Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 671428, 11 pages
http://dx.doi.org/10.1155/2013/671428

Research Article

The Departure Characteristics of Traffic Flow at the Signalized Intersection

Zhaowei Qu, Yan Xing, Hongyu Hu, Yuzhou Duan, Xianmin Song, TingTing Chai, and Hu Zhang

College of Transportation, Jilin University, Changchun 130025, China

Correspondence should be addressed to Hongyu Hu; huhongyu@jlu.edu.cn

Received 5 July 2013; Revised 22 August 2013; Accepted 25 August 2013

Academic Editor: Wuhong Wang

The motion characteristics of the leading vehicle and the following vehicles of the traffic flow at the typical urban intersections are qualitatively analyzed through the kinematical equation and the traffic wave theory. Then, the motion characteristic of the whole traffic flow during the dispersion process is also studied. Based on the spatiotemporal model of kinematics in the departure process and traffic wave model in the dispersion process proposed, the change of the leading vehicle of the departure process and the time of the following vehicles reaching to the stable speed as well as the relationship between the green time and the departure vehicle number at the intersection are acquired. Furthermore, according to the qualitative analysis and the quantitative calculation of the departure traffic flow at the signalized intersection, the dispersion characteristic of traffic flow at the signalized intersection was studied and analyzed, which provides reliable theoretical basis for traffic signal setting at the intersection.

1. Introduction

Study on the departure characteristic of traffic flow at the intersection is not only one of the important contents in research on the traffic theories at urban road but also directly affects the setting of signal timing program at the intersection and the optimization of traffic flow organization program. Moreover, it is closely related with the study on the traffic capacity of the intersection. Therefore, profound analysis on the traffic flow characteristics of starting traffic flow, dispersion traffic flow, and stably departure traffic flow in the departure process of traffic flow at the intersection are helpful to improve the level of service at intersection and relieve the traffic jam at the intersection. Based on this, many experts and scholars have conducted extensive researches on the departure characteristics of traffic flow at the intersection and achieved fruitful research results. For instance, for the micro characteristics of departure vehicles at the intersection, Gazis et al. [1, 2] first proposed stimulus-response model that sensitivity is inversely proportional to time headway on the basis of car-following theory analysis then conducted further researches and deduced the car-following GHR model based on nonlinear algorithm. Through analyzing the micro behavior of traffic flow moving, Yang et al. [3–7] put forward the concept of random degree of free flow to measure the interactive strength of vehicles in the traffic flow and obtained the traffic dispersion model suitable for the urban roads. From the aspects of drivers' psychological field features and visual attention features, Jin and Tao [8, 9] analyzed the influence of leading vehicles on the following vehicles in the process of vehicle moving then studied and improved the car-following model for the motion characteristics of following vehicles, respectively. Stokes et al. [10, 11] studied the departure characteristics of left-turn lanes at the intersection, obtained departure saturation flow of left-turn lanes by calculating the saturation time headway, and discovered that there are certain differences in various left-turn lanes. For macro characteristics of departure vehicles at the intersection, May et al. [12, 13] first transformed the traditional research methods, described the formation and dispersion of shock wave at the signalized intersection and bottleneck of pedestrian crosswalk by utilizing graphical method, and then analyzed the signalized traffic flow characteristics and its dispersion process. Bando et al. [14, 15] studied the overall features of traffic flow from the macro angle, then described the

macro features of traffic flow like the unstable phenomenon of traffic flow and blocking process, and obtained the mean-field equation for the relationship between the average speed of the stable traffic flow and the vehicle density. Through analyzing the kinematical characteristics of traffic starting wave at the intersection, Wang et al. [16–18] defined the significance of kinematics parameter of traffic flow in the traffic wave and built the kinematical model of traffic starting wave at the intersection. In addition, Guo et al. [19, 20] analyzed the dispersion process of traffic flow with graphical method, calibrated the moving speed, motion trajectory, and dispersion time in the departure process of traffic flow on the time-space rectangular coordinate system, and determined the queue length and overall dispersion time of vehicles at the intersection.

To a certain extent, the above research results improve the development of the study on the dispersion characteristics of traffic flow at the intersection. However, it needs further discussion on two aspects. First, the time from the beginning of departure of the traffic flow to stable moving should be measured, as well as the speed of stable moving and the traffic volumes from the departure to passing the stop lines stably. The relationship between the queue length and the platoon dispersion time was analyzed in the above researches, as well as the total time passed through the intersection and passing speed, although it has few discussions to the different stages of departure traffic flow in the intersection, which is difficult to offer accurate based data for signal timing at the intersection and the capacity optimization. Second, the relationships among the spreading speed of the whole traffic wave which produced by the arrived vehicles and the queued vehicles, green time, and the number of the passing vehicles during the queued traffic flow departing are analyzed. The results presented by the former researchers were only concentrated on the departure process of queued vehicles at the intersection. For the previous studies focus on the release of queuing traffic flow at the intersection, there are few studies on some situations, such as whether the follow-up vehicles could pass through the intersection in the queuing period, the traffic wave propagation situation after the follow-up flow overtakes the queuing flow, and the relationship between the follow-up vehicles and the green time at the intersection and so on. The departure and motion situations of the traffic flow arrived subsequently are closely related to the passing efficiency and signal timing at the intersection. As a result, it needs further research on these aspects.

By summarizing the above research experience, aiming at the traffic flow at the urban typical intersections, this paper applies kinematical equation and traffic wave theory to conduct qualitative analysis for the motion characteristic of the leading vehicle, the motion characteristic of the following vehicles, and the motion characteristic of the whole traffic flow at the intersection during the dispersion process at the signalized intersection. It further builds spatiotemporal model of kinematics in the departure process at the intersection and traffic wave model in the dispersion process at the intersection to determine the changing situations of the leading vehicle at the departure process, the time for the following vehicles to reach the stable speed, and the relationship between the green time at the intersection and the departure vehicle number at the intersection. In addition, in this paper we analyzed the traffic flow departure situations combined with the vehicle operation data at the signalized intersection. Through qualitative analysis and quantitative calculation, we analyzed the dispersion characteristic of traffic flow at the signalized intersection and provided reliable theoretical basis for the setting of traffic signals at the intersection.

