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Research Article

Delay-Dependent H_∞ Control for Networked Control Systems with Large Delays

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We consider the problems of robust stability and H_∞ control for a class of networked control systems with long-time delays. Firstly, a nonlinear discrete time model with mode-dependent time delays is proposed by converting the uncertainty of time delay into the uncertainty of parameter matrices. We consider a probabilistic case where the system is switched among different subsystems, and the probability of each subsystem being active is defined as its occurrence probability. For a switched system with a known subsystem occurrence probabilities, we give a stochastic stability criterion in terms of linear matrix inequalities (LMIs). Then, we extend the results to a more practical case where the subsystem occurrence probabilities are uncertain. Finally, a simulation example is presented to show the efficacy of the proposed method.

1. Introduction

Networked control systems (NCSs) are distributed systems in which communication between sensors, actuators, and controllers is supported by a shared real-time network. Compared with conventional point-to-point system connection, this new networked based control scheme reduces system wiring, low cost, high reliability, information sharing, and remote control [1, 2]. Nevertheless, the introduction of communication network also brings some new problems and challenges, such as time-delay, packet dropout, quantization, and band-limited channel [3–11], which all might be potential sources to poor performance, even instability.

Network-induced delay is one of the main problems in NCS [12, 13] and has attracted much research interest. Compared with the constant delays, the random or time-varying ones are more difficult to be dealt with, especially, when the delay is larger than one sampling period (long-time delays). Some important methods on dealing with time delay which were investigated, [14] have shown that the input-output technique is an effective way to deal with time delay; the delay partitioning approach is also advanced to deal with the time-varying delay [15, 16]. Reference [17] applies for the dynamic output feedback controller design of discrete-time systems with time-varying delays. Sliding mode control (SMC) is an effective robust control strategy to deal with the time-delay systems [18, 19]. In this paper, the uncertainties of the delays are transformed into those of the system models in the uncertain system approach. As has been mentioned above, it is difficult to deal with the NCS with long time-varying or random delays, and one aspect of the difficulties lies in providing an appropriate modeling method for such NCSs. Since the delay may be larger than one sampling period, more than one control signals may arrive at the actuator during one sampling interval. Moreover, the numbers of the arriving control signals vary over different sampling intervals; thus, the dynamic model of the overall closed-loop NCS varies from sampling period to sampling period. Reference [21] deals with the problem of stability and stabilization controller design for NCS with long time-varying delays. The NCS with time-varying delays is modeled as a discrete-time switched system with multiple state delays and with both stable subsystems and unstable subsystems [5].

In most models, NCSs are modeled as Markov jump linear systems (MJLSs). In an MJLS, the subsystem occurrence probability is called the stationary probability distribution of the Markovian states. According to the known transition matrix for an MJLS, we can readily infer the subsystem occurrence probability for each of the involved subsystems [20, 22–24]. Reference [25] focuses on studying the MJLS with partly unknown transition probabilities due to the complexity of network. Then, the MJLS can be converted to a switched system with known subsystem occurrence probabilities. Compared with the MJLS model, the advantage of the switched system model in this paper mainly lies in that one does not need to identify the transition probability from each mode to another.

Practically, there is an uncertainty on the information of the subsystem occurrence probability. However, most of the current works seldomly consider the uncertain environment of the NCS and the feature of stochastic long time-delay NCSs. With the motivation of the above reasons, it is natural to consider the stability and controller design problems for NCS with stochastic long delays, and less conservative results can be achieved by incorporating the available information of the occurrence probabilities, even when there are uncertainties on the information.

In this paper, we are interested in investigating the problems of the robust stochastic stability and H_∞ performance analysis for a class of NCSs with a continuous-time nonlinear plant, and the delay larger than the sampling period may be an arbitrary value in a finite interval; a discrete-time stochastic switched NCS model is proposed. A mode-dependent state feedback controller is designed by using a cone complementary linearization approach to ensure that closed-loop system is stochastically stable and achieves the disturbance attenuation level.

The remainder of the paper is organized as follows. In Section 2, we model the NCS as a discrete-time switched system by considering the subsystem occurrence probabilities. Further, the definition of the stochastic stability is introduced. The stochastic stability for switched systems with known and unknown subsystem occurrence probabilities is analyzed by the LMIs technology, and state feedback controller is designed to stabilize the NCS in Section 3. Simulation results are given in Section 4 to verify the proposed scheme. Finally, Section 5 concludes the paper.

Notation. Throughout the paper, the superscripts "–1" and "T" stand for the inverse and transpose of a matrix, respectively; R^n denotes the n-dimensional Euclidean space and the notation $P > 0$ means that P is real symmetric positive definite matrix. $E\{x\}$ is the expectation of the stochastic variable x. I and 0 represent identity matrix and zero matrix with appropriate dimensions in different place. In symmetric block matrices or complex matrix expressions, we use an asterisk $*$ to represent a term that is induced by symmetry, and diag$\{\cdots\}$ stands for a block diagonal matrix. $\|\cdot\|$ refers to the Euclidean norm for vectors and induced 2-norm for matrix. $\lambda_{\min}(Q)$ and $\lambda_{\max}(Q)$ are defined as the minimum and maximum eigenvalue of Q, respectively. The set of all positive integers is represented by Z^+.

2. Model for Networked Control System

The plant is a continuous-time system described by

$$\begin{aligned} \dot{x}(t) &= A_p x(t) + B_p u(t) + f(x,t) + H_p w(t), \\ z(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $z(t) \in R^q$ are the state vector, control input vector, and controlled output vector, respectively, $w(t) \in R^d$ is the exogenous disturbance signal belonging to $L_2[0,\infty)$. A_p, B_p, H_p, and C are known real matrices with appropriate dimensions. $f : \Omega \times [t_0,\infty) \to R^n(\Omega \subset R^n)$ is the nonlinear function vector, $f(0,t_0) = 0$. f satisfies the local Lipschitz condition, that is,

$$\begin{aligned} &\|f(x_1,t) - f(x_2,t)\|_2 \\ &\quad \le \alpha\|x_1 - x_2\|_2, \quad \forall x_1, x_2 \in \Omega \subset R^n,\ \forall t \in [t_0,\infty), \end{aligned} \tag{2}$$

where $\alpha > 0$ is a known constant.

In the considered NCS, time delays exist in both channels from sensor to controller and from controller to actuator. Sensor-to-controller delay and controller-to-actuator delay are denoted by τ^{sc} and τ^{ca}, respectively. The assumptions in the above NCS are as follows.

(1) The discrete-time state-feedback controller and the actuator are event-driven, and the sensor is time-driven with sampling period T.

