2 \int_{t-h_i}^t \bar{x}^T(s) \bar{Q}_i \bar{x}(s) ds \\
&+ \sum_{j=1}^{N-1} \int_{t-h^j}^t \bar{x}^T(s) \bar{Q}'_j \bar{x}(s) ds \\
&+ \int_{-h_2}^0 \int_{t+\theta}^t \dot{\bar{x}}^T(s) \bar{E}^T \bar{Z}_1 \bar{E} \dot{\bar{x}}(s) ds d\theta \\
&+ \int_{-h^1}^{-h^1} \int_{t+\theta}^t \dot{\bar{x}}^T(s) \bar{E}^T \bar{Z}_2 \bar{E} \dot{\bar{x}}(s) ds d\theta \\
&+ \sum_{l=1}^{N-2} \int_{-h^l}^{-h^{l+1}} \int_{t+\theta}^t \dot{\bar{x}}^T(s) \bar{E}^T \bar{Z}_{l+2} \bar{E} \dot{\bar{x}}(s) ds d\theta \\
&+ \int_{-h_2}^{-h^{N-1}} \int_{t+\theta}^t \dot{\bar{x}}^T(s) \bar{E}^T \bar{Z}_{N+1} \bar{E} \dot{\bar{x}}(s) ds d\theta
\end{aligned} \tag{47}$$

with $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Q}'_j > 0$, $j = 1, 2, \dots, N-1$, $\bar{Z}_1 > 0$, $\bar{Z}_2 > 0$ and $\bar{Z}_{l+2} > 0$, $l = 1, 2, \dots, N-2$. Then, by checking the variation of $\bar{V}(\bar{x}_t)$ for the case when $\tau(t) \in [h_1, h^1]$ or $\tau(t) \in [h^j, h^{j+1}]$ ($j = 1, 2, \dots, N-1$) or $\tau(t) \in [h^{N-1}, h_2]$, respectively, we can derive the delay-dependent condition, which can guarantee that $\bar{V}(\bar{x}_t) < 0$. Generally, increasing N may result in the reduction of conservatism of the obtained results. However, the corresponding computational complexity will be increased greatly since the dimensions and matrix variables of the involved LMIs will be sharply expanded. For example, in [36], a numerical example has shown that, with N changing from 1 to 3, the allowable upper bounds of h_2 increased 12.9%, but the consumed CPU time increased 9 times.

Theorem 10 presents a delay-range-dependent criterion for system (6) with time-varying delay $\tau(t)$ in a range. If we set $Q_1 = 0$ and $Z_2 = 0$, Theorem 10 yields the following delay-dependent stability criterion.

Corollary 14. Given scalars $h_2 > 0$, $h_1 = 0$ and μ , system (6) is regular, impulse free, and asymptotically stable if there exist positive-definite matrices P , Q_i , $i = 2, 3$, Z_1 , and matrix S with appropriate dimensions such that

$$\begin{bmatrix} T_{11} & T_{12} & 0 & h_2 A^T Z_1 \\ * & T_{22} & h_2^{-1} E^T Z_1 E & h_2 A_r^T Z_1 \\ * & * & T_{33} & 0 \\ * & * & * & -h_2 Z_1 \end{bmatrix}, \tag{48}$$

where

$$\begin{aligned}
 T_{11} &= E^T P A + A^T P E + S R^T A + A^T R S^T \\
 &\quad + \sum_{i=2}^3 Q_i - h_2^{-1} E^T Z_1 E, \\
 T_{12} &= E^T P A_\tau + S R^T A_\tau + h_2^{-1} E^T Z_1 E, \\
 T_{22} &= -(1 - \mu) Q_3 - 2h_2^{-1} E^T Z_1 E, \\
 T_{33} &= -Q_2 - h_2^{-1} E^T Z_1 E,
 \end{aligned} \tag{49}$$

and $R \in \mathbf{R}^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E^T R = 0$.

Now, we will present the delay-range-dependent robust stability conditions for the uncertain singular time-delay system (2) with $u(t) = 0$ via Theorem 10.

