Research Article

A Novel Approach to Sliding Mode Control of Time-Delay Systems

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This paper is concerned with the sliding mode control for a class of linear systems with time-varying delays. By utilizing a novel Lyapunov-Krasovskii functional and combining it with the delay fractioning approach as well as the free-weighting matrix technology, a sufficient condition is established such that the resulting sliding mode dynamics is asymptotically stable. Then, a sliding mode controller for reaching motion is synthesized to guarantee that the trajectories of the resulting closed-loop system can be driven onto a prescribed sliding surface and maintained there for all subsequent time. A numerical example is provided to illustrate the effectiveness of the proposed design approach.

1. Introduction

Since time delays often appear in various engineering, biological, and economical systems, which are frequently a source of instability or poor performance, the analysis and synthesis of time-delay systems have been one of the most active research areas in system science over the past decades; see, for example, [1–7] and the references therein. Time-delay systems are also called systems with dead-time or aftereffect, equations with deviating argument or differential-difference equations [5], belonging to the class of functional differential equations that are infinite dimensional and dependent on the past history of the dynamics [6–10]. In the literature, a great number of results have been published based on different design approaches such as characteristic equation method, Lyapunov technique, or state solution approach [8–12]. Those developed criteria can be roughly divided into two categories, namely, delay-independent stability criteria [8–10] and delay-dependent stability criteria [12–18]. In general, the delay-dependent stability criterion is less conservative than delay-independent one, particularly when the size of time-delay is small or time-varying [15–18].

In the process of stability analysis and synthesis, a Lyapunov function (or functional) is usually involved, and more or less tight techniques to bound some cross terms are used. Since the general form of this functional leads to a complicated system of partial differential equations [17–20], many researchers tried to construct special forms of Lyapunov functional and derived simpler sufficient conditions which are more conservative. Therefore, these choices of specific functional and overbounding techniques are the origin of conservatism [17, 18]. In order to reduce the conservatism, some efforts were made to improve the delay-dependent conditions. For instance, the delay-partitioning idea [18, 20] has been proven to be an effective approach to reduce the conservatism, and many researchers have employed and further improved the idea of delay-partitioning to analyze the stability of time-delay systems. More results can be found in the literature; see, for example, [15–18] and the references therein. What is more, Wu et al. [21] and He et al. [22] have developed new methods to obtain less conservative delay-dependent stability conditions. Those methods are quite different from the conventional ones of using model transformation which may introduce some superfluous dynamics

into the original system [15–18, 21–23]. Additional studies can be found in [12–19] and references cited therein.

On the other hand, the sliding mode control (SMC) scheme is well known for its robustness against incompletely modeled dynamics, disturbances, time delays, and nonlinearities [23–25]. In the past two decades, sliding mode control has successfully been applied to a wide variety of practical engineering systems such as power systems, cantilever beam systems, spacecrafts, underwater vehicles, electrical motors, and industrial boiler systems [25–36]. Sliding mode control utilizes a discontinuous control to drive the system state trajectories to some predefined sliding surfaces on which the system has the desired properties such as stability, disturbance rejection capability, and tracking ability [23–25]. Thus, sliding mode control approach displays good performances including fast response, good transient performance, and easy realization [24]. A great volume of the literature has been published on the general topic of sliding mode control problems for various systems; see, for example, uncertain systems [30–32, 37], Markovian jump systems [24, 32], singular systems [24, 36], time-delay systems [31, 33, 35, 36], sliding mode-based observers [26, 29], stochastic systems [35], and so forth. To mention a few, Wu et al. investigated the problem of sliding mode control of Markovian jump singular time-delay systems in [24], and Khan et al. presented a dynamic integral sliding mode controller which is based on the existing dynamic sliding mode control and integral sliding mode control techniques in [37]. However, when revisiting those problems, we find that there is still much room left for improvement, which motivates the present study.

In this context, we are interested in investigating the delay-dependent sliding mode control problem of a class of time-varying delay systems. By dividing the time-delay interval into multiple segments, using the Lyapunov functional technology combined with matrix inequality technology, a new delay-dependent sufficient condition for the existence of linear sliding surfaces is proposed. To obtain a less conservative delay-dependent condition, some slack matrix variables based on the Newton-Leibniz formula are introduced. And an explicit parameterization of the desired sliding surface is also given. Then, a sliding mode controller for reaching motion is synthesized to guarantee that the trajectories of the resulting closed-loop system can be driven onto a prescribed sliding surface and maintained there for all subsequent time.

The rest of the paper is organized as follows. Section 2 gives problem formulations and some necessary lemmas. In Section 3, we present our main results of sliding mode control problem. Numerical examples are given in Section 4, followed by the conclusions, which are presented in Section 5.

