Research Article

Modeling and Output Feedback Control of Networked Control Systems with Both Time Delays; and Packet Dropouts

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This paper is concerned with the problem of modeling and output feedback controller design for a class of discrete-time networked control systems (NCSs) with time delays and packet dropouts. A Markovian jumping method is proposed to deal with random time delays and packet dropouts. Different from the previous studies on the issue, the characteristics of networked communication delays and packet dropouts can be truly reflected by the unified model; namely, both sensor-to-controller (S-C) and controller-to-actuator (C-A) time delays, and packet dropouts are modeled and their history behavior is described by multiple Markov chains. The resulting closed-loop system is described by a new Markovian jump linear system (MJLS) with Markov delays model. Based on Lyapunov stability theory and linear matrix inequality (LMI) method, sufficient conditions of the stochastic stability and output feedback controller design method for NCSs with random time delays and packet dropouts are presented. A numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Networked control systems (NCSs) are systems in which control loops are closed over a real-time communication network. The fact that controllers, sensors, and actuators are not connected through point-to-point connections but through a multipurpose network offers advantages, such as low cost, decreased wiring, ease of installation and maintenance, and high efficiency, reliability, and flexibility [1–5]. Consequently, NCSs are applied in a broad range such as manufacturing plants, vehicles, aircrafts, spacecrafts, and remote surgery. However, incorporation of a network in control loops results in various constraints such as time delays and packet dropouts, due to limited bandwidth, quantization errors caused by hybrid nature of NCSs, variable sampling or transmission intervals due to multiple nodes, clock asynchronization among local and remote nodes, network security

and safety, and network security due to shared communication networks [6–8]. It is generally known that any of these networked-induced communication imperfections and constraints can degrade closed-loop performance or, even worse, can harm closed-loop stability of NCSs. Therefore, it is important to know how these effects influence the stability properties.

Recently, systematic approaches to analyze stability of NCSs subject to only one of these networked-induced imperfections are well developed. For instance, the effects of packet dropouts are studied in [9–14], and of time delays in [15–24], of quantization in [7, 25–28], of time-varying transmission intervals and communication constraints in [29–31] and [32–35], respectively. However, in NCSs all the aforementioned limitations and constraints are present simultaneously. Unfortunately, most of the results are available that study single problem of these imperfections. In this paper, we will

focus on the modeling and stability of NCSs with random time delays and packet dropouts.

In fact, packet dropout and time delay are both important issues that could severely destabilize NCSs. To study these issues, much research has been done on the effect of packet dropout and time delay on NCSs. As to the packet dropout, many approaches have been developed. Stabilization problem of NCSs with both arbitrary and Markovian packet losses is considered in [9]. The H_∞ control problem for NCSs with packet dropouts is studied in [10, 13]. A new NCSs model is considered for NCSs with single- and multiple-packet transmissions in [11]. The control problem for NCSs with Bernoulli-distributed stochastic packet losses is revisited in [14]. As to the time delay, great efforts have been made on stability analysis and controller design for NCSs with interval time-varying delay, uncertain time delay, large time delay, or random time delay, as presented in [15–23] (for more details, please refer to the literature therein).

So far, most of the results considered the network-induced delay and data packet loss separately, while in practice, the time delay and packet dropout are present simultaneously in NCSs. Fewer results are available that study combinations of time delay and packet dropout in the literature [36–42]. Furthermore, in [36–38], the switched system method is studied for NCSs with both delay and packet dropout. The two-mode-dependent state feedback controller design for NCSs with time delays and packet dropouts is addressed by augmenting the state variable approach in [39]. In [40] the robust H_∞ filtering problem for a class of uncertain nonlinear networked systems with both stochastic time-varying communication delays and packet dropouts is investigated. In [41] the problem of H_∞ controller design for NCSs with time delay and packet dropout by applying the linear estimation-based time delay and packet dropout compensation method is considered. The receding horizon H_∞ control problem for a class of NCSs with random delay and packet disordering is investigated by using the receding optimization principle in [42]. To the best of the authors' knowledge, up to now, little attention has been paid to the study of unified time delays and packet dropouts model and output feedback controller design for NCSs based on Markovian jump linear system (MJLS) with Markovian time delays model, which motivates the study of this paper.

In this paper, we address the unified model, stability analysis, and output feedback controller design of NCSs with the sensor-to-controller (S-C) and controller-to-actuator (C-A) random time delays and packet dropouts under an MJLS [43] framework. The present paper involves three contributions compared to the previous relevant works. The first is in the unified new model of NCSs with random network time delays and packet dropouts. The second one is that the Lyapunov functional method is used for stochastic stability analysis for NCSs. Compared with the extended matrix method, the Lyapunov functional avoids augmenting the dimension of the closed-loop system state, thus avoiding the complicated computation and conservatism caused by extended matrix method. The third one is the output feedback controller which is used to stabilize the closed-loop system, and the

FIGURE 1: The structure of the NCS with random delays and/or packet dropouts.

results are applied to a classical angular positioning system to illustrate the effectiveness of the approach.

Notations. In the sequel, matrices are assumed to have appropriate dimensions. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The notations $A > 0$ and $A < 0$ are used to denote a positive and negative definite matrices, respectively. $\text{diag}(A_1, \dots, A_n)$ refers to an $n \times n$ diagonal matrix with A_i as its i th diagonal entry. I and 0 denote identity matrix and zero matrix with appropriate dimensions, respectively. The superscript T denotes the transpose for vectors or matrices. $\mathbb{E}[\cdot]$ denotes the mathematical expectation operator. The symbol $*$ within a matrix represents the symmetric entries.

2. Problem Description

In this paper, we consider the following linear discrete-time system:

$$\begin{aligned} x_p((k+1)T) &= Ax_p(kT) + Bu(kT), \\ y(kT) &= Cx_p(kT), \end{aligned} \quad (1)$$

where $x_p(kT) \in \mathbb{R}^n$ is the system state, $u(kT) \in \mathbb{R}^m$ is the control input, and $y(kT) \in \mathbb{R}^p$ is the measurable output. kT and T are the sampling instant and the sampling period of the sensor. A , B , and C are known real constant matrices with appropriate dimensions. In this paper, for simplicity of presentation, kT is denoted by k .

Considering the same assumption in [16], the sensor, the controller, and the actuator are time driven and are connected over a network medium. The framework of the system over a network medium is depicted in Figure 1. The holder is chosen as zero-order hold (ZOH) holder. Under these assumptions, it is known that the controller and actuator update at the instant k only will always use the most recent data; otherwise, it will maintain the old data. In the NCS as in Figure 1, network-induced time delays and packet dropouts exist in the communication links between the S-C and C-A.