2. Model Building

When the traffic lights in the intersection turn green, the queued vehicles start to move. When the space headway between the leading vehicle and the second one is significantly greater than the average space headway of the queued vehicles, it indicates that the traffic flow is beginning to disperse. As the leading vehicle of the queue can accelerate continuously to the expected speed without the leading vehicle ahead, it can easily extend its space headway to the second vehicle. Moreover, the second vehicle will be stimulated to speed up to widen the space headway to the third vehicle. Accordingly, if the conditions are met, the change of the motion state will transfer to the following vehicles continuously so that the whole queued vehicles will participate in the dispersion process. If the green light time is long enough, the transfer process will last until all queued vehicles reach their expected speed by the stimulation of the leading vehicles ahead. Till then, the whole dispersion process of the queued vehicles at the intersection has been finished.

Based on the practical dispersion conditions in the intersection, the paper divides the vehicle motion characteristics during the whole dispersion process into three parts: the motion characteristic of the leading vehicle at the intersection during the departure process, the motion characteristic of the following vehicles at the intersection during the departure process, and the motion characteristic of the whole traffic flow at the intersection during the departure process. Moreover, we build a qualitative description model and make quantitative calculation analysis for these three parts, respectively, so as to analyze the departure characteristics of the traffic flow at urban road intersections accurately and thoroughly.

2.1. Assumed Conditions. To simplify the research environment, the following assumptions are proposed.

(1) There is no bus lane at the intersection or bus stop at the influence area.

(2) There is no new queue formed at the intersection.

(3) Take one single lane at the intersection as the research object.

(4) Ignore the individual differences of vehicles and disturbance of pedestrians and nonmotor vehicles.

(5) Take the traffic characteristics of straight-moving traffic flow at the intersection with four-phase traffic signal as the analysis object.

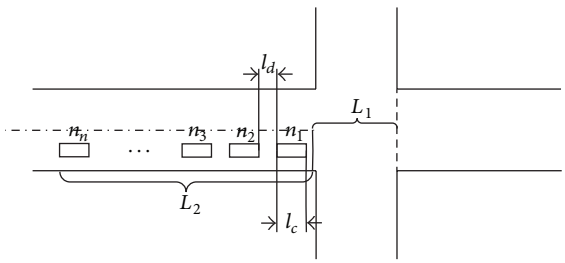

Figure 1: The traffic flow diagram at the intersection.

2.2. Solution Parameters. Based on kinematical equation and traffic wave theory, this paper analyzes the motion characteristics of various traffic flow stages at the departure process in the intersection and solves the following traffic parameters:

(1) the time from the beginning of departure of the queued vehicles to stable moving,

(2) the departure vehicle number from the beginning of departure of the queued vehicles to stable moving,

(3) the time headway when the queued vehicles accelerate and stably move,

(4) the total time needed in dispersion the queued vehicles,

(5) the spreading speed of the traffic wave after the confluence of the vehicles arrived subsequently and the queued vehicles,

(6) the relationship between green light time at the intersection and the number of the departure vehicles at different situations.

2.3. Descriptive Analysis on the Motion Characteristic of the Leading Vehicle. Different from other drivers who receive stimulation from the vehicles ahead, the drivers of the leading vehicles in the departure process receive stimulation mainly from the signal lights at the intersection. Therefore, this paper conducts independent study for them, builds a relationship model of moving speed-time-distance, determines the features of accelerating time, accelerating distance and the corresponding spatial positions at different time of the leading vehicle, and provides theoretical basis for the conflict analysis and the setting of red light time at the intersection.

(1) Qualitative Description. This paper takes the typical intersection as the example for analysis, defines the intersection length is L_1, the queued vehicle length at the intersection is L_2, and the vehicle distance of the queued vehicles is l_d, ignores the individual performance and size difference of vehicles, and takes the vehicle length as l_c. For details, please refer to Figure 1.

As the starting of the leading vehicle at the intersection is mainly influenced by the intersection signals, this paper considers the acceleration of the leading vehicle at the intersection as a, the stable speed of other vehicles as v_f, the starting time of other vehicles as t_{i0}, the initial position of various vehicles as x_i, the time for various vehicles to reach stop line as t_{i1}, the time for various vehicles to pass intersection as t_{i2}, and the time for various vehicles to achieve stable speed as t_{if}. In addition, it assumes the real position of other vehicles as x_i', the corresponding time of a vehicle at the real positions as t_1', the real speed of the vehicle moving as v_1', and the real distance of the Vehicle i as l_i. According to the kinematical equation, the relations of moving distance-time and speed-time of the leading vehicle at the intersection can be expressed in

$$l_i = x_1' - x_1 = \int_{t_{10}}^{t_1'} \int_{t_{10}}^{t_1'} adtdt,$$
$$v_1' = \int_{t_{10}}^{t_1'} adt. \tag{1}$$

As the leading vehicle has no obstructive factor ahead after being released, it can speed up till the stable speed for free travel. Hence, the accelerating time t_{1f} of the leading vehicle after being released and the distance covered during the accelerating process can be obtained as

$$v_{1f} = \int_{t_{10}}^{t_{1f}} adt,$$
$$s_1 = \int_{t_{10}}^{t_{1f}} \int_{t_{10}}^{t_{1f}} adtdt + v_{1f}\left(t_1' - t_{1f}\right). \tag{2}$$

If $s_1 < L_1$, it means that the leading vehicle reaches stable speed before passing the intersection; if $s_1 > L_1$, it means that the leading vehicle experiences the acceleration process during the whole motion process in the intersection; and if $s_1 = L_1$, it means that the speed of leading vehicle passing the intersection is exactly the stable speed of v_f, and the time for the leading vehicle to pass the intersection is the time needed for it to accelerate to the stable speed.

