Research Article

A Numerical Well-Balanced Scheme for One-Dimensional Heat Transfer in Longitudinal Triangular Fins

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The temperature profile for fins with temperature-dependent thermal conductivity and heat transfer coefficients will be considered. Assuming such forms for these coefficients leads to a highly nonlinear partial differential equation (PDE) which cannot easily be solved analytically. We establish a numerical balance rule which can assist in getting a well-balanced numerical scheme. When coupled with the zero-flux condition, this scheme can be used to solve this nonlinear partial differential equation (PDE) modelling the temperature distribution in a one-dimensional longitudinal triangular fin without requiring any additional assumptions or simplifications of the fin profile.

1. Introduction

There is a wide range of applications for extended surfaces, mostly called fins, in heat transfer which is more formally described as the temperature propagation or flow of the heat. Obvious examples may be found in several applications of mechanical engineering and in many home appliances [1]. In support of their use, Sparrow and Vemuri [2] have shown that with finned surfaces the heat transfer increases six times in comparison to unfinned surfaces. It has been found by many researchers that the fin orientation, height, length, and spacing in arrays play major roles in the manner and efficiency of heat transfer [3–8]. Given that these parameters play a fundamental role in the structure of the problem, and even though these interlinked factors increase the complexity of the problem, they should not be removed for the sake of simplicity. The consequence of this, however, is that we end up considering a nonlinear PDE, the solution of which is not always possible analytically. In fact, the use of analytical methods has often led to the consideration of a simplified model, especially for complex geometries, whereas this is not necessary when using certain numerical methods. However, as suggested by Wang et al. [9], when using numerical schemes as the solution method, it is fundamental

to recognize that a proper numerical treatment of the source term may eliminate possible spurious steady-state numerical solutions. In this paper we will follow this line of thought and thus specifically focus on an appropriate treatment of the source term of the problem under consideration. As such, the well-balancing approach will be considered and implemented as per the work of LeVeque [10]. More precisely, this approach is applied to triangular fins which have been characterized by singularities in the literature [11, 12].

When solving the problem of heat transfer in a triangular fin, it is essential to remember that triangular fin profiles have been classified by Kraus et al. [11] among singular profiles because it is analytically impossible to characterize them by any linear transformation. Kraus proposes that one assume the triangular profiles to be trapezoidal in nature so as to render the problem solvable. In this manner the original problem is oversimplified so as to guarantee solutions; however, this methodology may lead to inaccurate results given that fins with trapezoidal profiles have already been classified as such and solved accordingly. Therefore, the objective of this paper is to provide a numerical approach that effectively deals with the proper form of the triangular fin and solve, via a well-balancing numerical scheme, the problem of heat transfer in such a fin.

to vary linearly with the temperature [12] as is the case for many engineering applications. As such we find that

$$K(T) = k_a [1 + \beta(T - T_a)], \quad (7)$$

which in dimensionless variables gives $k(\theta) = 1 + B\theta$ where $B = \beta(T_b - T_a)$ is nonzero with β as the thermal conductivity gradient. Hence, the dimensionless heat transfer equation for a longitudinal one-dimensional fin is given by [12, 16, 22]

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial x} \left(f(x) (1 + B\theta) \frac{\partial \theta}{\partial x} \right) - \mathcal{M}^2 \theta^{n+1}, \quad 0 < x < 1, \quad \tau \geq 0, \quad (8)$$

where boundary conditions are as follows:

$$\left. \frac{\partial \theta}{\partial x} \right|_{x=0} = 0 \text{ at the fin tip,} \quad (9)$$

$$\theta(\tau, 1) = 1, \text{ at the base}$$

with initial condition

$$\theta(0, x) = 0. \quad (10)$$

3. Numerical Approach

3.1. The Finite Volume Method and Numerical Balance Law. At first we intend to briefly introduce the manner in which we will employ the finite volume method (FVM) and its advantages within the context of heat transfer problems. In this scenario, due to its integral approach, the FVM reduces the order of the spatial derivative by one. This motivates its use for the heat transfer equation, under consideration given the presence of a second derivative in its conduction term. If we were to consider the method for a more general structure of the heat transfer equation, we would consider the partial differential equation of the form

