Research Article

Exponential Synchronization for Neutral Complex Dynamical Networks with Interval Mode-Dependent Delays and Sampled Data

Xinghua Liu and Hongsheng Xi

Department of Auto, School of Information Science and Technology, University of Science and Technology of China, Anhui 230027, China

Correspondence should be addressed to Xinghua Liu; salxh@mail.ustc.edu.cn

Received 26 May 2013; Revised 7 August 2013; Accepted 8 August 2013

Academic Editor: Jun-Juh Yan

Copyright © 2013 X. Liu and H. Xi. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The exponential synchronization and sampled-data controller problem for a class of neutral complex dynamical networks (NCDNs) with Markovian jump parameters, partially unknown transition rates and delays, is investigated in this paper. Both the discrete and neutral delays are considered to be interval mode dependent and time varying, while the sampling period is assumed to be time varying and bounded. Based on a new augmented stochastic Lyapunov functional, the delay-range-dependent and rate-dependent exponential stability conditions for the closed-loop error system are obtained by the Lyapunov-Krasovskii stability theory and reciprocally convex lemma. Then according to the proposed exponential stability conditions, the sampled-data synchronization controllers are designed in terms of the solution to linear matrix inequalities that can be solved effectively by using Matlab. Finally, numerical examples are given to demonstrate the feasibility and effectiveness of the proposed methods.

1. Introduction

It is well known that many practical systems can be described as complex networks such as Internet networks, biological networks, epidemic spreading networks, collaborative networks, social networks and neural networks [1–4]. Thus, during the past decades, the research on the dynamics of complex dynamical networks (CDNs) has attracted extensive attention of scientific and engineering researchers in all fields domestic and overseas since the pioneering work of Watts and Strogatz [5]. As one of the most significant and important collective behaviors in CDNs, synchronization has received more attention; please refer to [6–10] and references therein for more details.

Since time delay inevitably exists and has become an important issue in studying the CDNs, synchronization problems for complex networks with time delays have gained increasing research attention, and considerable progress has been made; see, for example, [6–17] and references therein for more details. However, in some practical applications of communication networks, signal transmission channels often

involve network-induced delays, packet dropouts, bit errors, environment disturbances, and so on, which will cause the error of transmission from one system to another. Then past change rate of the state variables affects the dynamics of nodes in the networks. This kind of complex dynamical network is termed as neutral complex dynamical network (NCDN), which contains delays both in its states and the derivatives of its states. There are some results about the synchronization design problem for neutral systems [18–23]. In these works, [19, 20] had studied the synchronization control for a kind of master-response setup and were further extended to the case of neutral-type neural networks with stochastic perturbation. The authors of [18, 22] had researched the synchronization problem for a class of complex networks with neutral-type coupling delays. The authors in [21] had studied the global asymptotic stability of neural networks of neutral type with mixed delays, which include constant delay in the leakage term, time-varying delays, and continuously distributed delays. The authors in [23] had investigated the robust global exponential synchronization problem for an array of neutral-type neural networks. However, much fewer results have been

proposed for neutral complex dynamical networks (NCDNs) compared with the rich results for CDNs with only discrete delays.

On the other hand, network mode switching is also a universal phenomenon in CDNs of the actual systems, and sometimes the network has finite modes that switch from one to another with certain transition rate; then such switching can be governed by a Markovian chain. The stability and synchronization problems of complex networks and neural networks with Markovian jump parameters and delays are investigated in [16, 24–30], and references therein. The authors in [24] have established sufficient global exponential stability conditions on Markovian jump neural networks with impulse control and time varying delays. The authors in [27] have studied synchronization in an array of coupled neural networks with Markovian jumping and random coupling strength. Particularly, Ma et al. [25] have considered the stability and synchronization problems for Markovian jump delayed neural networks with partly unknown transition probabilities, which have not been fully investigated and need to propose more good results. Besides, sampled-data systems have attracted great attention because the digital signal processing methods require better reliability, accuracy, and stable performance with the rapid development in digital measurement and intelligent instrument. There are some important and essential results which have been reported in the literature [31–35]. What is worth mentioning is that the sampled-data synchronization control problem has been investigated for a class of general complex networks with time-varying coupling delays in [34, 35], where conditions have been presented to ensure the exponential stability of the closed-loop error system, and the desired sampled data feedback controllers have been designed. However, few results are available for neutral complex dynamical networks (NCDNs) with sampled data. To the best of the authors' knowledge, the NCDNs are difficult to treat and they are very challenging, especially in the presence of Markovian jump parameters, mode-dependent time-varying delays, and sampled data. Motivated by the above analysis, the exponential synchronization and sampled-data controller problem for a class of NCDNs with Markovian jump parameters and mode-dependent time-varying delay is investigated in this paper. The addressed NCDNs consist of M modes, and the networks switch from one mode to another according to a Markovian chain with partially known transition rate.

In this paper, the synchronization and sampled-data controller problem is studied for NCDNs with Markovian jump parameters and partially known transition rates. The sampling period considered here is assumed to be time varying and bounded, while the neutral and discrete delays are interval mode-dependent and time varying. Firstly, by constructing a new augmented stochastic Lyapunov functional, exponential stability conditions are derived based on the Lyapunov stability theory and reciprocally convex lemma. Then the design method of the desired sampled-data controllers is solved on the basis of the obtained conditions. Moreover, all the derived results are in terms of LMIs that can be solved numerically, which are proved to be less conservative than the existing results.

The remainder of the paper is organized as follows. Section 2 presents the problem and preliminaries. Section 3 gives the main results, which are then verified by numerical examples in Section 4. Section 5 concludes the paper.

Notations. The following notations are used throughout the paper. $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{m \times n}$ is the set of all $m \times n$ matrices. $X < Y$ ($X > Y$), where X and Y are both symmetric matrices, which means that $X - Y$ is negative (positive) definite. I is the identity matrix with proper dimensions. For a symmetric block matrix, we use $*$ to denote the terms introduced by symmetry. $\mathcal{E}$ stands for the mathematical expectation; $\|v\|$ is the Euclidean norm of vector v , $\|v\| = (v^T v)^{1/2}$, while $\|A\|$ is spectral norm of matrix A , $\|A\| = [\lambda_{\max}(A^T A)]^{1/2}$. $\lambda_{\max(\min)}(A)$ is the eigenvalue of matrix A with maximum (minimum) real part. The Kronecker product of matrices $P \in \mathbb{R}^{m \times n}$ and $Q \in \mathbb{R}^{p \times q}$ is a matrix in $\mathbb{R}^{mp \times nq}$ which is denoted as $P \otimes Q$. Let $\varsigma > 0$ and $C([- \varsigma, 0], \mathbb{R}^n)$ denote the family of continuous function φ , from $[- \varsigma, 0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-\varsigma \leq \theta \leq 0} \|\varphi(\theta)\|$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem Statement and Preliminaries

Given a complete probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbf{P}\}$ with a natural filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions, where Ω is the sample space, $\mathcal{F}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathcal{F}$. Let $\{r(t), t \geq 0\}$ be a right-continuous Markov chain taking values in a finite state space $S = \{1, 2, 3, \dots, M\}$ with a generator $\Upsilon = (\gamma_{ij})_{M \times M}$, $i, j \in S$, which is given by

$$P(r(t + \Delta t) = j \mid r(t) = i) = \begin{cases} \gamma_{ij} \Delta t + o(\Delta t) & i \neq j, \\ 1 + \gamma_{ii} \Delta t + o(\Delta t) & i = j, \end{cases} \quad (1)$$

where $\Delta t > 0$, $\lim_{\Delta t \rightarrow 0} (o(\Delta t)/\Delta t) = 0$, and $\gamma_{ij} \geq 0$ ($i, j \in S, i \neq j$) is the transition rate from mode i to j , and for any state or mode $i \in S$, it satisfies

$$\gamma_{ii} = - \sum_{j=1, j \neq i}^N \gamma_{ij}. \quad (2)$$

It is assumed that $r(t)$ is irreducible and available at time t , but the transition rates of the Markov chain are partially known in this paper, which means that some elements in matrix $\Upsilon = (\gamma_{ij})_{M \times M}$ are inaccessible. For instance, in the system with six operation modes, the jump rates matrix Υ may be viewed as

$$\begin{bmatrix} \gamma_{11}^0 & \gamma_{12} & \gamma_{13}^0 & \gamma_{14}^0 & \gamma_{15} & \gamma_{16}^0 \\ \gamma_{21} & \gamma_{22}^0 & \gamma_{23} & \gamma_{24} & \gamma_{25}^0 & \gamma_{26} \\ \gamma_{31}^0 & \gamma_{32}^0 & \gamma_{33} & \gamma_{34} & \gamma_{35}^0 & \gamma_{36}^0 \\ \gamma_{41}^0 & \gamma_{42} & \gamma_{43}^0 & \gamma_{44}^0 & \gamma_{45}^0 & \gamma_{46} \\ \gamma_{51} & \gamma_{52}^0 & \gamma_{53}^0 & \gamma_{54} & \gamma_{55} & \gamma_{56}^0 \\ \gamma_{61} & \gamma_{62}^0 & \gamma_{63}^0 & \gamma_{64} & \gamma_{65} & \gamma_{66}^0 \end{bmatrix}, \quad (3)$$

where $\gamma_{ij}^0, i, j \in S$ represents the unknown element. Furthermore, let γ_i^m, γ_i^M be lower and upper bound for the diagonal element γ_{ii} or γ_{ii}^0 , for all $i \in S$. For notation clarity, we denote that $\mathcal{S}^i = \mathcal{S}_k^i \cup \mathcal{S}_{uk}^i$, for all $i \in S$, and

$$\begin{aligned}\mathcal{S}_k^i &\triangleq \{j : \pi_{ij} \text{ is known for } j \in S\}, \\ \mathcal{S}_{uk}^i &\triangleq \{j : \pi_{ij} \text{ is unknown for } j \in S\}.\end{aligned}\quad (4)$$

If $\mathcal{S}_k^i \neq \emptyset$, then it is further described as

$$\mathcal{S}_k^i = \{k_1^i, k_2^i, \dots, k_m^i\}, \quad 1 \leq m \leq M, \quad (5)$$

where $k_j^i, (j = 1, 2, \dots, m)$ represent the j th known element of the set $\mathcal{S}_k^i$ in the i th row of the transition rate matrix Υ . It should be noted that if $\mathcal{S}_k^i = \emptyset, \mathcal{S}^i = \mathcal{S}_{uk}^i$ which means that any information between the i th mode and the other $M - 1$ modes is not accessible, then MJSs with M modes can be regarded as ones with $M - 1$ modes.

The following neutral complex dynamical network (NCDN) consisting of N identical nodes with Markovian jump parameters and interval time-varying delays over the space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbf{P}\}$ is investigated in this paper:

$$\begin{aligned}\dot{x}_k(t) &= \sum_{l=1}^N g_{kl}^{(1)}(r(t)) A(r(t)) x_l(t) \\ &+ \sum_{l=1}^N g_{kl}^{(2)}(r(t)) B(r(t)) x_l(t - d(t, r(t))) \\ &+ \sum_{l=1}^N g_{kl}^{(3)}(r(t)) C(r(t)) \dot{x}_l(t - \tau(t, r(t))) \\ &+ D(r(t)) f(x_k(t)) + u_k(t),\end{aligned}\quad (6)$$

where $x_k(t) \in \mathbb{R}^n, u_k(t) \in \mathbb{R}^n$ are state variable and the control input of the node $k \in \{1, 2, \dots, N\}$, respectively. $r(t)$ describes the evolution of the mode. $A(r(t)) \in \mathbb{R}^{n \times n}, B(r(t)) \in \mathbb{R}^{n \times n}$, and $C(r(t)) \in \mathbb{R}^{n \times n}$ represent the inner-coupling matrices linking between the subsystems in mode $r(t)$. $G^{(1)}(r(t)) = [g_{kl}^{(1)}]_{N \times N}$, $G^{(2)}(r(t)) = [g_{kl}^{(2)}]_{N \times N}$, and $G^{(3)}(r(t)) = [g_{kl}^{(3)}]_{N \times N}$ are the coupling configuration matrices of the networks representing the coupling strength and the topological structure of the NCDNs in mode $r(t)$, in which $g_{kl}^{(m)}$ is defined as follows: if there exists a connection between k th and l th ($k \neq l$) nodes, then $g_{kl}^{(m)}(r(t)) = g_{lk}^{(m)}(r(t)) = 1$; otherwise $g_{kl}^{(m)}(r(t)) = g_{lk}^{(m)}(r(t)) = 0$, and

$$\begin{aligned}g_{kk}^{(m)}(r(t)) &= - \sum_{l=1, l \neq k}^N g_{kl}^{(m)}(r(t)) \\ &= - \sum_{l=1, l \neq k}^N g_{lk}^{(m)}(r(t)), \\ m &= 1, 2, 3; \quad k = 1, 2, \dots, N.\end{aligned}\quad (7)$$

$\tau(t, r(t))$ and $d(t, r(t))$ denote the mode-dependent time-varying neutral delay and retarded delay, respectively. They are assumed to satisfy

$$\begin{aligned}0 &\leq \tau_i(t) \leq \bar{\tau}_i, \quad \dot{\tau}_i(t) \leq \nu_i < 1, \\ 0 &\leq d_{1i} \leq d_i(t) \leq d_{2i}, \quad \max_{i \in S} \{d_{1i}\} \leq \min_{j \in S} \{d_{2j}\}, \\ \dot{d}_i(t) &\leq \mu_i, \quad \text{when } r(t) = i,\end{aligned}\quad (8)$$

where $\bar{\tau}_i, 0 \leq \nu_i < 1, d_{1i}$, and $d_{2i}, 0 \leq \mu_i$, are real constant scalars.

$D(r(t)) \in \mathbb{R}^{n \times n}$ is a parametric matrix with real values in mode $r(t)$. $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous vector-valued nonlinear function. It is assumed to satisfy the following sector-bounded condition [36]:

$$\begin{aligned}&[f(x) - f(y) - F_1(x - y)]^T \\ &\times [f(x) - f(y) - F_2(x - y)] \\ &\leq 0, \quad \forall x, y \in \mathbb{R}^n,\end{aligned}\quad (9)$$

where F_1, F_2 are two constant matrices. Such a description of nonlinear functions has been exploited in [37–39] and is more general than the commonly used Lipschitz conditions. The sector-bounded condition would be possible to reduce the conservatism of the main results caused by quantifying the nonlinear functions via a matrix inequality technique.

Let $s(t) \in \mathbb{R}^n$ be the state trajectory of the unforced node $\dot{s}(t) = D(r(t))f(s(t))$; then the synchronization error is defined to be $e_k(t) = x_k(t) - s(t)$. So the error dynamics of NCDN (6) can be derived as follows:

$$\begin{aligned}\dot{e}_k(t) &= \sum_{l=1}^N g_{kl}^{(1)}(r(t)) A(r(t)) e_l(t) \\ &+ \sum_{l=1}^N g_{kl}^{(2)}(r(t)) B(r(t)) e_l(t - d(t, r(t))) \\ &+ \sum_{l=1}^N g_{kl}^{(3)}(r(t)) C(r(t)) \dot{e}_l(t - \tau(t, r(t))) \\ &+ D(r(t)) f_e(e_k(t)) + u_k(t),\end{aligned}\quad (10)$$

where $f_e(e_k(t)) = f(x_k(t)) - f(s(t)), k = 1, 2, \dots, N$.