(2) Quantitative Analysis. This paper then selects a typical intersection in Changchun and collects 16 groups of the moving data of the leading vehicles at the intersection by video, the collected data mainly includes the green light time, the starting time of the leading vehicle, and the time when the leading vehicle passes the mark points in the intersection. Then using these data, the parameters like responsive starting time, the speed variety, and the moving time of the leading vehicle in the intersection can be analyzed. Hereby the intersection length is 40 m. For details, please refer to Table 1.

Through analyzing the data in Table 1, we can get Figures 2, 3, and 4, from which we can find that the response time of various leading vehicles in the traffic flow at the intersection is quite different, lacking certain regularity. This is mainly caused by the different performances of vehicles and drivers. However, the moving speed and the moving time of the leading vehicle in the intersection are relatively stable. This is mainly because the leading vehicles are always at the accelerating stage after being released, and they have no interference from other vehicles. From the data statistics, it

Table 1: Moving data of the leading vehicle at the intersection.

Group	Green time	Starting time of the leading vehicle	Mark point 1	Mark point 2	Mark point 3	Mark point 4
Leading vehicle 1	21.40	22.44	26.24	27.72	28.92	30.04
Leading vehicle 2	21.40	22.92	26.72	28.72	30.20	31.44
Leading vehicle 3	21.40	24.00	28.20	29.92	31.32	32.60
Leading vehicle 4	21.40	23.40	28.04	29.80	31.32	32.44
Leading vehicle 5	19.72	21.84	26.60	28.00	29.28	30.32
Leading vehicle 6	19.72	23.20	27.20	29.04	30.32	31.44
Leading vehicle 7	19.72	23.88	28.64	30.24	31.64	32.72
Leading vehicle 8	19.72	22.28	27.32	29.16	30.76	32.08
Leading vehicle 9	24.44	25.08	29.60	31.20	32.48	33.52
Leading vehicle 10	24.44	25.60	30.00	31.84	33.20	34.40
Leading vehicle 11	24.44	27.04	33.20	35.80	37.48	39.08
Leading vehicle 12	24.44	26.20	31.00	32.60	34.12	35.32
Leading vehicle 13	26.00	26.72	30.16	31.40	32.56	33.40
Leading vehicle 14	26.00	27.64	32.56	34.28	35.64	36.76
Leading vehicle 15	26.00	27.56	33.24	35.08	36.52	37.96
Leading vehicle 16	26.00	29.56	33.40	34.84	36.24	37.28

Note: the mark points 1, 2, and 3 are random marks and the mark point 4 is the position of passing the intersection.

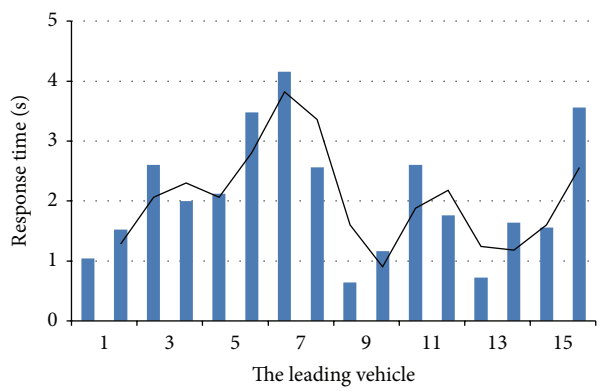

Figure 2: Responsive starting time variety of the leading vehicle at the intersection.

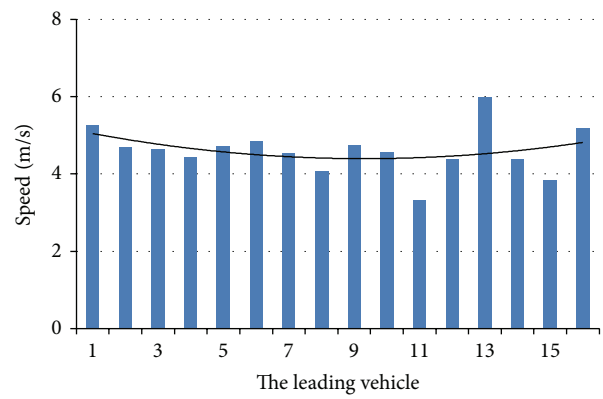

Figure 3: Speed variety of the leading vehicle in the intersection.

can be obtained that the average responsive starting time of the leading vehicle is 2.07 s, the average moving speed in the intersection is 4.6 m/s, and the average moving time of the leading vehicle in the intersection is 8.84 s. In addition, the leading vehicle in the intersection is always in the accelerating state, and the average accelerating speed of the investigated vehicles in the intersection is 0.52 m/s^2. Combined with the position-time-speed of the leading vehicle in the video data as well as (1)~(2), it can be concluded that the Spatiotemporal trajectory of the moving speed of the leading vehicle after being released at the intersection is as shown in Figure 5.

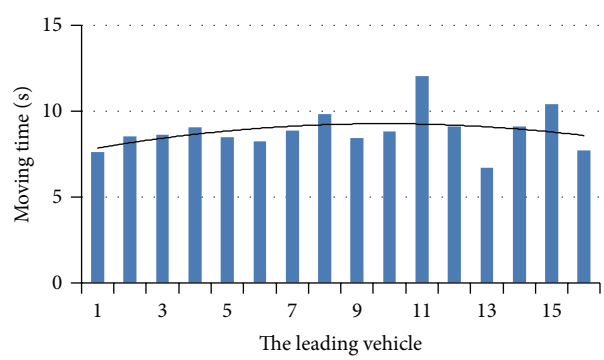

Figure 4: Moving time variety of the leading vehicle in the intersection.