$$\frac{\partial T}{\partial t} - \frac{A_p}{\rho c_v} \frac{\partial}{\partial x} \left(G(T, X) \frac{\partial T}{\partial x} \right) = -\frac{1}{\rho c_v} Q(H(T)), \quad (11)$$

where $G(T, X) = F(X)K(T)$ is a function of thermal conductivity K , involved in the convective term, and $Q(H(T))$ a function of the heat transfer coefficient H , which represents the source term from radiation. Furthermore, we define ρ as the density, c_v as the heat capacity, and T as the temperature.

In order to reduce the order of the spatial derivatives by one, we integrate (11) over the grid cell $[x_{i-1/2}, x_{i+1/2}]$ to obtain

$$\begin{aligned} \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial T}{\partial t} dx - \frac{A_p}{\rho c} \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial}{\partial x} \left(G(T, X) \frac{\partial T}{\partial x} \right) dx \\ = -\frac{1}{\rho c} \int_{x_{i-1/2}}^{x_{i+1/2}} Q(H(T)) dx. \end{aligned} \quad (12)$$

By cell averaging we find that

$$\Delta x_i \frac{d\bar{T}_i(t)}{dt} - \frac{A_p}{\rho c} \left(G(T, X) \frac{\partial T}{\partial x} \right) \Big|_{x_{i-1/2}}^{x_{i+1/2}} = -\Delta x_i \frac{1}{\rho c} \bar{Q}_i, \quad (13)$$

where

$$\bar{f}_i = \frac{1}{\Delta x_i} \int_{x_{i-1/2}}^{x_{i+1/2}} f dx \quad (14)$$

is the cell-averaged quantity of f over the grid cell $[x_{i-1/2}, x_{i+1/2}]$. It is obvious that the order of the partial differential equation under consideration has been reduced by one, and this increases the accuracy of the results we are to obtain.

In the next section, we will employ the numerical approach described above for (8) and in doing so develop a numerical balance law as given by (14). In this manner, we obtain a well-balanced scheme which preserves specific nontrivial steady-state solutions and may help to minimize some of the oscillations which occur around steady states [23]. Thus for the more general heat transfer equation (11) a well-balanced scheme can provide a solution that must satisfy

$$A_p \frac{\partial}{\partial x} \left(G(T, X) \frac{\partial T}{\partial x} \right) = Q(H(T)) \quad (15)$$

for steady states. An easily understandable and effective procedure has been established by Wang et al. [9] which will be implemented in this work for the one-dimensional heat transfer problem given by (8). It should also be kept in mind that this methodology may easily be extended to higher dimensions.

3.2. Numerical Well-Balanced Scheme. In considering (8) we find that the one-dimensional steady heat equation for regular fins is expressed by

$$\begin{aligned} \frac{d}{dx} \left(f(x) (1 + B\theta) \frac{d\theta}{dx} \right) &= \mathcal{M}^2 \theta^{n+1}, \quad 0 < x < 1, \\ \left. \frac{d\theta}{dx} \right|_{x=0} &= 0, \quad \theta(1) = 1. \end{aligned} \quad (16)$$

Integrating over the grid cell $[0, x + \Delta x/2]$, as discussed previously within the context of the FVM, we obtain

$$\int_0^{x+\Delta x/2} \left(\frac{d}{dx} \left(f(x) (1 + B\theta) \frac{d\theta}{dx} \right) \right) dx \quad (17)$$

$$= \mathcal{M}^2 \int_0^{x+\Delta x/2} \theta^{n+1} dx, \quad 0 < x < 1,$$

which is equivalent to

$$\begin{aligned} \left(f \left(x + \frac{\Delta x}{2} \right) \left(1 + B\theta \left(x + \frac{\Delta x}{2} \right) \right) \right) \frac{d\theta(x + \Delta x/2)}{dx} \\ = \mathcal{M}^2 \int_0^{x+\Delta x/2} \theta^{n+1} dx, \quad 0 < x < 1. \end{aligned} \quad (18)$$