(2) The network-induced delay $\tau_k \triangleq \tau_k^{sc} + \tau_k^{ca}$ satisfies $T \le \tau_k < NT$ during the kth sampling period, where N is a given positive number and $N \ge 2$.

(3) In order to improve the real-time ability of NCS, the data packet will be discarded and be held at previous value once the time delay of this data packet is longer than NT.

Define the indicator function

$$\xi(k) = [\xi_2(k), \ldots, \xi_l(k), \ldots, \xi_N(k)]^T \tag{3}$$

with $l = 2, \ldots, N$ and

$$\xi_l(k) = \begin{cases} 1, & \text{the } l\text{th subsystem is active,} \\ 0, & \text{the } l\text{th subsystem is not active.} \end{cases} \tag{4}$$

Remark 1. At any time instant k, if the long delay $\tau_k \in [(i-1)T, iT)$, then we can get $\xi_i = 1$, $\xi_j = 0$, where $j \ne i$, $i, j = 2, \ldots, N$, and $\sum_{i=2}^{N} \xi_i = 1$; that is, we can instantaneously identify which subsystem is active at any time instant k, so we can define the switching rule as follows $\sigma(k) : \mathbb{Z}^+ = \{0, 1, 2, \ldots\} \to \ell = \{2, 3, \ldots, N\}$.

Then, the system (1) can be written into discrete time model during the interval $[kT,(k+1)T)$, where k is a non-negative integer, we get

$$\begin{aligned} x(k+1) &= Ax(k) \\ &\quad + \sum_{l=2}^{N} \xi_l \left[B_{l0}(\tau_k) u(k-l+1) + B_{l1}(\tau_k) u(k-l) \right] \\ &\quad + \overline{f}(x,k) + Hw(k), \\ z(k) &= Cx(k), \end{aligned} \tag{5}$$

where $A = e^{A_p T}$, $B_{l0}(\tau_k) = \int_0^{lT-\tau_k} e^{A_p s} ds B_p$, $B_{l1}(\tau_k) = \int_{lT-\tau_k}^{T} e^{A_p s} ds B_p$, $H = \int_0^T e^{A_p s} ds H_p$, and

$$\overline{f}(x,k) = \int_0^T e^{A_p s} ds f(x,k). \tag{6}$$

According to Assumption 2, $\tau_k \in [T, NT)$ is stochastic variable; therefore, $B_{l0}(\tau_k)$ and $B_{l0}(\tau_k)$ are also stochastic matrices. Define a scalar τ_k', and the parameters in (6) could be transformed as follows:

$$\begin{aligned} B_{l0}(\tau_k) &= \int_0^{lT-\tau_k} e^{A_p s} ds B_p \\ &= \int_0^{lT-\tau_k'} e^{A_p s} ds B_p + e^{A_p (lT-\tau_k')} \int_0^{\tau_k'-\tau_k} e^{A_p s} ds B_p, \\ B_{l1}(\tau_k) &= \int_{lT-\tau_k}^{T} e^{A_p s} ds B_p \\ &= \int_{lT-\tau_k'}^{T} e^{A_p s} ds B_p - e^{A_p (lT-\tau_k')} \int_0^{\tau_k'-\tau_k} e^{A_p s} ds B_p. \end{aligned} \tag{7}$$

For convenience of the following proof, we can define $\Gamma_{l0} = \int_0^{lT-\tau_k'} e^{A_p s} ds B_p$, $\Gamma_{l1} = \int_{lT-\tau_k'}^{T} e^{A_p s} ds B_p$, $G_l = e^{A_p (lT-\tau_k')}$, $F(\tau_k') = \int_0^{\tau_k'-\tau_k} e^{A_p s} ds$, and $E = B_p$, where $F^T(\tau_k')F(\tau_k') \le I$, $l \in \ell$. Then, we get

$$\begin{aligned} B_{l0}(\tau_k) &= \Gamma_{l0} + G_l F(\tau_k') E, \\ B_{l1}(\tau_k) &= \Gamma_{l1} - G_l F(\tau_k') E. \end{aligned} \tag{8}$$

The discrete-time state feedback controller is

$$u(k) = K(k) x(k), \tag{9}$$

where $x(k) \in R^n$ and $u(k) \in R^m$ are the value of $x(t)$ and $u(t)$ at the sampling instant kT, respectively, and state feedback gain $K(k)$ corresponds to the subsystem at the sampling instant kT to be considered.

Consider the plant input:

$$u(k) = \begin{cases} \hat{u}(k), & \text{if } \hat{u}(k) \text{ and } x(k) \text{ is successfully transmitted,} \\ u(k-1), & \text{if } \hat{u}(k) \text{ or } x(k) \text{ is lost during transmission.} \end{cases} \tag{10}$$

Then the resulting closed-loop NCS is shown as follows:

$$\begin{aligned} x(k+1) &= Ax(k) \\ &\quad + \sum_{l=2}^{N} \xi_l \left[B_{l0}(\tau_k) K_l x(k-l+1) + B_{l1}(\tau_k) K_l x(k-l) \right] \\ &\quad + \overline{f}(x,k) + Hw(k). \end{aligned} \tag{11}$$

Let $\widetilde{x}(k) = \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}$, (11) leads to

$$\begin{aligned} \widetilde{x}(k+1) &= \widetilde{A}\widetilde{x}(k) + \sum_{l=2}^{N} \xi_l \widetilde{B}_l \widetilde{K}_l \widetilde{x}(k-l+1) \\ &\quad + \widetilde{f}(x,k) + \widetilde{H}w(k), \\ z(k) &= \widetilde{C}\widetilde{x}(k), \end{aligned} \tag{12}$$

where

$$\begin{aligned} &\widetilde{A} = \begin{bmatrix} A & 0 \\ I & 0 \end{bmatrix}, \qquad \widetilde{K}_l = \begin{bmatrix} K_l & 0 \\ 0 & K_l \end{bmatrix}, \\ &\widetilde{f}(x,k) = \begin{bmatrix} \overline{f}(x,k) \\ 0 \end{bmatrix}, \qquad \widetilde{H} = \begin{bmatrix} H \\ 0 \end{bmatrix}, \qquad \widetilde{C} = [C \ \ 0], \\ &\widetilde{B}_l = \begin{bmatrix} B_{l0} & B_{l1} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \Gamma_{l0} & \Gamma_{l1} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} G_l \\ 0 \end{bmatrix} F(\tau_k') [E \ \ -E] \\ &\quad = \widetilde{\Gamma}_l + \widetilde{G}_l F(\tau_k') \widetilde{E}. \end{aligned} \tag{13}$$

From (2)–(6), the nonlinear $\widetilde{f}(x,k)$ satisfies

$$\widetilde{f}^T(x,k)\widetilde{f}(x,k) = \overline{f}^T(x,k)\overline{f}(x,k) \le \widetilde{x}^T(k) U^T U \widetilde{x}(k) \tag{14}$$

with $U = \begin{bmatrix} \alpha\sqrt{\lambda_{\max}(\Pi)} I & 0 \\ 0 & 0 \end{bmatrix}$ is a known constant positive-definite matrix, where $\Pi = (\int_0^T e^{A_p s} ds)^T \int_0^T e^{A_p s} ds$.