2.4. Descriptive Analysis on the Motion Characteristics of following Vehicles. Generally speaking, the motion characteristics of the following vehicles largely depend on the leading vehicles. The secure time distance between the following vehicles and the leading vehicles determines the following-vehicle driver's sensibility degree for danger. The larger the time distance between the vehicles is, the lower the drivers' perceivable risk degree is, and the following vehicles

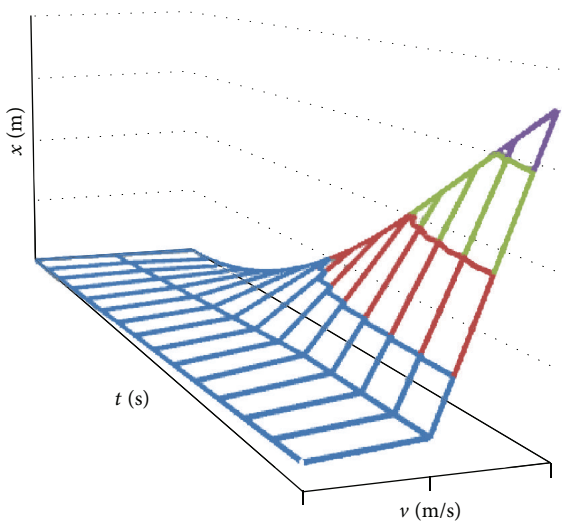

Figure 5: Spatiotemporal curve for the moving speed of the leading vehicle at the intersection.

can maintain the speed or accelerate to certain extent. Contrarily, the smaller the time distance between vehicles is, the higher the drivers' perceivable risk degree is, and the following vehicles should maintain the speed or decelerate to certain extent. Consequently, the speed variety (or acceleration) of the following vehicles is closely related to the influence degree of the leading vehicles on the following vehicles. Based on this relation, we can offer acceleration decision model of the following vehicles and establish car-following model and descriptive model for the motion characteristic of the following vehicles.

(1) Qualitative Description. In order to establish acceleration decision model of the following vehicles, we firstly need to confirm the functional relations between the acceleration of the following vehicles and their drivers' influence degree from the leading vehicles. Meanwhile, the influence degree of the leading vehicles on the following vehicles' drivers is closely related to the time distance between the following vehicles and leading vehicles. Based on such analysis, the paper primarily establishes the relations between influence degree of the leading vehicles on the following vehicles and the time distance between vehicles. Moreover, through confirming the influence degree of the leading vehicles on following vehicles, we provide the acceleration of the following vehicles whose concrete transformation models are showed as

$$f(h_{ti}) = h_0 - \frac{h_{ti}}{h_0} - h_{\min}, \tag{3}$$

$$\mathrm{IF} = 1 - \sum_{i=1}^{n} \frac{f(h_{ti})}{n}, \tag{4}$$

$$a_i = \omega * f(\mathrm{IF}). \tag{5}$$

In the equation, a_i is the acceleration of Vehicle i; $f(\cdot)$ is the transfer function of the influence degree of the leading vehicles on the following ones and the acceleration; IF is the influence degree of the leading vehicles on the following ones; R is the degree of randomness for the traffic flow; n is the number of the vehicles; $f(h_t)$ is the influence function of the vehicles; h_t is the time distance of the vehicles; $h_{\min}$ is the least necessary time distance for the vehicles.

When $i = 2$, IF is the influence degree between the following and leading vehicles; when $h \geq h_0$, $f(h) = 0$; when $h \leq h_{\min}$, $f(h) = 1$; or when IF = 1, the influence degree reaches the minimum; when IF = 0, it means that the time distance reaches its minimum while the influence degree reaches the maximum.

Then the established acceleration decision model for the following vehicles can be used to describe the motion characteristics of the following vehicles. First, it is supposed that the response time of the queued drivers is t_r, the green light time is t_0, the real-time moving speed is v_i^l before the queued vehicles reach the stable speed, and the time required for the following vehicles to reach the stable speed v_f with acceleration a_i is t_f. If the following vehicles have not reached the stable speed before reaching the stop line the relation of the time t_{i1} and the speed v_{i1} when the following vehicles arrive at the intersection stop line can be expressed by (6), while the relation of the distance l_{i1} between the following Vehicle i and the stop line, and the time t_{i1} when the vehicle reaches the stop line can be expressed by (7), thus we can get the length of time of the following car from start time to reaching the stop line time by (8):

$$l_{i1} = (i-1)(l_c + l_d), \tag{6}$$

$$v_{i1} = \int_{t_0+it_r}^{t_{i1}} \omega * f\left(1 - h_0 - h_{ti} + \frac{h_{t(i-1)}}{2}\right) dt, \tag{7}$$

$$t_{i1} - t_{i0} = \sqrt{\frac{2(i-1)(l_d + l_c)}{\omega} * f\left(1 - h_0 - h_{ti} + \frac{h_{t(i-1)}}{2}\right)}. \tag{8}$$

Meanwhile, the containable vehicles n and the density k of the traffic flow in the intersection can be worked out so as to calculate the traffic wave spreading situations when the traffic flow is departure. First, a relation between the moving time and distance of the following vehicles needs to be built. If the following Vehicle i has not reached the stable speed when moving out of the intersection, the expression can be shown as follows

$$x_{i2} - x_{i0} = \int_{t_0+it_r}^{t_{i2}} \int_{t_0+it_r}^{t_{i2}} \omega * f\left(1 - h_0 - h_{ti} + \frac{h_{t(i-1)}}{2}\right) dtdt. \tag{9}$$

In the equation, x_{i2} is the position when a vehicle passes the intersection.

If the vehicle has reached the stable speed when passing the intersection, the equation can be described as

$$x_{i2} - x_{i0} = \int_{t_0+it_r}^{t_f} \int_{t_0+it_r}^{t_f} \omega * f\left(1 - h_0 - h_{ti} + \frac{h_{t(i-1)}}{2}\right) dtdt + v_f\left(t_{i2} - t_f\right). \tag{10}$$

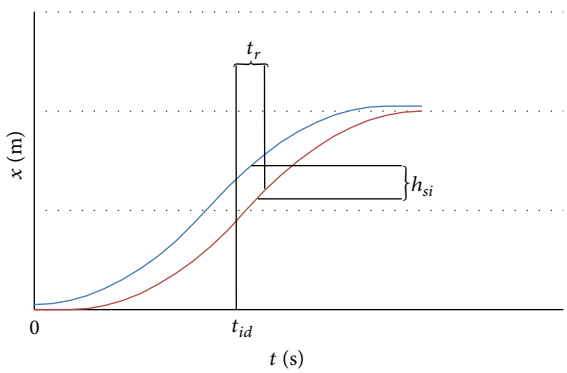

Figure 6: The relation diagram of motion trajectory between the following and the leading vehicles.