Random time delays exist in the S-C and C-A, as shown in Figure 1. Random but bounded scalar $d(k) \geq 0$ and $\tau(k) \geq 0$ are the random S-C and C-A time delays, respectively. It is assumed that both $d(k)$ and $\tau(k)$ are bounded, which are $0 \leq \underline{d} \leq d(k) \leq \bar{d}$, $0 \leq \underline{\tau} \leq \tau(k) \leq \bar{\tau}$, where $\underline{d} = \min\{d(k)\}$,

$\bar{d} = \max\{d(k)\}$, $\underline{\tau} = \min\{\tau(k)\}$, and $\bar{\tau} = \max\{\tau(k)\}$. One way to model the delays $d(k)$ and $\tau(k)$ is using the finite state Markov chains as shown in [19, 44–46]. The main advantages of the Markov model is as follows: the dependency between delays is taken into account since in real networks the current time delays are usually related with the previous delays [45]. In this paper $d(k)$ and $\tau(k)$ are modeled as two homogeneous Markov chains that take value in $\bar{S}_2 = \{0, 1, \dots, s_2\}$ and $\bar{S}_3 = \{0, 1, \dots, s_3\}$. $\lambda = \{\lambda_{mn}\}$ and $\gamma = \{\gamma_{qp}\}$ denote transition probability matrices of $d(k)$ and $\tau(k)$, respectively, with probabilities λ_{mn} and π_{ij} , which are defined in (7).

In Figure 1, S_1 and S_2 denote the network switches between the S-C and C-A, respectively. α and β ($\alpha = 0, 1$; $\beta = 0, 1$) denote the states of S_1 and S_2 . When $S_1(S_2)$ is in state $\alpha = 0$ ($\beta = 0$), the packet is received successfully and the $\bar{y}(k) = y(k - d(k))(u(k) = \bar{u}(k - \tau(k)))$. Whereas when $S_1(S_2)$ is in state $\alpha = 1$ ($\beta = 1$), the packet is lost and the switch output (control input) is held at the previous value $\bar{y}(k) = \bar{y}(k - 1)(u(k) = u(k - 1))$. The behavior of the S-C and C-A time delays and packet dropouts can be modeled as

$$\begin{aligned}\bar{y}(k) &= (1 - \alpha) y(k - d(k)) + \alpha \bar{y}(k - 1), \\ u(k) &= (1 - \beta) \bar{u}(k - \tau(k)) + \beta u(k - 1),\end{aligned}\quad (2)$$

where

$$\begin{aligned}\alpha &= \begin{cases} 0, & \text{if } S_1 \text{ is closed and the packet is received} \\ 1, & \text{otherwise,} \end{cases} \\ \beta &= \begin{cases} 0, & \text{if } S_2 \text{ is closed the packet is received} \\ 1, & \text{otherwise.} \end{cases}\end{aligned}\quad (3)$$

From the above analysis, the random data packet dropouts in S-C and C-A can be modeled a discrete-time homogeneous Markov chain $\mu(k)$ with four modes. Four modes of Markov chain $\mu(k) = 1$, $\mu(k) = 2$, $\mu(k) = 3$, and $\mu(k) = 4$ are corresponding to four states ($\alpha = 0, \beta = 0$), ($\alpha = 0, \beta = 1$), ($\alpha = 1, \beta = 0$), and ($\alpha = 1, \beta = 1$) of network switches S_1 and S_2 .

This paper studies the stability of system (1) under an output feedback controller; that is,

$$\bar{u}(k) = K \bar{y}(k), \quad (4)$$

where K is the output feedback controller gain.

Remark 1. There are two ways to design the controller (4). One is adopting a mode-independent controller, and another one is using a mode-dependent controller (the controller gain dependent on the information of time delays and packet losses). However, the mode-independent controller (4) can avoid the controller gain switching in high frequency following the multimodes of Markov chains $\mu(k)$, $\tau(k)$, and $d(k)$ in system (5). Hence, in this paper, we choose the mode-independent controller.

Let $x(k) = [x_p^T(k) \ \bar{y}^T(k-1) \ u^T(k-1)]^T$ be the augmented state vector. Applying the controller (4) to (1) and (2), we can

obtain the closed-loop system for the NCS with random time delays and packet dropouts in Figure 1 as

$$\begin{aligned}x(k+1) &= \bar{A}(\mu(k)) x(k) + \sum_{s=1}^3 \bar{B}_s(\mu(k)) H_s x(k - \eta_s(k)), \\ x(k) &= \varphi(k), \quad k = -\bar{d} - \bar{\tau}, -\bar{d} - \bar{\tau} + 1, \dots, 0,\end{aligned}\quad (5)$$

where

$$\begin{aligned}\bar{A}(\mu(k)) &= \begin{bmatrix} A & 0 & \beta B \\ 0 & \alpha I & 0 \\ 0 & 0 & \beta I \end{bmatrix}, \\ \bar{B}_1(\mu(k)) &= \begin{bmatrix} 0 \\ (1 - \alpha) C \\ 0 \end{bmatrix}, \\ \bar{B}_2(\mu(k)) &= \bar{B}_2(\mu(k)) K, \\ \bar{B}_3(\mu(k)) &= \bar{B}_3(\mu(k)) K C, \\ \bar{B}_2(\mu(k)) &= \begin{bmatrix} \alpha(1 - \beta) B \\ 0 \\ \alpha(1 - \beta) I \end{bmatrix}, \\ \bar{B}_3(\mu(k)) &= \begin{bmatrix} (1 - \alpha)(1 - \beta) B \\ 0 \\ (1 - \alpha)(1 - \beta) I \end{bmatrix}, \\ H_1 &= [I \ 0 \ 0], \quad H_2 = [0 \ I \ 0], \\ H_3 &= [I \ 0 \ 0], \\ \eta_1(k) &= d(k), \quad \eta_2(k) = \tau(k), \\ \eta_3(k) &= d(k - \tau(k)) + \tau(k) \approx d(k) + \tau(k), \\ \bar{d} &= \max\{d(k)\}, \quad \underline{d} = \min\{d(k)\}, \\ \bar{\tau} &= \max\{\tau(k)\}, \quad \underline{\tau} = \min\{\tau(k)\},\end{aligned}\quad (6)$$

$\varphi(k)$, $k = -\bar{d} - \bar{\tau}, -\bar{d} - \bar{\tau} + 1, \dots, 0$ is the system initial value.