The control signal is assumed to be generated by using a zero-order-hold (ZOH) function with a sequence of hold times $0 = t_0 < t_1 < \dots < t_s < \dots$. Thus the state feedback controller takes the following form:

$$u_k(t) = K_k e_k(t_s), \quad k = 1, 2, \dots, N, \quad t_s \leq t < t_{s+1}, \quad (11)$$

where K_k is sampled-data feedback controller gain matrix to be determined. $e_k(t_s)$ is discrete measurement of $e_k(t)$ at the sampling instant t_s satisfying $\lim_{s \rightarrow +\infty} t_s = +\infty$. It is assumed that $t_{s+1} - t_s = h_s \leq \bar{h}$ for any integer $s \geq 0$, where $\bar{h} > 0$ represents the largest sampling interval.

With (10) and (11), the error dynamics of NCDN (6) can be replaced as

$$\begin{aligned} \dot{e}_k(t) = & \sum_{l=1}^N g_{kl}^{(1)}(r(t)) A(r(t)) e_l(t) \\ & + \sum_{l=1}^N g_{kl}^{(2)}(r(t)) B(r(t)) e_l(t - d(t, r(t))) \\ & + \sum_{l=1}^N g_{kl}^{(3)}(r(t)) C(r(t)) \dot{e}_l(t - \tau(t, r(t))) \\ & + D(r(t)) f_e(e_k(t)) + K_k e_k(t - h(t)), \end{aligned} \quad (12)$$

where $h(t) = t - t_s$ and $0 \leq h(t) \leq \bar{h}$.

For simplicity of notations, we denote $A(r(t))$, $B(r(t))$, $C(r(t))$, $D(r(t))$, $G^{(m)}(r(t))$, ($m = 1, 2, 3$) by A_i , B_i , C_i , D_i , $G_i^{(m)}$, ($m = 1, 2, 3$) for $r_t = i \in S$. By utilizing the Kronecker product of the matrices, (12) can be written in a more compact form as

$$\begin{aligned} \dot{e}(t) = & \mathbb{A}_i e(t) + \mathbb{B}_i e(t - d_i(t)) \\ & + \mathbb{C}_i \dot{e}(t - \tau_i(t)) + \mathbb{D}_i F_e(e(t)) + \mathbb{K} e(t - h(t)), \end{aligned} \quad (13)$$

where $\mathbb{A}_i = G_i^{(1)} \otimes A_i$, $\mathbb{B}_i = G_i^{(2)} \otimes B_i$, $\mathbb{C}_i = G_i^{(3)} \otimes C_i$, $\mathbb{D}_i = I_N \otimes D_i$, and

$$\begin{aligned} e(t) = & \text{col}\{e_1(t), e_2(t), \dots, e_N(t)\}, \\ e(t - d_i(t)) = & \text{col}\{e_1(t - d_i(t)), e_2(t - d_i(t)), \\ & \dots, e_N(t - d_i(t))\}, \\ \dot{e}(t - \tau_i(t)) = & \text{col}\{\dot{e}_1(t - \tau_i(t)), \dot{e}_2(t - \tau_i(t)), \\ & \dots, \dot{e}_N(t - \tau_i(t))\}, \\ F_e(e(t)) = & \text{col}\{f_e(e_1(t)), \\ & f_e(e_2(t)), \dots, f_e(e_N(t))\}, \\ e(t - h(t)) = & \text{col}\{e_1(t - h(t)), e_2(t - h(t)), \\ & \dots, e_N(t - h(t))\}, \\ \mathbb{K} = & \text{diag}\{K_1, K_2, \dots, K_N\}. \end{aligned} \quad (14)$$

The purpose of this paper is to design a serial of sampled-data controllers (11) to ensure the exponential synchronization of NCDN (6). Before proceeding with the main results, we present the following definitions and lemmas.

Definition 1 (see [40]). Define operator $\mathfrak{D}: C([- \rho, 0], \mathbb{R}^n) \rightarrow \mathbb{R}^n$ as $\mathfrak{D}(x_t) = x(t) - Cx(t - \tau)$. $\mathfrak{D}$ is said to be stable if the homogeneous difference equation

$$\begin{aligned} \mathfrak{D}(x_t) = & 0, \quad t \geq 0, \\ x_0 = & \psi \in \{\phi \in C([- \rho, 0], \mathbb{R}^n) : \mathfrak{D}\phi = 0\} \end{aligned} \quad (15)$$

is uniformly asymptotically stable. In this paper, that is, $\|G_i^{(3)} \otimes C_i\| < 1$.

Definition 2 (see [41]). Define the stochastic Lyapunov-Krasovskii function of the error system (13) as $V(e(t), r(t) = i, t > 0) = V(e(t), i, t)$ where its infinitesimal generator is defined as

$$\begin{aligned} \mathcal{I}V(e(t), i, t) &= \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} [\mathcal{E}\{V(e(t + \Delta t), r(t + \Delta t), t + \Delta t) | e(t) \\ &= e, r(t) = i\} - V(e(t), i, t)] \\ &= \frac{\partial}{\partial t} V(e(t), i, t) + \frac{\partial}{\partial e} V(e(t), i, t) \dot{e}(t) \\ &+ \sum_{j=1}^N \pi_{ij} V(e(t), j, t). \end{aligned} \quad (16)$$

Definition 3. The NCDN (6) is said to be exponentially synchronized if the error system (13) is exponentially stable; that is, [42] there exist two constants $\varepsilon > 0$ and $\kappa \geq 1$ such that for all, $e(t)$,

$$\|e(t)\| \leq \kappa \exp\{-\varepsilon(t - t_0)\} \sup_{\theta \in [-\varsigma, t_0]} \|e(\theta)\|, \quad (17)$$

where ε is the exponential decay rate; $\varsigma = \max\{\bar{h}, \max_{i \in S}\{\bar{\tau}_i\}, \max_{i \in S}\{d_{2i}\}\}$.

Lemma 4 (see [43]). Given matrices A , B , C , and D with appropriate dimensions and scalar α , by the definition of the Kronecker product, the following properties hold:

$$\begin{aligned} (\alpha A) \otimes B &= A \otimes (\alpha B), \\ (A + B) \otimes C &= A \otimes C + B \otimes C, \\ (A \otimes B)(C \otimes D) &= (AC) \otimes (BD), \\ (A \otimes B)^T &= A^T \otimes B^T. \end{aligned} \quad (18)$$

Lemma 5. For any constant matrix $Q = Q^T > 0$, continuous functions $0 \leq h_1(t) \leq h_2(t)$, constant scalars $0 \leq \tau_1 < \tau_2$, and constant $\varepsilon > 0$ such that the following integrations are well defined:

$$\begin{aligned} \text{(a)} \quad & \frac{\exp\{2\varepsilon h_2(t)\} - \exp\{2\varepsilon h_1(t)\}}{2\varepsilon} \\ & \times \int_{t-h_2(t)}^{t-h_1(t)} \exp\{2\varepsilon(s-t)\} x^T(s) Q x(s) ds \\ & \geq \left[\int_{t-h_2(t)}^{t-h_1(t)} x^T(s) ds \right] Q \left[\int_{t-h_2(t)}^{t-h_1(t)} x(s) ds \right], \\ \text{(b)} \quad & \frac{\tau_2^2 - \tau_1^2}{2} \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \exp\{2\varepsilon(s-t)\} x^T(s) Q x(s) ds d\theta \\ & \geq \exp\{-2\varepsilon\tau_2\} \left[\int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t x^T(s) ds d\theta \right] Q \\ & \times \left[\int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t x(s) ds d\theta \right]. \end{aligned} \quad (19)$$

Proof. (a) is directly obtained from [44]. In addition, from $-\tau_2 \leq \theta \leq -\tau_1$ and $t + \theta \leq s \leq t$, it is held that $-\tau_2 \leq \theta \leq s - t \leq 0$. Then,

$$\begin{aligned} & \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \exp \{2\varepsilon(s-t)\} x^T(s) Q x(s) ds d\theta \\ & \geq \exp \{-2\varepsilon\tau_2\} \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t x^T(s) Q x(s) ds d\theta; \end{aligned} \quad (20)$$

(b) is thus true by [45]. $\square$

Lemma 6 (see [46]). For functions $\lambda_1(t), \lambda_2(t) \in [0, 1]$, $\lambda_1(t) + \lambda_2(t) = 1$, and $\eta_1 = 0$ with $\lambda_1(t) = 0$ and $\eta_2 = 0$ with $\lambda_2(t) = 0$ and matrices $P > 0, Q > 0$, then there exists matrix T such that

$$\begin{bmatrix} P & T \\ T^T & Q \end{bmatrix} > 0, \quad (21)$$

and the following inequality holds:

$$\frac{1}{\lambda_1(t)} \eta_1 P \eta_1^T + \frac{1}{\lambda_2(t)} \eta_2 Q \eta_2^T \geq [\eta_1 \ \eta_2] \begin{bmatrix} P & T \\ T^T & Q \end{bmatrix} \begin{bmatrix} \eta_1^T \\ \eta_2^T \end{bmatrix}. \quad (22)$$

According to Definition 3, the aim will be achieved if we obtain the gain matrices $\mathbb{K}$ such that the error system (13) is exponentially stable. So we give the main results as follows.

3. Main Results

In this section, sufficient conditions are presented to ensure that the error system (13) is exponentially stable. Then, we propose a design method of the sampled-data controllers for NCDN (6).

3.1. Exponential Stability for the Error System

Theorem 7. For given scalars $\gamma_i^m, \gamma_i^M, \varepsilon, \bar{\tau}_i, \nu_i, \bar{h}, d_{1i}, d_{2i}$, and μ_i and constant scalar d_{mi} satisfying $d_{1i} < d_{mi} < d_{2i}$, the error system (13) with partially known transition rates and sector-bounded condition (9) is exponentially stable with decay rate ε and $\kappa = \sqrt{\lambda/\lambda}$ if $\|G_i^{(3)} \otimes C_i\| < 1$, and there exist symmetric positive matrices $P_i > 0, Q_{1i} > 0, Q_{2i} > 0, R_{1i} > 0, (i \in S), Q_j > 0, (j = 3, 4, 5), R_k > 0, (k = 2, 3, 4), T_l > 0, (l = 1, 2, 3), U_m > 0, V_n > 0, W_s > 0, (m, n, s = 1, 2, 3, 4, 5)$ and matrices $M_{k_1}, N_{k_2}, (k_1, k_2 = 1, 2, 3, 4)$, for any scalar $\delta > 0$, and symmetric matrices $X_i, Y_{1i}, Y_{2i}, Z_i, (i \in S)$ such that the following matrix inequalities hold.

When $i \in \mathcal{S}_k^i$; then

$$\begin{aligned} & \sum_{j \in \mathcal{S}_k^i, j \neq i} \gamma_{ij} (Q_{1j} - Y_{1i}) - \gamma_{ii} Y_{1i} \leq 0, \\ & \sum_{j \in \mathcal{S}_k^i, j \neq i} \gamma_{ij} (Q_{2j} - Y_{2i}) - \gamma_{ii} Y_{2i} \leq 0, \\ & \sum_{j \in \mathcal{S}_k^i, j \neq i} \gamma_{ij} (R_{1j} - Z_i) - \gamma_{ii} Z_i \leq 0, \end{aligned}$$

$$\begin{aligned} Q_{1j} - Y_{1i} &\leq 0, \quad j \in \mathcal{S}_{uk}^i, & Q_{2j} - Y_{2i} &\leq 0, \quad j \in \mathcal{S}_{uk}^i, \\ R_{1j} - Z_i &\leq 0, \quad j \in \mathcal{S}_{uk}^i, & P_j - X_i &\leq 0, \quad j \in \mathcal{S}_{uk}^i. \end{aligned} \quad (23)$$

When $i \in \mathcal{S}_{uk}^i$, then

$$\begin{aligned} & \sum_{j \in \mathcal{S}_k^i} \gamma_{ij} (Q_{1j} - Y_{1i}) \leq 0, \\ & \sum_{j \in \mathcal{S}_k^i} \gamma_{ij} (Q_{2j} - Y_{2i}) \leq 0, \end{aligned} \quad (24)$$

$$\begin{aligned} & \sum_{j \in \mathcal{S}_k^i} \gamma_{ij} (R_{1j} - Z_i) \leq 0, \\ & P_j - X_i \leq 0, \quad j \in \mathcal{S}_{uk}^i, \quad j \neq i, \end{aligned} \quad (25)$$

$$P_j - X_i \geq 0, \quad j \in \mathcal{S}_{uk}^i, \quad j = i,$$

$$\begin{aligned} & - \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \gamma_i^M (Q_{1j} - Y_{1i}) - \gamma_i^m Y_{1i} \leq 0, \\ & - \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \gamma_i^M (Q_{2j} - Y_{2i}) - \gamma_i^m Y_{2i} \leq 0, \end{aligned} \quad (26)$$

$$- \sum_{j \in \mathcal{S}_{uk}^i, j \neq i} \gamma_i^M (R_{1j} - Z_i) - \gamma_i^m Z_i \leq 0, \quad (27)$$

and

$$\begin{bmatrix} U_1^T & M_1 \\ M_1^T & U_1 \end{bmatrix} > 0, \quad \begin{bmatrix} V_1^T & M_2 \\ M_2^T & V_1 \end{bmatrix} > 0, \quad (28)$$

$$\begin{bmatrix} U_2^T & M_3 \\ M_3^T & U_2 \end{bmatrix} > 0, \quad \begin{bmatrix} V_2^T & M_4 \\ M_4^T & V_2 \end{bmatrix} > 0,$$

$$(i) \quad \begin{bmatrix} U_4^T & N_1 \\ N_1^T & U_4 \end{bmatrix} > 0, \quad \begin{bmatrix} V_4^T & N_2 \\ N_2^T & V_4 \end{bmatrix} > 0, \quad (29)$$

$$(ii) \quad \Omega_{i0}^\varepsilon + \Omega_{i1}^\varepsilon + \Lambda^T J \Lambda < 0,$$

$$(iii) \quad \begin{bmatrix} U_5^T & N_3 \\ N_3^T & U_5 \end{bmatrix} > 0, \quad \begin{bmatrix} V_5^T & N_4 \\ N_4^T & V_5 \end{bmatrix} > 0, \quad (30)$$

$$(iv) \quad \Omega_{i0}^\varepsilon + \Omega_{i2}^\varepsilon + \Lambda^T J \Lambda < 0,$$