Notations. The notations used throughout the paper are standard except where otherwise stated. The superscript “ T ” stands for matrix transposition. “ I ” and “ 0 ” denote the identity matrix and a zero matrix with compatible dimensions, respectively. The notation “ $P > 0$ ($P \geq 0$)” means that P is real symmetric and positive definite (positive semidefinite); $\text{diag}\{M_1, M_2, \dots, M_n\}$ stands for a block-diagonal matrix with matrices $M_1, M_2, \dots, M_n$ on the diagonal. “ $*$ ” represents the elements below the main diagonal of a symmetric

matrix. $\|\cdot\|$ denotes the Euclidean norm of a vector or the spectral norm of a matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulations and Preliminaries

Consider a class of linear time-delay systems given by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_d x(t-d(t)) + Bu(t) + f(t), \quad t > 0 \\ x(t) &= \phi(t), \quad t \in [-h, 0],\end{aligned}\quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the controlled input vector, and $f(t)$ represents the neglected dynamics and external disturbance; the matrices A , A_d , and B are known constant matrices of appropriate dimensions. $\phi(t)$ ($t \in [-h, 0]$) is the system's initial function which is continuously differentiable on $[-h, 0]$. Time delay $d(t)$ is a continuously differentiable function, satisfying the conditions as the following:

$$\begin{aligned}0 &\leq d(t) \leq h \\ \dot{d}(t) &\leq \mu,\end{aligned}\quad (2)$$

where h, μ are known positive real constants.

The following preliminary assumptions are made for system (1) [23].

Assumption 1. The pair $(A + A_d, B)$ is controllable.

Assumption 2. The perturbation term f satisfies the classical matching conditions, that is,

$$f(t) = Bw(t) \quad \text{or} \quad Bu(t) + f(t) = B(u(t) + w(t)), \quad (3)$$

and it is bounded, as follows, by a known functional $\rho(t)$:

$$\|f(t)\| < \rho(t). \quad (4)$$

Assumption 3. Matrix B is of full column rank; that is, $\text{rank}(B) = m$.

The aim of this paper is to design a SMC controller $u(t)$ such that the resulting closed-loop system is asymptotically stable for all exogenous disturbance. Basically, the design procedure of a sliding mode control system is a two-stage process. The first phase is to choose a set of sliding manifolds such that the original system restricted to the intersection of this sliding manifold has a desired behavior. The second phase is to design a sliding mode control law that forces the system's trajectories onto the sliding surface and maintains them on it.

We first transform the original system (1) into a special form (classically called regular form) that is appropriate for sliding mode control. A nonsingular matrix T can be chosen such that

$$TB = \begin{bmatrix} 0_{(n-m) \times m} \\ B_2 \end{bmatrix}, \quad (5)$$

where $B_2 \in R^{m \times m}$ is nonsingular. Thus, by a state transformation $z(t) = Tx(t)$, the system (1) has the regular form

$$\begin{aligned} \dot{z}(t) &= \bar{A}z(t) + \bar{A}_d z(t-d(t)) + \bar{B}(u(t) + w(t)) \\ z(t) &= \bar{\phi}(t), \quad t \in [-r, 0], \end{aligned} \quad (6)$$

where $\bar{A} = TAT^{-1}$, $\bar{A}_d = TA_dT^{-1}$, $\bar{B} = \begin{bmatrix} 0_{(n-m) \times m} \\ B_2 \end{bmatrix}$, and $\bar{\phi}(t) = T\phi(t)$.

Let $z(t) \triangleq \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}$, $\bar{A} \triangleq \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$, and $\bar{A}_d \triangleq \begin{bmatrix} \bar{A}_{d11} & \bar{A}_{d12} \\ \bar{A}_{d21} & \bar{A}_{d22} \end{bmatrix}$; then system (6) can be rewritten as

$$\begin{aligned} \dot{z}_1(t) &= \bar{A}_{11}z_1(t) + \bar{A}_{d11}z_1(t-d(t)) \\ &\quad + \bar{A}_{12}z_2(t) + \bar{A}_{d12}z_2(t-d(t)) \\ \dot{z}_2(t) &= \bar{A}_{21}z_1(t) + \bar{A}_{d21}z_1(t-d(t)) + \bar{A}_{22}z_2(t) \\ &\quad + \bar{A}_{d22}z_2(t-d(t)) + B_2(u(t) + w(t)) \\ z_1(t) &= \bar{\phi}_1(t) \\ z_2(t) &= \bar{\phi}_2(t), \end{aligned} \quad (7)$$

where $z_1(t) \in R^{(n-m)}$, $z_2(t) \in R^m$, $\bar{\phi}_1(t) \in R^{(n-m)}$ and $\bar{\phi}_2(t) \in R^m$ are subblocks of $\bar{\phi}(t)$.