Now, for each subsystem, we define the subsystem occurrence probability β_l. Subsequently, we have a priori information about ξ_l, for all $l \in \ell$. As discussed previously, the probability of $\xi_l = 1$ is represented by β_l, that is,

$$\begin{aligned} \text{Prob}\{\xi_l = 1\} &= \beta_l, \\ \text{Prob}\{\xi_l = 0\} &= 1 - \beta_l. \end{aligned} \tag{15}$$

We consider two cases: (1) the value of β_l is precisely known; (2) β_l is subject to an uncertainty. A general form for ξ_l is depicted as follows

$$\text{Prob}\{\xi_l = 1\} = \beta_l \in [\beta_{1l}, \beta_{2l}], \tag{16}$$

where β_{1l} and β_{2l} are two constants satisfying $0 \le \beta_{1l} \le \beta_{2l} \le 1$.

Lemma 2 (see [26]). *The stochastic stability in discrete-time implies the stochastic stability in continuous time.*

Lemma 3 (see [27] (Schur complement)). *For a given matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, *where* S_{11}, S_{22} *are square matrixes, then the following conditions are equivalent:*

(1) $S < 0$;

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 4 (see [28]). *Let D, E, and F be matrices with appropriate dimensions. If $F^T F \le I$, then for any scalar $\varepsilon > 0$, one has*

$$DFE + E^T F^T D^T \le \varepsilon D D^T + \varepsilon^{-1} E^T E. \tag{17}$$

Lemma 5 (see [29]). *Assume that $i \in \ell = \{2, 3, \ldots, N\}$, and N is a given positive number and $N \ge 2$, then for any positive-definite matrix $P \in R^{n\times n}$, one has*

$$\begin{aligned}(N-1) &\sum_{s=k-N+1}^{k-1} x^T(s) P x(s) \\ &\ge \left(\sum_{s=k-i+1}^{k-1} x^T(s)\right) P \left(\sum_{s=k-i+1}^{k-1} x(s)\right), \quad k \in Z^+.\end{aligned} \tag{18}$$

Definition 6 (see [30]). The system (12) with $w(k) \equiv 0$ is said to be stochastically stable if for every finite $\tilde{x}_0 = \tilde{x}(0)$, and the following inequality holds:

$$E\left\{\sum_{k=0}^{\infty} \|\tilde{x}(k)\|^2 \mid \tilde{x}_0\right\} < \infty. \tag{19}$$

Definition 7 (see [31]). The closed-loop system (12) is robustly stable with H_∞ performance γ if there exists a state feed-back controller $u(k) = K(k)x(k)$, and the following conditions are satisfied.

(a) The closed-loop system (12) with $\omega(k) \equiv 0$ is stochastically stable.

(b) Under the zero-initial condition, it holds that

$$\begin{aligned}&\sum_{k=0}^{\infty} E\left\{z^T(k) z(k)\right\} \\ &\quad < \gamma^2 \sum_{k=0}^{\infty} E\left\{w^T(k) w(k)\right\}, \quad \forall w(k) \in L_2[0,\infty).\end{aligned} \tag{20}$$

3. Main Results

3.1. Stochastic Stability Analysis. With Lemma 2, the stability of system (1) can be converted into the stability of system (12). Then a sufficient condition for stochastic stability of system (12) with $w(k) = 0$ is given in the following theorems.

3.1.1. Stability with the Known Occurrence Probability

Theorem 8. *Suppose that the occurrence probability for each subsystem is known. The networked control system* (12) *is stochastically stable if there exist positive definite matrices $P_i \in R^{2n\times 2n}$, $Q \in R^{2n\times 2n}$, $R \in R^{2n\times 2n}$ and $S_i \in R^{2n\times 2n}$, $i \in \ell$ with appropriate dimensions satisfying the following LMI:*

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ * & \Phi_{22} & 0 \\ * & * & \Phi_{33} \end{bmatrix} < 0, \quad i, j \in \ell, \tag{21}$$

where

$$\Phi_{11} = \begin{bmatrix} \Omega_i & 0 & \cdots & -S_i^T & \cdots & 0 & 0 & -S_i^T \\ * & -Q & \cdots & 0 & \cdots & 0 & 0 & 0 \\ * & * & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\ * & * & * & -Q & \cdots & 0 & 0 & 0 \\ * & * & * & * & \ddots & \vdots & \vdots & \vdots \\ * & * & * & * & * & -Q & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -R \end{bmatrix},$$

$$\Phi_{12} = \begin{bmatrix} \sqrt{\beta_2}\widetilde{A}^T P_j & \cdots & \sqrt{\beta_i}\widetilde{A}^T P_j & \cdots & \sqrt{\beta_N}\widetilde{A}^T P_j \\ \sqrt{\beta_2}\left(\widetilde{B}_2\widetilde{K}_2\right)^T P_j & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\beta_i}\left(\widetilde{B}_i\widetilde{K}_i\right)^T P_j & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \sqrt{\beta_N}\left(\widetilde{B}_N\widetilde{K}_N\right)^T P_j \\ \sqrt{\beta_2}P_j & \cdots & \sqrt{\beta_i}P_j & \cdots & \sqrt{\beta_N}P_j \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix},$$

$$\Phi_{13} = \begin{bmatrix} \sqrt{\beta_2}\left(\widetilde{A}-I\right)^T R & \cdots & \sqrt{\beta_i}\left(\widetilde{A}-I\right)^T R & \cdots & \sqrt{\beta_N}\left(\widetilde{A}-I\right)^T R \\ \sqrt{\beta_2}\left(\widetilde{B}_2\widetilde{K}_2\right)^T R & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\beta_i}\left(\widetilde{B}_i\widetilde{K}_i\right)^T R & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \sqrt{\beta_N}\left(\widetilde{B}_N\widetilde{K}_N\right)^T R \\ \sqrt{\beta_2}R & \cdots & \sqrt{\beta_i}R & \cdots & \sqrt{\beta_N}R \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix},$$

$$\begin{aligned}\Phi_{22} &= -\operatorname{diag}\underbrace{\left\{P_j, \ldots, P_j, \ldots, P_j\right\}}_{N-1}, \\ \Phi_{33} &= -\frac{1}{(N-1)^2}\operatorname{diag}\underbrace{\{R, \ldots, R, \ldots, R\}}_{N-1}, \\ \Omega_i &= -P_i + S_i + S_i^T + U^T U + (N-1)Q.\end{aligned} \tag{22}$$