Remark 2. In (5), we assume that delay steps $\eta_3(k) = d(k - \tau(k)) + \tau(k) \approx d(k) + \tau(k)$. There are three reasons to support the ideal hypothesis: the first one is that $d(k)$ and $d(k - \tau(k))$ take values in the same finite set $\bar{S}_2 = \{0, 1, \dots, d_{\max}\}$; the second one is that $\eta_3(k) = d(k - \tau(k)) + \tau(k)$ and $d(k) + \tau(k)$ take values in the same finite set $\{0, 1, 2, \dots, d_{\max} + \tau_{\max}\}$; and the third one is that for system (5), $\eta_3(k)$ denotes the delay steps of system state $x(k)$; using $(d(k) + \tau(k))$ approximation equal to $(d(k - \tau(k)) + \tau(k))$ will not affect the performance of system (5). Therefore, $\eta_3(k) = d(k - \tau(k)) + \tau(k)$ will be replaced by $\eta_3(k) = d(k) + \tau(k)$ in the following.

In system (5), $\{\mu(k), k \in \mathbb{Z}\}$, $\{d(k), k \in \mathbb{Z}\}$ and $\{\tau(k), k \in \mathbb{Z}\}$, are three finite state discrete-time homogeneous Markov chains. The three Markov chains take values in the finite sets

$\bar{S}_1 = \{1, 2, \dots, s_1\}$, $\bar{S}_2 = \{0, 1, \dots, s_2\}$, and $\bar{S}_3 = \{0, 1, \dots, s_3\}$ with transition probabilities

$$\begin{aligned} \Pr\{\mu(k+1) = j \mid \mu(k) = i\} &= \pi_{ij}, & \pi_i &= \Pr(\pi_0 = i), \\ \Pr\{d(k+1) = n \mid d(k) = m\} &= \lambda_{mn}, & \lambda_m &= \Pr(\lambda_0 = m), \\ \Pr\{\tau(k+1) = q \mid \tau(k) = p\} &= \gamma_{pq}, & \gamma_p &= \Pr(\gamma_0 = p), \end{aligned} \quad (7)$$

where $\pi_{ij} \geq 0$, $\lambda_{mn} \geq 0$, and $\gamma_{pq} \geq 0$ for all $i, j \in \bar{S}_1$, $m, n \in \bar{S}_2$, $p, q \in \bar{S}_3$ and

$$\begin{aligned} \sum_{j=1}^{s_1} \pi_{ij} &= 1, & \sum_{n=0}^{s_2} \lambda_{mn} &= 1, \\ \sum_{q=0}^{s_3} \gamma_{pq} &= 1. \end{aligned} \quad (8)$$

For $\mu(k) = i$, $i \in \bar{S}_1$, when $i = 1$, $i = 2$, $i = 3$, and $i = 4$, the α and β in (5) take values $(\alpha = 0, \beta = 0)$, $(\alpha = 0, \beta = 1)$, $(\alpha = 1, \beta = 0)$, and $(\alpha = 1, \beta = 1)$, respectively. $\bar{A}(\mu(k))$, $\bar{B}_1(\mu(k))$, $\bar{B}_2(\mu(k))$, and $\bar{B}_3(\mu(k))$ are known constant matrices of appropriate dimensions.

Remark 3. The closed-loop system (5) is a Markovian jump linear system with multiple Markov chains, which describe the behavior of the S-C and C-A time delays and packet dropouts, respectively. This enables us to analyze and synthesize such NCSs by applying Markovian jump linear system theory. Note that the problem of unified modeling and output feedback control for NCSs with both S-C and C-A time delays and packet dropouts modeled by multiple Markov chains has not been done in the literature.

Definition 4 (see [11, 47]). System (5) is stochastically stable if there exists a constant C such that

$$\begin{aligned} \sum_{k=0}^{\infty} \mathbb{E}[\|x(k)\|^2 \mid x_0, \mu_0, d_0, \tau_0] \\ \leq CY(x_0, \varphi_i, -d_{\max} - \tau_{\max} \\ \leq i \leq 0, \mu_0, d_0, \tau_0), \end{aligned} \quad (9)$$

where $Y(x_0, \varphi_i, -\bar{d} - \bar{\tau} \leq i \leq 0, \mu_0, d_0, \tau_0)$ is a nonnegative function and the system initial values satisfy $Y(0, 0, \dots, 0) = 0$.

3. Main Results

By applying a new Lyapunov functional, sufficient conditions for the stochastic stability, the synthesis of controller design for system (5) will be established in this section.

Theorem 5. For system (5), given random but bounded scalar $d(k) \in [\underline{d}, \bar{d}]$ and $\tau(k) \in [\underline{\tau}, \bar{\tau}]$, if for each mode $i \in \bar{S}_1$, $m \in \bar{S}_2$, and $p \in \bar{S}_3$, there exist matrices $P_{i,m,p} > 0$, $Q_{1s} > 0$, $Q_{2s} > 0$,

$Q_{3s} > 0$, $R_{1s} > 0$, and $R_{2s} > 0$, $s = 1, 2, 3$, such that the following matrix inequalities:

$$\begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 \\ * & -\bar{P}_{i,m,p} & 0 & 0 \\ * & * & -\bar{R}_1 & 0 \\ * & * & * & -\bar{R}_2 \end{bmatrix} < 0, \quad (10)$$

where

$$\Xi_1 = \begin{bmatrix} \Pi_{i,m,p} & \Omega_{12} & 0 & 0 \\ * & -\bar{Q}_3 - \Omega_{22} - 2\Psi & \Psi & \Omega_{24} + \Psi \\ * & * & -\bar{Q}_2 - \Psi & 0 \\ * & * & * & -\bar{Q}_1 - \Omega_{44} - \Psi \end{bmatrix},$$

$$\Xi_2 = [\bar{P}_{i,m,p} \bar{A}_i \quad \bar{P}_{i,m,p} \Lambda \quad 0 \quad 0]^T,$$

$$\Xi_3 = [\Xi_{31} \quad \Xi_{32} \quad \Xi_{33}], \quad \Xi_4 = [\Xi_{41} \quad \Xi_{42} \quad \Xi_{43}],$$

$$\Xi_{31} = [\bar{t}_1 R_{11} H_1 (\bar{A}_i - I) \quad \bar{t}_1 R_{11} \bar{\Lambda} \quad 0 \quad 0]^T,$$

$$\Xi_{32} = [\bar{t}_2 R_{12} H_2 (\bar{A}_i - I) \quad \bar{t}_2 R_{12} \bar{\Lambda} \quad 0 \quad 0]^T, k$$

$$\Xi_{33} = [\bar{t}_3 R_{13} H_3 (\bar{A}_i - I) \quad \bar{t}_3 R_{13} \bar{\Lambda} \quad 0 \quad 0]^T,$$

$$\Xi_{41} = [(\bar{t}_1 - \underline{t}_1) R_{21} H_1 (\bar{A}_i - I) \quad (\bar{t}_1 - \underline{t}_1) R_{21} \bar{\Lambda} \quad 0 \quad 0]^T,$$

$$\Xi_{42} = [(\bar{t}_2 - \underline{t}_2) R_{22} H_2 (\bar{A}_i - I) \quad (\bar{t}_2 - \underline{t}_2) R_{22} \bar{\Lambda} \quad 0 \quad 0]^T,$$