It is obvious that the first equation of system (7) represents the sliding motion dynamics of (6); therefore, the corresponding sliding surface can be chosen as follows:

$$S(t) = [C \ I] z(t) = Cz_1(t) + z_2(t) = 0, \quad (8)$$

where $C \in R^{(n-m)}$ is the parameter to be designed.

When the system trajectories reach onto the sliding surface $S(t) = \dot{S}(t) = 0$, that is, $z_2(t) = -Cz_1(t)$, the sliding mode dynamics is attained. By combining (7) and (8), we obtain the sliding motion

$$\begin{aligned} \dot{z}_1(t) &= (\bar{A}_{11} - \bar{A}_{12}C)z_1(t) + (\bar{A}_{d11} - \bar{A}_{d12}C)z_1(t-d(t)), \\ z_1(t) &= \bar{\phi}_1(t). \end{aligned} \quad (9)$$

In this paper, we are interested in designing a reaching motion control law $u(t)$ such that the following requirements are satisfied.

- (i) The corresponding sliding motion (9) is asymptotically stable.
- (ii) The system (7) is asymptotically stable with the reaching control law $u(t)$.

To facilitate further developments, we introduce the following lemma that will be frequently used in deriving the main results.

Lemma 4 (Schur complement). *For symmetrical matrix $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22} \end{bmatrix}$, the following inequalities are equivalent:*

- (1) $L < 0$,
- (2) $L_{11} < 0, L_{22} - L_{12}^T L_{11}^{-1} L_{12} < 0$,
- (3) $L_{22} < 0, L_{11} - L_{12} L_{22}^{-1} L_{12}^T < 0$.

3. Main Results

3.1. Sliding Mode Dynamics Analysis. Due to the influence of the sliding manifold on system stability and transient performance, the design and analysis of the sliding manifold has become one of the main issues in the sliding mode control [27–30], which is shown in this subsection. In the following theorem, we first present a new delay-dependent stability condition by delay partitioning approach and free-weighting matrices technique.

Theorem 5. *For given positive scalars $h > 0$ and $\mu > 0$, the reduced order system (9) is asymptotically stable if there exist matrices $P = P^T > 0$, $Q_i = Q_i^T \geq 0$ ($i = 1, \dots, 4$), $Z = Z^T > 0$,*

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ * & X_{22} & X_{23} & X_{24} & X_{25} \\ * & * & X_{33} & X_{34} & X_{35} \\ * & * & * & X_{44} & X_{45} \\ * & * & * & * & X_{55} \end{bmatrix} \geq 0, \quad (10)$$

and free-weighting matrices N_i ($i = 1, 2, \dots, 5$) with appropriate dimensions, such that the following LMIs hold:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & 0 \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} \\ * & * & * & \Phi_{44} & \Phi_{45} & 0 \\ * & * & * & * & \Phi_{55} & 0 \\ * & * & * & * & * & -hZ \end{bmatrix} < 0, \quad (11)$$

$$\Psi = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & N_1 \\ * & X_{22} & X_{23} & X_{24} & X_{25} & N_2 \\ * & * & X_{33} & X_{34} & X_{35} & N_3 \\ * & * & * & X_{44} & X_{45} & N_4 \\ * & * & * & * & X_{55} & N_5 \\ * & * & * & * & * & Z \end{bmatrix} \geq 0, \quad (12)$$

where

$$\begin{aligned} \Phi_{11} &= P(\bar{A}_{11} - \bar{A}_{12}C) + (\bar{A}_{11} - \bar{A}_{12}C)^T P \\ &\quad + Q_1 + N_1 + N_1^T + hX_{11}, \end{aligned}$$