Similarly over $[0, x - \Delta x/2]$, we find that

$$\begin{aligned} & \left(f \left(x - \frac{\Delta x}{2} \right) \left(1 + B\theta \left(x - \frac{\Delta x}{2} \right) \right) \right) \frac{d\theta(x - \Delta x/2)}{dx} \\ &= \mathcal{M}^2 \int_0^{x-\Delta x/2} \theta^{n+1} dx, \quad 0 < x < 1. \end{aligned} \quad (19)$$

By cell averaging over $[0, x + \Delta x/2]$ and $[0, x - \Delta x/2]$ and by using the first-order Taylor approximation around x equations (18) and (19), after simple algebraic transformations, give the cell-averaged quantity

$$\begin{aligned} & \int_{x-\Delta x/2}^{x+\Delta x/2} \theta^{n+1} dx \\ &= \Delta x \left(\theta^{n+1}(x) \right. \\ & \quad \left. + \frac{(n+1) \mathcal{M}^2 x \Delta x \theta^{2n+1}(x)}{2f(x)(1+B\theta(x)) - (n+1) \mathcal{M}^2 x^2 \theta^n(x)} \right). \end{aligned} \quad (20)$$

Equation (20) constitutes our numerical balance law.

Integrating (8) over $[x - \Delta x/2, x + \Delta x/2]$ and incorporating expression (20) into the resulting expression we obtain

$$\begin{aligned} \frac{\partial \theta}{\partial \tau} &= \frac{1}{(\Delta x)^2} \\ & \times \left\{ f \left(x + \frac{\Delta x}{2} \right) \left(1 + B\theta \left(x + \frac{\Delta x}{2} \right) \right) \frac{\partial \theta(x + \Delta x/2)}{\partial x} \right. \\ & \quad \left. - f \left(x - \frac{\Delta x}{2} \right) \left(1 + B\theta \left(x - \frac{\Delta x}{2} \right) \right) \frac{\partial \theta(x - \Delta x/2)}{\partial x} \right\} \\ & - \mathcal{M}^2 \left(\theta^{n+1}(x) \right. \\ & \quad \left. + \frac{(n+1) \mathcal{M}^2 x \Delta x \theta^{2n+1}(x)}{2f(x)(1+B\theta(x)) - (n+1) \mathcal{M}^2 x^2 \theta^n(x)} \right), \end{aligned} \quad (21)$$

where we can see that the order of our equation has been reduced as expected. We now substitute finite difference approximations to our derivatives into (21). We consider $[x_i - \Delta x/2, x_i + \Delta x/2]$ for a particular time t_j which provides us with the following approximations:

$$\begin{aligned} \frac{\partial \theta(x_i + \Delta x/2)}{\partial x} \Big|_j &= \frac{\theta^j(x_i + \Delta x) - \theta^j(x_i)}{\Delta x} = \frac{\theta_{i+1}^j - \theta_i^j}{\Delta x}, \\ \frac{\partial \theta(x_i - \Delta x/2)}{\partial x} \Big|_j &= \frac{\theta^j(x_i) - \theta^j(x_i - \Delta x)}{\Delta x} = \frac{\theta_i^j - \theta_{i-1}^j}{\Delta x}, \\ \frac{\partial \theta}{\partial \tau} \Big|_i &= \frac{\theta_i^{j+1} - \theta_i^j}{\Delta t}. \end{aligned} \quad (22)$$

Hence, our well-balanced numerical scheme is given by the following recurrence relation:

$$\begin{aligned} \theta_i^{j+1} &= \theta_i^j + \frac{\Delta t}{(\Delta x)^2} \left[f_{i+1/2} \left(1 + B\theta_{i+1/2}^j \right) (\theta_{i+1}^j - \theta_i^j) \right. \\ & \quad \left. - f_{i-1/2} \left(1 + B\theta_{i-1/2}^j \right) (\theta_i^j - \theta_{i-1}^j) \right] \\ & \quad - \Delta t \mathcal{M}^2 \left((\theta_i^j)^{n+1} + \frac{(n+1) \mathcal{M}^2 x_i \Delta x (\theta_i^j)^{2n+1}}{2f_i (1 + B\theta_i^j) - (n+1) \mathcal{M}^2 x_i^2 (\theta_i^j)^n} \right), \end{aligned} \quad (23)$$

where a linear interpolation is used to determine $f_{i+1/2}$, $f_{i-1/2}$, $\theta_{i+1/2}$, and $\theta_{i-1/2}$.