Proof. Construct Lyapunov function candidates for closed-loop system (12) as follows:

$$V(\tilde{x}(k)) = V_1(\tilde{x}(k)) + V_2(\tilde{x}(k)) + (N-1)V_3(\tilde{x}(k)), \tag{23}$$

where

$$V_1(\tilde{x}(k)) = \tilde{x}^T(k) P_{\sigma(k)} \tilde{x}(k),$$
$$V_2(\tilde{x}(k)) = \sum_{\theta=-N}^{-2} \sum_{s=k+\theta+1}^{k-1} \tilde{x}^T(s) Q \tilde{x}(s), \tag{24}$$
$$V_3(\tilde{x}(k)) = \sum_{\theta=-N}^{-2} \sum_{s=k+\theta+1}^{k-1} e^T(s) \operatorname{Re}(s).$$

$P_{\sigma(k)} > 0$, $Q > 0$, and $R > 0$ are matrices to be determined. The new variable $e(k)$ satisfies the following equation:

$$e(k) = \tilde{x}(k+1) - \tilde{x}(k). \tag{25}$$

Then, we can further write (25) in the equivalent descriptor form

$$e(k) = \left(\widetilde{A} - I\right) \tilde{x}(k) + \sum_{l=2}^{N} \xi_l \widetilde{B}_l \widetilde{K}_l \tilde{x}(k-l+1) + \tilde{f}(x,k). \tag{26}$$

Assume that the ith and jth modes are active at times k and $k+1$, respectively. That is, $\tau_k \in [(i-1)T, iT)$ and $\tau_{k+1} \in [(j-1)T, jT)$ for any $i, j \in \ell$. Along the solution of the system (12) with $w(k) = 0$ and using Lemma 5, we have

$$\begin{aligned}
\Delta V_1(\tilde{x}(k)) &= E\left[V_1(\tilde{x}(k+1))\right] - V_1(\tilde{x}(k)) \\
&= E\left[\tilde{x}^T(k+1) P_j \tilde{x}(k+1)\right] - \tilde{x}^T(k) P_i \tilde{x}(k) \\
&= E\left[\widetilde{A}\tilde{x}(k) + \sum_{l=2}^{N} \xi_l \widetilde{B}_l \widetilde{K}_l \tilde{x}(k-l+1) + \tilde{f}(x,k)\right]^T \\
&\quad \times P_j \left[\widetilde{A}\tilde{x}(k) + \sum_{l=2}^{N} \xi_l \widetilde{B}_l \widetilde{K}_l \tilde{x}(k-l+1) + \tilde{f}(x,k)\right] \\
&\quad - \tilde{x}^T(k) P_i \tilde{x}(k) \\
&\quad + 2\tilde{x}^T(k) S_i^T \left[\tilde{x}(k) - \tilde{x}(k-i+1) - \sum_{s=k-i+1}^{k-1} e(s)\right],
\end{aligned}$$

$$\begin{aligned}
\Delta V_2(\tilde{x}(k)) &= E\left(V_2(\tilde{x}(k+1))\right) - V_2(\tilde{x}(k)) \\
&= \sum_{\theta=-N}^{-2} \tilde{x}^T(k) Q \tilde{x}(k) - \sum_{s=k-N+1}^{k-1} \tilde{x}^T(s) Q \tilde{x}(s) \\
&\le (N-1) \tilde{x}^T(k) Q \tilde{x}(k) - \sum_{s=k-N+1}^{k-1} \tilde{x}^T(s) Q \tilde{x}(s),
\end{aligned}$$

$$\begin{aligned}
\Delta V_3(\tilde{x}(k)) &= E\left(V_3(\tilde{x}(k+1))\right) - V_3(\tilde{x}(k)) \\
&= \sum_{\theta=-N}^{-2} e^T(k) \operatorname{Re}(k) - \sum_{s=k-N+1}^{k-1} e^T(s) \operatorname{Re}(s) \\
&\le (N-1) e^T(k) \operatorname{Re}(k) \\
&\quad - \frac{1}{N-1} \left(\sum_{s=k-i+1}^{k-1} e^T(s)\right) R \left(\sum_{s=k-i+1}^{k-1} e(s)\right).
\end{aligned} \tag{27}$$

Denote $v(k) = \sum_{s=k-i+1}^{k-1} e(s)$, for all $i \in \ell$ and combine (27) with (14), then we can obtain

$$\begin{aligned}
\Delta V(\tilde{x}(k)) &= \Delta V_1(\tilde{x}(k)) + \Delta V_2(\tilde{x}(k)) + (N-1)\Delta V_3(\tilde{x}(k)) \\
&\le \Delta V_1(\tilde{x}(k)) + \Delta V_2(\tilde{x}(k)) + (N-1)\Delta V_3(\tilde{x}(k)) \\
&\quad + \tilde{x}^T(k) U^T U \tilde{x}(k) - \tilde{f}^T(x,k) \tilde{f}(x,k) \\
&= \eta^T(k) \Phi(i,j) \eta(k),
\end{aligned} \tag{28}$$

where $\eta^T(k) = [\tilde{x}^T(k)\ \tilde{x}^T(k-1) \cdots \tilde{x}^T(k-i+1) \cdots \tilde{x}^T(k-N+1)\ \tilde{f}^T(x,k)\ v^T(k)]$, and

$$\Phi(i,j) = \Phi_{11} + \sum_{l=2}^{N} \beta_l \widetilde{\Phi}_{1l}^T P_j \widetilde{\Phi}_{1l} + (N-1)^2 \sum_{l=2}^{N} \beta_l \widetilde{\Phi}_{2l}^T R \widetilde{\Phi}_{2l}, \tag{29}$$

with

$$\widetilde{\Phi}_{1l} = \left[\widetilde{A}\ \ \underbrace{0\cdots0}_{l-2}\ \ \widetilde{B}_l\widetilde{K}_l\ \ \underbrace{0\cdots0}_{N-l}\ \ I\ \ 0\right],$$
$$\widetilde{\Phi}_{2l} = \left[\left(\widetilde{A} - I\right)\ \ \underbrace{0\cdots0}_{l-2}\ \ \widetilde{B}_l\widetilde{K}_l\ \ \underbrace{0\cdots0}_{N-l}\ \ I\ \ 0\right], \quad \forall i, j, l \in \ell. \tag{30}$$