$$\Phi_{12} = N_2^T + hX_{12},$$

$$\Phi_{13} = P(\bar{A}_{d11} - \bar{A}_{d12}C) - N_1 + N_3^T + hX_{13},$$

$$\Phi_{14} = N_4^T + hX_{14}, \quad \Phi_{15} = N_5^T + hX_{15},$$

$$\Phi_{16} = h(\bar{A}_{11} - \bar{A}_{12}C)^T Z,$$

$$\Phi_{22} = -\left(1 - \frac{\mu}{2}\right)Q_1 + Q_2 + hX_{22},$$

$$\Phi_{23} = -N_2 + hX_{23}, \quad \Phi_{24} = hX_{24},$$

$$\Phi_{25} = hX_{25},$$

$$\Phi_{33} = -(1 - \mu)Q_2 + Q_3 - N_3 - N_3^T + hX_{33},$$

$$\Phi_{34} = -N_4^T + hX_{34}, \quad \Phi_{35} = -N_5^T + hX_{35},$$

$$\begin{aligned}
\Phi_{36} &= h(\bar{A}_{d11} - \bar{A}_{d12}C)^T Z, \\
\Phi_{44} &= -\left(1 - \frac{\mu}{2}\right) Q_3 + Q_4 + hX_{44}, \\
\Phi_{45} &= hX_{45}, \quad \Phi_{55} = -Q_4 + hX_{55}.
\end{aligned} \tag{13}$$

Proof. We define the following Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) \tag{14}$$

with

$$\begin{aligned}
V_1(t) &= z_1^T(t) P z_1(t) \\
V_2(t) &= \int_{t-d(t)/2}^t z_1^T(s) Q_1 z_1(s) ds \\
V_3(t) &= \int_{t-d(t)}^{t-d(t)/2} z_1^T(s) Q_2 z_1(s) ds \\
V_4(t) &= \int_{t-((d(t)+h)/2)}^{t-d(t)} z_1^T(s) Q_3 z_1(s) ds \\
V_5(t) &= \int_{t-h}^{t-((d(t)+h)/2)} z_1^T(s) Q_4 z_1(s) ds \\
V_6(t) &= \int_{-h}^0 \int_{t+\theta}^t \dot{z}_1^T(s) Z \dot{z}_1(s) ds d\theta,
\end{aligned} \tag{15}$$

where $P = P^T > 0$, $Q_i = Q_i^T \geq 0$ ($i = 1, \dots, 4$), and $Z = Z^T > 0$ are matrices to be determined. By using the Leibniz-Newton formula, one has

$$z_1(t-d(t)) = z_1(t) - \int_{t-d(t)}^t \dot{z}_1(s) ds. \tag{16}$$

Due to this relation, one can introduce the following zero equation:

$$\begin{aligned}
&2 \left[z_1^T(t) N_1 + z_1^T\left(t - \frac{d(t)}{2}\right) N_2 + z_1^T(t-d(t)) N_3 \right. \\
&\quad \left. + z_1^T\left(t - \frac{d(t)+h}{2}\right) N_4 + z_1^T(t-h) N_5 \right] \\
&\quad \times \left[z_1(t) - z_1(t-d(t)) - \int_{t-d(t)}^t \dot{z}_1(s) ds \right] = 0,
\end{aligned} \tag{17}$$

where N_i ($i = 1, \dots, 5$) are unknown constant matrices with appropriate dimensions.

Moreover, it follows from (2) that for any appropriate dimensional matrix $X \geq 0$, (18) is always satisfied as follows:

$$h\eta^T(t) X \eta(t) - \int_{t-d(t)}^t \eta^T(s) X \eta(s) ds \geq 0, \tag{18}$$

where

$$\eta(t) = \begin{bmatrix} z_1^T(t) & z_1^T\left(t - \frac{d(t)}{2}\right) & z_1^T(t-d(t)) & z_1^T\left(t - \frac{d(t)+h}{2}\right) & z_1^T(t-h) \end{bmatrix}^T. \tag{19}$$

According to (16)–(19), and by taking the time derivative of the Lyapunov-Krasovskii functional in (14), we have

$$\begin{aligned}
\dot{V}(t) &= 2z_1^T(t) P \dot{z}_1(t) + z_1^T(t) Q_1 z_1(t) \\
&\quad - \left(1 - \frac{\dot{d}(t)}{2}\right) z_1^T\left(t - \frac{d(t)}{2}\right) Q_1 z_1\left(t - \frac{d(t)}{2}\right) \\
&\quad + \left(1 - \frac{\dot{d}(t)}{2}\right) z_1^T\left(t - \frac{d(t)}{2}\right) Q_2 z_1\left(t - \frac{d(t)}{2}\right) \\
&\quad - (1 - \dot{d}(t)) z_1^T(t-d(t)) Q_2 z_1(t-d(t)) \\
&\quad + (1 - \dot{d}(t)) z_1^T(t-d(t)) Q_3 z_1(t-d(t)) \\
&\quad - \left(1 - \frac{\dot{d}(t)}{2}\right) z_1^T\left(t - \frac{d(t)+h}{2}\right) Q_3 z_1\left(t - \frac{d(t)+h}{2}\right)
\end{aligned}$$