Theorem 15. Given scalars $0 \leq h_1 < h_2$ and μ , the uncertain singular time-delay system (2) with $u(t) = 0$ is regular, impulse free, and robustly asymptotically stable if there exist positive-definite matrices $P, Q_i, i = 1, 2, 3, Z_j, j = 1, 2$, and matrix S with appropriate dimensions and a scalar $\eta > 0$ such that the following LMI holds:

$$\begin{bmatrix}
 \Lambda_{11} & \Lambda_{12} & 0 & 0 & h_2 A^T Z_1 & h_{12} A^T Z_2 & \eta E_a^T & (E^T P + S R^T) D \\
 * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & h_2 A_\tau^T Z_1 & h_{12} A_\tau^T Z_2 & \eta E_\tau^T & 0 \\
 * & * & \Lambda_{33} & 0 & 0 & 0 & 0 & 0 \\
 * & * & * & \Lambda_{44} & 0 & 0 & 0 & 0 \\
 * & * & * & * & -h_2 Z_1 & 0 & 0 & h_2 Z_1 D \\
 * & * & * & * & * & -h_{12} Z_2 & 0 & h_{12} Z_2 D \\
 * & * & * & * & * & * & -\eta I & \eta J \\
 * & * & * & * & * & * & * & -\eta I
 \end{bmatrix} < 0, \tag{50}$$

where $\Lambda_{11}, \Lambda_{12}, \Lambda_{22}, \Lambda_{23}, \Lambda_{24}, \Lambda_{33}, \Lambda_{44}$, and h_{12} are defined in (11).

Proof. Suppose (50) to be true. Let $\varepsilon = 1/\sqrt{\eta}$. Pre- and postmultiplying the left-hand side matrix of (50) by $\text{diag}\{I, I, I, I, I, \varepsilon, \varepsilon\}$ and its transpose, respectively, we obtain

$$\Lambda + \begin{bmatrix} \varepsilon^{-1} \Xi^T & \varepsilon \Gamma \end{bmatrix} \begin{bmatrix} I & -J \\ -J^T & I \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon^{-1} \Xi \\ \varepsilon \Gamma^T \end{bmatrix} < 0, \tag{51}$$

where

$$\begin{aligned}
 \Gamma &= \begin{bmatrix} D^T(P E + R S^T) & 0 & 0 & 0 & h_2 D^T Z_1 & h_{12} D^T Z_2 \end{bmatrix}^T, \\
 \Xi &= \begin{bmatrix} E_a & E_\tau & 0 & 0 & 0 & 0 \end{bmatrix},
 \end{aligned} \tag{52}$$

and Λ is defined in (11). Thus, $\Theta = \Lambda + \Gamma \Delta(t) \Xi + \Xi^T \Delta^T(t) \Gamma^T < 0$ holds according to Lemma 9. It can be verified that Θ is exactly the left-hand side of (11) when A and A_τ are replaced with $A + D \Delta(t) E_a$ and $A_\tau + D \Delta(t) E_\tau$ in (11), respectively. The result then follows from Theorem 10. $\square$

4. Control Design

On the basis of the previous stability conditions, we will present a design method of robustly stabilizing controllers in this section. For simplicity, we first consider system (6).

Theorem 16. Given scalars $0 \leq h_1 < h_2$ and μ , if there exist scalar $\alpha > 0$, positive-definite matrices $P, Q_i, i = 1, 2, 3, Z_j, j = 1, 2, X$, and matrices R, S, U with appropriate dimensions such that

$$\Psi = \begin{bmatrix}
 \Psi_{11} & \Psi_{12} & \Psi_{13} & 0 & 0 & 0 & \Psi_{17} & 0 & \Psi_{19} \\
 * & \Psi_{22} & X A_\tau^T & 0 & 0 & h_2 Z_1 & -h_2 X & h_{12} Z_2 & -h_{12} X \\
 * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & 0 & h_2 A_\tau X & 0 & h_{12} A_\tau X \\
 * & * & * & \Psi_{44} & 0 & 0 & 0 & 0 & 0 \\
 * & * & * & * & \Psi_{55} & 0 & 0 & 0 & 0 \\
 * & * & * & * & * & -h_2 Z_1 & 0 & 0 & 0 \\
 * & * & * & * & * & * & -h_2 X & 0 & 0 \\
 * & * & * & * & * & * & * & -h_{12} Z_2 & 0 \\
 * & * & * & * & * & * & * & * & -h_{12} X
 \end{bmatrix} < 0, \tag{53}$$