where

$$\begin{aligned}
\Omega_{i0}^\varepsilon = & \sum_{m=1}^{24} E_m \Pi_m E_m^T + \mathfrak{K} \\
& - \delta [E_1 \ E_{24}] \begin{bmatrix} \widehat{F}_1 & \widehat{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} E_1^T \\ E_{24}^T \end{bmatrix} \\
& - \frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\} - 1} (E_1 - E_3) V_3 (E_1^T - E_3^T) \\
& - \exp\{-2\varepsilon \bar{\tau}_i\} (\bar{\tau}_i E_1 - E_{19}) W_1 (\bar{\tau}_i E_1^T - E_{19}^T) \\
& - \exp\{-2\varepsilon \bar{h}\} (\bar{h} E_1 - E_{23}) W_2 (\bar{h} E_1^T - E_{23}^T) \\
& - \exp\{-2\varepsilon d_{1i}\} (d_{1i} E_1 - E_9) W_3 (d_{1i} E_1^T - E_9^T) \\
& - \exp\{-2\varepsilon d_{mi}\} (q_{1i} E_1 - E_{12}) W_4 (q_{1i} E_1^T - E_{12}^T) \\
& - \exp\{-2\varepsilon d_{2i}\} (q_{2i} E_1 - E_{13}) W_5 (q_{2i} E_1^T - E_{13}^T) \\
& - \frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} [E_{18} \ E_{19} - E_{18}] \\
& \times \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} \begin{bmatrix} E_{18}^T \\ E_{19}^T - E_{18}^T \end{bmatrix} \\
& - \frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} [E_1 - E_{16} \ E_{16} - E_{14}] \\
& \times \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} \begin{bmatrix} E_1^T - E_{16}^T \\ E_{16}^T - E_{14}^T \end{bmatrix} \\
& - \frac{2\varepsilon \bar{h}}{\exp\{2\varepsilon \bar{h}\} - 1} [E_{22} \ E_{23} - E_{22}] \\
& \times \begin{bmatrix} U_2 & M_3 \\ M_3^T & U_2 \end{bmatrix} \begin{bmatrix} E_{22}^T \\ E_{23}^T - E_{22}^T \end{bmatrix} \\
& - \frac{2\varepsilon \bar{h}}{\exp\{2\varepsilon \bar{h}\} - 1} [E_1 - E_{21} \ E_{21} - E_{20}] \\
& \times \begin{bmatrix} V_2 & M_4 \\ M_4^T & V_2 \end{bmatrix} \begin{bmatrix} E_1^T - E_{21}^T \\ E_{21}^T - E_{20}^T \end{bmatrix}, \tag{31}
\end{aligned}$$

where

$$\begin{aligned}
\mathfrak{K} = & E_1 P_i (G_i^{(2)} \otimes B_i) E_2^T + E_1 P_i (G_i^{(3)} \otimes C_i) E_{17} \\
& + E_1 P_i \mathbb{K} E_{21}^T + E_1 P_i (I_N \otimes D_i) E_{24}^T \\
& + E_2 (G_i^{(2)} \otimes B_i)^T P_i E_1^T + E_{17} (G_i^{(3)} \otimes C_i)^T P_i E_1^T \\
& + E_{21} \mathbb{K}^T P_i E_1^T + E_{24} (I_N \otimes D_i)^T P_i E_1^T \\
& \widehat{F}_1 = \frac{I_N \otimes (F_1^T F_2 + F_2^T F_1)}{2},
\end{aligned}$$

$$\begin{aligned}
\widehat{F}_2 = & - \frac{(I_N \otimes F_1)^T + (I_N \otimes F_2)^T}{2}, \\
\Pi_1 = & (G_i^{(1)} \otimes A_i)^T P_i + P_i (G_i^{(1)} \otimes A_i) \\
& + \sum_{j \in \mathcal{S}_k^i} \gamma_{ij} (P_j - X_i) + 2\varepsilon P_i \\
& + Q_{1i} + Q_3 + Q_5 \\
& + R_2 + \bar{\tau}_i^2 U_1 + \bar{h}^2 U_2 \\
& + d_{1i}^2 U_3 + q_{1i}^2 U_4 + q_{2i}^2 U_5, \\
\Pi_2 = & \omega(\varepsilon, \mu_i) R_{1i}, \\
\Pi_3 = & \exp\{-2\varepsilon d_{1i}\} (R_{1i} + R_3 - R_2), \\
\Pi_4 = & \exp\{-2\varepsilon d_{mi}\} (R_4 - R_3), \\
\Pi_5 = & -\exp\{-2\varepsilon d_{2i}\} R_4, \\
\Pi_6 = & \exp\{-2\varepsilon d_{1i}\} (T_2 - T_1), \\
\Pi_7 = & \exp\{-2\varepsilon d_{mi}\} (T_3 - T_2), \\
\Pi_8 = & -\exp\{-2\varepsilon d_{2i}\} T_3, \\
\Pi_9 = & -\frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\} - 1} U_3, \\
\Pi_{14} = & -\exp\{-2\varepsilon \bar{\tau}_i\} Q_3, \\
\Pi_{15} = & -\exp\{-2\varepsilon \bar{\tau}_i\} Q_4, \\
\Pi_{16} = & -(1 - \nu_i) \exp\{-2\varepsilon \bar{\tau}_i\} Q_{1i}, \\
\Pi_{17} = & -(1 - \nu_i) \exp\{-2\varepsilon \bar{\tau}_i\} Q_{2i}, \\
\Pi_{20} = & -\exp\{-2\varepsilon \bar{h}\} Q_5,
\end{aligned}$$

$$\Pi_m = 0, \quad (m = 10, 11, 12, 13, 18, 19, 21, \dots, 24),$$

$$\begin{aligned}
\Omega_{i1}^\varepsilon = & - \frac{2\varepsilon q_{2i}}{\exp\{2\varepsilon d_{2i}\} - \exp\{2\varepsilon d_{mi}\}} E_{13} U_5 E_{13}^T \\
& - \frac{2\varepsilon q_{2i}}{\exp\{2\varepsilon d_{2i}\} - \exp\{2\varepsilon d_{mi}\}} (E_4 - E_5) V_5 (E_4^T - E_5^T) \\
& - \frac{2\varepsilon q_{1i}}{\exp\{2\varepsilon d_{mi}\} - \exp\{2\varepsilon d_{1i}\}} [E_{10} \ E_{12} - E_{10}] \\
& \times \begin{bmatrix} U_4 & N_1 \\ N_1^T & U_4 \end{bmatrix} \begin{bmatrix} E_{10}^T \\ E_{12}^T - E_{10}^T \end{bmatrix} \\
& - \frac{2\varepsilon q_{1i}}{\exp\{2\varepsilon d_{mi}\} - \exp\{2\varepsilon d_{1i}\}} [E_3 - E_2 \ E_2 - E_4] \\
& \times \begin{bmatrix} V_4 & N_2 \\ N_2^T & V_4 \end{bmatrix} \begin{bmatrix} E_3^T - E_2^T \\ E_2^T - E_4^T \end{bmatrix},
\end{aligned}$$

$$\begin{aligned} & \times \left\{ \sum_{m=1}^5 \lambda_{\max}(U_m) + \sum_{n=1}^5 \lambda_{\max}(V_n) \right\} \\ & + \frac{1 - \exp\{-2\varepsilon\zeta\} + 2\varepsilon^2\zeta^2 - 2\varepsilon\zeta}{8\varepsilon^3} \\ & \times \left\{ \sum_{s=1}^5 \lambda_{\max}(W_s) \right\}, \\ & \underline{\lambda} = \min_{i \in S} \{\lambda_{\min}(P_i)\}, \end{aligned}$$

$$\varrho_{1i} = d_{mi} - d_{1i}, \quad \varrho_{2i} = d_{2i} - d_{mi},$$

$$\varrho_{3i} = \frac{1}{2} (d_{mi}^2 - d_{1i}^2), \quad \varrho_{4i} = \frac{1}{2} (d_{2i}^2 - d_{mi}^2). \quad (33)$$

Proof. Construct the following stochastic Lyapunov functional:

$$V(e(t), i, t) = \sum_{k=1}^7 V_k(e(t), i, t), \quad (34)$$

where

$$V_1(e(t), i, t) = e^T(t) P_i e(t), \quad (35)$$

$$\begin{aligned}
V_2(e(t), i, t) = & \int_{t-\tau_i(t)}^t \exp\{2\varepsilon(s-t)\} e^T(s) Q_{1i} e(s) ds \\
& + \int_{t-\tau_i(t)}^t \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) Q_{2i} \dot{e}(s) ds \\
& + \int_{t-\bar{\tau}_i}^t \exp\{2\varepsilon(s-t)\} e^T(s) Q_3 e(s) ds \\
& + \int_{t-\bar{\tau}_i}^t \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) Q_4 \dot{e}(s) ds \\
& + \int_{t-h}^t \exp\{2\varepsilon(s-t)\} e^T(s) Q_5 e(s) ds,
\end{aligned} \tag{36}$$

$$\begin{aligned}
V_3(e(t), i, t) = & \int_{t-d_{1i}(t)}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} e^T(s) R_{1i} e(s) ds \\
& + \int_{t-d_{1i}}^t \exp\{2\varepsilon(s-t)\} e^T(s) R_2 e(s) ds \\
& + \int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} e^T(s) R_3 e(s) ds \\
& + \int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\} e^T(s) R_4 e(s) ds,
\end{aligned} \tag{37}$$

where E_i $\{i = 1, 2, \dots, 24\}$ are block entry matrices; that is,

$$E_4^T = [0 \ 0 \ 0 \ I \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0],$$

$$\varpi(\varepsilon, \mu_i) = \begin{cases} (\mu_i - 1) \exp \{-2\varepsilon d_{1i}\} & \text{with } \mu_i \geq 1, \\ (\mu_i - 1) \exp \{-2\varepsilon d_{2i}\} & \text{with } \mu_i < 1, \end{cases}$$

$$\begin{aligned} \bar{\lambda} = & \max_{i \in S} \{\lambda_{\max}(P_i)\} + \frac{1 - \exp\{-2\varepsilon\zeta\}}{2\varepsilon} \\ & \times \left\{ \sum_{j=3}^5 \lambda_{\max}(Q_j) + \sum_{k=2}^4 \lambda_{\max}(R_k) + \sum_{l=1}^3 \lambda_{\max}(T_l) \right\} \\ & + \frac{1 - \exp\{-2\varepsilon\zeta\}}{2\varepsilon} \left\{ \max_{i \in S} \{\lambda_{\max}(Q_{1i})\} + \max_{i \in S} \{\lambda_{\max}(Q_{2i})\} \right. \\ & \quad \left. + \max_{i \in S} \{\lambda_{\max}(R_{1i})\} \right\} \\ & + \frac{\exp\{-2\varepsilon\zeta\} + 2\varepsilon\zeta - 1}{4\varepsilon^2} \end{aligned}$$

$$\begin{aligned}
V_4(e(t), i, t) = & \int_{t-d_{1i}}^t \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) T_1 \dot{e}(s) ds \\
& + \int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) T_2 \dot{e}(s) ds \\
& + \int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) T_3 \dot{e}(s) ds,
\end{aligned} \quad (38)$$

$$\begin{aligned}
V_5(e(t), i, t) = & \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t \bar{\tau}_i \exp\{2\varepsilon(s-t)\} e^T(s) U_1 e(s) ds d\theta \\
& + \int_{-\bar{h}}^0 \int_{t+\theta}^t \bar{h} \exp\{2\varepsilon(s-t)\} e^T(s) U_2 e(s) ds d\theta \\
& + \int_{-d_{1i}}^0 \int_{t+\theta}^t d_{1i} \exp\{2\varepsilon(s-t)\} e^T(s) U_3 e(s) ds d\theta \\
& + \int_{-d_{mi}}^{-d_{1i}} \int_{t+\theta}^t \varrho_{1i} \exp\{2\varepsilon(s-t)\} e^T(s) U_4 e(s) ds d\theta \\
& + \int_{-d_{2i}}^{-d_{mi}} \int_{t+\theta}^t \varrho_{2i} \exp\{2\varepsilon(s-t)\} e^T(s) U_5 e(s) ds d\theta,
\end{aligned} \quad (39)$$

$$\begin{aligned}
V_6(e(t), i, t) = & \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t \bar{\tau}_i \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_1 \dot{e}(s) ds d\theta \\
& + \int_{-\bar{h}}^0 \int_{t+\theta}^t \bar{h} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_2 \dot{e}(s) ds d\theta \\
& + \int_{-d_{1i}}^0 \int_{t+\theta}^t d_{1i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_3 \dot{e}(s) ds d\theta \\
& + \int_{-d_{mi}}^{-d_{1i}} \int_{t+\theta}^t \varrho_{1i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_4 \dot{e}(s) ds d\theta \\
& + \int_{-d_{2i}}^{-d_{mi}} \int_{t+\theta}^t \varrho_{2i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_5 \dot{e}(s) ds d\theta,
\end{aligned} \quad (40)$$

$$\begin{aligned}
V_7(e(t), i, t) = & \int_{-\bar{\tau}_i}^0 \int_{\theta}^0 \int_{t+\lambda}^t \frac{\bar{\tau}_i^2}{2} \exp\{2\varepsilon(s-t)\} \dot{e}^T \\
& \times (s) W_1 \dot{e}(s) ds d\lambda d\theta \\
& + \int_{-\bar{h}}^0 \int_{\theta}^0 \int_{t+\lambda}^t \frac{\bar{h}^2}{2} \exp\{2\varepsilon(s-t)\} \dot{e}^T \\
& \times (s) W_2 \dot{e}(s) ds d\lambda d\theta \\
& + \int_{-d_{1i}}^0 \int_{\theta}^0 \int_{t+\lambda}^t \frac{d_{1i}^2}{2} \exp\{2\varepsilon(s-t)\} \dot{e}^T \\
& \times (s) W_3 \dot{e}(s) ds d\lambda d\theta
\end{aligned}$$

$$\begin{aligned}
& + \int_{-d_{mi}}^{-d_{1i}} \int_{\theta}^0 \int_{t+\lambda}^t \varrho_{3i} \exp\{2\varepsilon(s-t)\} \dot{e}^T \\
& \times (s) W_4 \dot{e}(s) ds d\lambda d\theta \\
& + \int_{-d_{2i}}^{-d_{mi}} \int_{\theta}^0 \int_{t+\lambda}^t \varrho_{4i} \exp\{2\varepsilon(s-t)\} \dot{e}^T \\
& \times (s) W_5 \dot{e}(s) ds d\lambda d\theta.
\end{aligned} \quad (41)$$