$$\begin{aligned}
&+ \left(1 - \frac{\dot{d}(t)}{2}\right) z_1^T\left(t - \frac{d(t)+h}{2}\right) Q_4 z_1\left(t - \frac{d(t)+h}{2}\right) \\
&\quad - z_1^T(t-h) Q_4 z_1(t-h) + h\dot{z}_1^T(t) Z \dot{z}_1(t) \\
&\quad - \int_{t-h}^t \dot{z}_1^T(s) Z \dot{z}_1(s) ds \\
&\leq 2z_1^T(t) P (\bar{A}_{11} - \bar{A}_{12}C) z_1(t) \\
&\quad + (\bar{A}_{d11} - \bar{A}_{d12}C) z_1(t-d(t)) + z_1^T(t) Q_1 z_1(t) \\
&\quad - \left(1 - \frac{\mu}{2}\right) z_1^T\left(t - \frac{d(t)}{2}\right) Q_1 z_1\left(t - \frac{d(t)}{2}\right) \\
&\quad + \left(1 - \frac{\dot{d}(t)}{2}\right) z_1^T\left(t - \frac{d(t)}{2}\right) Q_2 z_1\left(t - \frac{d(t)}{2}\right) \\
&\quad - (1 - \mu) z_1^T(t-d(t)) Q_2 z_1(t-d(t))
\end{aligned}$$

$$\begin{aligned}
& + (1 - \dot{d}(t)) z_1^T(t-d(t)) Q_3 z_1(t-d(t)) \\
& - \left(1 - \frac{\mu}{2}\right) z_1^T\left(t - \frac{d(t)+h}{2}\right) Q_3 z_1\left(t - \frac{d(t)+h}{2}\right) \\
& + \left(1 - \frac{\dot{d}(t)}{2}\right) z_1^T\left(t - \frac{d(t)+h}{2}\right) Q_4 z_1\left(t - \frac{d(t)+h}{2}\right) \\
& - z_1^T(t-h) Q_4 z_1(t-h) + h z_1^T(t) Z \dot{z}_1(t) \\
& - \int_{t-d(t)}^t \dot{z}_1^T(s) Z \dot{z}_1(s) ds \\
& + 2 \left[z_1^T(t) N_1 + z_1^T\left(t - \frac{d(t)}{2}\right) N_2 + z_1^T(t-d(t)) N_3 \right. \\
& \quad \left. + z_1^T\left(t - \frac{d(t)+h}{2}\right) N_4 + z_1^T(t-h) N_5 \right] \\
& \times \left[z_1(t) - z_1(t-d(t)) - \int_{t-d(t)}^t \dot{z}_1(s) ds \right] \\
& + h \eta^T(t) X \eta(t) - \int_{t-d(t)}^t \eta^T(t) X \eta(t) ds \\
& = \eta^T(t) \Xi \eta(t) - \int_{t-d(t)}^t \xi^T(t, s) \Psi \xi(t, s) ds - Y,
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
\xi(t) &= [\eta^T(t) \quad \dot{z}_1^T(s)]^T, \\
Y &= \frac{\dot{d}(t)}{2} z_1^T\left(t - \frac{d(t)}{2}\right) Q_2 z_1\left(t - \frac{d(t)}{2}\right) \\
&+ \dot{d}(t) z_1^T(t-d(t)) Q_3 z_1(t-d(t)) \\
&+ \frac{\dot{d}(t)}{2} z_1^T\left(t - \frac{d(t)+h}{2}\right) Q_4 z_1\left(t - \frac{d(t)+h}{2}\right), \\
\Xi &= \begin{bmatrix} \bar{\Phi}_{11} & \Phi_{12} & \bar{\Phi}_{13} & \Phi_{14} & \Phi_{15} \\ * & \Phi_{22} & \bar{\Phi}_{23} & \Phi_{24} & \Phi_{25} \\ * & * & \bar{\Phi}_{33} & \Phi_{34} & \Phi_{35} \\ * & * & * & \Phi_{44} & \Phi_{45} \\ * & * & * & * & \Phi_{55} \end{bmatrix}, \\
\bar{\Phi}_{11} &= \Phi_{11} + (\bar{A}_{11} - \bar{A}_{12}C)^T H (\bar{A}_{11} - \bar{A}_{12}C), \\
\bar{\Phi}_{13} &= \Phi_{13} + (\bar{A}_{11} - \bar{A}_{12}C)^T H (\bar{A}_{d11} - \bar{A}_{d12}C), \\
\bar{\Phi}_{33} &= \Phi_{33} + (\bar{A}_{d11} - \bar{A}_{d12}C)^T H (\bar{A}_{d11} - \bar{A}_{d12}C),
\end{aligned} \tag{21}$$

and Ψ is denoted in (12).