where

$$\begin{aligned}
 \Psi_{11} &= A X + X A^T + B U + U^T B^T + \sum_{i=1}^3 Q_i - h_2^{-1} E Z_1 E^T, \\
 \Psi_{12} &= E P + S R^T - X + A X + B U, \\
 \Psi_{13} &= X A_\tau^T + h_2^{-1} E Z_1 E^T, \\
 \Psi_{17} &= h_2 A X + h_2 B U, \quad \Psi_{19} = h_{12} A X + h_{12} B U, \\
 \Psi_{22} &= -2X + 3aI, \\
 \Psi_{33} &= -(1 - \mu) Q_3 - h_2^{-1} E Z_1 E^T - h_{12}^{-1} E (Z_1 + Z_2) E^T \\
 &\quad - h_{12}^{-1} E Z_2 E^T, \\
 \Psi_{34} &= h_{12}^{-1} E Z_2 E^T, \quad \Psi_{35} = h_{12}^{-1} E (Z_1 + Z_2) E^T, \\
 \Psi_{44} &= -Q_1 - h_{12}^{-1} E Z_2 E^T, \\
 \Psi_{55} &= -Q_2 - h_{12}^{-1} E (Z_1 + Z_2) E^T
 \end{aligned} \tag{54}$$

and $R_1 \in \mathbf{R}^{n \times (n-r)}$ is any matrix with full column rank and satisfying $E^T R_1 = 0$, then there exists a state feedback controller (7) such that the resulting closed-loop system of system (6) is regular, impulse free, and asymptotically stable. In this case, a suitable controller gain is given by

$$u(t) = U X^{-1} x(t). \tag{55}$$

Proof. With the control law $u(t) = K x(t)$, the resultant closed-loop system of system (6) is

$$E \dot{x}(t) = (A + B K) x(t) + A_\tau x(t - \tau(t)). \tag{56}$$

Following the same philosophy as that in [37], we represent system (56) as the following form,

$$\begin{aligned}
 \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} &= \begin{bmatrix} 0 & I \\ A + B K & -I \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A_\tau & 0 \end{bmatrix} \begin{bmatrix} x(t - \tau(t)) \\ y(t - \tau(t)) \end{bmatrix}, \\
 &\tag{57}
 \end{aligned}$$

where $y(t) = E x(t)$.

For notational convenience, we introduce

$$\begin{aligned}\bar{E} &= \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, & \bar{A} &= \begin{bmatrix} 0 & I \\ A+BK & -I \end{bmatrix}, \\ \bar{A}_\tau &= \begin{bmatrix} 0 & 0 \\ A_\tau & 0 \end{bmatrix}.\end{aligned}\quad (58)$$

Then, by Theorem 10, we can show that system (57) is regular, impulse free, and asymptotically stable if (11) holds, where E , A , A_τ , P , Q_i , and $i = 1, 2, 3$, Z_j , $j = 1, 2$, R , S are replaced by $\bar{E}$, $\bar{A}$, $\bar{A}_\tau$, $\bar{P}$, $\bar{Q}_i$, $i = 1, 2, 3$, $\bar{Z}_j$, and $j = 1, 2$, $\bar{R}$, $\bar{S}$, respectively. For a special issue, we choose $\bar{P}$, $\bar{Q}_i$, $\bar{Z}_j$, $\bar{R}$, and $\bar{S}$ as

$$\begin{aligned}\bar{P} &= \begin{bmatrix} P & 0 \\ 0 & \alpha I \end{bmatrix}, & \bar{Q}_1 &= \begin{bmatrix} Q_1 & 0 \\ 0 & \alpha I \end{bmatrix}, \\ \bar{Q}_2 &= \begin{bmatrix} Q_2 & 0 \\ 0 & \alpha I \end{bmatrix}, & \bar{Q}_3 &= \begin{bmatrix} Q_3 & 0 \\ 0 & \alpha I \end{bmatrix}, \\ \bar{Z}_1 &= \begin{bmatrix} Z_1 & 0 \\ 0 & X \end{bmatrix}, & \bar{Z}_2 &= \begin{bmatrix} Z_2 & 0 \\ 0 & X \end{bmatrix}, \\ \bar{R} &= \begin{bmatrix} R & 0 \\ 0 & X \end{bmatrix}, & \bar{S} &= \begin{bmatrix} S & I \\ 0 & I \end{bmatrix},\end{aligned}\quad (59)$$

where $P \in \mathbf{R}^{n \times n}$, $Q_i \in \mathbf{R}^{n \times n}$, $i = 1, 2, 3$, $Z_j \in \mathbf{R}^{n \times n}$, and $j = 1, 2$, $X \in \mathbf{R}^{n \times n}$ are symmetrical positive-definite matrices, $R \in \mathbf{R}^{n \times (n-r)}$ is with full column rank and satisfies $E^T R = 0$, $S \in \mathbf{R}^{n \times (n-r)}$ is any matrix and $\alpha > 0$ is a scalar. It is easy to verify that $\bar{R}$ is with full column rank and satisfies $\bar{E}^T \bar{R} = 0$. Then, the following LMI can be obtained:

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & 0 & 0 & 0 & \Phi_{17} & 0 & \Phi_{19} \\ * & \Phi_{22} & XA_\tau & 0 & 0 & h_2 Z_1 & -h_2 X & h_{12} Z_2 & -h_{12} X \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & 0 & h_2 A_\tau^T X & 0 & h_{12} A_\tau^T X \\ * & * & * & \Phi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -h_2 Z_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -h_2 X & 0 & 0 \\ * & * & * & * & * & * & * & -h_{12} Z_2 & 0 \\ * & * & * & * & * & * & * & * & -h_{12} X \end{bmatrix} < 0, \quad (60)$$

where

$$\Phi_{11} = X(A+BK) + (A+BK)^T X + \sum_{i=1}^3 Q_i - h_2^{-1} E^T Z_1 E,$$

$$\Phi_{12} = E^T P + S_1 R_1^T - X + (A+BK)^T X,$$

$$\Phi_{13} = XA_\tau + h_2^{-1} E^T Z_1 E, \quad \Phi_{17} = h_2(A+BK)^T X,$$

$$\Phi_{19} = h_{12}(A+BK)^T X, \quad \Phi_{22} = -2X + 3\alpha I,$$

$$\Phi_{33} = -(1-\mu)Q_3 - h_2^{-1} E^T Z_1 E - h_{12}^{-1} E^T (Z_1 + Z_2) E$$

$$- h_{12}^{-1} E^T Z_2 E,$$

$$\Phi_{34} = h_{12}^{-1} E^T Z_2 E, \quad \Phi_{35} = h_{12}^{-1} E^T (Z_1 + Z_2) E,$$

$$\Phi_{44} = -Q_1 - h_{12}^{-1} E^T Z_2 E, \quad \Phi_{55} = -Q_2 - h_{12}^{-1} E^T (Z_1 + Z_2) E. \quad (61)$$

Note that the pairs $(E, (A+BK))$ and $(E, (A+BK+A_\tau))$ are regular, causal if and only if the pairs $(E^T, (A+BK)^T)$ and $(E^T, (A+BK+A_\tau)^T)$ are regular, causal. Furthermore, $\det(sE - (A+BK) - e^{-\tau(t)s} A_\tau) = \det(sE^T - (A+BK)^T - e^{-\tau(t)s} A_\tau^T)$. Then, as long as the regularity, causality, and stability problems are concerned, we can consider the following system instead of (56):

$$E^T \dot{\zeta}(t) = (A+BK)^T \zeta(t) + A_\tau^T \zeta(t - \tau(t)), \quad (62)$$

where $\zeta(t) \in \mathbf{R}^n$ is the state vector.

In this sense, (53) can be obtained by replacing E , $(A+BK)$, A_τ in (60) by E^T , $(A+BK)^T$, A_τ^T , respectively, and introducing a matrix $U = KX$. $\square$

The robust stabilizability result for uncertain singular system (2) is presented in the following theorem.

Theorem 17. Given scalars $0 \leq h_1 < h_2$ and μ , if there exist scalar $\alpha > 0$, positive-definite matrices P , Q_i , $i = 1, 2, 3$, Z_j , $j = 1, 2$, X , and matrices R , S , U with appropriate dimensions and scalars $\eta_1 > 0$, $\eta_2 > 0$ such that

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & 0 & 0 & 0 & \Psi_{17} & 0 & \Psi_{19} & \Psi_{110} & \eta_1 D & \Psi_{112} & 0 \\ * & \Psi_{22} & XA_\tau^T & 0 & 0 & h_2 Z_1 & -h_2 X & h_{12} Z_2 & \Psi_{29} & \Psi_{210} & 0 & \Psi_{212} & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & 0 & h_2 A_\tau^T X & 0 & \Psi_{39} & 0 & 0 & 0 & \eta_2 D \\ * & * & * & \Psi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -h_2 Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -h_2 X & 0 & 0 & \Psi_{710} & 0 & \Psi_{712} & 0 \\ * & * & * & * & * & * & * & -h_{12} Z_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -h_{12} X & \Psi_{910} & 0 & \Psi_{912} & 0 \\ * & * & * & * & * & * & * & * & * & -\eta_1 I & \eta_1 J & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\eta_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\eta_2 I & \eta_2 J \\ * & * & * & * & * & * & * & * & * & * & * & * & -\eta_2 I \end{bmatrix} < 0, \quad (63)$$