Taking Γ as its infinitesimal generator along the trajectory of error system (13), we obtain the following from Definition 3 and (34)-(35):

$$\Gamma V(e(t), i, t) = \sum_{k=1}^7 \Gamma V_k(e(t), i, t), \quad (42)$$

$$\begin{aligned}
\Gamma V_1(e(t), i, t) = & 2 \left[e^T(t) \mathbb{A}_i^T + e^T(t - d_i(t)) \mathbb{B}_i^T \right. \\
& + \dot{e}^T(t - \tau_i(t)) \mathbb{C}_i^T + F_e^T(e(t)) \mathbb{D}_i^T \\
& \left. + e^T(t - h(t)) \mathbb{K}^T \right] P_i e(t)
\end{aligned} \quad (43)$$

$$\begin{aligned}
& + e^T(t) \left[\sum_{j \in \mathcal{S}_k^i} \gamma_{ij} P_j + \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} P_j \right] e(t) \\
& + 2\varepsilon e^T(t) P_i e(t) - 2\varepsilon V_1(e(t), i, t),
\end{aligned}$$

$$\begin{aligned}
\Gamma V_2(e(t), i, t) = & e^T(t) [Q_{1i} + Q_3 + Q_5] e(t) \\
& + \dot{e}^T(t) [Q_{2i} + Q_4] \dot{e}(t) - 2\varepsilon V_2(e(t), i, t) \\
& - (1 - \dot{\tau}_i(t)) \exp\{-2\varepsilon \tau_i(t)\} \\
& \times e^T(t - \tau_i(t)) Q_{1i} e(t - \tau_i(t)) \\
& - (1 - \dot{\tau}_i(t)) \exp\{-2\varepsilon \tau_i(t)\} \\
& \times \dot{e}^T(t - \tau_i(t)) Q_{2i} \dot{e}(t - \tau_i(t)) \\
& - \exp\{-2\varepsilon \bar{\tau}_i\} e^T(t - \bar{\tau}_i) Q_3 e(t - \bar{\tau}_i) \\
& - \exp\{-2\varepsilon \bar{\tau}_i\} \dot{e}^T(t - \bar{\tau}_i) Q_4 \dot{e}(t - \bar{\tau}_i) \\
& + \sum_{j \in \mathcal{S}} \gamma_{ij} \int_{t-\tau_j(t)}^t \exp\{2\varepsilon(s-t)\} \\
& \times [e^T(s) Q_{1j} e(s) + \dot{e}^T(s) Q_{2j} \dot{e}(s)] ds \\
& - \exp\{-2\varepsilon \bar{h}\} e^T(t - \bar{h}) Q_5 e(t - \bar{h})
\end{aligned}$$

$$\begin{aligned}
&\leq e^T(t) [Q_{1i} + Q_3 + Q_5] e(t) \\
&\quad + \dot{e}^T(t) [Q_{2i} + Q_4] \dot{e}(t) - 2\varepsilon V_2(e(t), i, t) \\
&\quad - (1 - \dot{\tau}_i(t)) \exp\{-2\varepsilon\tau_i(t)\} \\
&\quad \times e^T(t - \tau_i(t)) Q_{1i} e(t - \tau_i(t)) \\
&\quad - (1 - \dot{\tau}_i(t)) \exp\{-2\varepsilon\tau_i(t)\} \\
&\quad \times \dot{e}^T(t - \tau_i(t)) Q_{2i} \dot{e}(t - \tau_i(t)) \\
&\quad - \exp\{-2\varepsilon\bar{\tau}_i\} e^T(t - \bar{\tau}_i) Q_3 e(t - \bar{\tau}_i) \\
&\quad - \exp\{-2\varepsilon\bar{\tau}_i\} \dot{e}^T(t - \bar{\tau}_i) Q_4 \dot{e}(t - \bar{\tau}_i) \\
&\quad + \sum_{j \neq i} \gamma_{ij} \int_{t-\bar{\tau}_j}^t \exp\{2\varepsilon(s-t)\} \\
&\quad \times [e^T(s) Q_{1j} e(s) + \dot{e}^T(s) Q_{2j} \dot{e}(s)] ds \\
&\quad - \exp\{-2\varepsilon\bar{h}\} e^T(t - \bar{h}) Q_5 e(t - \bar{h}),
\end{aligned} \tag{44}$$

$$\Gamma V_3(e(t), i, t)$$

$$\begin{aligned}
&= e^T(t) R_2 e(t) \\
&\quad + \exp\{-2\varepsilon d_{1i}\} e^T(t - d_{1i}) \\
&\quad \times [R_{1i} + R_3 - R_2] e(t - d_{1i}) \\
&\quad + \exp\{-2\varepsilon d_{mi}\} e^T(t - d_{mi}) \\
&\quad \times [R_4 - R_3] e(t - d_{mi}) \\
&\quad - \exp\{-2\varepsilon d_{2i}\} e^T(t - d_{2i}) \\
&\quad \times R_4 e(t - d_{2i}) - 2\varepsilon V_3(e(t), i, t) \\
&\quad - (1 - \dot{d}_i(t)) \exp\{-2\varepsilon d_i(t)\} \\
&\quad \times e^T(t - d_i(t)) R_{1i} e(t - d_i(t)) \\
&\quad + \sum_{j \in S} \gamma_{ij} \int_{t-d_j(t)}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} e^T(s) R_{1j} e(s) ds \\
&\leq e^T(t) R_2 e(t) + \exp\{-2\varepsilon d_{1i}\} \\
&\quad \times e^T(t - d_{1i}) [R_{1i} + R_3 - R_2] e(t - d_{1i}) \\
&\quad + \exp\{-2\varepsilon d_{mi}\} e^T(t - d_{mi}) \\
&\quad \times [R_4 - R_3] e(t - d_{mi}) \\
&\quad - \exp\{-2\varepsilon d_{2i}\} e^T(t - d_{2i}) \\
&\quad \times R_4 e(t - d_{2i}) - 2\varepsilon V_3(e(t), i, t) \\
&\quad - (1 - \dot{d}_i(t)) \exp\{-2\varepsilon d_i(t)\}
\end{aligned}$$

$$\begin{aligned}
&\times e^T(t - d_i(t)) R_{1i} e(t - d_i(t)) \\
&\quad + \sum_{j \neq i} \gamma_{ij} \int_{t-d_{2j}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} e^T(s) R_{1j} e(s) ds,
\end{aligned} \tag{45}$$

$$\Gamma V_4(e(t), i, t)$$

$$\begin{aligned}
&= \dot{e}^T(t) T_1 \dot{e}(t) \\
&\quad + \exp\{-2\varepsilon d_{1i}\} \dot{e}^T(t - d_{1i}) [T_2 - T_1] \\
&\quad \times \dot{e}(t - d_{1i}) - 2\varepsilon V_4(e(t), i, t) \\
&\quad + \exp\{-2\varepsilon d_{mi}\} \dot{e}^T(t - d_{mi}) \\
&\quad \times [T_3 - T_2] \dot{e}(t - d_{mi}) \\
&\quad - \exp\{-2\varepsilon d_{2i}\} \dot{e}^T(t - d_{2i}) T_3 \dot{e}(t - d_{2i}),
\end{aligned} \tag{46}$$

$$\Gamma V_5(e(t), i, t)$$

$$\begin{aligned}
&= e^T(t) [\bar{\tau}_i^2 U_1 + \bar{h}^2 U_2 \\
&\quad + d_{1i}^2 U_3 + \varrho_{1i}^2 U_4 + \varrho_{2i}^2 U_5] e(t) \\
&\quad - 2\varepsilon V_5(e(t), i, t) \\
&\quad - \int_{t-\bar{\tau}_i}^t \bar{\tau}_i \exp\{2\varepsilon(s-t)\} e^T(s) U_1 e(s) ds \\
&\quad - \int_{t-\bar{h}}^t \bar{h} \exp\{2\varepsilon(s-t)\} e^T(s) U_2 e(s) ds \\
&\quad - \int_{t-d_{1i}}^t d_{1i} \exp\{2\varepsilon(s-t)\} e^T(s) U_3 e(s) ds \\
&\quad - \int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp\{2\varepsilon(s-t)\} e^T(s) U_4 e(s) ds \\
&\quad - \int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} e^T(s) U_5 e(s) ds,
\end{aligned} \tag{47}$$

$$\Gamma V_6(e(t), i, t)$$

$$\begin{aligned}
&= \dot{e}^T(t) [\bar{\tau}_i^2 V_1 + \bar{h}^2 V_2 + d_{1i}^2 V_3 + \varrho_{1i}^2 V_4 + \varrho_{2i}^2 V_5] \dot{e}(t) \\
&\quad - 2\varepsilon V_6(e(t), i, t) \\
&\quad - \int_{t-\bar{\tau}_i}^t \bar{\tau}_i \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_1 \dot{e}(s) ds \\
&\quad - \int_{t-\bar{h}}^t \bar{h} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_2 \dot{e}(s) ds \\
&\quad - \int_{t-d_{1i}}^t d_{1i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_3 \dot{e}(s) ds \\
&\quad - \int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_4 \dot{e}(s) ds \\
&\quad - \int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_5 \dot{e}(s) ds,
\end{aligned} \tag{48}$$

$\Gamma V_7(e(t), i, t)$

$$= \dot{e}^T(t) \left[\frac{\bar{\tau}_i^4}{4} W_1 + \frac{\bar{h}^4}{4} W_2 + \frac{d_{1i}^4}{4} W_3 + \varrho_{3i}^2 W_4 + \varrho_{4i}^2 W_5 \right] \dot{e}(t) \\ - 2\varepsilon V_7(e(t), i, t) \\ - \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t \frac{\bar{\tau}_i^2}{2} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) W_1 \dot{e}(s) ds d\theta \\ - \int_{-\bar{h}}^0 \int_{t+\theta}^t \frac{\bar{h}^2}{2} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) W_2 \dot{e}(s) ds d\theta$$

$$- \int_{-d_{1i}}^0 \int_{t+\theta}^t \frac{d_{1i}^2}{2} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) W_3 \dot{e}(s) ds d\theta \\ - \int_{-d_{mi}}^{-d_{1i}} \int_{t+\theta}^t \varrho_{3i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) W_4 \dot{e}(s) ds d\theta \\ - \int_{-d_{2i}}^{-d_{mi}} \int_{t+\theta}^t \varrho_{4i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) W_5 \dot{e}(s) ds d\theta. \quad (49)$$

Define

$$\xi(t) = \text{col} \left\{ e(t) e(t-d_i(t)) e(t-d_{1i}) e(t-d_{mi}) e(t-d_{2i}) \dot{e}(t-d_{1i}) \dot{e}(t-d_{mi}) \dot{e}(t-d_{2i}) \right. \\ \int_{t-d_{1i}}^t e(s) ds \int_{t-d_{1i}(t)}^{t-d_{1i}} e(s) ds \int_{t-d_{2i}}^{t-d_{1i}(t)} e(s) ds \int_{t-d_{mi}}^{t-d_{1i}} e(s) ds \int_{t-d_{2i}}^{t-d_{mi}} e(s) ds \\ e(t-\bar{\tau}_i) \dot{e}(t-\bar{\tau}_i) e(t-\tau_i(t)) \dot{e}(t-\tau_i(t)) \int_{t-\tau_i(t)}^t e(s) ds \int_{t-\bar{\tau}_i}^t e(s) ds \\ \left. e(t-\bar{h}) e(t-h(t)) \int_{t-h(t)}^t e(s) ds \int_{t-\bar{h}}^t e(s) ds F_e(e(t)) \right\}. \quad (50)$$

Due to $\sum_{j=1}^N \gamma_{ij} = 0$, the following zero equations hold for arbitrary matrices:

$$X_i = X_i^T, \quad Y_{1i} = Y_{1i}^T, \quad (51) \\ Y_{2i} = Y_{2i}^T, \quad Z_i = Z_i^T, \quad i \in S;$$

that is,

$$-e^T(t) \left[\sum_{j \in \mathcal{S}_k^i} \gamma_{ij} X_i + \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} X_i \right] e(t) = 0, \\ - \int_{t-\bar{\tau}_j}^t \exp\{2\varepsilon(s-t)\} e^T(s) \\ \times \left[\sum_{j \in \mathcal{S}_k^i} \gamma_{ij} Y_{1i} + \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} Y_{1i} \right] e(s) ds = 0, \\ - \int_{t-\bar{\tau}_j}^t \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) \\ \times \left[\sum_{j \in \mathcal{S}_k^i} \gamma_{ij} Y_{2i} + \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} Y_{2i} \right] \dot{e}(s) ds = 0,$$

$$- \int_{t-d_{2j}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} e^T(s) \\ \times \left[\sum_{j \in \mathcal{S}_k^i} \gamma_{ij} Z_i + \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} Z_i \right] e(s) ds = 0. \quad (52)$$

Based on (9), we can obtain the following inequality for any $\delta > 0$:

$$\delta \left[e^T(t) \quad F_e^T(e(t)) \right] \begin{bmatrix} \widehat{F}_1 & \widehat{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} e(t) \\ F_e(e(t)) \end{bmatrix} \\ = \delta \xi^T(t) \begin{bmatrix} E_1 & E_{24} \end{bmatrix} \begin{bmatrix} \widehat{F}_1 & \widehat{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} E_1^T \\ E_{24}^T \end{bmatrix} \xi(t) \leq 0. \quad (53)$$

From (42) and (53), we have

$$\Gamma V(e(t), i, t) \leq \sum_{k=1}^7 \Gamma V_k(e(t), i, t) \\ - \delta \xi^T(t) \begin{bmatrix} E_1 & E_{24} \end{bmatrix} \begin{bmatrix} \widehat{F}_1 & \widehat{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} E_1^T \\ E_{24}^T \end{bmatrix} \xi(t). \quad (54)$$

From (8) we easily obtain that

$$-(1-\dot{\tau}_i(t)) \exp\{-2\varepsilon\tau_i(t)\} \leq -(1-\nu_i) \exp\{-2\varepsilon\bar{\tau}_i\}, \\ -(1-\dot{d}_i(t)) \exp\{-2\varepsilon d_i(t)\} \leq \omega(\varepsilon, \mu_i). \quad (55)$$

Notice (a) of Lemma 5; then,

$$\begin{aligned}
& - \int_{t-d_{1i}}^t d_{1i} \exp \{2\varepsilon(s-t)\} e^T(s) U_3 e(s) ds \\
& \leq - \frac{2\varepsilon d_{1i}}{\exp \{2\varepsilon d_{1i}\} - 1} \xi^T(t) E_9 U_3 E_9^T \xi(t), \\
& - \int_{t-d_{1i}}^t d_{1i} \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) V_3 \dot{e}(s) ds \\
& \leq - \frac{2\varepsilon d_{1i}}{\exp \{2\varepsilon d_{1i}\} - 1} \xi^T(t) (E_1 - E_3) V_3 (E_1^T - E_3^T) \xi(t).
\end{aligned} \tag{56}$$

Notice (b) of Lemma 5; then,

$$\begin{aligned}
& - \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t \frac{\bar{\tau}_i^2}{2} \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) W_1 \dot{e}(s) ds d\theta \\
& \leq - \exp \{-2\varepsilon \bar{\tau}_i\} \xi^T(t) (\bar{\tau}_i E_1 - E_{19}) W_1 \\
& \quad \times (\bar{\tau}_i E_1^T - E_{19}^T) \xi(t), \\
& - \int_{-\bar{h}}^0 \int_{t+\theta}^t \frac{\bar{h}^2}{2} \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) W_2 \dot{e}(s) ds d\theta \\
& \leq - \exp \{-2\varepsilon \bar{h}\} \xi^T(t) (\bar{h} E_1 - E_{23}) W_2 \\
& \quad \times (\bar{h} E_1^T - E_{23}^T) \xi(t), \\
& - \int_{-d_{1i}}^0 \int_{t+\theta}^t \frac{d_{1i}^2}{2} \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) W_3 \dot{e}(s) ds d\theta \\
& \leq - \exp \{-2\varepsilon d_{1i}\} \xi^T(t) (d_{1i} E_1 - E_9) W_3 \\
& \quad \times (d_{1i} E_1^T - E_9^T) \xi(t), \\
& - \int_{-d_{mi}}^{-d_{1i}} \int_{t+\theta}^t \varrho_{3i} \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) W_4 \dot{e}(s) ds d\theta \\
& \leq - \exp \{-2\varepsilon d_{mi}\} \xi^T(t) (\varrho_{1i} E_1 - E_{12}) W_4 \\
& \quad \times (\varrho_{1i} E_1^T - E_{12}^T) \xi(t), \\
& - \int_{-d_{2i}}^{-d_{mi}} \int_{t+\theta}^t \varrho_{4i} \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) W_5 \dot{e}(s) ds d\theta \\
& \leq - \exp \{-2\varepsilon d_{2i}\} \xi^T(t) (\varrho_{2i} E_1 - E_{13}) W_5 \\
& \quad \times (\varrho_{2i} E_1^T - E_{13}^T) \xi(t).
\end{aligned} \tag{57}$$