By Schur complement (Lemma 4) and some matrices primary manipulations, it is easy to conclude that the above-mentioned matrix $\Xi < 0$ in (21) is equivalent to (11). Since $Q_2 \geq 0$, $Q_3 \geq 0$, and $Q_4 \geq 0$, we have $Y \geq 0$. Therefore, if $\Xi < 0$ and $\Psi \geq 0$, there exists a positive scalar ε such that $\dot{V}(t) \leq -\varepsilon \|z_1(t)\|^2$, which guarantees that system (7) is asymptotically stable.

Thus, the proof is completed. $\square$

Remark 6. It should be pointed out that Theorem 5 presents a delay-dependent admissible criterion for sliding mode dynamics by utilizing a novel Lyapunov-Krasovskii functional (LKF), which is much less conservative than most results in the literature. The reduced conservatism of Theorem 5 benefits from the construction of the new LKF in (14) based on the idea of delay partitioning. In the general delay-partitioning method, the time delay should be modeled and represented as a constant part and a time-varying part, and then, the idea of delay partitioning is applied to the constant part [16–19]. However, it is not suitable to the case when the time delay cannot be modeled as those two parts. According to this, in this brief, we divided the whole time varying delay interval $[0, h]$ into multiparts

$$\begin{aligned}
& \left[0, \frac{d(t)}{2}\right], \quad \left[\frac{d(t)}{2}, d(t)\right], \\
& \left[d(t), \frac{d(t)+h}{2}\right], \quad \left[\frac{d(t)+h}{2}, h\right],
\end{aligned} \tag{22}$$

and a novel LKF is constructed with every delay component. Thus, we obtain a time varying LKF, since it is dependent on the time varying delay, which constitutes the major difference from most existing results in the literature.

Remark 7. Note that the criteria for sliding mode dynamics analysis problem in Theorem 5 are not LMI conditions due to the term $C^T \bar{A}_{12}^T Z$ in (11). Then, based on Theorem 5, by carrying out some algebraic manipulations, we can obtain the following condition which is characterized in terms of a set of linear matrix inequalities and can be easily solved by using the existing Matlab LMI toolbox.

Theorem 8. For given positive scalars $h > 0$ and $\mu > 0$, the reduced order system (9) is asymptotically stable if there exist matrices $\bar{P} = \bar{P}^T > 0$, $\bar{Q}_i = \bar{Q}_i^T \geq 0$ ($i = 1, \dots, 4$), $\bar{Z} = \bar{Z}^T > 0$,

$$\bar{X} = \begin{bmatrix} \bar{X}_{11} & \bar{X}_{12} & \bar{X}_{13} & \bar{X}_{14} & \bar{X}_{15} \\ * & \bar{X}_{22} & \bar{X}_{23} & \bar{X}_{24} & \bar{X}_{25} \\ * & * & \bar{X}_{33} & \bar{X}_{34} & \bar{X}_{35} \\ * & * & * & \bar{X}_{44} & \bar{X}_{45} \\ * & * & * & * & \bar{X}_{55} \end{bmatrix} \geq 0, \tag{23}$$

and free-weighting matrices $\tilde{N}_i$ ($i = 1, \dots, 5$) with appropriate dimensions, such that the following LMIs hold:

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} & \tilde{\Phi}_{14} & \tilde{\Phi}_{15} & \tilde{\Phi}_{16} \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} & \tilde{\Phi}_{24} & \tilde{\Phi}_{25} & 0 \\ * & * & \tilde{\Phi}_{33} & \tilde{\Phi}_{34} & \tilde{\Phi}_{35} & \tilde{\Phi}_{36} \\ * & * & * & \tilde{\Phi}_{44} & \tilde{\Phi}_{45} & 0 \\ * & * & * & * & \tilde{\Phi}_{55} & 0 \\ * & * & * & * & * & \tilde{\Phi}_{66} \end{bmatrix} < 0, \quad (24)$$

$$\tilde{\Psi} = \begin{bmatrix} \tilde{X}_{11} & \tilde{X}_{12} & \tilde{X}_{13} & \tilde{X}_{14} & \tilde{X}_{15} & \tilde{N}_1 \\ * & \tilde{X}_{22} & \tilde{X}_{23} & \tilde{X}_{24} & \tilde{X}_{25} & \tilde{N}_2 \\ * & * & \tilde{X}_{33} & \tilde{X}_{34} & \tilde{X}_{35} & \tilde{N}_3 \\ * & * & * & \tilde{X}_{44} & \tilde{X}_{45} & \tilde{N}_4 \\ * & * & * & * & \tilde{X}_{55} & \tilde{N}_5 \\ * & * & * & * & * & \tilde{Z} \end{bmatrix} \geq 0, \quad (25)$$