By Schur complement lemma, the inequality (21) guarantees $\Phi(i,j) < 0$. Thus, with the above relations, the inequality (28) can be rewritten as follows:

$$\begin{aligned}
\Delta V(\tilde{x}(k)) &= E\left\{V(\tilde{x}(k+1) \mid \sigma(k+1) = j)\right\} \\
&\quad - V(\tilde{x}(k) \mid \sigma(k) = i) \\
&\le -\lambda_{\min}(-\Phi(i,j)) \|\tilde{x}(k)\|^2 \\
&= -\mu \|\tilde{x}(k)\|^2.
\end{aligned} \tag{31}$$

From the previous inequalities, we can obtain

$$\begin{aligned}
&E\left\{V(\tilde{x}(k+1)) \mid \sigma(k+1) = j\right\} - V(\tilde{x}(0) \mid \sigma(0)) \\
&\quad \le -\mu E\left\{\sum_{k=0}^{k} \|\tilde{x}(k)\|^2\right\}
\end{aligned} \tag{32}$$

which implies that

$$E\left\{\sum_{k=0}^{\infty}\|\tilde{x}(k)\|^2\right\} \le \frac{1}{\mu}E\{V(\tilde{x}(0))\} \le \infty. \tag{33}$$

Therefore, by Definition 6, it can be obtained that the closed-loop system (12) is stochastically stable. The proof is completed. □

3.1.2. Stability with Uncertain Active Probabilities. In Section 3.1.1, we studied the stochastic stability for the switched networked control system with known subsystem occurrence probabilities. As a matter of fact, there is an uncertainty on the information of the subsystem occurrence probability. Next, the stochastic stability criterion of the closed-loop system (12) with uncertain subsystem occurrence probabilities is given in the following theorem.

Theorem 9. *Suppose that the range of occurrence probability for each subsystem is known. The networked control system* (12) *is stochastically stable if there exist positive definite matrices* $P_i \in R^{2n\times 2n}$, $Q \in R^{2n\times 2n}$, $R \in R^{2n\times 2n}$ *and* $S_i \in R^{2n\times 2n}$, $i \in \ell$ *with appropriate dimensions satisfying the following LMI:*

$$\begin{bmatrix} \Phi_{11} & \Phi'_{12} & \Phi'_{13} \\ * & \Phi_{22} & 0 \\ * & * & \Phi_{33} \end{bmatrix} < 0, \quad i, j \in \ell, \tag{34}$$

where

$$\Phi'_{12} = \begin{bmatrix} \sqrt{\beta_{22}}\tilde{A}^T P_j & \cdots & \sqrt{\beta_{2i}}\tilde{A}^T P_j & \cdots & \sqrt{\beta_{2N}}\tilde{A}^T P_j \\ \sqrt{\beta_{22}}\left(\tilde{B}_2\tilde{K}_2\right)^T P_j & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\beta_{2i}}\left(\tilde{B}_i\tilde{K}_i\right)^T P_j & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \sqrt{\beta_{2N}}\left(\tilde{B}_N\tilde{K}_N\right)^T P_j \\ \sqrt{\beta_{22}}P_j & \cdots & \sqrt{\beta_{2i}}P_j & \cdots & \sqrt{\beta_{2N}}P_j \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix},$$

$$\Phi'_{13} = \begin{bmatrix} \sqrt{\beta_{22}}\left(\tilde{A}-I\right)^T R & \cdots & \sqrt{\beta_{2i}}\left(\tilde{A}-I\right)^T R & \cdots & \sqrt{\beta_{2N}}\left(\tilde{A}-I\right)^T R \\ \sqrt{\beta_{22}}\left(\tilde{B}_2\tilde{K}_2\right)^T R & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\beta_{2i}}\left(\tilde{B}_i\tilde{K}_i\right)^T R & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \sqrt{\beta_{2N}}\left(\tilde{B}_N\tilde{K}_N\right)^T R \\ \sqrt{\beta_{22}}R & \cdots & \sqrt{\beta_{2i}}R & \cdots & \sqrt{\beta_{2N}}R \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}. \tag{35}$$

Proof. By Schur complement lemma, (34) implies

$$\Phi_{11} + \sum_{l=2}^{N}\beta_{2l}\tilde{\Phi}_{1l}^T P_j \tilde{\Phi}_{1l} + (N-1)^2\sum_{l=2}^{N}\beta_{2l}\tilde{\Phi}_{2l}^T R\tilde{\Phi}_{2l} < 0. \tag{36}$$

Note that, for positive definite matrices P_j and R, and $0 \le \beta_{1l} \le \beta_l \le \beta_{2l} \le 1$,

$$\begin{gathered} \sum_{l=2}^{N}\beta_l\tilde{\Phi}_{1l}^T P_j \tilde{\Phi}_{1l} \le \sum_{l=2}^{N}\beta_{2l}\tilde{\Phi}_{1l}^T P_j \tilde{\Phi}_{1l}, \\ \sum_{l=2}^{N}\beta_l\tilde{\Phi}_{2l}^T R\tilde{\Phi}_{2l} \le \sum_{l=2}^{N}\beta_{2l}\tilde{\Phi}_{2l}^T R\tilde{\Phi}_{2l}, \quad \forall j = \ell. \end{gathered} \tag{37}$$

Thus, if the LMI (34) holds, then we obtain

$$\Phi_{11} + \sum_{l=2}^{N}\beta_l\tilde{\Phi}_{1l}^T P_j \tilde{\Phi}_{1l} + (N-1)^2\sum_{l=2}^{N}\beta_l\tilde{\Phi}_{2l}^T R\tilde{\Phi}_{2l} < 0. \tag{38}$$

According to Theorem 8, the discrete-time stochastic switched system (12) with uncertain subsystem occurrence probabilities is stochastically stable. This completes the proof. □

3.2. H_∞ Controller Synthesis with Uncertain Active Probabilities. This section is devoted to synthesizing a controller given in the form $u(k) = K_i x(k)$, for all $i \in \ell$ that guarantees the closed-loop system is robustly stable with the noise attenuation level γ.