For $d_i(t) \in [d_{1i}, d_{mi}]$, the following is held from (a) of Lemma 5:

$$\begin{aligned}
& - \int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) V_4 \dot{e}(s) ds \\
& = - \left\{ \int_{t-d_i(t)}^{t-d_{1i}} + \int_{t-d_{mi}}^{t-d_i(t)} \right\} \varrho_{1i} \\
& \quad \times \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) V_4 \dot{e}(s) ds \\
& \leq - \frac{2\varepsilon \varrho_{1i}}{\exp \{2\varepsilon d_i(t)\} - \exp \{2\varepsilon d_{1i}\}} \\
& \quad \times \left[\int_{t-d_i(t)}^{t-d_{1i}} \dot{e}^T(s) ds \right] V_4 \left[\int_{t-d_i(t)}^{t-d_{1i}} \dot{e}(s) ds \right] \\
& \quad - \frac{2\varepsilon \varrho_{1i}}{\exp \{2\varepsilon d_{mi}\} - \exp \{2\varepsilon d(t)\}} \\
& \quad \times \left[\int_{t-d_{mi}}^{t-d_i(t)} \dot{e}^T(s) ds \right] V_4 \left[\int_{t-d_{mi}}^{t-d_i(t)} \dot{e}(s) ds \right] \\
& = - \frac{2\varepsilon \varrho_{1i}}{\exp \{2\varepsilon d_{mi}\} - \exp \{2\varepsilon d_{1i}\}} \\
& \quad \times \xi^T(t) \left[\frac{1}{\lambda_{1i}(t)} (E_3 - E_2) V_4 (E_3^T - E_2^T) \right. \\
& \quad \left. + \frac{1}{\lambda_{2i}(t)} (E_2 - E_4) V_4 (E_2^T - E_4^T) \right] \xi(t),
\end{aligned} \tag{58}$$

where

$$\begin{aligned}
\lambda_{1i}(t) &= \frac{\exp \{2\varepsilon d_i(t)\} - \exp \{2\varepsilon d_{1i}\}}{\exp \{2\varepsilon d_{mi}\} - \exp \{2\varepsilon d_{1i}\}}, \\
\lambda_{2i}(t) &= \frac{\exp \{2\varepsilon d_{mi}\} - \exp \{2\varepsilon d_i(t)\}}{\exp \{2\varepsilon d_{mi}\} - \exp \{2\varepsilon d_{1i}\}}.
\end{aligned} \tag{59}$$

By Lemma 6, there exists matrix N_2 with appropriate dimensions such that

$$\begin{aligned}
& - \int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp \{2\varepsilon(s-t)\} \dot{e}^T(s) V_4 \dot{e}(s) ds \\
& \leq - \frac{2\varepsilon \varrho_{1i}}{\exp \{2\varepsilon d_{mi}\} - \exp \{2\varepsilon d_{1i}\}} \xi^T(t) \\
& \quad \times [E_3 - E_2 \quad E_2 - E_4] \begin{bmatrix} V_4 & N_2 \\ N_2^T & V_4 \end{bmatrix} \begin{bmatrix} E_3^T - E_2^T \\ E_2^T - E_4^T \end{bmatrix} \xi(t), \\
& \quad \begin{bmatrix} V_4 & N_2 \\ N_2^T & V_4 \end{bmatrix} > 0.
\end{aligned} \tag{60}$$

Similarly, considering $-\int_{t-d_{mi}}^{t-d_{li}} \varrho_{li} \exp\{2\varepsilon(s-t)\} x^T(s) U_4 x(s) ds$ and following the same procedure, there exists matrix N_1 with appropriate dimensions such that

$$\begin{aligned} & -\int_{t-d_{mi}}^{t-d_{li}} \varrho_{li} \exp\{2\varepsilon(s-t)\} e^T(s) U_4 e(s) ds \\ & \leq -\frac{2\varepsilon \varrho_{li}}{\exp\{2\varepsilon d_{mi}\} - \exp\{2\varepsilon d_{li}\}} \xi^T(t) \\ & \quad \times [E_{10} \ E_{12} - E_{10}] \begin{bmatrix} U_4 & N_1 \\ N_1^T & U_4 \end{bmatrix} \begin{bmatrix} E_{10}^T & E_{12}^T \\ E_{12}^T - E_{10}^T \end{bmatrix} \xi(t), \\ & \quad \begin{bmatrix} U_4 & N_1 \\ N_1^T & U_4 \end{bmatrix} > 0. \end{aligned} \quad (61)$$

For $\tau_i(t) \in [0, \bar{\tau}_i]$, with the same matrix inequalities technique, we obtain the following:

$$\begin{aligned} & -\int_{t-\bar{\tau}_i}^t \bar{\tau}_i \exp\{2\varepsilon(s-t)\} e^T(s) U_1 e(s) ds \\ & \leq -\frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} \xi^T(t) [E_{18} \ E_{19} - E_{18}] \\ & \quad \times \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} \begin{bmatrix} E_{18}^T \\ E_{19}^T - E_{18}^T \end{bmatrix} \xi(t), \\ & \quad \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} > 0, \\ & -\int_{t-\bar{\tau}_i}^t \bar{\tau}_i \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_1 \dot{e}(s) ds \\ & \leq -\frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} \xi^T(t) [E_1 - E_{16} \ E_{16} - E_{14}] \\ & \quad \times \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} \begin{bmatrix} E_1^T - E_{16}^T \\ E_{16}^T - E_{14}^T \end{bmatrix} \xi(t), \\ & \quad \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} > 0. \end{aligned} \quad (62)$$

For $h(t) \in [0, \bar{h}]$, we obtain the following:

$$\begin{aligned} & -\int_{t-\bar{h}}^t \bar{h} \exp\{2\varepsilon(s-t)\} e^T(s) U_2 e(s) ds \\ & \leq -\frac{2\varepsilon \bar{h}}{\exp\{2\varepsilon \bar{h}\} - 1} \xi^T(t) [E_{22} \ E_{23} - E_{22}] \\ & \quad \times \begin{bmatrix} U_2 & M_3 \\ M_3^T & U_2 \end{bmatrix} \begin{bmatrix} E_{22}^T \\ E_{23}^T - E_{22}^T \end{bmatrix} \xi(t), \end{aligned}$$

$$\begin{bmatrix} U_2 & M_3 \\ M_3^T & U_2 \end{bmatrix} > 0,$$

$$\begin{aligned} & -\int_{t-\bar{h}}^t \bar{h} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_2 \dot{e}(s) ds \\ & \leq -\frac{2\varepsilon \bar{h}}{\exp\{2\varepsilon \bar{h}\} - 1} \xi^T(t) [E_1 - E_{21} \ E_{21} - E_{20}] \\ & \quad \times \begin{bmatrix} V_2 & M_4 \\ M_4^T & V_2 \end{bmatrix} \begin{bmatrix} E_1^T - E_{21}^T \\ E_{21}^T - E_{20}^T \end{bmatrix} \xi(t), \\ & \quad \begin{bmatrix} V_2 & M_4 \\ M_4^T & V_2 \end{bmatrix} > 0. \end{aligned} \quad (63)$$

Consider $-\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} e^T(s) U_5 e(s) ds$ and $-\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_5 \dot{e}(s) ds$, which are directly estimated by (a) of Lemma 5; that is,

$$\begin{aligned} & -\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} e^T(s) U_5 e(s) ds \\ & \leq -\frac{2\varepsilon \varrho_{2i}}{\exp\{2\varepsilon d_{2i}\} - \exp\{2\varepsilon d_{mi}\}} \xi^T(t) E_{13} U_5 E_{13}^T \xi(t), \\ & -\int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) V_5 \dot{e}(s) ds \\ & \leq -\frac{2\varepsilon \varrho_{2i}}{\exp\{2\varepsilon d_{2i}\} - \exp\{2\varepsilon d_{mi}\}} \\ & \quad \times \xi^T(t) (E_4 - E_5) V_5 (E_4^T - E_5^T) \xi(t). \end{aligned} \quad (64)$$

In addition, with the error system (53), we obtain $\dot{e}(t) = \Lambda \xi(t)$ and

$$\dot{e}^T(t) J \dot{e}(t) = \xi^T(t) \Lambda^T J \Lambda \xi(t), \quad (65)$$

where Λ and J have been defined in Theorem 7.

Substituting (43)–(3.1) and (55)–(65) into (54), we obtain

$$\begin{aligned} & \Gamma V(e(t), i, t) + 2\varepsilon V(e(t), i, t) \\ & \leq \xi^T(t) (\Omega_{i0}^\varepsilon + \Omega_{i1}^\varepsilon + \Lambda^T J \Lambda) \xi(t) + \bar{U}, \end{aligned} \quad (66)$$

where

$$\begin{aligned}
\mathcal{U} = & \int_{t-\bar{\tau}_j}^t \exp \{2\varepsilon (s-t)\} e^T (s) \\
& \times \left\{ \sum_{j \neq i} \gamma_{ij} Q_{1j} - \sum_{j \in \mathcal{S}_k^i} \gamma_{ij} Y_{2i} - \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} Y_{1i} \right\} e(s) ds \\
& + \int_{t-\bar{\tau}_j}^t \exp \{2\varepsilon (s-t)\} \dot{e}^T (s) \\
& \times \left\{ \sum_{j \neq i} \gamma_{ij} Q_{2j} - \sum_{j \in \mathcal{S}_k^i} \gamma_{ij} Y_{2i} - \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} Y_{2i} \right\} \dot{e}(s) ds \\
& + \int_{t-d_{2j}}^{t-d_{1i}} \exp \{2\varepsilon (s-t)\} e^T (s) \\
& \times \left\{ \sum_{j \neq i} \gamma_{ij} R_{1j} - \sum_{j \in \mathcal{S}_k^i} \gamma_{ij} Z_i - \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} Z_i \right\} e(s) ds \\
& + e^T (t) \left\{ \sum_{j \in \mathcal{S}_{uk}^i} \gamma_{ij} (P_j - X_i) \right\} e(t).
\end{aligned} \tag{67}$$

On the other hand, for $d_i(t) \in [d_{mi}, d_{2i}]$, the integral terms

$$\begin{aligned}
& - \int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp \{2\varepsilon (s-t)\} e^T (s) U_5 e(s) ds, \\
& - \int_{t-d_{2i}}^{t-d_{mi}} \varrho_{2i} \exp \{2\varepsilon (s-t)\} \dot{e}^T (s) V_5 \dot{e}(s) ds
\end{aligned} \tag{68}$$

are disposed and estimated by Lemma 6.

$$\begin{aligned}
& - \int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp \{2\varepsilon (s-t)\} e^T (s) U_4 e(s) ds, \\
& - \int_{t-d_{mi}}^{t-d_{1i}} \varrho_{1i} \exp \{2\varepsilon (s-t)\} \dot{e}^T (s) V_4 \dot{e}(s) ds
\end{aligned} \tag{69}$$

are directly estimated by (a) of Lemma 5. Therefore,

$$\begin{aligned}
& \Gamma V(e(t), i, t) + 2\varepsilon V(e(t), i, t) \\
& \leq \xi^T(t) (\Omega_{i0}^\varepsilon + \Omega_{i2}^\varepsilon + \Lambda^T J \Lambda) \xi(t) + \mathcal{U}.
\end{aligned} \tag{70}$$

With (66) and (70), the following inequality is held for $d_i(t) \in [d_{1i}, d_{2i}]$ if (23)–(28) are satisfied:

$$\Gamma V(e(t), i, t) + 2\varepsilon V(e(t), i, t) < 0. \tag{71}$$

From the stochastic Lyapunov functional (34) and (71), it is held that

$$V(e(t), i, t) \geq \min_{i \in \mathcal{S}} \{\lambda_{\min}(P_i)\} \|e(t)\|^2 = \underline{\lambda} \|e(t)\|^2, \tag{72}$$

$$V(e(t), i, t) < \exp \{-2\varepsilon (t-t_0)\} V(e(t_0), r(t_0), t_0).$$

Moreover, we have

$$\begin{aligned}
V(e(t_0), r(t_0), t_0) = & \sum_{k=1}^7 V_k(e(t_0), r(t_0), t_0), \\
& \sum_{k=1}^7 V_k(e(t_0), r(t_0), t_0) \\
& \leq \int_{t_0-\varsigma}^{t_0} \exp \{2\varepsilon (s-t_0)\} e^T (s) \\
& \times \left\{ Q_{1i} + Q_3 + Q_5 + R_{1i} + \sum_{k=2}^4 R_k \right\} e(s) ds \\
& + \int_{t_0-\varsigma}^{t_0} \exp \{2\varepsilon (s-t_0)\} \dot{e}^T (s) \\
& \times \left\{ Q_{2i} + Q_4 + \sum_{l=1}^3 T_l \right\} \dot{e}(s) ds \\
& + \int_{-\varsigma}^0 \int_{t_0+\theta}^{t_0} \varsigma \exp \{2\varepsilon (s-t_0)\} e^T \\
& \times (s) \left(\sum_{m=1}^5 U_m \right) e(s) ds d\theta \\
& + \int_{-\varsigma}^0 \int_{t_0+\theta}^{t_0} \varsigma \exp \{2\varepsilon (s-t_0)\} \dot{e}^T \\
& \times (s) \left(\sum_{n=1}^5 V_n \right) \dot{e}(s) ds d\theta \\
& + \int_{-\varsigma}^0 \int_{t_0+\lambda}^0 \int_{t_0+\lambda}^{t_0} \frac{\varsigma^2}{2} \exp \{2\varepsilon (s-t_0)\} \dot{e}^T \\
& \times (s) \left(\sum_{s=1}^5 W_s \right) \dot{e}(s) ds d\lambda d\theta \\
& + e^T(t_0) P_i e(t_0) \\
& \leq \bar{\lambda} \|e(t_0)\|_\varsigma^2.
\end{aligned} \tag{73}$$

Then from (72) and (73), it is readily seen that

$$\|e(t)\| \leq \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}} \exp \{-\varepsilon (t-t_0)\} \|e(t_0)\|_\varsigma, \tag{74}$$

where $\kappa = \sqrt{\bar{\lambda}/\underline{\lambda}} \geq 1$.

Therefore, the error system (13) with partially known transition rates and sector-bounded condition (9) is exponentially stable with a decay rate ε . This completes the proof. $\square$

In some situations, due to the complexity of NCDN (6), the information on the transition rates may be completely

unknown, which can be viewed as switched complex dynamical networks with arbitrary switching. The following corollary is therefore given to guarantee the exponential stability for this case.