$$\Xi_{43} = [(\bar{t}_3 - \underline{t}_3) R_{23} H_3 (\bar{A}_i - I) \quad (\bar{t}_3 - \underline{t}_3) R_{23} \bar{\Lambda} \quad 0 \quad 0]^T,$$

$$\bar{P}_{i,m,p} = \sum_{j=1}^{s_1} \sum_{n=0}^{s_2} \sum_{q=0}^{s_3} \pi_{ij} \lambda_{mn} \gamma_{pq} P_{j,n,q},$$

$$\bar{R}_1 = \text{diag}\{R_{11}, R_{12}, R_{13}\}, \quad \bar{R}_2 = \text{diag}\{R_{21}, R_{22}, R_{23}\},$$

$$\Pi_{i,m,p} = -P_{i,m,p}$$

$$\begin{aligned} &+ \sum_{s=1}^3 [H_s^T (Q_{1s} + Q_{2s} + (\bar{t}_s - \underline{t}_s + 1) Q_{3s}) H_s] \\ &- \Omega_{11}, \end{aligned}$$

$$\Omega_{11} = \sum_{s=1}^3 H_s^T R_{1s} H_s, \quad \Omega_{12} = [H_1^T R_{11} \quad H_2^T R_{12} \quad H_3^T R_{13}],$$

$$\Omega_{22} = \text{diag}\{2R_{11}, 2R_{12}, 2R_{13}\},$$

$$\Omega_{24} = \Omega_{44} = \text{diag}\{R_{11}, R_{12}, R_{13}\},$$

$$\Psi = \text{diag}\{R_{21}, R_{22}, R_{23}\}, \quad \bar{Q}_1 = \text{diag}\{Q_{11}, Q_{12}, Q_{13}\},$$

$$\bar{Q}_2 = \text{diag}\{Q_{21}, Q_{22}, Q_{23}\}, \quad \bar{Q}_3 = \text{diag}\{Q_{31}, Q_{32}, Q_{33}\},$$

$$\Lambda = [\bar{B}_{i1} \quad \bar{B}_{i2} \quad \bar{B}_{i3}], \quad \bar{\Lambda} = [H_1 \bar{B}_{i1} \quad H_2 \bar{B}_{i2} \quad H_3 \bar{B}_{i3}],$$

$$\bar{A}_i = \bar{A}(\mu(k)), \quad \bar{B}_{i1} = \bar{B}_1(\mu(k)),$$

$$\begin{aligned}
\bar{B}_{i2} &= \bar{B}_2(\mu(k)), \\
\bar{B}_{i3} &= \bar{B}_3(\mu(k)), \\
\bar{t}_1 &= \bar{d}, \quad \underline{t}_1 = \underline{d}, \quad \bar{t}_2 = \bar{\tau}, \quad \underline{t}_2 = \underline{\tau}, \\
\bar{t}_3 &= \bar{d} + \bar{\tau}, \quad \underline{t}_3 = \underline{d} + \underline{\tau},
\end{aligned} \tag{11}$$

$\bar{A}(\mu(k))$, $\bar{B}_1(\mu(k))$, $\bar{B}_2(\mu(k))$, and $\bar{B}_3(\mu(k))$ defined in (5) hold for all $i, j \in \bar{S}_1$, $m, n \in \bar{S}_2$, and $p, q \in \bar{S}_3$, then system (5) is stochastically stable.

Proof. It is given in the appendix. $\square$

Theorem 5 gives a sufficient condition for the stochastic stability of system (5). However, it should be noted that the conditions (10) are no more LMI conditions. To handle this, the equivalent LMI conditions are given in Theorem 6 by cone complementarity linearization (CCL) algorithm.

Theorem 6. Consider system (5) with random but bounded scalar $d(k) \in [\underline{d} \ \bar{d}]$ and $\tau(k) \in [\underline{\tau} \ \bar{\tau}]$. There exists an output feedback controller (4) such that the resulting closed-loop system is stochastically stable if for each mode $i \in \bar{S}_1$, $m \in \bar{S}_2$, and $p \in \bar{S}_3$, there exist matrices $P_{i,m,p} > 0$, $X_{i,m,p} > 0$, $Q_{1s} > 0$, $Q_{2s} > 0$, $Q_{3s} > 0$, $R_{1s} > 0$, $R_{2s} > 0$, $U_{1s} > 0$, and $U_{2s} > 0$, $s = 1, 2, 3$ and K such that

$$\begin{aligned}
\begin{bmatrix} \Xi_1 & \hat{\Xi}_2 & \hat{\Xi}_3 & \hat{\Xi}_4 \\ * & -X_{j,n,q} & 0 & 0 \\ * & * & -U_{1s} & 0 \\ * & * & * & -U_{2s} \end{bmatrix} < 0, \tag{12} \\
P_{i,m,p} X_{i,m,p} = I, \quad R_{1s} U_{1s} = I, \quad R_{2s} U_{2s} = I, \\
s = 1, 2, 3, \tag{13}
\end{aligned}$$

where

$$\begin{aligned}
\hat{\Xi}_2 &= [\mathcal{L}_{i,m,p} \bar{A}_i \ \mathcal{L}_{i,m,p} \Lambda \ 0 \ 0]^T, \\
\hat{\Xi}_3 &= [\hat{\Xi}_{31} \ \hat{\Xi}_{32} \ \hat{\Xi}_{33}], \quad \hat{\Xi}_4 = [\hat{\Xi}_{41} \ \hat{\Xi}_{42} \ \hat{\Xi}_{43}], \\
\hat{\Xi}_{31} &= [\bar{t}_1 H_1 (\bar{A}_i - I) \ \bar{t}_1 \bar{\Lambda} \ 0 \ 0]^T, \\
\hat{\Xi}_{32} &= [\bar{t}_2 H_2 (\bar{A}_i - I) \ \bar{t}_2 \bar{\Lambda} \ 0 \ 0]^T, \\
\hat{\Xi}_{33} &= [\bar{t}_3 H_3 (\bar{A}_i - I) \ \bar{t}_3 \bar{\Lambda} \ 0 \ 0]^T, \\
\hat{\Xi}_{41} &= [(\bar{t}_1 - \underline{t}_1) H_1 (\bar{A}_i - I) \ (\bar{t}_1 - \underline{t}_1) \bar{\Lambda} \ 0 \ 0]^T, \\
\hat{\Xi}_{42} &= [(\bar{t}_2 - \underline{t}_2) H_2 (\bar{A}_i - I) \ (\bar{t}_2 - \underline{t}_2) \bar{\Lambda} \ 0 \ 0]^T, \\
\hat{\Xi}_{43} &= [(\bar{t}_3 - \underline{t}_3) H_3 (\bar{A}_i - I) \ (\bar{t}_3 - \underline{t}_3) \bar{\Lambda} \ 0 \ 0]^T, \\
X_{j,n,q} &= \text{diag} \{ \underbrace{X_{1,0,0}, X_{1,0,1}, \dots, X_{j,n,q}, \dots, X_{s_1, s_2, s_3}}_{(s_1(1+s_2)(1+s_3)) \text{ items}} \},
\end{aligned}$$