Theorem 10. *Suppose that the range of occurrence probability for each subsystem is known. The closed-loop system* (12) *is stochastically stable and achieves the given disturbance attenuation performance if there exist constants* $\varepsilon_i > 0$, *and positive definite matrices* $P_i \in R^{2n\times 2n}$, $Q \in R^{2n\times 2n}$, $R \in R^{2n\times 2n}$, $X_i \in R^{2n\times 2n}$, $Y \in R^{2n\times 2n}$ *and* $S_i \in R^{2n\times 2n}$, $i \in \ell$ *with appropriate dimensions, and feedback gain matrix* K_i *satisfying matrix inequalities:*

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & 0 & 0 \\ * & * & \Theta_{33} & 0 \\ * & * & * & \Theta_{44} \end{bmatrix} < 0, \tag{39}$$

$$P_j X_j = I, \quad RY = I, \quad \forall i, j \in \ell,$$

where

$$\Theta_{11}=\begin{bmatrix} \mho_i & 0 & \cdots & -S_i^T & \cdots & 0 & 0 & -S_i^T & 0 \\ * & -Q & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 \\ * & * & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & -Q & \cdots & 0 & 0 & 0 & 0 \\ * & * & * & * & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & * & -Q & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & \vdots \\ * & * & * & * & * & * & * & -R & 0 \\ * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Theta_{12}=\begin{bmatrix} \sqrt{\beta_{22}}\widetilde{A}^T & \cdots & \sqrt{\beta_{2i}}\widetilde{A}^T & \cdots & \sqrt{\beta_{2N}}\widetilde{A}^T \\ \sqrt{\beta_{22}}\left(\widetilde{\Gamma}_2\widetilde{K}_2\right)^T & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\beta_{2i}}\left(\widetilde{\Gamma}_i\widetilde{K}_i\right)^T & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \sqrt{\beta_{2N}}\left(\widetilde{\Gamma}_N\widetilde{K}_N\right)^T \\ \sqrt{\beta_{22}}I & \cdots & \sqrt{\beta_{2i}}I & \cdots & \sqrt{\beta_{2N}}I \\ 0 & \cdots & 0 & \cdots & 0 \\ \sqrt{\beta_{22}}\widetilde{H}^T & \cdots & \sqrt{\beta_{2i}}\widetilde{H}^T & \cdots & \sqrt{\beta_{2N}}\widetilde{H}^T \end{bmatrix},$$

$$\Theta_{13}=\begin{bmatrix} \sqrt{\beta_{22}}\left(\widetilde{A}-I\right)^T & \cdots & \sqrt{\beta_{2i}}\left(\widetilde{A}-I\right)^T & \cdots & \sqrt{\beta_{2N}}\left(\widetilde{A}-I\right)^T \\ \sqrt{\beta_{22}}\left(\widetilde{\Gamma}_2\widetilde{K}_2\right)^T & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\beta_{2i}}\left(\widetilde{\Gamma}_i\widetilde{K}_i\right)^T & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \sqrt{\beta_{2N}}\left(\widetilde{\Gamma}_N\widetilde{K}_N\right)^T \\ \sqrt{\beta_{22}}I & \cdots & \sqrt{\beta_{2i}}I & \cdots & \sqrt{\beta_{2N}}I \\ 0 & \cdots & 0 & \cdots & 0 \\ \sqrt{\beta_{22}}\widetilde{H}^T & \cdots & \sqrt{\beta_{2i}}\widetilde{H}^T & \cdots & \sqrt{\beta_{2N}}\widetilde{H}^T \end{bmatrix},$$

$$\Theta_{14}=\begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \sqrt{\beta_{22}}\left(\widetilde{E}\widetilde{K}_2\right)^T & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{\beta_{2i}}\left(\widetilde{E}\widetilde{K}_i\right)^T & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \sqrt{\beta_{2N}}\left(\widetilde{E}\widetilde{K}_N\right)^T \\ 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix},$$

$$\Theta_{22}=\operatorname{diag}\underbrace{\{\delta_2,\ldots,\delta_i,\ldots,\delta_N\}}_{N-1},\qquad \delta_i=-X_j+\varepsilon_i\widetilde{G}_i\widetilde{G}_i^T,$$

$$\Theta_{33}=\operatorname{diag}\underbrace{\{\theta_2,\ldots,\theta_i,\ldots,\theta_N\}}_{N-1},$$

$$\theta_i=-\frac{1}{(N-1)^2}Y+\varepsilon_i\widetilde{G}_i\widetilde{G}_i^T,$$

$$\Theta_{44}=\operatorname{diag}\underbrace{\{-\varepsilon_2 I,\ldots,-\varepsilon_i I,\ldots,-\varepsilon_N I\}}_{N-1},$$

$$\mho_i=-P_i+S_i+S_i^T+(N-1)Q+U^TU+\widetilde{C}^T\widetilde{C}. \tag{40}$$

Proof. From Theorem 8, the closed-loop system (12) with $w(k)=0$ is stochastically stable.

Assume that the ith and jth modes are activated at instants k and $k+1$, respectively. For nonzero $w(k)$, using the same Lyapunov function candidates as in Theorem 8, we have

$$\begin{aligned}
&\Delta V_1(\widetilde{x}(k))\\
&\quad=E\left[V_1(\widetilde{x}(k+1))\right]-V_1(\widetilde{x}(k))\\
&\quad=E\left[\widetilde{x}^T(k+1)P_j\widetilde{x}(k+1)\right]-\widetilde{x}^T(k)P_i\widetilde{x}(k)\\
&\quad=E\Bigg[\Bigg(\widetilde{A}\widetilde{x}(k)+\sum_{l=2}^{N}\xi_l\widetilde{B}_l\widetilde{K}_l\widetilde{x}(k-l+1)\\
&\qquad\qquad+\widetilde{f}(x,k)+\widetilde{H}w(k)\Bigg)^T\\
&\qquad\times P_j\Bigg(\widetilde{A}\widetilde{x}(k)+\sum_{l=2}^{N}\xi_l\widetilde{B}_l\widetilde{K}_l\widetilde{x}(k-l+1)\\
&\qquad\qquad+\widetilde{f}(x,k)+\widetilde{H}w(k)\Bigg)\Bigg]\\
&\qquad-\widetilde{x}^T(k)P_i\widetilde{x}(k)\\
&\qquad+2\widetilde{x}^T(k)S_i^T\left[\widetilde{x}(k)-\widetilde{x}(k-i+1)-\sum_{s=k-i+1}^{k-1}e(s)\right],\\
&\Delta V_2(\widetilde{x}(k))\\
&\quad=E\left(V_2(\widetilde{x}(k+1))\right)-V_2(\widetilde{x}(k))\\
&\quad=\sum_{\theta=-N}^{-2}\widetilde{x}^T(k)Q\widetilde{x}(k)-\sum_{s=k-N+1}^{k-1}\widetilde{x}^T(s)Q\widetilde{x}(s)\\
&\quad\le(N-1)\widetilde{x}^T(k)Q\widetilde{x}(k)-\sum_{s=k-N+1}^{k-1}\widetilde{x}^T(s)Q\widetilde{x}(s),\\
&\Delta V_3(\widetilde{x}(k))\\
&\quad=E\left(V_3(\widetilde{x}(k+1))\right)-V_3(\widetilde{x}(k))\\
&\quad=\sum_{\theta=-N}^{-2}e^T(k)\operatorname{Re}(k)-\sum_{s=k-N+1}^{k-1}e^T(s)\operatorname{Re}(s)\\
&\quad\le(N-1)e^T(k)\operatorname{Re}(k)\\
&\qquad-\frac{1}{N-1}\left(\sum_{s=k-i+1}^{k-1}e^T(s)\right)R\left(\sum_{s=k-i+1}^{k-1}e(s)\right).
\end{aligned}\tag{41}$$