where

$$\begin{aligned} \tilde{\Phi}_{11} &= (\bar{A}_{11}\tilde{P} - \bar{A}_{12}\tilde{C}) + (\bar{A}_{11}\tilde{P} - \bar{A}_{12}\tilde{C})^T \\ &\quad + \tilde{Q}_1 + \tilde{N}_1 + \tilde{N}_1^T + h\tilde{X}_{11}, \\ \tilde{\Phi}_{12} &= \tilde{N}_2^T + h\tilde{X}_{12}, \\ \tilde{\Phi}_{13} &= (\bar{A}_{d11}\tilde{P} - \bar{A}_{d12}\tilde{C}) - \tilde{N}_1 + \tilde{N}_3^T + h\tilde{X}_{13}, \\ \tilde{\Phi}_{14} &= \tilde{N}_4^T + h\tilde{X}_{14}, \quad \tilde{\Phi}_{15} = \tilde{N}_5^T + h\tilde{X}_{15}, \\ \tilde{\Phi}_{16} &= (\bar{A}_{11}\tilde{P} - \bar{A}_{12}\tilde{C})^T, \\ \tilde{\Phi}_{22} &= -\left(1 - \frac{\mu}{2}\right)\tilde{Q}_1 + \tilde{Q}_2 + h\tilde{X}_{22}, \\ \tilde{\Phi}_{23} &= -\tilde{N}_2 + h\tilde{X}_{23}, \quad \tilde{\Phi}_{24} = h\tilde{X}_{24}, \\ \tilde{\Phi}_{25} &= h\tilde{X}_{25}, \\ \tilde{\Phi}_{33} &= -(1 - \mu)\tilde{Q}_2 + \tilde{Q}_3 - \tilde{N}_3 - \tilde{N}_3^T + h\tilde{X}_{33}, \\ \tilde{\Phi}_{34} &= -\tilde{N}_4^T + h\tilde{X}_{34}, \\ \tilde{\Phi}_{35} &= -\tilde{N}_5^T + h\tilde{X}_{35}, \quad \tilde{\Phi}_{36} = (\bar{A}_{d11}\tilde{P} - \bar{A}_{d12}\tilde{C})^T, \\ \tilde{\Phi}_{44} &= -\left(1 - \frac{\mu}{2}\right)\tilde{Q}_3 + \tilde{Q}_4 + h\tilde{X}_{44}, \\ \tilde{\Phi}_{45} &= h\tilde{X}_{45}, \quad \tilde{\Phi}_{55} = -\tilde{Q}_4 + h\tilde{X}_{55}, \\ \tilde{\Phi}_{66} &= h^{-1}(\tilde{Z} - 2\tilde{P}). \end{aligned} \quad (26)$$

Moreover, if the conditions mentioned above are feasible, the matrix C in (8) can be given by $C = \tilde{C}\tilde{P}^{-1}$; that is, the sliding surface of (7) can be designed as

$$S(t) = (\tilde{C}\tilde{P}^{-1})z_1(t) + z_2(t) = 0. \quad (27)$$

Proof. By performing a congruence transformation $\text{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}\}$ to both sides of (10), and defining

matrices $\tilde{X}_{mn} \triangleq P^{-1}X_{mn}P^{-1}$ ($m, n = 1, 2, 3, 4, 5$), thus, (23) is obtained.

Similarly, performing a congruence transformation to (12) by $\text{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}\}$ and defining matrices $\tilde{N}_k \triangleq P^{-1}N_kP^{-1}$, ($k = 1, 2, 3, 4, 5$), $\tilde{Z} \triangleq P^{-1}ZP^{-1}$, we have (25).

Performing a congruence transformation to (11) with $\text{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, Z^{-1}\}$ and defining matrices $\tilde{P} \triangleq P^{-1}$, $\tilde{Q}_i \triangleq P^{-1}Q_iP^{-1}$, ($i = 1, 2, 3, 4$), $\tilde{C} = CP^{-1}$, we have

$$\begin{aligned} &\tilde{\Phi} \\ &= \begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} & \tilde{\Phi}_{14} & \tilde{\Phi}_{15} & h(\bar{A}_{11}\tilde{P} - \bar{A}_{12}\tilde{C})^T \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} & \tilde{\Phi}_{24} & \tilde{\Phi}_{25} & 0 \\ * & * & \tilde{\Phi}_{33} & \tilde{\Phi}_{34} & \tilde{\Phi}_{35} & h(\bar{A}_{d11}\tilde{P} - \bar{A}_{d12}\tilde{C})^T \\ * & * & * & \tilde{\Phi}_{44} & \tilde{\Phi}_{45} & 0 \\ * & * & * & * & \tilde{\Phi}_{55} & 0 \\ * & * & * & * & * & -h\tilde{P}\tilde{Z}^{-1}\tilde{P} \end{bmatrix} < 0. \end{aligned} \quad (28)$$