$$\begin{aligned}
(\mathcal{L}_{i,m,p} \bar{A}_i)^T &= \underbrace{\begin{bmatrix} \sqrt{\pi_{i1} \lambda_{m0} \gamma_{p0}} \bar{A}_i^T & \sqrt{\pi_{i1} \lambda_{m0} \gamma_{p1}} \bar{A}_i^T & \dots \\ \sqrt{\pi_{ij} \lambda_{mn} \gamma_{pq}} \bar{A}_i^T & \dots & \sqrt{\pi_{is_1} \lambda_{ms_2} \gamma_{ps_3}} \bar{A}_i^T \end{bmatrix}}_{(s_1(1+s_2)(1+s_3)) \text{ items}}, \\
(\mathcal{L}_{i,m,p} \Lambda)^T &= \underbrace{\begin{bmatrix} \sqrt{\pi_{i1} \lambda_{m0} \gamma_{p0}} \Lambda^T & \sqrt{\pi_{i1} \lambda_{m0} \gamma_{p1}} \Lambda^T & \dots \\ \sqrt{\pi_{ij} \lambda_{mn} \gamma_{pq}} \Lambda^T & \dots & \sqrt{\pi_{is_1} \lambda_{ms_2} \gamma_{ps_3}} \Lambda^T \end{bmatrix}}_{(s_1(1+s_2)(1+s_3)) \text{ items}}, \tag{14}
\end{aligned}$$

Ξ_1 , $\bar{A}_i$, Λ , and $\bar{\Lambda}$ are defined in Theorem 5. Moreover, if (12) and (13) have solutions, the controller gain is given by K .

Proof. By Schur complement, (12) is equivalent to

$$\begin{bmatrix} \Xi_1 & \hat{\Xi}_2 & \hat{\Xi}_3 & \hat{\Xi}_4 \\ * & -P_{j,n,q}^{-1} & 0 & 0 \\ * & * & -R_{1s}^{-1} & 0 \\ * & * & * & -R_{2s}^{-1} \end{bmatrix} < 0, \quad s = 1, 2, 3. \tag{15}$$

Let $P_{j,n,q}^{-1} = X_{j,n,q}$, $R_{1s}^{-1} = U_{1s}$, and $R_{2s}^{-1} = U_{2s}$; then we can obtain (12) and (13). This completes the proof. $\square$

The conditions in Theorem 6 are a set of LMIs with some matrix inverse constraints. Although they are nonconvex, which prevents us from solving them using the existing convex optimization tool, we can use the CCL algorithm to transform this problem into the nonlinear minimization problem with LMI constraints as follows:

$$\min \text{Trace} \left(\sum_{s=1}^3 (R_{1s} U_{1s} + R_{2s} U_{2s}) + \sum_{j=1}^{s_1} \sum_{n=0}^{s_2} \sum_{q=0}^{s_3} P_{i,m,p} X_{i,m,p} \right), \tag{16}$$

$$\text{s.t.} \begin{cases} \text{(i)} \text{ (12)} \\ \text{(ii)} \begin{bmatrix} R_{1s} & I \\ * & U_{1s} \end{bmatrix} > 0, \quad s = \{1, 2, 3\}, \\ \text{(iii)} \begin{bmatrix} R_{2s} & I \\ * & U_{2s} \end{bmatrix} > 0, \quad s = \{1, 2, 3\}, \\ \text{(iv)} \begin{bmatrix} P_{i,m,p} & I \\ * & X_{i,m,p} \end{bmatrix} > 0, \quad i \in \bar{S}_1, m \in \bar{S}_2, p \in \bar{S}_3. \end{cases} \tag{17}$$

The above nonlinear minimization problem can be solved by the CCL algorithm presented in the following.

Algorithm 7. Consider the following steps.

Step 1. Find a feasible solution satisfying LMIs (i), (ii), (iii), and (iv) in (17), set as $(R_{1s}^0, U_{1s}^0, R_{2s}^0, U_{2s}^0, P_{i,m,p}^0, X_{i,m,p}^0)$, $s = \{1, 2, 3\}$ and $k = 0$.

Step 2. Solve the following LMI optimization problem for variables $(R_{1s}, U_{1s}, R_{2s}, U_{2s}, s = \{1, 2, 3\}, P_{i,m,p}, X_{i,m,p}, K^0)$: Minimize trace $\{\sum_{s=1}^3 ((R_{1s}^k U_{1s} + R_{1s} U_{1s}^k) + (R_{2s}^k U_{2s} + R_{2s} U_{2s}^k)) + \sum_{j=1}^{s_1} \sum_{n=0}^{s_2} \sum_{q=0}^{s_3} (P_{i,m,p}^k X_{i,m,p} + P_{i,m} X_{i,m,p}^k)\}$ subject to LMIs (17).

FIGURE 2: The angular positioning system.

FIGURE 3: Values of $\mu(k)$.

Set $R_{1s}^{k+1} = R_{1s}$, $U_{1s}^{k+1} = U_{1s}$, $R_{2s}^{k+1} = R_{2s}$, $U_{2s}^{k+1} = U_{2s}$, $s = \{1, 2, 3\}$, $P_{i,m,p}^{k+1} = P_{i,m,p}$, $X_{i,m,p}^{k+1} = X_{i,m,p}$, and $K^{k+1} = K$.

Step 3. If LMI (i) in (17) is satisfied, then exit the iteration. If LMI (i) in (17) is not satisfied, let $k = k + 1$ and then return to Step 2.

4. Numerical Example

In this section, a simulation example is given to illustrate the usefulness of the developed method. Consider a classical angular positioning system [42, 48, 49] in Figure 2, which consists of a rotating antenna at the origin of the plane, driven by an electric motor. Assume that the angular position of the antenna θ (rad), the angular position of the moving object θ_r (rad), and the angular velocity of the antenna $\dot{\theta}$ (rad s⁻¹) are measurable. The state variables are chosen as $[\theta \ \dot{\theta}]^T$ and the output is $y = \theta^T$. The control problem is to use the input voltage to the motor to rotate the antenna so that it always point in the direction of a moving object in the plant. The output feedback controller is designed for the following values of the matrices A , B , and C :

$$A = \begin{bmatrix} 1 & 0.0995 \\ 0 & 0.99 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0039 \\ 0.0783 \end{bmatrix}, \quad (18)$$

$$C = \begin{bmatrix} 1.4 & 0.8 \\ -0.2 & 0.4 \end{bmatrix}.$$

FIGURE 4: S-C random delays $d(k)$. FIGURE 5: C-A random delays $\tau(k)$.