Corollary 8. For given scalars $\gamma_i^m, \gamma_i^M, \varepsilon, \bar{\tau}_i, \nu_i, \bar{h}, d_{1i}, d_{2i}$, and μ_i and constant scalar d_{mi} satisfying $d_{1i} < d_{mi} < d_{2i}$, the error system (13) with entirely unknown transition rates and sector-bounded condition (9) is exponentially stable with decay rate ε and $\kappa = \sqrt{\bar{\lambda}/\lambda}$ if $\|G_i^{(3)} \otimes C_i\| < 1$, and there exist symmetric positive matrices $P > 0, Q_j > 0, (j = 1, 2, 3, 4, 5), R_k > 0, (k = 1, 2, 3, 4), T_l > 0, (l = 1, 2, 3), U_m > 0, V_n > 0, W_s > 0, (m, n, s = 1, 2, 3, 4, 5)$ and matrices $M_{k_1}, N_{k_2}, (k_1, k_2 = 1, 2, 3, 4)$, for any scalar $\delta > 0$, and symmetric matrices Y_1, Y_2, Z such that (28) and the following matrix inequalities hold:

$$- \sum_{j \in \mathcal{S}_{ik}^i, j \neq i} \gamma_i^M (Q_1 - Y_1) - \gamma_i^m Y_1 \leq 0, \quad (75)$$

$$- \sum_{j \in \mathcal{S}_{ik}^i, j \neq i} \gamma_i^M (Q_2 - Y_2) - \gamma_i^m Y_2 \leq 0,$$

$$- \sum_{j \in \mathcal{S}_{ik}^i, j \neq i} \gamma_i^M (R_1 - Z) - \gamma_i^m Z \leq 0, \quad (76)$$

$$(i) \quad \begin{bmatrix} U_4 & N_1 \\ N_1^T & U_4 \end{bmatrix} > 0, \quad \begin{bmatrix} V_4 & N_2 \\ N_2^T & V_4 \end{bmatrix} > 0, \quad (77)$$

$$(ii) \quad \bar{\Omega}_{i0}^\varepsilon + \bar{\Omega}_{i1}^\varepsilon + \bar{\Lambda}^T \bar{J} \bar{\Lambda} < 0,$$

$$(iii) \quad \begin{bmatrix} U_5 & N_3 \\ N_3^T & U_5 \end{bmatrix} > 0, \quad \begin{bmatrix} V_5 & N_4 \\ N_4^T & V_5 \end{bmatrix} > 0, \quad (78)$$

$$(iv) \quad \bar{\Omega}_{i0}^\varepsilon + \bar{\Omega}_{i2}^\varepsilon + \bar{\Lambda}^T \bar{J} \bar{\Lambda} < 0,$$

where

$$\bar{J} = Q_2 + Q_4 + T_1 + \bar{\tau}_i^2 V_1 + \bar{h}^2 V_2$$

$$+ d_{1i}^2 V_3 + \varrho_{1i}^2 V_4 + \varrho_{2i}^2 V_5 + \frac{\bar{\tau}_i^4}{4} W_1 + \frac{\bar{h}^4}{4} W_2$$

$$+ \frac{d_{1i}^4}{4} W_3 + \varrho_{3i}^2 W_4 + \varrho_{4i}^2 W_5,$$

$$\bar{\Lambda} = \Lambda, \quad \bar{\Omega}_{i1}^\varepsilon = \Omega_{i1}^\varepsilon, \quad \bar{\Omega}_{i2}^\varepsilon = \Omega_{i2}^\varepsilon,$$

$$\bar{\Omega}_{i0}^\varepsilon = \sum_{m=1}^{24} E_m \bar{\Pi}_m E_m^T + \mathfrak{K} - \delta$$

$$\times [E_1 \ E_{24}] \begin{bmatrix} \widehat{F}_1 & \widehat{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} E_1^T \\ E_{24}^T \end{bmatrix}$$

$$- \frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\} - 1} (E_1 - E_3) V_3 (E_1^T - E_3^T)$$

$$- \exp\{-2\varepsilon \bar{\tau}_i\} (\bar{\tau}_i E_1 - E_{19}) W_1 (\bar{\tau}_i E_1^T - E_{19}^T)$$

$$- \exp\{-2\varepsilon \bar{h}\} (\bar{h} E_1 - E_{23}) W_2 (\bar{h} E_1^T - E_{23}^T)$$

$$- \exp\{-2\varepsilon d_{1i}\} (d_{1i} E_1 - E_9) W_3 (d_{1i} E_1^T - E_9^T)$$

$$- \exp\{-2\varepsilon d_{mi}\} (\varrho_{1i} E_1 - E_{12}) W_4 (\varrho_{1i} E_1^T - E_{12}^T)$$

$$- \exp\{-2\varepsilon d_{2i}\} (\varrho_{2i} E_1 - E_{13}) W_5 (\varrho_{2i} E_1^T - E_{13}^T)$$

$$- \frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} [E_{18} \ E_{19} - E_{18}]$$

$$\times \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} \begin{bmatrix} E_{18}^T \\ E_{19}^T - E_{18}^T \end{bmatrix}$$

$$- \frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} [E_1 - E_{16} \ E_{16} - E_{14}]$$

$$\times \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} \begin{bmatrix} E_1^T - E_{16}^T \\ E_{16}^T - E_{14}^T \end{bmatrix}$$

$$- \frac{2\varepsilon \bar{h}}{\exp\{2\varepsilon \bar{h}\} - 1} [E_{22} \ E_{23} - E_{22}]$$

$$\times \begin{bmatrix} U_2 & M_3 \\ M_3^T & U_2 \end{bmatrix} \begin{bmatrix} E_{22}^T \\ E_{23}^T - E_{22}^T \end{bmatrix}$$

$$- \frac{2\varepsilon \bar{h}}{\exp\{2\varepsilon \bar{h}\} - 1} [E_1 - E_{21} \ E_{21} - E_{20}]$$

$$\times \begin{bmatrix} V_2 & M_4 \\ M_4^T & V_2 \end{bmatrix} \begin{bmatrix} E_1^T - E_{21}^T \\ E_{21}^T - E_{20}^T \end{bmatrix},$$

$$\bar{\Pi}_1 = (G_i^{(1)} \otimes A_i)^T P + P (G_i^{(1)} \otimes A_i)$$

$$+ 2\varepsilon P + Q_1 + Q_3 + Q_5 + R_2$$

$$+ \bar{\tau}_i^2 U_1 + \bar{h}^2 U_2 + d_{1i}^2 U_3 + \varrho_{1i}^2 U_4 + \varrho_{2i}^2 U_5,$$

$$\bar{\Pi}_2 = \omega(\varepsilon, \mu_i) R_1, \quad \bar{\Pi}_3 = \exp\{-2\varepsilon d_{1i}\} (R_1 + R_3 - R_2),$$

$$\bar{\Pi}_{16} = -(1 - \nu_i) \exp\{-2\varepsilon \bar{\tau}_i\} Q_1,$$

$$\bar{\Pi}_{17} = -(1 - \nu_i) \exp\{-2\varepsilon \bar{\tau}_i\} Q_2,$$

$$\bar{\Pi}_4 = \Pi_4, \quad \bar{\Pi}_5 = \Pi_5, \quad \bar{\Pi}_6 = \Pi_6,$$

$$\bar{\Pi}_7 = \Pi_7, \quad \bar{\Pi}_8 = \Pi_8,$$

$$\bar{\Pi}_9 = \Pi_9, \quad \bar{\Pi}_{14} = \Pi_{14}, \quad \bar{\Pi}_{15} = \Pi_{15}, \quad \bar{\Pi}_{20} = \Pi_{20},$$

$$\bar{\Pi}_m = 0, \quad (m = 10, 11, 12, 13, 18, 19, 21, \dots, 24),$$

$$\bar{\lambda} = \lambda_{\max}(P) + \frac{1 - \exp\{-2\varepsilon \varsigma\}}{2\varepsilon}$$

$$\times \left\{ \sum_{j=1}^5 \lambda_{\max}(Q_j) + \sum_{k=1}^4 \lambda_{\max}(R_k) + \sum_{l=1}^3 \lambda_{\max}(T_l) \right\}$$

$$+ \frac{\exp\{-2\varepsilon \varsigma\} + 2\varepsilon \varsigma - 1}{4\varepsilon^2}$$

$$\begin{aligned} & \times \left\{ \sum_{m=1}^5 \lambda_{\max}(U_m) + \sum_{n=1}^5 \lambda_{\max}(V_n) \right\} \\ & + \frac{1 - \exp\{-2\varepsilon\zeta\} + 2\varepsilon^2\zeta^2 - 2\varepsilon\zeta}{8\varepsilon^3} \\ & \times \left\{ \sum_{s=1}^5 \lambda_{\max}(W_s) \right\}. \end{aligned} \quad (79)$$

Other notations are the same as those in Theorem 7.

Proof. Since the elements of transition rates are entirely unknown, we choose the Lyapunov functional as follows:

$$V(e(t), i, t) = e^T(t) P e(t) + \sum_{k=2}^7 V_k(e(t), i, t), \quad (80)$$

where

$$\begin{aligned} V_2(e(t), i, t) &= \int_{t-\tau_i(t)}^t \exp\{2\varepsilon(s-t)\} e^T(s) Q_1 e(s) ds \\ &+ \int_{t-\tau_i(t)}^t \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) Q_2 \dot{e}(s) ds \\ &+ \int_{t-\bar{\tau}_i}^t \exp\{2\varepsilon(s-t)\} e^T(s) Q_3 e(s) ds \\ &+ \int_{t-\bar{\tau}_i}^t \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) Q_4 \dot{e}(s) ds \\ &+ \int_{t-\bar{h}}^t \exp\{2\varepsilon(s-t)\} e^T(s) Q_5 e(s) ds, \\ V_3(e(t), i, t) &= \int_{t-d_i(t)}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} e^T(s) R_1 e(s) ds \\ &+ \int_{t-d_{1i}}^t \exp\{2\varepsilon(s-t)\} e^T(s) R_2 e(s) ds \\ &+ \int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} e^T(s) R_3 e(s) ds \\ &+ \int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\} e^T(s) R_4 e(s) ds, \\ V_4(e(t), i, t) &= \int_{t-d_{1i}}^t \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) T_1 \dot{e}(s) ds \\ &+ \int_{t-d_{mi}}^{t-d_{1i}} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) T_2 \dot{e}(s) ds \\ &+ \int_{t-d_{2i}}^{t-d_{mi}} \exp\{2\varepsilon(s-t)\} \dot{e}^T(s) T_3 \dot{e}(s) ds, \end{aligned} \quad (81)$$

and $V_5(e(t), i, t)$, $V_6(e(t), i, t)$, $V_7(e(t), i, t)$ are the same as (39), (40), (41).

Then we follow a similar line as in proof of Theorem 7 and obtain the result. $\square$

3.2. The Sampled-Data Controllers for NCDN. In this subsection, we give the design method of the desired sampled-data controllers to ensure the NCDN (6) exponentially synchronized on the basis of Theorem 7.

Theorem 9. The NCDN (6) with partially known transition rates and sector-bounded condition (9) is exponentially synchronized by controllers of the form (11) if there exist symmetric positive definite matrices $P_i = \text{diag}\{P_i^{(1)}, P_i^{(2)}, \dots, P_i^{(N)}\} > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_{ki} > 0$, ($i \in S = \{1, 2, \dots, M\}$), $Q_j > 0$, ($j = 3, 4, 5$), $R_k > 0$, ($k = 2, 3, 4$), $T_l > 0$, ($l = 1, 2, 3$), $U_m > 0$, $V_n > 0$, $W_s > 0$, ($m, n, s = 1, 2, 3, 4, 5$), matrices $\mathbb{Y}_i = \text{diag}\{Y_i^{(1)}, Y_i^{(2)}, \dots, Y_i^{(N)}\}$ and matrices M_{k_1} , N_{k_2} , ($k_1, k_2 = 1, 2, 3, 4$), for any scalar $\delta > 0$, and symmetric matrices X_p , Y_{1p} , Y_{2p} , Z_p , ($i \in S$) such that (23)–(28) and the following linear matrix inequalities hold:

$$(i) \quad \begin{bmatrix} U_4 & N_1 \\ N_1^T & U_4 \end{bmatrix} > 0, \quad \begin{bmatrix} V_4 & N_2 \\ N_2^T & V_4 \end{bmatrix} > 0, \quad (82)$$

$$(ii) \quad \begin{bmatrix} \Theta_{i0}^\varepsilon + \Omega_{i1}^\varepsilon & \Lambda^T P_i \\ P_i \Lambda & -2P_i + J \end{bmatrix} < 0,$$

$$(iii) \quad \begin{bmatrix} U_5 & N_3 \\ N_3^T & U_5 \end{bmatrix} > 0, \quad \begin{bmatrix} V_5 & N_4 \\ N_4^T & V_5 \end{bmatrix} > 0, \quad (83)$$

$$(iv) \quad \begin{bmatrix} \Theta_{i0}^\varepsilon + \Omega_{i2}^\varepsilon & \Lambda^T P_i \\ P_i \Lambda & -2P_i + J \end{bmatrix} < 0.$$

In addition, the desired sampled-data controllers gain matrices are given by

$$K_k = (P_i^{(k)})^{-1} Y_i^{(k)}, \quad k = 1, 2, \dots, N, \quad (84)$$

where

$$\begin{aligned} \Theta_{i0}^\varepsilon &= \sum_{m=1}^{24} E_m \Pi_m E_m^T + \mathfrak{K}_\Theta - \delta [E_1 \ E_{24}] \\ &\times \begin{bmatrix} \widehat{F}_1 & \widehat{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} E_1^T \\ E_{24}^T \end{bmatrix} - \frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\} - 1} \\ &\times (E_1 - E_3) V_3 (E_1^T - E_3^T) \\ &- \exp\{-2\varepsilon \bar{\tau}_i\} (\bar{\tau}_i E_1 - E_{19}) W_1 (\bar{\tau}_i E_1^T - E_{19}^T) \\ &- \exp\{-2\varepsilon \bar{h}\} (\bar{h} E_1 - E_{23}) W_2 (\bar{h} E_1^T - E_{23}^T) \\ &- \exp\{-2\varepsilon d_{1i}\} (d_{1i} E_1 - E_9) W_3 (d_{1i} E_1^T - E_9^T) \\ &- \exp\{-2\varepsilon d_{mi}\} (d_{mi} E_1 - E_{12}) W_4 (d_{mi} E_1^T - E_{12}^T) \\ &- \exp\{-2\varepsilon d_{2i}\} (d_{2i} E_1 - E_{13}) W_5 (d_{2i} E_1^T - E_{13}^T) \\ &- \frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} [E_{18} \ E_{19} - E_{18}] \end{aligned}$$

$$\begin{aligned}
& \times \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} \begin{bmatrix} E_{18}^T \\ E_{19}^T - E_{18}^T \end{bmatrix} \\
& - \frac{2\bar{\varepsilon}\bar{\tau}_i}{\exp\{2\bar{\varepsilon}\bar{\tau}_i\} - 1} [E_1 - E_{16} \quad E_{16} - E_{14}] \\
& \times \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} \begin{bmatrix} E_{16}^T - E_{14}^T \\ E_{16}^T - E_{14}^T \end{bmatrix} \\
& - \frac{2\bar{\varepsilon}\bar{h}}{\exp\{2\bar{\varepsilon}\bar{h}\} - 1} [E_{22} \quad E_{23} - E_{22}] \\
& \times \begin{bmatrix} U_2 & M_3 \\ M_3^T & U_2 \end{bmatrix} \begin{bmatrix} E_{22}^T \\ E_{23}^T - E_{22}^T \end{bmatrix} \\
& - \frac{2\bar{\varepsilon}\bar{h}}{\exp\{2\bar{\varepsilon}\bar{h}\} - 1} [E_1 - E_{21} \quad E_{21} - E_{20}] \\
& \times \begin{bmatrix} V_2 & M_4 \\ M_4^T & V_2 \end{bmatrix} \begin{bmatrix} E_{21}^T - E_{20}^T \\ E_{21}^T - E_{20}^T \end{bmatrix}, \\
\mathfrak{K}_\Theta &= E_1 P_i (G_i^{(2)} \otimes B_i) E_2^T \\
& + E_1 P_i (G_i^{(3)} \otimes C_i) E_{17} \\
& + E_1 \mathbb{Y}_i E_{21}^T + E_1 P_i (I_N \otimes D_i) E_{24}^T \\
& + E_2 (G_i^{(2)} \otimes B_i)^T P_i E_1^T \\
& + E_{17} (G_i^{(3)} \otimes C_i)^T P_i E_1^T + E_{21} \mathbb{Y}_i^T E_1^T \\
& + E_{24} (I_N \otimes D_i)^T P_i E_1^T.
\end{aligned} \tag{85}$$

Other notations are the same as those in Theorem 7.