where $\Psi_{11}, \Psi_{12}, \Psi_{13}, \Psi_{17}, \Psi_{19}, \Psi_{22}, \Psi_{33}, \Psi_{34}, \Psi_{35}, \Psi_{44}$, and Ψ_{55} are defined in (53), $\Psi_{29} = -h_{12}X$, $\Psi_{110} = \Psi_{210} = XE_a^T + U^T E_b^T$, $\Psi_{112} = \Psi_{212} = XE_\tau^T$, $\Psi_{39} = h_{12}A_\tau X$, $\Psi_{710} = h_2(XE_a^T + U^T E_b^T)$, $\Psi_{910} = h_{12}(XE_a^T + U^T E_b^T)$, $\Psi_{712} = h_2XE_\tau^T$, $\Psi_{912} = h_{12}XE_\tau^T$, and $R_1 \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column rank and satisfying $E^T R_1 = 0$; then there exists a state feedback controller (7) such that the resulting closed-loop system of system (2) is regular, impulse free, and robustly asymptotically stable. In this case, a suitable controller gain is given by

$$u(t) = UX^{-1}x(t). \quad (64)$$

Proof. Replacing A by $A + D\Delta(t)E_a$, A_τ by $A_\tau + D\Delta(t)E_\tau$ and B by $B + D\Delta(t)E_b$ in (53), respectively, results in the following condition:

$$\Psi + \Gamma_1 \Delta(t) \Xi_1 + \Xi_1^T \Delta^T(t) \Gamma_1^T + \Gamma_2 \Delta(t) \Xi_2 + \Xi_2^T \Delta^T(t) \Gamma_2^T < 0, \quad (65)$$

where

$$\begin{aligned} \Gamma_1 &= [D^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\ \Xi_1 &= [E_a X + E_b U \ E_a X + E_b U \ 0 \ 0 \ 0 \ 0 \ h_2(E_a X + E_b U) \ 0 \ h_{12}(E_a X + E_b U)], \\ \Gamma_2 &= [0 \ 0 \ D^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\ \Xi_2 &= [E_\tau U \ E_\tau U \ 0 \ 0 \ 0 \ 0 \ h_2 E_\tau X \ 0 \ h_{12} E_\tau X], \end{aligned} \quad (66)$$

and Ψ is defined in (53).

By Lemma 9, (65) holds for $\Delta(t)$ satisfying (3), if there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that

$$\begin{aligned} \Psi + [\varepsilon_1^{-1} \Xi_1^T \ \varepsilon_1 \Gamma_1] \begin{bmatrix} I & -J \\ -J^T & I \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_1^{-1} \Xi_1 \\ \varepsilon_1 \Gamma_1^T \end{bmatrix} \\ + [\varepsilon_2^{-1} \Xi_2^T \ \varepsilon_2 \Gamma_2] \begin{bmatrix} I & -J \\ -J^T & I \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_2^{-1} \Xi_2 \\ \varepsilon_2 \Gamma_2^T \end{bmatrix} < 0. \end{aligned} \quad (67)$$

Suppose (63) to be true. Let $\varepsilon_i = \sqrt{\eta_i}$, $i = 1, 2$. Pre- and postmultiplying the left-hand side matrix of (63) by $\text{diag}\{I, I, I, I, I, \varepsilon_1^{-1}, \varepsilon_1^{-1}, \varepsilon_2^{-1}, \varepsilon_2^{-1}\}$ and its transpose, respectively, and using Schur complement equivalence to (63) yields (67). The result then follows from Theorem 16. $\square$

5. Numerical Examples

In this section, some examples are provided to illustrate the benefits of our results.

Example 1. Consider the nominal unforced part of system (2) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix}, \quad A_\tau = \begin{bmatrix} -1.1 & 1 \\ 0 & 0.5 \end{bmatrix}. \quad (68)$$

The case for $h_1 = 0$ and $\mu = 0$ has been studied in [12]. We choose $R = [0 \ 1]^T$. The comparison among Corollary 14 in this note and those in [6–9, 11, 12, 19] is listed in Table 1 for $h_1 = 0$. It should be pointed out that the results of [7] fail to deal with the system because the matrix describing the relationship between the fast and slow variables cannot be chosen beforehand. It can be seen that our method is less conservative than those in [6, 9] and gives the same results as that in [8, 11, 12, 19]. However, when the time delay is

a varying delay, our method gives better results than that in [19] for $\mu = 0.3$ and $\mu = 0.75$ since the relationship among h_2 , $\tau(t)$, $h_2 - \tau(t)$, and $\tau(t) - h_1$ has been taken into account in Corollary 14. On the other hand, for system with time-varying delay in a range, Table 1 also lists the allowable maximum upper bounds of h_2 for different μ with $h_1 = 0.3$ and $h_1 = 0.6$.