3.2.1. No Heat Flux Flow at the Fin Tip. In order to implement our well-balanced numerical scheme, we need to first incorporate the relevant boundary conditions. According to the work by Kraus et al. [11], some fins' shapes require special interpretation—a clear example thereof is the triangular fin profile. Longitudinal fins of triangular profile have been classified among singular fins that cannot be characterized by any linear transformations. As such it is important to remember that the fin profile tapers to zero thickness at the tip, and hence, there will be zero flux at this point. This means that

$$\left(f(x)(1+B\theta) \frac{\partial \theta}{\partial x} \right) \Big|_{x=0} = 0. \quad (24)$$

We now implement a time forward discretisation at the origin and employ the zero-flux condition given by (24) to obtain

$$\theta_0^{j+1} = \theta_0^j - \Delta \tau \mathcal{M}^2 (\theta_0^j)^{n+1}. \quad (25)$$

As one can see the physical reality of zero thickness at the tip complicates the solution of the problem. If one were to only employ the zero-flux condition, given the initial condition of zero temperature, one would always have a zero temperature at the origin as per (25). This does not make physical sense, however, given that after a considerable time the temperature would be expected to increase at the tip of the fin. At this stage we turn to the well-balancing principle as a means of overcoming this problem.

We employ the well-balancing principle at the origin as a means of incorporating the expression of θ_0^j into (25). As such, we consider the steady-state equation as follows:

$$\begin{aligned} & \int_{x_0}^{x_0+\Delta x/2} \frac{d}{dx} \left(f(x)(1+B\theta) \frac{d\theta}{dx} \right) dx = \frac{\Delta x}{2} \mathcal{M}^2 \theta^{n+1}(x_0), \\ & f \left(x_0 + \frac{\Delta x}{2} \right) \left(1 + B\theta \left(x_0 + \frac{\Delta x}{2} \right) \right) \frac{d}{dx} \theta \left(x_0 + \frac{\Delta x}{2} \right) \\ &= \frac{\Delta x}{2} \mathcal{M}^2 \theta^{n+1}(x_0). \end{aligned} \quad (26)$$

FIGURE 2: A triangular fin profile with $B = 1$, $n = 1$, $\mathcal{M} = 0.01$, $\mathcal{M} = 1$, $\mathcal{M} = 1.5$, and $\mathcal{M} = 5$ for $\tau = 0.0001$ (a) and $\tau = 0.001$ (b).

Through the use of a central difference approximation, we then obtain

$$\theta_0^j = \theta_1^j - \frac{\Delta x^2}{2f(x_{1/2})(1 + B\theta_{1/2}^j)} \mathcal{M}^2(\theta_0^j)^{n+1}. \quad (27)$$

The coupled equations (25) and (27) provide a numerical well-balanced discretisation for a triangular fin profile at the origin.

3.2.2. Heat Flux Flow at the Fin Tip. For regular fin profiles, the flux at the origin is a finite nonzero value in which case there would be heat flow from the tip. It is difficult to impose such a boundary condition because the nonzero value is not known. Instead we impose an adiabatic boundary condition which allows for this nonzero-flux value at the tip to come out as part of the numerical solution obtained. Thus, at the origin we employ the condition

$$\left. \frac{\partial \theta}{\partial x} \right|_{x=0} = 0 \quad (28)$$

which upon substitution into the equation under consideration gives

$$\left. \frac{\partial \theta}{\partial \tau} - f(x)(1 + B\theta) \frac{\partial^2 \theta}{\partial x^2} + \mathcal{M}^2(\theta)^{n+1} \right|_{x=0} = 0. \quad (29)$$