The stochastic jumping parameter $\mu(k) \in \{1, 2, 3, 4\}$ and the random delays involved in system (5) which are $d(k) = m$, $m \in \{0, 1, 2\}$ and $\tau(k) = p$, $p \in \{0, 1\}$, and their transition matrices are given by

$$\pi = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.2 & 0.1 \\ 0.3 & 0.2 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.2 & 0.5 \end{bmatrix}, \quad \lambda = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.1 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}, \quad (19)$$

$$\gamma = \begin{bmatrix} 0.5 & 0.5 \\ 0.8 & 0.2 \end{bmatrix}.$$

Figures 3, 4, and 5 show part of the simulation of the stochastic jumping parameter $\mu(k)$, S-C delay $d(k)$, and C-A delay $\tau(k)$ governed by their corresponding transition probability matrices, respectively.

The initial value $x(0) = [0.01 \ 0]^T$. By Theorem 6, we can obtain the gain matrices K of controller (4) are constructed as $K = [-0.0088 \ -0.0117]$.

The state trajectories are shown in Figure 6, where two curves represent state trajectories under the controller gains K and Figure 6 indicates that system (5) is stochastically stable.

5. Conclusion

The output feedback stabilization problem for NCSs with both S-C and C-A random time delays and packet dropouts is investigated in this paper. The S-C and C-A random time delays and packet dropouts are modeled by independent

FIGURE 6: State trajectories under K .

multiple Markov chains. Then the resulting closed-loop system is modeled as a Markovian jump linear system with Markov delays. Sufficient conditions on stochastic stability and stabilization are obtained by Lyapunov stability theory and linear matrix inequalities method. The CCL algorithm is employed to obtain the output feedback controller. Finally, an example is presented to illustrate the effectiveness of the approach.

Appendix

A. Proof of Theorem 5

Proof. For the closed-loop system (5), construct a Lyapunov-Krasovskii functional as

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k), \quad (\text{A.1})$$

where

$$\begin{aligned} V_1(k) &= x(k)^T P(\mu(k), d(k), \tau(k)) x(k), \\ V_2(k) &= \sum_{s=1}^3 \sum_{l=k-\bar{t}_s}^{k-1} x(l)^T H_s^T Q_{1s} H_s x(l) \\ &\quad + \sum_{s=1}^3 \sum_{l=k-\underline{t}_s}^{k-1} x(l)^T H_s^T Q_{2s} H_s x(l), \\ V_3(k) &= \sum_{s=1}^3 \sum_{\theta=-\bar{\eta}_s+2}^{-\eta_s+1} \sum_{l=k+\theta-1}^{k-1} x(l)^T H_s^T Q_{3s} H_s x(l) \\ &\quad + \sum_{s=1}^3 \sum_{l=k-\eta_s(k)}^{k-1} x(l)^T H_s^T Q_{3s} H_s x(l), \end{aligned}$$

$$\begin{aligned} V_4(k) &= \sum_{s=1}^3 \sum_{\theta=-\bar{\eta}_s+1}^0 \sum_{l=k+\theta-1}^{k-1} \bar{\eta}_s \delta(l)^T H_s^T R_{1s} H_s \delta(l) \\ &\quad + \sum_{s=1}^3 \sum_{\theta=-\bar{\eta}_s+1}^{-\eta_s} \sum_{l=k+\theta-1}^{k-1} (\bar{\eta}_s - \underline{\eta}_s) \delta(l)^T H_s^T R_{2s} H_s \delta(l), \end{aligned} \quad (\text{A.2})$$

$\delta(l) = x(l+1) - x(l)$, $P(\mu(k), d(k), \tau(k))$, Q_{1s} , Q_{2s} , Q_{3s} , R_{1s} , $R_{2s} > 0$, $s = \{1, 2, 3\}$, $\bar{t}_1$, $\bar{t}_2$, $\bar{t}_3$, and $\underline{t}_3$ are defined in Theorem 5. In the following, when $\mu(k) = i$, $d(k) = m$, and $\tau(k) = p$, we will denote $P(\mu(k), d(k), \tau(k))$, $\bar{A}(\mu(k))$ and $\bar{B}_s(\mu(k))$ by $P_{i,m,p}$, $\bar{A}_i$ and $\bar{B}_{is}$, respectively.

$$\text{Let } \xi(k) = [x(k)^T \eta(k)^T \bar{\eta}(k)^T \underline{\eta}(k)^T]^T,$$

$$\begin{aligned} \eta(k) &= [(H_1 x(k-m))^T (H_2 x(k-p))^T (H_3 x(k-m-p))^T]^T, \\ \underline{\eta}(k) &= [(H_1 x(k-\underline{d}))^T (H_2 x(k-\underline{\tau}))^T (H_3 x(k-\underline{d}-\underline{\tau}))^T]^T, \\ \bar{\eta}(k) &= [(H_1 x(k-\bar{d}))^T (H_2 x(k-\bar{\tau}))^T (H_3 x(k-\bar{d}-\bar{\tau}))^T]^T. \end{aligned} \quad (\text{A.3})$$

Then, along the solution of system (5) we have

$$\begin{aligned} \mathbb{E} [\Delta V_1(k)] &= x(k+1)^T \left[\sum_{j=1}^{s_1} \sum_{n=0}^{s_2} \sum_{q=0}^{s_3} \pi_{ij} \lambda_{mn} \gamma_{pq} P_{j,n,q} \right] \\ &\quad \times x(k+1) - x^T(k) P_{i,m,p} x(k) \\ &= \xi^T(k) \begin{bmatrix} \bar{A}_i^T \\ \Lambda^T \\ 0 \\ 0 \end{bmatrix} \bar{P}_{i,m,p} [\bar{A}_i \quad \Lambda \quad 0 \quad 0] \\ &\quad \times \xi(k) - x^T(k) P_{i,m,p} x(k), \end{aligned} \quad (\text{A.4})$$

where $\bar{P}_{i,m,p}$ and Λ are defined in Theorem 5.