Proof. Considering (27) and (28), by applying the Schu Complement Lemma, we have

$$(i) \quad \begin{bmatrix} U_4 & N_1 \\ N_1^T & U_4 \end{bmatrix} > 0, \quad \begin{bmatrix} V_4 & N_2 \\ N_2^T & V_4 \end{bmatrix} > 0, \tag{86}$$

$$(ii) \quad \begin{bmatrix} \Omega_{i0}^\varepsilon + \Omega_{i1}^\varepsilon & \Lambda^T J \\ J\Lambda & -J \end{bmatrix} < 0,$$

$$(iii) \quad \begin{bmatrix} U_5 & N_3 \\ N_3^T & U_5 \end{bmatrix} > 0, \quad \begin{bmatrix} V_5 & N_4 \\ N_4^T & V_5 \end{bmatrix} > 0, \tag{87}$$

$$(iv) \quad \begin{bmatrix} \Omega_{i0}^\varepsilon + \Omega_{i2}^\varepsilon & \Lambda^T J \\ J\Lambda & -J \end{bmatrix} < 0.$$

Define the block diagonal matrix $\mathbb{X} = \begin{bmatrix} H & \\ & P_i J^{-1} \end{bmatrix}$, where $H = \sum_{m=1}^{24} E_m E_m^T$. Noting that $E_i^T E_j = \begin{cases} 0 & i \neq j \\ I & i = j \end{cases}$, with (ii)

of (86) and (iv) of (87), we define $\mathbb{Y}_i = P_i \mathbb{K}$ and obtain the following:

$$\begin{aligned}
& \mathbb{X} \begin{bmatrix} \Omega_{i0}^\varepsilon + \Omega_{i1}^\varepsilon & \Lambda^T J \\ J\Lambda & -J \end{bmatrix} \mathbb{X}^T = \begin{bmatrix} \Theta_{i0}^\varepsilon + \Omega_{i1}^\varepsilon & \Lambda^T P_i \\ P_i \Lambda & -P_i J^{-1} P_i \end{bmatrix} < 0, \\
& \mathbb{X} \begin{bmatrix} \Omega_{i0}^\varepsilon + \Omega_{i2}^\varepsilon & \Lambda^T J \\ J\Lambda & -J \end{bmatrix} \mathbb{X}^T = \begin{bmatrix} \Theta_{i0}^\varepsilon + \Omega_{i2}^\varepsilon & \Lambda^T P_i \\ P_i \Lambda & -P_i J^{-1} P_i \end{bmatrix} < 0.
\end{aligned} \tag{88}$$

Due to $J > 0$ and $(P_i - J)J^{-1}(P_i - J) \geq 0$, we know that $-P_i J^{-1} P_i \leq -2P_i + J$. Thus (88) are held if (ii) of (82) and (iv) of (83) are satisfied, which implies that (ii) of (27) and (iv) of (28) are held.

According to Definition 3, by the sampled-data controllers gain matrices (84), the NCDN (6) with partially known transition rates and sector-bounded condition (9) is exponentially synchronized. This completes the proof. $\square$

Remark 10. It should be mentioned that the sampled-data synchronization problem has been solved for NCDN (6) in Theorem 9, and the desired controllers can be obtained when LMIs (23)–(26), (82), and (83) are feasible. In this paper, we construct the novel stochastic Lyapunov functional (34) containing some triple-integral terms; which is very effective in the reduction of conservatism [45]. Besides, we apply Lemma 5 to the corresponding terms, the method by using reciprocally convex lemma [46] can achieve less conservative results. Thus the obtained delay-range-dependent and decay rate-dependent stability condition for NCDN (6) in Theorem 7 is less conservative than the previous ones, which can achieve larger maximum value of sampling period $\bar{h}$ and will be verified in Section 4.

Remark 11. It should be pointed out that the given results can be extended to more general neutral delay in the form of NCDN (6). For instance, $\tau_{1i} \leq \tau_i(t) \leq \tau_{2i}$; we can choose τ_{mi} satisfying $\tau_{1i} \leq \tau_{mi} \leq \tau_{2i}$ and construct the stochastic Lyapunov functional on $[t - \tau_i(t), t - \tau_{1i}]$, $[t - \tau_{1i}, t]$, $[t - \tau_{2i}, t - \tau_{mi}]$, and $[t - \tau_{mi}, t - \tau_{1i}]$. Following the same thought, the time delay $h(t)$ in the control input can also be extended to be more general.

Corollary 12. *The NCDN (6) with completely unknown transition rates and sector-bounded condition (9) is exponentially synchronized by controllers of the form (11) if there exist symmetric positive definite matrices $P = \text{diag}\{P^{(1)}, P^{(2)}, \dots, P^{(N)}\} > 0$, $Q_j > 0$, ($j = 1, 2, 3, 4, 5$), $R_k > 0$, ($k = 1, 2, 3, 4$), $T_l > 0$, ($l = 1, 2, 3$), $U_m > 0$, $V_n > 0$, $W_s > 0$, ($m, n, s = 1, 2, 3, 4, 5$), matrices $\mathbb{Y} = \text{diag}\{Y^{(1)}, Y^{(2)}, \dots, Y^{(N)}\}$ and matrices M_{k_1}, N_{k_2} , ($k_1, k_2 = 1, 2, 3, 4$), for any scalar $\delta > 0$, and symmetric matrices Y_1, Y_2, Z such that (28), (75), (76), and the following linear matrix inequalities hold:*

$$\begin{aligned}
(i) \quad & \begin{bmatrix} U_4 & N_1 \\ N_1^T & U_4 \end{bmatrix} > 0, \quad \begin{bmatrix} V_4 & N_2 \\ N_2^T & V_4 \end{bmatrix} > 0, \\
(ii) \quad & \begin{bmatrix} \tilde{\Theta}_{i0}^\varepsilon + \tilde{\Omega}_{i1}^\varepsilon & \Lambda^T P \\ P\Lambda & -2P + J \end{bmatrix} < 0,
\end{aligned} \tag{89}$$

$$\begin{aligned}
\text{(iii)} \quad & \begin{bmatrix} U_5 & N_3 \\ N_3^T & U_5 \end{bmatrix} > 0, \quad \begin{bmatrix} V_5 & N_4 \\ N_4^T & V_5 \end{bmatrix} > 0, \\
\text{(iv)} \quad & \begin{bmatrix} \widetilde{\Theta}_{i0}^\varepsilon + \widetilde{\Omega}_{i2}^\varepsilon & \Lambda^T P \\ P \Lambda & -2P + J \end{bmatrix} < 0.
\end{aligned} \tag{90}$$

In addition, the desired sampled-data controllers gain matrices are given by

$$K_k = (P^{(k)})^{-1} Y^{(k)}, \quad k = 1, 2, \dots, N, \tag{91}$$

where

$$\begin{aligned}
\widetilde{\Theta}_{i0}^\varepsilon = & \sum_{m=1}^{24} E_m \widetilde{\Pi}_m E_m^T + \widetilde{\mathfrak{F}}_\Theta \\
& - \delta [E_1 \ E_{24}] \begin{bmatrix} \widehat{F}_1 & \widehat{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} E_1^T \\ E_{24}^T \end{bmatrix} \\
& - \frac{2\varepsilon d_{1i}}{\exp\{2\varepsilon d_{1i}\} - 1} (E_1 - E_3) V_3 (E_1^T - E_3^T) \\
& - \exp\{-2\varepsilon \bar{\tau}_i\} (\bar{\tau}_i E_1 - E_{19}) W_1 (\bar{\tau}_i E_1^T - E_{19}^T) \\
& - \exp\{-2\varepsilon \bar{h}\} (\bar{h} E_1 - E_{23}) W_2 (\bar{h} E_1^T - E_{23}^T) \\
& - \exp\{-2\varepsilon d_{1i}\} (d_{1i} E_1 - E_9) W_3 (d_{1i} E_1^T - E_9^T) \\
& - \exp\{-2\varepsilon d_{mi}\} (q_{1i} E_1 - E_{12}) W_4 (q_{1i} E_1^T - E_{12}^T) \\
& - \exp\{-2\varepsilon d_{2i}\} (q_{2i} E_1 - E_{13}) W_5 (q_{2i} E_1^T - E_{13}^T) \\
& - \frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} [E_{18} \ E_{19} - E_{18}] \\
& \times \begin{bmatrix} U_1 & M_1 \\ M_1^T & U_1 \end{bmatrix} \begin{bmatrix} E_{18}^T \\ E_{19}^T - E_{18}^T \end{bmatrix} \\
& - \frac{2\varepsilon \bar{\tau}_i}{\exp\{2\varepsilon \bar{\tau}_i\} - 1} [E_1 - E_{16} \ E_{16} - E_{14}] \\
& \times \begin{bmatrix} V_1 & M_2 \\ M_2^T & V_1 \end{bmatrix} \begin{bmatrix} E_1^T - E_{16}^T \\ E_{16}^T - E_{14}^T \end{bmatrix} \\
& - \frac{2\varepsilon \bar{h}}{\exp\{2\varepsilon \bar{h}\} - 1} [E_{22} \ E_{23} - E_{22}] \\
& \times \begin{bmatrix} U_2 & M_3 \\ M_3^T & U_2 \end{bmatrix} \begin{bmatrix} E_{22}^T \\ E_{23}^T - E_{22}^T \end{bmatrix} \\
& - \frac{2\varepsilon \bar{h}}{\exp\{2\varepsilon \bar{h}\} - 1} [E_1 - E_{21} \ E_{21} - E_{20}] \\
& \times \begin{bmatrix} V_2 & M_4 \\ M_4^T & V_2 \end{bmatrix} \begin{bmatrix} E_1^T - E_{21}^T \\ E_{21}^T - E_{20}^T \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\widetilde{\mathfrak{F}}_\Theta = & E_1 P (G_i^{(2)} \otimes B_i) E_2^T + E_1 P (G_i^{(3)} \otimes C_i) E_{17} \\
& + E_1 \mathbb{Y}_i E_{21}^T + E_1 P (I_N \otimes D_i) E_{24}^T \\
& + E_2 (G_i^{(2)} \otimes B_i)^T P E_1^T + E_{17} (G_i^{(3)} \otimes C_i)^T P E_1^T \\
& + E_{21} \mathbb{Y}_i^T E_1^T + E_{24} (I_N \otimes D_i)^T P E_1^T.
\end{aligned} \tag{92}$$

Other notations are the same as those in Corollary 8.

4. Numerical Examples

In this section, numerical examples are used to illustrate the effectiveness of the results derived above.

Example 1. As shown in the example, a three-node NCDN (6) with Markovian switching between two modes is taken into consideration; that is, $N = 3$ and $M = 2$. The parametric matrices of the NCDN are given as follows:

$$\begin{aligned}
A_1 &= \begin{bmatrix} -0.50 & -0.35 \\ 0.10 & -0.50 \end{bmatrix}, & A_2 &= \begin{bmatrix} -0.30 & 0.09 \\ 0.20 & -0.40 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 0.20 & -0.15 \\ 0.50 & -0.50 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.31 & 0.23 \\ -0.12 & 0.17 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.28 & 0.02 \\ -0.06 & 0.11 \end{bmatrix}, & C_2 &= \begin{bmatrix} 0.51 & 0.24 \\ 0.02 & -0.44 \end{bmatrix}, \\
D_1 &= \begin{bmatrix} 0.20 & 0 \\ 0 & -0.15 \end{bmatrix}, & D_2 &= \begin{bmatrix} 0.30 & 0 \\ -0.10 & 0.23 \end{bmatrix}, \\
G_1^{(1)} &= \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}, & G_2^{(1)} &= \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}, \\
G_1^{(2)} &= \begin{bmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}, & G_2^{(2)} &= \begin{bmatrix} -2 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & -2 \end{bmatrix}, \\
G_1^{(3)} &= \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 1 & 0 & -1 \end{bmatrix}, & G_2^{(3)} &= \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}.
\end{aligned} \tag{93}$$

The partially known transition rate matrix is considered as in the following two cases:

$$\Upsilon_1 = \begin{bmatrix} -2 & 2 \\ \gamma_{21}^0 & \gamma_{22}^0 \end{bmatrix}, \quad \Upsilon_2 = \begin{bmatrix} \gamma_{11}^0 & \gamma_{12}^0 \\ \gamma_{21}^0 & \gamma_{22}^0 \end{bmatrix}. \tag{94}$$

Furthermore, the nonlinear function $f(x_k(t))$ is given by

$$f(x_k(t)) = [0.4x_{k1}(t) - \tanh(0.1x_{k1}(t)) \ 0.1x_{k2}(t)]^T. \tag{95}$$

Then, it is easy to verify that

$$F_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}. \quad (96)$$

The interval mode-dependent time-varying neutral delays and discrete delays are, respectively, assumed to be

$$\begin{aligned} \tau_1(t) &= 0.5(1 + \sin t), & \tau_2(t) &= 0.6(1 + \cos t), \\ d_1(t) &= 0.4(1 + \sin^4(t)), & d_2(t) &= 0.8(1 + \cos^4(t)). \end{aligned} \quad (97)$$

They are governed by the Markov process $\{r(t), t \geq 0\}$ and shown in Figures 1 and 2. It can be readily obtained that

$$\begin{aligned} \bar{\tau}_1 &= 1, & \bar{\tau}_2 &= 1.2, & \nu_1 &= 0.5, & \nu_2 &= 0.6, \\ d_{11} &= 0.4, & d_{21} &= 0.8, & d_{12} &= 0.8, \\ d_{22} &= 1.6, & \mu_1 &= \frac{3\sqrt{3}}{5}, & \mu_2 &= \frac{3\sqrt{3}}{10}. \end{aligned} \quad (98)$$

Given decay rate $\varepsilon = 0.3$ and the maximum value of sampling period $\bar{h} = 0.2$, we choose $d_{m1} = 0.5$ and $d_{m2} = 1.0$ and seek to solve the sampled-data controllers.

For the case of Y_1 , with the above parameters, by Theorem 9, we can obtain the sampled-data controllers as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.7246 & -0.0345 \\ -0.0253 & -0.6358 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -0.5149 & -0.0326 \\ -0.0117 & -0.5048 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -0.3557 & -0.0263 \\ -0.0046 & -0.4356 \end{bmatrix}. \end{aligned} \quad (99)$$

Furthermore, the state trajectories of the error system (13) are given in Figure 3, where $x_1(0) = [0.2 \ 0.1]^T$, $x_2(0) = [0.3 \ -0.2]^T$, $x_3(0) = [-0.1 \ 0.1]^T$, and $s_0 = [0.1 \ 0.2]^T$.

For another case of Y_2 , the sampled-data controllers can be readily obtained by Corollary 12.

Example 2. Consider NCDN (6) with three nodes but only one mode; that is, $N = 3$ and $M = 1$. So the subscript of the mode i is omitted here. The outer-coupling matrices are assumed to be

$$G^{(1)} = G^{(2)} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}, \quad G^{(3)} = 0. \quad (100)$$

In fact, the NCDN in consideration here has become a common CDN.

The nonlinear function $f(x_k(t))$ is taken as

$$f(x_k(t)) = [-0.5x_{k1}(t) + \tanh(0.2x_{k1}(t)) + 0.2x_{k2}(t) - 0.95x_{k2}(t) - \tanh(0.75x_{k2}(t))]^T, \quad (101)$$

(101) can be easily verified that

$$F_1 = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}. \quad (102)$$

The time-varying delay is chosen as $d(t) = 0.2 + 0.05 \sin(10t)$; then we know that $d_1 = 0.15$, $d_2 = 0.25$, and $\mu = 0.5$. Besides, we choose $D = I_2$, $d_m = 0.18$ satisfying $d_1 < d_m < d_2$ and consider the following two cases.