Implementing the forward difference approximation for time and the central difference approximation for space, we find that

$$\theta_0^{j+1} = \theta_0^j + \frac{2\Delta\tau f(x_0)(1 + B\theta_0^j)}{\Delta x^2} (\theta_1^j - \theta_0^j) - \Delta\tau \mathcal{M}^2(\theta_0^j)^{n+1}, \quad (30)$$

and using similar finite difference approximations on the steady-state equation of regular fins, we obtain

$$\theta_0^j = \theta_1^j - \frac{\Delta x^2}{2f(x_0)(1 + B\theta_0^j)} \mathcal{M}^2(\theta_0^j)^{n+1}. \quad (31)$$

Equations (30) and (31) summarize the discretisation at the origin for regular fin profiles.

4. Results and Discussion

4.1. Triangular Fin Profiles. As stated earlier, previous researches have proposed that one approximates the shape of triangular fins by considering the trapezoidal profile as a means of facilitating linear transformations. Aside from proposed simplifications, work has also been done while maintaining the profile in its original triangular form. In [12] for instance, numerical solutions were obtained for the heat transfer in a triangular fin which did not maintain the adiabatic condition—this was thought to be due to thermal instability within the fin as discussed by Yeh and Liaw [21].

The importance of the work conducted here is that the numerical scheme developed did not rely on any simplifying assumptions as proposed by Kraus et al. [11]. Furthermore, the results obtained in [12] are shown to be due to an inaccurate methodology, specifically related to the boundary conditions for profiles which lead to singularities. In applying the zero-flux condition in a novel manner, we were able to obtain a recursive scheme able to capture the true behaviour of the model under consideration.

We obtained numerical solutions via our well-balanced scheme to (8) for a triangular fin profile with $B = 1$, $n = 1$, $\mathcal{M} = 0.01, 1, 1.5$, and $\mathcal{M} = 5$ at different values of τ . Figures 2 and 3 indicate that the temperature decreases from the base to

FIGURE 3: A triangular fin profile with $B = 1$, $n = 1$, $\mathcal{M} = 0.01$, $\mathcal{M} = 1$, $\mathcal{M} = 1.5$, and $\mathcal{M} = 5$ for $\tau = 0.1$ (a) and $\tau = 0.5$ (b).

FIGURE 4: A triangular fin profile with $B = 1$, $n = 1$, $\mathcal{M} = 0.01$, $\mathcal{M} = 1$, $\mathcal{M} = 1.5$, and $\mathcal{M} = 5$ for $\tau = 0.75$ (a) and $\tau = 1$ (b).

tip of the fin and that the temperature at the tip increases with time. These results make physical sense and also match the behaviour of the temperature distribution found for other fin profiles. Interestingly, for small values of time τ , the response temperature is virtually independent of the value of $\mathcal{M}$, and this is why a single curve is shown for $\tau = 0.0001$, $\tau = 0.001$, and $\tau = 0.01$. This has been explained by Suryanarayana [16] where it is said that at small values of τ , the bulk of the thermal energy entering at the base remains stored in the fin with only a small fraction available for dissipation through surface convection. Thus, the heat transfer coefficient has very little

direct impact on the temperature profile—rather its impact may be related to the length of the fin which in turn influences the temperature profile [12].

In turn, as τ increases, it is seen that the role of convection and hence $\mathcal{M}$ become progressively significant as shown in Figures 4 and 5. Another point is that the steady state is reached quicker for longer fins or fins with higher values of $\mathcal{M}$ as shown in Figure 3 for $\tau = 0.5$. This same Figure 3 shows clearly that at $\tau = 0.5$ only a stationary state has been reached for the fin profile where $\mathcal{M} = 5$, which is the largest value chosen. This is a consequence of the fact that the

FIGURE 5: A triangular fin profile with $B = 1$, $n = 1$, $M = 0.01$, $M = 1$, $M = 1.5$, and $M = 5$ for $\tau = 5$ (a) and $\tau = 10$ (b).

FIGURE 6: A rectangular fin profile with $B = 1$, $n = 1$, $M = 1$, $M = 3$, $M = 5$, and $M = 8$ for $\tau = 2.5$.