The secure space between any two vehicles during the departure process can be worked out, on the basis of the exact position and speed of all the vehicles in different time. If the vehicle is accelerating, the calculating model of secure space between the following and the leading vehicles can be shown as (11), while the motion trajectory of the decelerating process (to zero) of the two vehicles is shown in Figure 6:

$$h_{si} = \int_{t_{(i-1)0}}^{t_{id}+t_r} \int_{t_{(i-1)0}}^{t_{id}+t_r} \omega * f\left(1 - h_0 - h_{ti} + \frac{h_{t(i-1)}}{2}\right) dtdt - \int_{t_{i0}}^{t_{id}} \int_{t_{i0}}^{t_{id}} adtdt. \tag{11}$$

In Figure 6, t_{id} and $t_{id} + t_r$ are the time and position when the vehicles i and $i - 1$ start to decelerate.

If the neighboring vehicles are both moving at a stable speed, the secure space of the two vehicles is the distance covered by the following vehicle within the response time. The expression is as follows

$$h_{si} = v_f t_r. \tag{12}$$

Then, the time headway between the neighboring vehicles can be worked out; this paper analyzes the mentioned time headway h_{ti}, with l_c being the standard length of small-sized cars, summarizing (13) as follows:

$$h_{ti} = \frac{(l_c + h_{si})}{v_{i1}}. \tag{13}$$

Based on the above analysis, the paper can work out the number of the containable vehicles in the intersection n and the traffic flow density k, as shown in

$$n \leq \frac{(L_1 - \sum_{i=1}^{n} h_{si})}{l_c}, \tag{14}$$

$$k = L_1 - \sum_{i=1}^{n} \frac{h_{si}}{l_c L_1}. \tag{15}$$

(2) Quantitative Analysis. Based on the video-collected sixteen groups of the straight-moving traffic flow data at a typical intersection as in shown in Section 2.3 quantitative analysis, the paper has chosen eight groups and marked the time when the vehicles reach the stop line and the relevant crossing speed. Then the paper analyzes the motion characteristics of the following vehicles at the intersection in the departure process, and the detailed data is shown in Table 2.

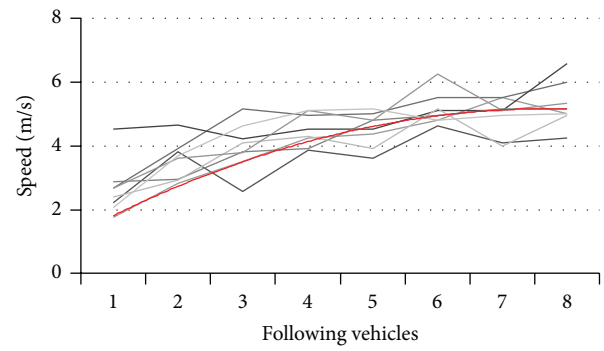

Figure 7: Speed variety of the following vehicles.

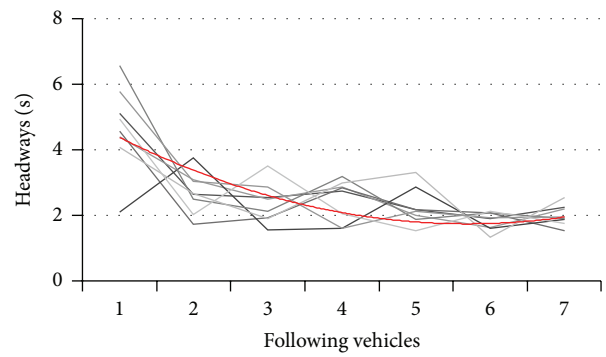

Figure 8: Headway variety of the following vehicles.

According to Figures 7 and 8, the following vehicles speed up gradually in the initial departure stage at the intersection, and it is until the sixth vehicle that the speed is leveling off. The time headway decreases gradually in the initial departure stage, and it is until the fifth vehicle that the time headway is leveling off. At this time, the stable average speed and time headway of the following vehicles are 5.28 m/s and 1.99 s, respectively. Based on the data above, the paper gives the qualitative description of the following vehicles. According to the earlier quantitative analysis, the average acceleration of the vehicles is $a = 0.52\,\mathrm{m/s^2}$, the average response time of the drivers is $t_r = 1.5$ s, the average length of the vehicle is 4.8 meters, and the average space of the queued vehicles is 2 meters. Hence, the paper concludes that the moving speed of the sixth vehicle arriving at the intersection stop line is 6.35 m/s, and the time headway when the traffic flow is moving steadily is 2.08 s. Through the comparison, the paper finds that the errors of calculated value and observed average value in speed and time headway are 20% and 4.5%, respectively. So the error values are acceptable, proving that (3) and (8) have a good imitative effect to the moving speed and the time headway of the following vehicles.

Table 2: Time and speed of the vehicles in each group when passing the stop line.