Consider that

$$\begin{aligned} \mathbb{E} [\Delta V_2(k)] &= \sum_{s=1}^3 \left((H_s x(k))^T (Q_{1s} + Q_{2s}) H_s x(k) \right. \\ &\quad - (H_s x(k-\bar{t}_s))^T Q_{1s} H_s x(k-\bar{t}_s) \\ &\quad \left. - (H_s x(k-\underline{t}_s))^T Q_{2s} H_s x(k-\underline{t}_s) \right), \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned}
\mathbb{E} [\Delta V_3(k)] &= \sum_{s=1}^3 \left((\bar{t}_s - \underline{t}_s + 1) x(k)^T H_s^T Q_{3s} H_s x(k) \right. \\
&\quad - \sum_{l=k-\bar{t}_s+1}^{k-\underline{t}_s} x(l)^T H_s^T Q_{3s} H_s x(l) \\
&\quad + \left(\sum_{l=k-\eta_s(k+1)+1}^{k-1} - \sum_{l=k-\eta_s(k)+1}^{k-1} \right) \\
&\quad \times x(l)^T H_s^T Q_{3s} H_s x(l) \\
&\quad - x(k - \eta_s(k))^T \\
&\quad \times H_s^T Q_{3s} H_s x(k - \eta_s(k)) \Big). \tag{A.6}
\end{aligned}$$

Note that

$$\begin{aligned}
&\sum_{s=1}^3 \sum_{l=k-\eta_s(k+1)+1}^{k-1} x^T(l) H_s^T Q_{3s} H_s x(l) \\
&= \sum_{s=1}^3 \left[\sum_{l=k-\underline{t}_s+1}^{k-1} + \sum_{l=k-\eta_s(k+1)+1}^{k-\underline{t}_s} \right] x^T(l) H_s^T Q_{3s} H_s x(l) \\
&\leq \sum_{s=1}^3 \left[\sum_{l=k-\eta_s(k)+1}^{k-1} + \sum_{l=k-\bar{t}_s+1}^{k-\underline{t}_s} \right] x^T(l) H_s^T Q_{3s} H_s x(l). \tag{A.7}
\end{aligned}$$

By combining (A.6) and (A.7), we have

$$\begin{aligned}
\mathbb{E} [\Delta V_3] &= \sum_{s=1}^3 \left((\bar{t}_s - \underline{t}_s + 1) x(k)^T H_s^T Q_{3s} H_s x(k) \right. \\
&\quad \left. - x(k - \eta_s(k))^T H_s^T Q_{3s} H_s x(k - \eta_s(k)) \right), \tag{A.8}
\end{aligned}$$

$$\begin{aligned}
\mathbb{E} [\Delta V_4] &= \sum_{s=1}^3 \left(\bar{t}_s^2 \delta^T(k) H_s^T R_{1s} H_s \delta(k) \right. \\
&\quad - \sum_{l=k-\bar{t}_s}^{k-1} \bar{t}_s \delta^T(l) H_s^T R_{1s} H_s \delta(l) \\
&\quad + (\bar{t}_s - \underline{t}_s)^2 \delta^T(k) H_s^T R_{2s} H_s \delta(k) \\
&\quad \left. - \sum_{l=k-\bar{t}_s}^{k-\underline{t}_s-1} (\bar{t}_s - \underline{t}_s) \delta^T(l) H_s^T R_{2s} H_s \delta(l) \right). \tag{A.9}
\end{aligned}$$

By Jensen's inequality, we can get

$$\begin{aligned}
&\sum_{s=1}^3 \sum_{l=k-\bar{t}_s}^{k-1} \bar{t}_s \delta^T(l) H_s^T R_{1s} H_s \delta(l) \\
&= \sum_{s=1}^3 \left(\sum_{l=k-\bar{t}_s}^{k-\eta_k-1} + \sum_{l=k-\eta_k}^{k-1} \right) \\
&\quad \times (\bar{t}_s - \eta_k + \eta_k) \delta^T(l) H_s^T R_{1s} H_s \delta(l) \\
&\geq \sum_{s=1}^3 \left((x(k - \bar{t}_s) - x(k - \eta_k))^T H_s^T R_{1s} H_s \right. \\
&\quad \times (x(k - \bar{t}_s) - x(k - \eta_k)) \\
&\quad + (x(k - \eta_k) - x(k))^T \\
&\quad \times H_s^T R_{1s} H_s (x(k - \eta_k) - x(k)) \Big) \\
&\geq \xi(k)^T \begin{bmatrix} \Omega_{11} & -\Omega_{12} & 0 & 0 \\ * & \Omega_{22} & 0 & -\Omega_{24} \\ * & * & 0 & 0 \\ * & * & * & \Omega_{44} \end{bmatrix} \xi(k), \tag{A.10}
\end{aligned}$$

where Ω_{11} , Ω_{12} , Ω_{22} , Ω_{24} , and Ω_{44} are defined in Theorem 5.

Similarly, we have

$$\begin{aligned}
&\sum_{s=1}^3 \sum_{l=k-\bar{t}_s}^{k-\underline{t}_s-1} (\bar{t}_s - \underline{t}_s) \delta^T(l) H_s^T R_{2s} H_s \delta(l) \\
&= \sum_{s=1}^3 \left(\sum_{l=k-\bar{t}_s}^{k-\eta_k-1} + \sum_{l=k-\eta_k}^{k-\underline{t}_s-1} \right) \\
&\quad \times (\bar{t}_s - \eta_k + \eta_k - \underline{t}_s) \delta^T(l) H_s^T R_{2s} H_s \delta(l) \\
&\geq \sum_{s=1}^3 \left((\bar{t}_s - \eta_k) \sum_{l=k-\bar{t}_s}^{k-\eta_k-1} \delta^T(l) H_s^T R_{2s} H_s \delta(l) \right. \\
&\quad \left. + (\eta_k - \underline{t}_s) \sum_{l=k-\eta_k}^{k-\underline{t}_s-1} \delta^T(l) H_s^T R_{2s} H_s \delta(l) \right) \\
&\geq \sum_{s=1}^3 \left(\left(\sum_{l=k-\bar{t}_s}^{k-\eta_k-1} \delta(l) \right)^T H_s^T R_{2s} H_s \left(\sum_{l=k-\bar{t}_s}^{k-\eta_k-1} \delta(l) \right) \right. \\
&\quad \left. + \left(\sum_{l=k-\eta_k}^{k-\underline{t}_s-1} \delta(l) \right)^T H_s^T R_{2s} H_s \left(\sum_{l=k-\eta_k}^{k-\underline{t}_s-1} \delta(l) \right) \right) \\
&\geq \xi(k)^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 2\Psi & -\Psi & -\Psi \\ * & * & \Psi & 0 \\ * & * & * & \Psi \end{bmatrix} \xi(k), \tag{A.11}
\end{aligned}$$

where Ψ is defined in Theorem 5.