Case 1. The inner-coupling matrices are given as $A = 0$ and $B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$. According to the above parameters, the maximum value of sampling period $\bar{h}$, which satisfies LMIs (23)–(26), (82), and (83) in Theorem 9, can be calculated by solving a quasiconvex optimization problem. The degraded NCDN has also been considered in [34, 35]. The results on the maximum value of sampling period $\bar{h}$ are compared in Table 1, where it can be seen that our method is less conservative than [34, 35].

Case 2. The inner-coupling matrices are given as $A = \text{diag}\{0.3, 0.3\}$ and $B = \text{diag}\{0.4, 0.4\}$. The maximum value of sampling period $\bar{h}$ also can be solved by Theorem 9. The

results are listed in Table 2. In addition, the proposed method in [34] is not applicable here. From Table 2, it also can be seen that our method is less conservative than existing ones.

5. Conclusions

In this paper, the sampled-data synchronization problem has been solved for a class of NCDNs with Markovian jump parameters and partially known transition rates. The discrete and neutral delays are considered to be interval mode-dependent and time varying, while the sampling period here is bounded and time-varying. With a novel stochastic Lyapunov functional, the delay-range-dependent and rate-dependent exponential synchronization conditions have been proposed by Lyapunov theory and better technique of matrix inequalities. Then the sampled-data controllers have been designed on the basis of the obtained conditions. These theoretical results are successfully verified through numerical examples, whose simulation results are less conservative than the previous results. Finally, the main contributions of this

TABLE 1: Maximum value of sampling period $\bar{h}$ with $d_m = 0.18$ and $\varepsilon = 0$.

Methods		Sampled-data controllers		
Li et al. [34]	$\bar{h} = 0.5409$	$K_1 = \begin{bmatrix} -0.4856 & -0.1435 \\ -0.0987 & -1.5662 \end{bmatrix}$	$K_2 = \begin{bmatrix} -0.4857 & -0.1435 \\ -0.0987 & -1.5664 \end{bmatrix}$	$K_3 = \begin{bmatrix} -0.2435 & -0.1036 \\ -0.0063 & -1.0367 \end{bmatrix}$
Wu et al. [35]	$\bar{h} = 0.5573$	$K_1 = \begin{bmatrix} -0.4201 & -0.1614 \\ 0.0001 & -1.1698 \end{bmatrix}$	$K_2 = \begin{bmatrix} -0.4201 & -0.1614 \\ 0.0001 & -1.1698 \end{bmatrix}$	$K_3 = \begin{bmatrix} 0.1221 & -0.2073 \\ -0.0024 & -1.0093 \end{bmatrix}$
Theorem 9	$\bar{h} = 0.5764$	$K_1 = \begin{bmatrix} -0.4096 & -0.1523 \\ 0.0012 & -1.1577 \end{bmatrix}$	$K_2 = \begin{bmatrix} -0.4096 & -0.1523 \\ 0.0012 & -1.1577 \end{bmatrix}$	$K_3 = \begin{bmatrix} 0.1176 & -0.2135 \\ -0.0046 & -0.9438 \end{bmatrix}$

(a)

(b)

FIGURE 1: Time-varying neutral delay $\tau_i(t)$ at mode 1 and mode 2.

(a)

(b)

FIGURE 2: Time-varying discrete delay $d_i(t)$ at mode 1 and mode 2.

TABLE 2: Maximum value of sampling period $\bar{h}$ with $d_m = 0.18$ and $\varepsilon = 0$.

Methods	Sampled-data controllers		
Li et al. [34]	—	—	—
Wu et al. [35]	$\bar{h} = 0.0500$	$K_1 = \begin{bmatrix} -0.4330 & -0.2074 \\ -0.0631 & -1.5627 \end{bmatrix}, K_2 = \begin{bmatrix} -0.4330 & -0.2071 \\ -0.0631 & -1.5627 \end{bmatrix}, K_3 = \begin{bmatrix} -0.1631 & -0.1744 \\ -0.0355 & -1.0803 \end{bmatrix}$	
Theorem 9	$\bar{h} = 0.0617$	$K_1 = \begin{bmatrix} -0.4252 & -0.1836 \\ 0.0094 & -1.1847 \end{bmatrix}, K_2 = \begin{bmatrix} -0.4252 & -0.1845 \\ 0.0092 & -1.1847 \end{bmatrix}, K_3 = \begin{bmatrix} -0.1466 & -0.1746 \\ -0.0243 & -1.0125 \end{bmatrix}$

FIGURE 3: State trajectories of the error system in Example 1.

paper can be summarized as follows. (i) To achieve exponential synchronization for NCDN (6), the desired sampled-data feedback controllers have been designed in terms of the solution to certain LMIs. The proposed results are expressed in a new representation, which are theoretically and numerically proved to be less conservative than some existing ones. (ii) The constructed stochastic Lyapunov functional contains some triple-integral terms, which are very effective in the reduction of conservativeness and have not appeared in the context of NCDNs. (iii) The bound of the delay is fully utilized in this paper; that is, improved bounding technique is used to reduce the conservativeness. (iv) The reciprocally convex lemma is used to derive the delay-range-dependent and rate-dependent stability conditions, which can well reduce the conservativeness of the investigated systems.

Acknowledgments

This work was supported in part by the National Key Scientific Research Project (61233003), the National Natural Science Foundation of China (60935001, 61174061, and 61074033), the Doctoral Fund of the Ministry of Education

of China (20093402110019), the Anhui Provincial Natural Science Foundation (11040606M143), the Fundamental Research Funds for the Central Universities, and the Program for New Century Excellent Talents in University.

References

- [1] B. A. Huberman and L. A. Adamic, "Growth dynamics of the world-wide web," *Nature*, vol. 401, no. 6749, p. 131, 1999.
- [2] R. Pastor-Satorras and A. Vespignani, "Epidemic spreading in scale-free networks," *Physical Review Letters*, vol. 86, no. 14, pp. 3200–3203, 2001.
- [3] R. Pastor-Satorras, E. Smith, and R. V. Solé, "Evolving protein interaction networks through gene duplication," *Journal of Theoretical Biology*, vol. 222, no. 2, pp. 199–210, 2003.
- [4] X. F. Wang and G. Chen, "Synchronization in scale-free dynamical networks: robustness and fragility," *IEEE Transactions on Circuits and Systems. I. Fundamental Theory and Applications*, vol. 49, no. 1, pp. 54–62, 2002.
- [5] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.
- [6] C. P. Li, W. G. Sun, and J. Kurths, "Synchronization of complex dynamical networks with time delays," *Physica A*, vol. 361, no. 1, pp. 24–34, 2006.
- [7] H. Gao, J. Lam, and G. Chen, "New criteria for synchronization stability of general complex dynamical networks with coupling delays," *Physics Letters A*, vol. 360, no. 2, pp. 263–273, 2006.
- [8] X. Liu and T. Chen, "Exponential synchronization of nonlinear coupled dynamical networks with a delayed coupling," *Physica A*, vol. 381, no. 1-2, pp. 82–92, 2007.
- [9] Y. Wang, Z. Wang, and J. Liang, "A delay fractioning approach to global synchronization of delayed complex networks with stochastic disturbances," *Physics Letters A*, vol. 372, no. 39, pp. 6066–6073, 2008.
- [10] Y. Wang, Z. Wang, and J. Liang, "Global synchronization for delayed complex networks with randomly occurring nonlinearities and multiple stochastic disturbances," *Journal of Physics A*, vol. 42, no. 13, Article ID 135101, 2009.
- [11] H. Li, D. Yue, and Z. Gu, "Synchronization stability of complex dynamical networks with probabilistic time-varying delays," in *Proceedings of the 48th IEEE Conference on Decision and Control Held Jointly with 28th Chinese Control Conference (CDC/CCC '09)*, pp. 621–625, December 2009.
- [12] H. Li, W. K. Wong, and Y. Tang, "Global synchronization stability for stochastic complex dynamical networks with probabilistic interval time-varying delays," *Journal of Optimization Theory and Applications*, vol. 152, no. 2, pp. 496–516, 2012.

- [13] J. Cao and L. Li, "Cluster synchronization in an array of hybrid coupled neural networks with delay," *Neural Networks*, vol. 22, no. 4, pp. 335–342, 2009.
- [14] W. Yu, J. Cao, and J. Lü, "Global synchronization of linearly hybrid coupled networks with time-varying delay," *SIAM Journal on Applied Dynamical Systems*, vol. 7, no. 1, pp. 108–133, 2008.
- [15] Y. Tang, J.-A. Fang, M. Xia, and D. Yu, "Delay-distribution-dependent stability of stochastic discrete-time neural networks with randomly mixed time-varying delays," *Neurocomputing*, vol. 72, no. 16–18, pp. 3830–3838, 2009.
- [16] Y. Tang, J.-A. Fang, and Q.-Y. Miao, "Synchronization of stochastic delayed neural networks with markovian switching and its application," *International Journal of Neural Systems*, vol. 19, no. 1, pp. 43–56, 2009.
- [17] Y. Wang, H. Zhang, X. Wang, and D. Yang, "Networked synchronization control of coupled dynamic networks with time-varying delay," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 40, no. 6, pp. 1468–1479, 2010.
- [18] D. H. Ji, D. W. Lee, J. H. Koo, S. C. Won, S. M. Lee, and J. H. Park, "Synchronization of neutral complex dynamical networks with coupling time-varying delays," *Nonlinear Dynamics*, vol. 65, no. 4, pp. 349–358, 2011.
- [19] J. H. Park, "Synchronization of cellular neural networks of neutral type via dynamic feedback controller," *Chaos, Solitons and Fractals*, vol. 42, no. 3, pp. 1299–1304, 2009.
- [20] J. H. Park and O. M. Kwon, "Synchronization of neural networks of neutral type with stochastic perturbation," *Modern Physics Letters B*, vol. 23, no. 14, pp. 1743–1751, 2009.
- [21] X. D. Li and J. D. Cao, "Delay-dependent stability of neural networks of neutral type with time delay in the leakage term," *Nonlinearity*, vol. 23, no. 7, pp. 1709–1726, 2010.
- [22] Y. Dai, Y. Cai, and X. Xu, "Synchronization criteria for complex dynamical networks with neutral-type coupling delay," *Physica A*, vol. 387, no. 18, pp. 4673–4682, 2008.
- [23] Y. Zhang, S. Xu, Y. Chu, and J. Lu, "Robust global synchronization of complex networks with neutral-type delayed nodes," *Applied Mathematics and Computation*, vol. 216, no. 3, pp. 768–778, 2010.
- [24] H. B. Bao and J. D. Cao, "Stochastic global exponential stability for neutral-type impulsive neural networks with mixed time-delays and Markovian jumping parameters," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 9, pp. 3786–3791, 2011.
- [25] Q. Ma, S. Xu, and Y. Zou, "Stability and synchronization for Markovian jump neural networks with partly unknown transition probabilities," *Neurocomputing*, vol. 74, no. 17, pp. 3404–3411, 2011.
- [26] X. S. Yang, J. D. Cao, and J. Q. Lu, "Synchronization of markovian coupled neural networks with nonidentical node-delays and random coupling strengths," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 1, pp. 60–71.
- [27] X. Yang, J. Cao, and J. Lu, "Synchronization of randomly coupled neural networks with Markovian jumping and time-delay," *IEEE Transactions on Circuits and Systems I*, vol. 60, no. 2, pp. 363–376, 2013.
- [28] Y. Liu, Z. Wang, J. Liang, and X. Liu, "Stability and synchronization of discrete-time Markovian jumping neural networks with mixed mode-dependent time delays," *IEEE Transactions on Neural Networks*, vol. 20, no. 7, pp. 1102–1116, 2009.
- [29] Y. Liu, Z. Wang, and X. Liu, "Exponential synchronization of complex networks with Markovian jump and mixed delays," *Physics Letters A*, vol. 372, no. 22, pp. 3986–3998, 2008.
- [30] Y. Tang, J.-A. Fang, and Q. Miao, "On the exponential synchronization of stochastic jumping chaotic neural networks with mixed delays and sector-bounded non-linearities," *Neurocomputing*, vol. 72, no. 7–9, pp. 1694–1701, 2009.
- [31] E. Fridman, A. Seuret, and J.-P. Richard, "Robust sampled-data stabilization of linear systems: an input delay approach," *Automatica*, vol. 40, no. 8, pp. 1441–1446, 2004.
- [32] J.-w. Wen, F. Liu, and S. K. Nguang, "Sampled-data predictive control for uncertain jump systems with partly unknown jump rates and time-varying delay," *Journal of the Franklin Institute. Engineering and Applied Mathematics*, vol. 349, no. 1, pp. 305–322, 2012.
- [33] M. Liu, J. You, and X. Ma, " H_∞ filtering for sampled-data stochastic systems with limited capacity channel," *Signal Processing*, vol. 91, no. 8, pp. 1826–1837, 2011.
- [34] N. Li, Y. Zhang, J. Hu, and Z. Nie, "Synchronization for general complex dynamical networks with sampled-data," *Neurocomputing*, vol. 74, no. 5, pp. 805–811, 2011.
- [35] Z.-G. Wu, J. H. Park, H. Su, B. Song, and J. Chu, "Exponential synchronization for complex dynamical networks with sampled-data," *Journal of the Franklin Institute*, vol. 349, no. 9, pp. 2735–2749, 2012.
- [36] Z. Wang, Y. Liu, and X. Liu, " H_∞ filtering for uncertain stochastic time-delay systems with sector-bounded nonlinearities," *Automatica*, vol. 44, no. 5, pp. 1268–1277, 2008.
- [37] B. Shen, Z. Wang, and X. Liu, "Bounded H_∞ synchronization and state estimation for discrete time-varying stochastic complex networks over a finite horizon," *IEEE Transactions on Neural Networks*, vol. 22, no. 1, pp. 145–157, 2011.
- [38] J. Liang, Z. Wang, and X. Liu, "State estimation for coupled uncertain stochastic networks with missing measurements and time-varying delays: the discrete-time case," *IEEE Transactions on Neural Networks*, vol. 20, no. 5, pp. 781–793, 2009.
- [39] J. Liang, Z. Wang, and X. Liu, "Distributed state estimation for discrete-time sensor networks with randomly varying nonlinearities and missing measurements," *IEEE Transactions on Neural Networks*, vol. 22, no. 3, pp. 486–496, 2011.
- [40] J. K. Hale and S. M. Verduyn Lunel, *Introduction to Functional-Differential Equations*, vol. 99 of *Applied Mathematical Sciences*, Springer, New York, NY, USA, 1993.
- [41] X. Mao, "Stability of stochastic differential equations with Markovian switching," *Stochastic Processes and their Applications*, vol. 79, no. 1, pp. 45–67, 1999.
- [42] X.-M. Sun, J. Zhao, and D. J. Hill, "Stability and L_2 -gain analysis for switched delay systems: a delay-dependent method," *Automatica*, vol. 42, no. 10, pp. 1769–1774, 2006.
- [43] A. N. Langville and W. J. Stewart, "The Kronecker product and stochastic automata networks," *Journal of Computational and Applied Mathematics*, vol. 167, no. 2, pp. 429–447, 2004.
- [44] Y. G. Sun, L. Wang, and G. Xie, "Exponential stability of switched systems with interval time-varying delay," *IET Control Theory & Applications*, vol. 3, no. 8, pp. 1033–1040, 2009.
- [45] J. Sun, G. P. Liu, and J. Chen, "Delay-dependent stability and stabilization of neutral time-delay systems," *International Journal of Robust and Nonlinear Control*, vol. 19, no. 12, pp. 1364–1375, 2009.
- [46] P. G. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.