From inequations (41), and combining the nonlinear condition (14), we have

$$\begin{aligned}\Delta V(\tilde{x}(k)) &= \Delta V_1(\tilde{x}(k)) + \Delta V(\tilde{x}(k))_2 + (N-1)\Delta V_3(\tilde{x}(k)) \\ &\le \Delta V_1(\tilde{x}(k)) + \Delta V_2(\tilde{x}(k))_2 + (N-1)\Delta V_3(\tilde{x}(k)) \\ &\quad + \tilde{x}^T(k)U^TU\tilde{x}(k) - \tilde{f}^T(x,k)\tilde{f}(x,k) \\ &\quad + E\left\{z^T(k)z(k)\right\} - \gamma^2 E\left\{w^T(k)w(k)\right\} \\ &\quad - E\left\{z^T(k)z(k)\right\} + \gamma^2 E\left\{w^T(k)w(k)\right\} \\ &= \zeta^T(k)\Theta(i,j)\zeta(k) - E\left\{z^T(k)z(k)\right\} \\ &\quad + \gamma^2 E\left\{w^T(k)w(k)\right\},\end{aligned} \tag{42}$$

where $\zeta^T(k) = [\tilde{x}^T(k)\ \tilde{x}^T(k-1)\cdots\tilde{x}^T(k-i+1)\cdots\tilde{x}^T(k-N+1)\ \tilde{f}^T(x,k)\ v^T(k)\ w(k)]$, and

$$\begin{aligned}\Theta(i,j) &= \Theta_{11} + \sum_{l=2}^{N}\beta_l\widetilde{\Theta}_{1l}^T P_j\widetilde{\Theta}_{1l} \\ &\quad + (N-1)^2\sum_{l=2}^{N}\beta_l\widetilde{\Theta}_{2l}^T R\widetilde{\Theta}_{2l}, \quad \forall l\in\ell.\end{aligned} \tag{43}$$

with

$$\begin{aligned}\widetilde{\Theta}_{1l} &= \left[\widetilde{A}\ \ \underbrace{0\cdots0}_{l-2}\ \ \widetilde{B}_l\widetilde{K}_l\ \ \underbrace{0\cdots0}_{N-l}\ \ I\ \ 0\ \ \widetilde{H}\right], \\ \widetilde{\Theta}_{2l} &= \left[\left(\widetilde{A}-I\right)\ \ \underbrace{0\cdots0}_{l-2}\ \ \widetilde{B}_l\widetilde{K}_l\ \ \underbrace{0\cdots0}_{N-l}\ \ I\ \ 0\ \ \widetilde{H}\right].\end{aligned} \tag{44}$$

Using the condition (13) and by Lemmas 3 and 4, we can obtain

$$\begin{aligned}&\sum_{l=2}^{N}\phi_l^T F^T\left(\tau_k'\right)\varphi_l + \sum_{l=2}^{N}\varphi_l^T F\left(\tau_k'\right)\phi_l \\ &\quad \le \sum_{l=2}^{N}\varepsilon_l^{-1}\phi_l^T\phi_l + \sum_{l=2}^{N}\varepsilon_l\varphi_l^T\varphi_l,\end{aligned} \tag{45}$$

where

$$\begin{aligned}\phi_l &= \left[\underbrace{0\cdots0}_{l-1}\ \ \sqrt{\beta_l}\widetilde{E}\widetilde{K}_l\ \ \underbrace{0\cdots0}_{3N-l+1}\right], \\ \varphi_l &= \left[\underbrace{0\cdots0}_{N+l+1}\ \ \widetilde{G}_l^T\ \ \underbrace{0\cdots0}_{N-2}\ \ \widetilde{G}_l^T\ \ \underbrace{0\cdots0}_{N-1}\right], \quad \forall l\in\ell.\end{aligned} \tag{46}$$

Using the similar analysis methods as in Theorem 9, the inequality (39) guarantees $\Theta(i,j) < 0$.

From Theorem 8, the following inequality can be obtained:

$$\begin{aligned}\Delta V(\tilde{x}(k)) &= E\{V(\tilde{x}(k+1))\} - V(\tilde{x}(k)) \\ &\le -E\left\{z^T(k)z(k)\right\} + \gamma^2 E\left\{w^T(k)w(k)\right\} \\ &\quad + \min\left\{\lambda_{\min}\left(-\Theta(i,j)\right)\right\}\|\tilde{x}(k)\|^2 \\ &\le -E\left\{z^T(k)z(k)\right\} + \gamma^2 E\left\{w^T(k)w(k)\right\}.\end{aligned} \tag{47}$$

Taking expectation and summing up from $k = 0$ to ∞ on both sides of inequality (47), it can be obtained that the above inequality (47) is equivalent to

$$\begin{aligned}E[V(\tilde{x}(\infty))] - V(\tilde{x}(0)) &\le -\sum_{k=0}^{\infty}E\left\{z^T(k)z(k)\right\} \\ &\quad + \gamma^2\sum_{k=0}^{\infty}E\left\{w^T(k)w(k)\right\}\end{aligned} \tag{48}$$

which implies that

$$\sum_{k=0}^{\infty}E\left\{z^T(k)z(k)\right\} \le \gamma^2\sum_{k=0}^{\infty}E\left\{w^T(k)w(k)\right\}. \tag{49}$$

Therefore, the closed-loop system (12) is stochastically stable with disturbance attenuation level γ. This completes the proof. □

Remark 11. It should be pointed out that the sufficient conditions proposed in Theorem 10 are not standard LMI condition anymore. The subsystem occurrence probabilities are not coupled with the Lyapunov weighting matrix P_j and R. In this paper, it is suggested to use the cone complementarity linearization (CCL) algorithm in [32], and a nonlinear constraint can be converted to a linear optimization problem with a rank constraint.