Group	Data of the vehicles in each group (v: m/s)							
	1	2	3	4	5	6	7	8
1								
t	22.07	27.17	29.8	32.33	35.07	37.23	39.13	41.37
v	2.22	3.81	2.57	3.87	3.61	4.62	4.10	4.25
2								
t	21.33	25.73	28.8	31.3	34.17	36.17	37.8	40
v	1.75	2.82	3.50	4.25	4.36	4.80	5.52	6.00
3								
t	25.5	30.07	31.8	33.73	36.57	38.73	40.8	42.33
v	2.67	3.90	5.16	4.95	5.00	5.52	5.52	6.00
4								
t	29.67	31.77	35.53	37.07	38.67	41.53	43.13	45
v	4.53	4.66	4.21	4.53	4.53	5.11	5.11	6.58
5								
t	31	37.57	40.07	42.2	45.37	47.23	49.3	51.2
v	2.87	2.94	3.81	3.90	4.80	4.95	5.16	5.16
6								
t	38.57	42.63	45.3	47.2	50.17	53.47	54.8	57.33
v	2.40	2.93	4.10	2.29	3.90	5.16	4.00	4.95
7								
t	40.40	46.17	49.20	52.06	53.67	55.80	57.73	59.67
v	2.67	3.61	3.78	5.11	4.80	6.23	5.11	5.33
8								
t	41.17	45.10	47.13	50.63	52.67	54.20	56.33	58.07
v	2.06	3.69	4.62	5.11	5.16	4.80	4.95	5.00

2.5. Analysis of the Departure Characteristics of Traffic Flow at the Intersection. When the traffic lights began to turn green, queued vehicles formed during red light and early green light period will be released at a relatively stable rate (saturation flow rate). The vehicles will accelerate to a stable speed, while the vehicles arrive after the vehicles ahead are dispersed will cross the intersection at a stable speed. Subsequently, vehicles will queue up again when the traffic lights are not green. The specific Spatiotemporal diagram of the departure vehicles at the intersection is shown in Figure 9.

According to the departure trajectory diagram of traffic flow at the intersection, the departure traffic flow at the intersection mainly includes starting and moving of the traffic flow, which are in correspondence with the start wave and kinematical wave in the traffic wave model. Hence, the traffic wave can be used to study the departure characteristics of the traffic flow at the intersection. Traditional wave model is mainly built on the basis of traffic flow conservation on a certain section within a fixed time. For example, given two neighboring areas with different densities A and B (the densities being k_A and k_B, resp.), and the density boundary is S, the spreading speed of the boundary with the density variety is u_S, and the average moving speeds of the traffic flow in two different areas are u_A and u_B. Figure 10 shows the specific results.

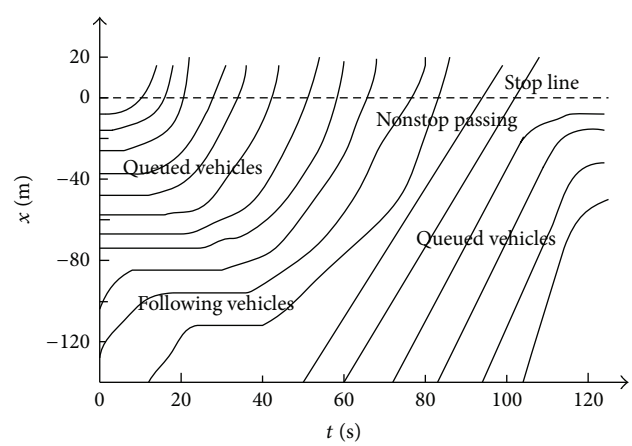

Figure 9: Departure trajectory of the traffic flow at the intersection.

According to Greenshield's model, the basic model of traffic flow, the speed of the traffic flow increases as the density decreases. Based on the conservative traffic flow N, (16) can be formed referring to Figure 10:

$$N = (u_A - u_S)\, k_A t = (u_B - u_S)\, k_B t. \tag{16}$$

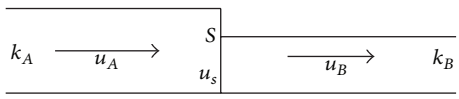

Figure 10: Fundamental diagram of the traffic wave.

For there is the relation between traffic flow, traffic density, and moving speed: $= ku$, which put into (16) we have

$$u_S = \frac{(q_A - q_B)}{(k_A - k_B)}. \tag{17}$$

Combined with flow-density-speed relation of Greenshield's model:

$$u = u_f\left(1 - \frac{k}{k_j}\right). \tag{18}$$

In the equation, u_f is free flow speed and k_j is the jam density. Putting (18) into (17), the wave speed expression can be obtained as follows.

Through analysis we can obtain

$$\left[u_f\left(1 - \frac{k_A}{k_j}\right) - u_S\right]k_A = \left[u_f\left(1 - \frac{k_B}{k_j}\right) - u_S\right]k_B. \tag{19}$$

Rearranging the equation one can get

$$u_S = \frac{u_f\left[k_A\left(1 - k_A/k_j\right) - k_B\left(1 - k_B/k_j\right)\right]}{(k_A - k_B)}. \tag{20}$$

When just being released, the traffic flow at the intersection is at the saturation flow rate. If calculating the traffic density of the area from L_3 in head of the intersection stop line, the following results with (15) can be obtained:

$$k_A = L_3 - \sum_{i=1}^{n}\frac{h_{si}}{l_c L_3}. \tag{21}$$

Meanwhile, when the traffic flow in area A is about to start, the traffic density in area B is approaching $k_B \approx 0$. Then the spreading speed of the traffic wave is as shown in

$$u_S = u_f\left(1 - \frac{L_3}{k_j} - \sum_{i=1}^{n}\frac{h_{si}}{l_c L_3 k_j}\right). \tag{22}$$