By combining (A.9), (A.10), and (A.11), we have

$$\mathbb{E} [\Delta V_4]$$

$$\begin{aligned} &\leq \xi(k)^T \left\{ \begin{bmatrix} \bar{t}_1 (H_1(\bar{A}_i - I))^T \\ \bar{t}_1 \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \right. \\ &\quad \times R_{11} [\bar{t}_1 H_1 (\bar{A}_i - I) \quad \bar{t}_1 \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} \bar{t}_2 (H_2(\bar{A}_i - I))^T \\ \bar{t}_2 \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{12} [\bar{t}_2 H_2 (\bar{A}_i - I) \quad \bar{t}_1 \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} \bar{t}_3 (H_3(\bar{A}_i - I))^T \\ \bar{t}_3 \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{13} [\bar{t}_3 H_3 (\bar{A}_i - I) \quad \bar{t}_1 \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} (\bar{t}_1 - \underline{t}_1) (H_1(\bar{A}_i - I))^T \\ (\bar{t}_1 - \underline{t}_1) \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{21} [(\bar{t}_1 - \underline{t}_1) H_1 (\bar{A}_i - I) \quad (\bar{t}_1 - \underline{t}_1) \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} (\bar{t}_2 - \underline{t}_2) (H_2(\bar{A}_i - I))^T \\ (\bar{t}_2 - \underline{t}_2) \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{22} [(\bar{t}_2 - \underline{t}_2) H_2 (\bar{A}_i - I) \quad (\bar{t}_2 - \underline{t}_2) \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} (\bar{t}_3 - \underline{t}_3) (H_3(\bar{A}_i - I))^T \\ (\bar{t}_3 - \underline{t}_3) \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{23} [(\bar{t}_3 - \underline{t}_3) H_3 (\bar{A}_i - I) \quad (\bar{t}_3 - \underline{t}_3) \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} -\Omega_{11} & \Omega_{12} & 0 & 0 \\ * & -\Omega_{22} & 0 & \Omega_{24} \\ * & * & 0 & 0 \\ * & * & * & -\Omega_{44} \end{bmatrix} \end{aligned}$$

$$+ \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & -2\Psi & \Psi & \Psi \\ * & * & -\Psi & 0 \\ * & * & * & -\Psi \end{bmatrix} \right\} \xi(k),$$

(A.12)

where $\bar{\Lambda}$ is defined in Theorem 5.

By combining (A.4), (A.5), (A.8), and (A.12), we have

$$\mathbb{E} [\Delta V]$$

$$\leq \xi^T(k)$$

$$\begin{aligned} &\times \left\{ \begin{bmatrix} \Pi_{i,m,p} & \Omega_{12} & 0 & 0 \\ * & -\bar{Q}_3 - \Omega_{22} - 2\Psi & \Psi & \Omega_{24} + \Psi \\ * & * & -\bar{Q}_2 - \Psi & 0 \\ * & * & * & -\bar{Q}_1 - \Omega_{44} - \Psi \end{bmatrix} \right. \\ &\quad + \begin{bmatrix} \bar{A}_i^T \\ \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \bar{P}_{i,m,p} [\bar{A}_i \quad \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} \bar{t}_1 (H_1(\bar{A}_i - I))^T \\ \bar{t}_1 \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{11} [\bar{t}_1 H_1 (\bar{A}_i - I) \quad \bar{t}_1 \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} \bar{t}_2 (H_2(\bar{A}_i - I))^T \\ \bar{t}_2 \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{12} [\bar{t}_2 H_2 (\bar{A}_i - I) \quad \bar{t}_2 \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} \bar{t}_3 (H_3(\bar{A}_i - I))^T \\ \bar{t}_3 \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{13} [\bar{t}_3 H_3 (\bar{A}_i - I) \quad \bar{t}_3 \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} (\bar{t}_1 - \underline{t}_1) (H_1(\bar{A}_i - I))^T \\ (\bar{t}_1 - \underline{t}_1) \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \\ &\quad \times R_{21} [(\bar{t}_1 - \underline{t}_1) H_1 (\bar{A}_i - I) \quad (\bar{t}_1 - \underline{t}_1) \bar{\Lambda} \quad 0 \quad 0] \\ &\quad + \begin{bmatrix} (\bar{t}_2 - \underline{t}_2) (H_2(\bar{A}_i - I))^T \\ (\bar{t}_2 - \underline{t}_2) \bar{\Lambda}^T \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
& \times R_{22} \left[\begin{pmatrix} \bar{t}_2 - \underline{t}_2 \\ \bar{t}_3 - \underline{t}_3 \end{pmatrix} H_2 (\bar{A}_i - I) \begin{pmatrix} \bar{t}_2 - \underline{t}_2 \\ \bar{t}_3 - \underline{t}_3 \end{pmatrix} \bar{\Lambda} \ 0 \ 0 \right] \\
& + \left[\begin{pmatrix} \bar{t}_3 - \underline{t}_3 \\ \bar{t}_3 - \underline{t}_3 \end{pmatrix} \begin{pmatrix} H_3 (\bar{A}_i - I) \\ \bar{\Lambda}^T \end{pmatrix}^T \right. \\
& \quad \left. \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right] \\
& \times R_{23} \left[\begin{pmatrix} \bar{t}_3 - \underline{t}_3 \\ \bar{t}_3 - \underline{t}_3 \end{pmatrix} H_3 (\bar{A}_i - I) \begin{pmatrix} \bar{t}_3 - \underline{t}_3 \\ \bar{t}_3 - \underline{t}_3 \end{pmatrix} \bar{\Lambda} \ 0 \ 0 \right] \Bigg\} \\
& \times \xi(k) \\
& = \xi^T(k) \Theta_{i,m,p} \xi(k), \tag{A.13}
\end{aligned}$$

where $\bar{Q}_1, \bar{Q}_2, \bar{Q}_3$, and $\Pi_{i,m,p}$ are defined in Theorem 5.

By Schur complement and from (10), we have $\Theta_{i,m,p} < 0$. Therefore,

$$\mathbb{E} [\Delta V] \leq -\lambda_{\min}(-\Theta_{i,m,p}) \xi(k)^T \xi(k) \leq -\eta x(k)^T x(k), \tag{A.14}$$

where $\lambda_{\min}(-\Theta_{i,m,p})$ denotes the minimal eigenvalue of $-\Theta_{i,m,p}$ and $\eta = \inf\{\lambda_{\min}(-\Theta_{i,m,p})\}$. From (A.14), it is seen that for any $t > 0$

$$\mathbb{E} [V(k+1)] - \mathbb{E} [V(0)] \leq -\eta \sum_{k=0}^t \mathbb{E} [x(k)^T x(k)]. \tag{A.15}$$

Furthermore

$$\sum_{k=0}^t \mathbb{E} [x(k)^T x(k)] \leq \frac{1}{\eta} \mathbb{E} [V(0)]. \tag{A.16}$$

By taking limit as $t \rightarrow \infty$, we have

$$\sum_{k=0}^{\infty} \mathbb{E} [x(k)^T x(k)] \leq \frac{1}{\eta} \mathbb{E} [V(0)] < \infty. \tag{A.17}$$

According to Definition 4, the closed-loop system (5) is stochastically stable. This completes the proof. $\square$

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