Remark 12. The CCL algorithm has been used to solve the nonconvex feasibility problems by formulating them into some sequential optimization problems subject to LMI constraints, and the work in [33] has used the CCL algorithm to solve the model reduction problem recently.

Remark 13. In Theorems 9 and 10, the upper bound of occurrence probability for each subsystem is used to establish the sufficient conditions. In order to reduce the computation complexity, we can employ some constraints of the subsystem occurrence probability to narrow the range of the upper bound. Then by using the CCL algorithm, we can obtain the upper bound of subsystem occurrence probability.

4. Numerical Example

In this section, we present an example to illustrate the effectiveness of the proposed approach. Consider system (1) with parameters as follows:

$$\dot{x}(t) = \begin{bmatrix} 0 & -2 \\ 3 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 0.4 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0.4x_1 \sin x_1 \\ 0.4x_2 \cos x_2 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix} w(t), \quad z(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t). \tag{50}$$

Here, the nonlinear function $f(x,t)$ satisfies sector condition (2), and we can obtain that $\alpha = 0.4$. Choose the sampling period $T = 0.1s$ and suppose $0.1 \le \tau_k < 0.3s$, so we can know that the upper bound of the long time delays is $N = 3$, and the NCS can be modeled with two subsystems, where the corresponding system matrices are obtained as follows:

$$A = \begin{bmatrix} 0.9746 & -0.1558 \\ 0.2337 & 0.5850 \end{bmatrix}, \quad \Gamma_{20} = \begin{bmatrix} 0.0473 \\ 0.1342 \end{bmatrix},$$
$$\Gamma_{21} = \begin{bmatrix} -0.0162 \\ -0.0512 \end{bmatrix}, \quad \Gamma_{30} = \begin{bmatrix} 0.0564 \\ 0.1726 \end{bmatrix},$$
$$\Gamma_{31} = \begin{bmatrix} -0.0252 \\ -0.0896 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.9205 & -0.2367 \\ 0.3550 & 0.3289 \end{bmatrix}, \tag{51}$$
$$G_3 = \begin{bmatrix} 0.8418 & -0.2819 \\ 0.4228 & 0.1371 \end{bmatrix}, \quad E = \begin{bmatrix} 0.4 \\ 1 \end{bmatrix},$$
$$H = \begin{bmatrix} 0.0190 \\ 0.0103 \end{bmatrix}, \quad U = \operatorname{diag}\{0.2, 0.2, 0, 0\}.$$

By using iterative algorithm-CCL and solving the constraint conditions in Theorem 10, we can obtain the upper bounds of the occurrence probability for the first subsystem and the second subsystem that are $\beta_{22} = 0.4366$ and $\beta_{23} = 0.7035$, respectively.

Then, the controller can be obtained as follows:

$$K_2 = \begin{bmatrix} -0.1610 & -0.5509 \end{bmatrix}, \quad K_3 = \begin{bmatrix} -2.3499 & -3.0658 \end{bmatrix}. \tag{52}$$

Then the range of the subsystem occurrence probability is $\beta_2 \in [0.2965, 0.4366]$ and $\beta_3 \in [0.5634, 0.7035]$. Assume that the actual subsystem occurrence probabilities for the subsystem are $\beta_2 = 0.4$ and $\beta_3 = 0.6$, the initial state of the system is $x_0 = [-1 \;\; 1]^T$ and $w(k) = 0.05\exp(-0.01k)$, and the state trajectories of the NCS and the corresponding switching signal are shown in Figures 1 and 2, respectively.

From simulation results, it can be seen that the NCS is stochastically stable and has H_∞ disturbance attenuation level $\gamma = 1.2$.

It should be pointed that the methods proposed in the literature [22–25] cannot be used to deal with the H_∞ control of the given NCS because of the uncertain environment and the feature of stochastic long-time delays.

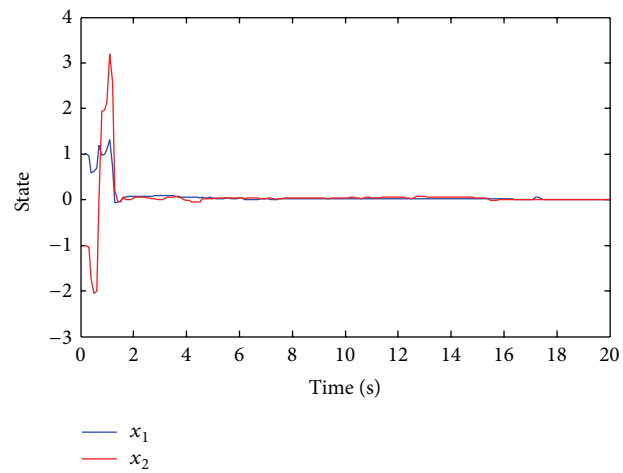

Figure 1: State trajectories of the closed-loop system.

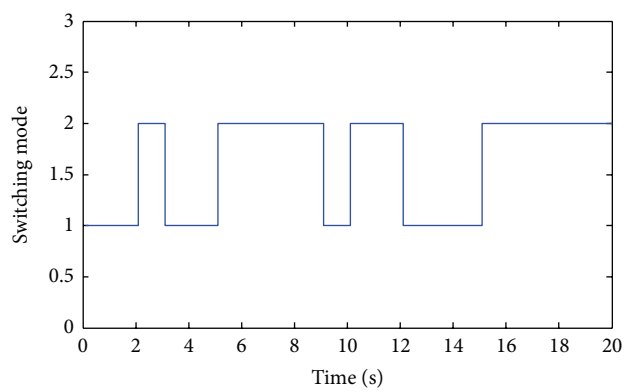

Figure 2: The stochastic switching signal.

5. Conclusion

In this paper, the problems of H_∞ control have been studied for a class of nonlinear NCS with long-time delays. We first model NCS as a switched system with a priori information on the subsystem occurrence probabilities. Sufficient condition for the existence of the stochastic stability is established for the case where subsystem occurrence probabilities are known. Then, the obtained result is generalized to a more practical scenario when the subsystem occurrence probabilities are subject to uncertainties. The controller design method can be used to design a mode-dependent controller such that the closed-loop system is stochastically stable and achieves H_∞ disturbance attenuation level. Finally, a numerical example is provided to show the correctness and effectiveness of the proposed method. Our further work will focus on extending the proposed method to solve the tracking control problem of NCS with communication constraints.

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