According to the traffic wave speed, the exact time when the traffic wave is approaching every vehicle in the traffic flow at the intersection, namely the starting time of the vehicles, can be calculated by using the following equation:

$$t_{(i+1)0} = t_{10} + \sum_{i=1}^{n-1}\frac{h_{si}}{u_s}. \tag{23}$$

Referring to (23) and kinematical equation, this paper can work out not only the time t_{if} when the vehicles reach a stable speed in the departure process of traffic flow but also the exact position x_{if} when they reach the stable speed v_f, as shown in

$$t_{if} = t_{10} + \frac{v_f}{a} + \sum_{i=1}^{n-1}\frac{h_{si}}{u_s}, \tag{24}$$

$$x_{if} = x_i + \int_0^{v_f/a}\int_0^{v_f/a} adtdt. \tag{25}$$

Combining (24) and (25), if the rth vehicle is the first vehicle to arrive at the intersection with a stable speed, the time t_{im} when the ith vehicle comes to the stop line will be as follows:

$$r = \frac{\int_0^{v_f/a}\int_0^{v_f/a} adtdt}{h_{si}} + 1, \tag{26}$$

$$t_{im} = t_{if} + \frac{v_f}{a}. \tag{27}$$

Referring to (26) and (27), the relation between t_g the green light time at the intersection and N_g the saturated departure number of the queued vehicles can be founded as follows:

$$N_g \le \sum_{i=1}^{r} i + \frac{v_f\left(t_g - t_k\right)}{h_{si}} \tag{28}$$

However, when the saturated departure of the queued vehicles at the intersection is over, the vehicles arrived subsequently from the upstream intersection are always queuing up, the saturated departure vehicles are supposed to encounter the follow-up traffic flow, resulting in that the original traffic wave disappears and a new one is formed. Given that the saturated departure rate at the intersection is q_m, the saturated departure density is k_m, the flow rate of the vehicles arrived subsequently is q_a, and the relative density is k_a, the following relation based on (16) can be worked out:

$$k_a\left(\frac{q_a}{k_a} - u_s'\right) = k_m\left(u_s - u_s'\right). \tag{29}$$

The newly formed traffic wave speed can be expressed by

$$u_s' = \frac{(q_m - q_a)}{(k_m - k_a)}. \tag{30}$$

Suppose that the meeting time and place is t_a and x_a, and the moving directions of the new traffic wave and the previous traffic flow are consistent, the time t'' when the new traffic wave comes to the stop line can be worked out through the employment of kinematical equation:

$$t'' = t_a + \frac{(x_a - x_{10})}{u_s'}. \tag{31}$$

Given that the green light time at the intersection is t_g, and the red light time is t_r, the spatial distance of each queued vehicle is l_s, the maximum queue length is $l_{\max}$, and

Table 3: Relevant departure data of traffic flow at the intersection through investigation.

	The starting time of leading vehicle	The starting time of the last vehicle	The number of queued vehicles	Length of the queue	Speed of traffic wave (m/s)	Stabilized time headway (s)	Stabilized space headway	Green light time (s)	The number of departure vehicles in green light time (s)
1	00.59.09	01.22.11	17	110.5	4.78	2.02	15.23	42	19
2	03.38.13	04.04.37	18	115.2	4.39	1.76	17.6	42	22
3	06.14.17	06.40.67	18	111.6	4.21	1.96	14.92	42	20
4	08.53.50	09.17.22	15	98.3	3.96	1.8	16.17	42	22
5	11.32.77	11.52.60	15	94.5	4.76	1.93	15.44	42	18

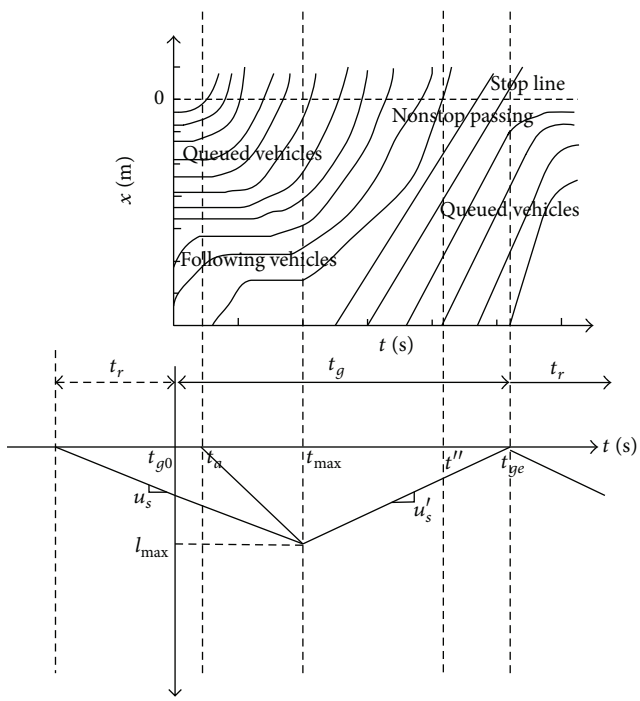

Figure 11: Diagram of traffic wave spreading at the intersection.

the corresponding time is $t_{\max}$, the traffic wave spreading situation is shown as in Figure 11.

Through the analysis of Figure 11, (27), (30), and (31), the following equation can be obtained:

$$\begin{gathered} t_{ge} = t'', \\ t_g = t_{ge} - t_{g0}, \\ t_{ge} - t_{\max} = \frac{l_{\max}}{u'_s}, \\ N_{\max} = \sum_{i=1}^{r} i + q_m \left(t_{ge} - t_{g0} - t_{km}\right), \\ \frac{l_{\max}}{l_s} = N_g. \end{gathered} \tag{32}$$

Figure 12: Departure data by video capture at the intersection.

Then the relation between the maximum departure length of the queued vehicles at the intersection and the critical green light time at the intersection is expressed in

$$l_{\max} = l_s \left[\sum_{i=1}^{r} i + q_m \left(t_{ge} - t_{g0} - t_{km}\right) \right]. \tag{33}$$

To sum up, when $t_g > t'' - t_{g0}$, the traffic flow can be completely released within a cycle; when $t_g = t'' - t_{g0}$, the traffic flow is on the critical point where no new queue is formed; and when $t_g < t'' - t_{g0}$, a new queue will be formed relating to the traffic flow at the intersection.