Example 2. Consider the unforced part of uncertain system (2) with

$$\begin{aligned} E &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.4 & 0.2 \\ 0 & -1 \end{bmatrix}, \\ A_\tau &= \begin{bmatrix} -1 & 1 \\ 0 & 0.3 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ E_a &= \begin{bmatrix} -0.05 & 0 \\ 0 & 0.05 \end{bmatrix}, \quad E_\tau = \begin{bmatrix} 0.05 & 0 \\ 0 & -0.05 \end{bmatrix}. \end{aligned} \quad (69)$$

We choose $R = [0 \ 1]^T$. For $h_1 = 0$, the comparison among Theorem 15 in this note and that in [19] is listed in Table 2. It is clear that the result in this note is better than that in [19]. The corresponding maximum upper bounds of h_2 for different h_1 and μ derived by Theorem 15 are also listed in Table 2.

Example 3. Consider system (2) with

$$\begin{aligned} E &= \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1.5 & 0.5 & 1 \\ -1 & 0 & 1 \\ 0.5 & 0 & 1 \end{bmatrix}, \\ A_\tau &= \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0.5 \\ 0.3 & 0.5 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \end{aligned}$$

TABLE 1: Allowable upper bounds of h_2 with given h_1 for different μ .

h_1	Methods	$\mu = 0$	$\mu = 0.3$	$\mu = 0.75$
0	[7]	—	—	—
	[9]	0.5567	—	—
	[6]	0.9680	—	—
	[8, 11, 12]	1.0660	—	—
	[19]	1.0660	1.0130	0.6496
	Corollary 14	1.0660	1.0263	0.7141
0.3	Theorem 10	1.0660	1.0315	0.7376
0.6	Theorem 10	1.0660	1.0421	0.7426

TABLE 2: Allowable upper bounds of h_2 with given h_1 for different μ .

h_1	Methods	$\mu = 0$	$\mu = 0.3$	$\mu = 0.75$
0	[19]	1.1836	1.1217	1.0307
	Theorem 15 ($J = 0$)	1.1836	1.1348	1.1095
0.3	Theorem 15 ($J = 0$)	1.1836	1.1394	1.1197
	Theorem 15 ($J = 0.1$)	1.1842	1.1400	1.1200
0.6	Theorem 15 ($J = 0$)	1.1836	1.1489	1.1317
	Theorem 15 ($J = 0.1$)	1.1842	1.1494	1.1321

TABLE 3: Allowable upper bounds of h_2 with given h_1 .

h_1	Methods	h_2	K
0	Theorem 17 ($J = -0.1$)	0.9062	$\begin{bmatrix} -0.5515 & -0.1800 & -0.7849 \\ -1.0962 & -0.7997 & -0.2503 \end{bmatrix}$
	Theorem 17 ($J = -0.1$)	1.0482	$\begin{bmatrix} -0.5845 & -0.1547 & -0.8065 \\ -1.1257 & -0.7953 & -0.2363 \end{bmatrix}$
0.6	Theorem 17 ($J = -0.1$)	1.1919	$\begin{bmatrix} -0.6093 & -0.1360 & -0.8218 \\ -1.1582 & -0.7942 & -0.2136 \end{bmatrix}$

$$D = [0.1 \ 0.1 \ 0.1]^T, \quad E_a = [0.2 \ 0.1 \ 0.3],$$

$$E_\tau = [0.1 \ 0.2 \ 0.1], \quad E_b = [0.1 \ 0.1], \quad \mu = 0.1. \quad (70)$$

We choose $R = [-1 \ 1 \ 2]^T$. According to Theorem 17, Table 3 shows the allowed maximum upper bounds of h_2 and the corresponding state feedback gain K for different h_1 .

6. Conclusions

In this note, the delay-range-dependent robust stability and stabilization for singular time-delay systems with linear fractional uncertainty and time-varying delay in a range are studied. The results are obtained by using the strict LMI approach and constructing an appropriate Lyapunov-Krasovskii functional. Numerical examples have been given to demonstrate the effectiveness of the presented criteria and their improvement over existing results.

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