FIGURE 7: A rectangular fin profile with $B = 1$, $n = 1$, $M = 1$, and varying τ .

dimensionless time $\tau = k_a t / \rho c L^2$ should decrease with an increase in M .

While the numerical results we have obtained for the heat transfer in triangular fins match those obtained by Suryanarayana [16] for other types of fin profiles in linear cases, we still require further verification of our results. The results provided in [16] in and of itself cannot justify the accuracy of the results obtained via our well-balanced numerical scheme given the fact that no other concrete analysis currently exists and that all previous attempts at obtaining solutions for the

triangular case were done with reservations regarding the results obtained [12]. For this reason, our model has been applied to the rectangular case, where we do have confirmed results, as a means of validating the scheme implemented.

4.2. Model Validation via a Consideration of the Rectangular Fin Profile. For a rectangular fin profile, the solution profiles from our well-balanced numerical scheme are depicted by Figures 6, 7, and 8. It is clearly visible that the temperature

FIGURE 8: A rectangular fin profile with $B = 1$, $\mathcal{M} = 6$, $\tau = 2.5$, and varying n .

is an increasing function of time and that it decreases from the base to the tip. Figure 6 depicts the effect of the thermogeometric fin parameter on the temperature. We can see that the temperature is a decreasing function of $\mathcal{M}$. In contrast, the temperature distribution is an increasing function of parameter n as shown in Figure 8. What is important to realise is that the results we have obtained for the rectangular case via the well-balanced scheme employed for the triangular case verify the benchmarks results of [12, 16] and hence act as a validation of our well-balanced numerical scheme.

5. Conclusion

The well-balanced numerical scheme which we have established in this work has been shown to effectively and efficiently obtain results for the rectangular fin profile, matching previous results found in the literature [12, 15, 16]. Our discretisation incorporates the flux condition for the rectangular case as is appropriate; however, for the triangular fin profile, we have incorporated the zero-flux condition into our established well-balanced numerical scheme. It is this latter scheme and the manner in which it is employed which constitute the originality of our work.

Several researchers [15, 20] have proposed some exact solutions, but the main problem was that the problem was being simplified through an adjustment of the geometric form of the fin as a means of guaranteeing analytical solutions. Kraus et al. [11], for example, suggested that one assumes triangular profiles to be trapezoidal so as to guarantee the existence of linear transformations.

In our work however, such simplifications are not needed. The well-balanced numerical scheme which we have developed is able to handle the triangular case without any additional assumptions due to the incorporation of an appropriate boundary condition, namely, the zero-flux condition. This approach can easily be extended to other singular profiles, such as the convex and concave parabolic profiles, and hence, it constitutes a clear path to a generalized numerical scheme for the solution of problems in heat transfer.

Nomenclature

A_p :	Profile area, m^2
B :	Thermal conductivity parameter
c :	Specific heat capacity, J/KgK
c_v :	Volumetric heat capacity $2c/(\delta_b A_p)$
$F(X)$:	Fin profile, m^2
$f(x)$:	Dimensionless fin profile
H :	Heat transfer coefficient, $\text{W/m}^2\text{K}$
h :	Dimensionless heat transfer coefficient
h_b :	Heat transfer at the base, $\text{W/m}^2\text{K}$
K :	Thermal conductivity, W/mK
k :	Dimensionless thermal conductivity
k_a :	Thermal conductivity of the fin at the ambient temperature, W/mK
L :	Length of the fin, m
n :	Exponent
P :	Fin perimeter, m
Q :	Heat flux, W/m^2
T :	Temperature distribution, K
T_b :	Fin base temperature, K
T_a :	Surrounding temperature, K
t :	Time, S
X :	Spatial variable, m
x :	Dimensionless spatial variable.

Greek Symbols

β :	Thermal conductivity gradient
δ :	Fin thickness, m
δ_b :	Fin thickness at the base, m
η :	Fin efficiency
θ :	Dimensionless temperature
$\mathcal{M}$:	Thermogeometric fin parameter
τ :	Dimensionless time.

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