On the analysis of the straight-moving traffic flow departure characteristics of the southing entrance at a typical intersection in Changchun, the relevant traffic flow parameters have been collected, such as the starting time of the head and last vehicles in the queued vehicles at the intersection, the number, the length, and the moving speed of the queued vehicles, as shown in Figure 12 and Table 3, respectively, so as to have a qualitative analysis of the departure characteristics of the traffic flow at the intersections and test the effectiveness of the model in quantitative description. Besides, qualitative analysis mentioned above is employed to analyze and calculate the traffic wave spreading speed, the relation between green light time, and the number of the departure vehicles, as well as the space and time headway when the traffic flow is dispersing steadily. During the calculation, suppose that the stop line of the intersection is the starting point, the average length of vehicles is 4.8 meters, the average acceleration of

Table 4: Relevant departure data of traffic flow at the intersection through calculation.

	The starting time of leading vehicle	The starting time of the last vehicle	The number of queued vehicles	Length of the queue	Speed of traffic wave	Stabilized time of traffic flow	Stabilized space headway	Green light time	The number of departure vehicles in green light time
1	00.59.09	01.22.11	17	110.5	5.62	01.17.80	18.07	42	24
2	03.38.13	04.04.37	18	115.2	4.76	03.50.67	19.36	42	27
3	06.14.17	06.40.67	18	111.6	4.7	06.28.23	15.24	42	21
4	08.53.50	09.17.22	15	98.3	4.57	09.06.07	17.36	42	24
5	11.32.77	11.52.60	15	94.5	5.04	11.44.17	14.34	42	21

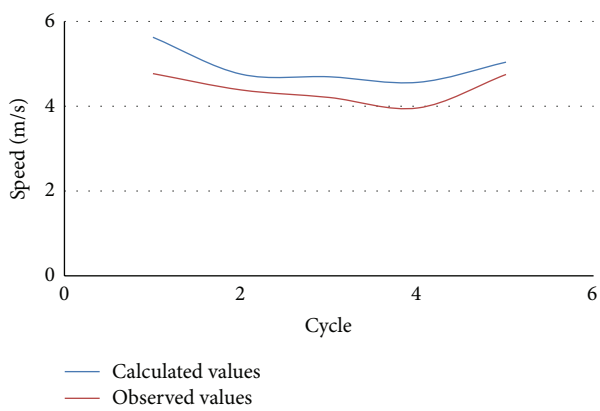

Figure 13: Comparison of the calculated and observed values of traffic wave speed.

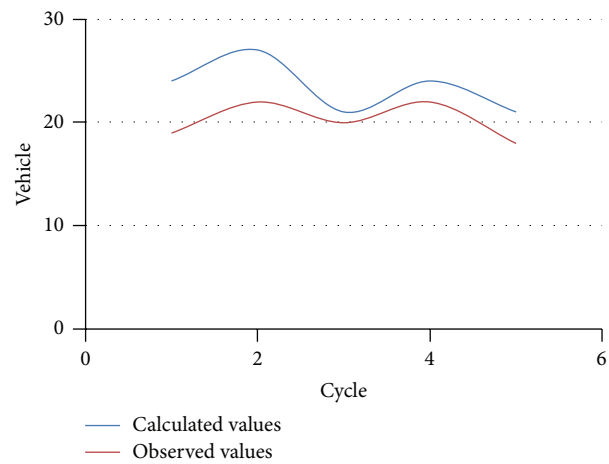

Figure 14: Comparison of the calculated and observed values of the passing vehicles in green time.

the vehicles through observation is $0.52\,\mathrm{m/s^2}$, the average distance of queued vehicles is 2 meters, and the intersection distance ahead is $L_3 = 62\,\mathrm{m} = L_1$, it can be found that the average space headway of the queued vehicles at the intersection $\overline{h}_d$ and the jam density k_j are as follows:

$$\overline{h}_d = 4.8 + 2 = 6.8\,\mathrm{m},$$
$$k_j = \frac{1}{\overline{h}_d} = \frac{1}{6.8} \approx 0.147\,\mathrm{veh/m}. \tag{34}$$

Based on (6), (15), (16), (18), and (26), along with the survey data, relevant data relating to the departure process of traffic flow at the intersection is worked out as in Table 4.

Through the analysis of the data in Tables 3 and 4, it can be found from the result of the video survey that, during the five cycles, the average spreading speed at the intersection is 4.42 m/s, the number of the average passing vehicles in green light time is 20.2, and the average time headway and space headway when the traffic flow is departure steadily are 1.89 s and 15.87 m, respectively. While through calculation, the relative data are 4.94 m/s, 23.4, and 13.77 s, respectively. Comparing Figures 13 with 14, it can be found that the calculated value is higher than the observed value of the wave spreading speed during traffic flow departure and the passing vehicles during green light time. For example, the maximum error and the average error of traffic wave spreading speed are 17.5% and 11.7%, respectively, while the relative numbers of the passing vehicles in green light time are 26.3% and 15.9%, respectively. The main cause for the deviation is that the calculation is based on the conditions that all the vehicles are small-sized cars, and they are moving with secure distance, while the fact is that there are large and medium vehicles and the drivers are extremely different in their driving behaviors.

3. Conclusions

In this paper, we applied kinematical equation and traffic wave theory to conduct qualitative analysis for the motion characteristic of the leading vehicle, the motion characteristic of the following vehicles, and the motion characteristic of the whole traffic flow at the intersection during the dispersion process at the signalized intersection. It further built spatiotemporal model of kinematics in the departure process at the intersection and traffic wave model in the dispersion process at the intersection to determine the changing situations of the leading vehicle at the departure process, the time for the following vehicles to reach the stable speed, and the relationship between the green light time at the intersection and the departure vehicle number at the intersection. In addition, combined with the vehicle operation data by video acquisition, we can know that the error is mostly less than

20% between the model calculation and the actual statistical analysis; therefore, the error is acceptable, which verified effectiveness of the model.

Acknowledgments

This work is supported by the National Science Foundation of China (nos. 51278220, 51278454, 51278520, and 51108208) and the Fundamental Research Funds for the Central Universities of China (no. 201103146). the Postdoctoral Science Foundation Funded Project of China (nos. 20110491307, 2013T60330), the Science and Technology Development Project of Jilin Province (no. 20130522121JH).

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