According to the definition of $\tilde{P}$ and $\tilde{Z}$, the following equation always holds:

$$(\tilde{P} - \tilde{Z})^T \tilde{Z}^{-1} (\tilde{P} - \tilde{Z}) \geq 0 \quad (29)$$

which implies that

$$-\tilde{P}\tilde{Z}^{-1}\tilde{P} \leq \tilde{Z} - 2\tilde{P}. \quad (30)$$

Thus, we can obtain (24).

Hence, the proof is completed. $\square$

After switching surface design, the next important aspect of sliding mode control is to guarantee the existence of a sliding mode.

3.2. Sliding Mode Controller Design. Now, we are in the position to design a SMC law, by which the trajectories of the time-delay system (7) can be driven onto the predesigned sliding surface $S(t) = 0$ in a finite time.

Theorem 9. Suppose that the conditions of (23)–(25) in Theorem 8 are feasible and the linear sliding surface is given by (8). Then, the trajectory of the closed-loop system (7) can be driven onto the sliding surface in limited time with the control:

$$\begin{aligned} u(t) &= u_{eq}(t) + u_N(t) \\ u_{eq}(t) &= -B_2^{-1} [\bar{C}\bar{A}z(t) + \bar{C}\bar{A}_d z(t - d(t))] \\ u_N(t) &= -[\rho(t) + \varepsilon] \text{sgn}(B_2^T s(t)), \end{aligned} \quad (31)$$

where $\varepsilon > 0$ and $\rho(t)$ is the upper bound of $w(t)$.

Proof. We will complete the proof by showing that the control law (31) can not only drive the system trajectories onto the liner sliding surface but also keep it there for all subsequent time.

According to SMC theory, when the system trajectories reach onto the sliding surface, it follows that $S(t) = 0$ and $\dot{S}(t) = 0$.

From the sliding surface $S = [C \ I] z = \bar{C}z$, we have

$$\begin{aligned}\dot{S} &= [C \ I] \dot{z} = \bar{C}(\dot{z}_1 + \dot{z}_2) \\ &= \bar{C}A z(t) + \bar{C}A_d z(t - d(t)) + B_2(u(t) + w(t)).\end{aligned}\quad (32)$$

Therefore, by $\dot{S}(t) = 0$, we get the equivalent control as follows:

$$u_{eq}(t) = -B_2^{-1} [\bar{C}A z(t) + \bar{C}A_d z(t - d(t))]. \quad (33)$$

Choose the following Lyapunov functional

$$V(t) = \frac{1}{2} S^T(t) S(t). \quad (34)$$

By taking the derivative of V , it gives

$$\begin{aligned}\dot{V}(t) &= S^T(t) \dot{S}(t) \\ &= S^T(t) [\bar{C}A z(t) + \bar{C}A_d z(t - d(t)) + B_2(u(t) + w(t))].\end{aligned}\quad (35)$$

Combining (31) and (35), we have

$$\begin{aligned}\dot{V}(t) &= S^T(t) [B_2(u_N(t) + w(t))] \\ &= S^T(t) B_2 [-(\rho(t) + \varepsilon) \operatorname{sgn}(B_2^T S(t)) + w(t)] \\ &= -\varepsilon \|B_2^T S(t)\| - S^T(t) B_2 (\rho(t) \operatorname{sgn}(B_2^T S(t)) - w(t)) \\ &\leq -\varepsilon \|B_2^T S(t)\| < 0\end{aligned}\quad (36)$$

which implies that the trajectories of (7) can be driven onto the predefined sliding surface in a finite time by the control law (31) and be maintained there for all subsequent time. Thus, the proof is completed. $\square$

4. Numerical Example

In this section, a simulation example is used to demonstrate the effectiveness of our proposed theoretical results. Consider the following time-delay system (1) with

$$A = \begin{bmatrix} -4 & 1.5 \\ 4 & -2 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.7 & 0.8 \\ 0.6 & 0.9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (37)$$

and $w(t) = \sin(t)$, the time delay in system (1) is selected as $h = 0.8, \mu = 0.2$. It is easy to check that the above system with $u(t) = 0$ is unstable, and the states of the open-loop system are shown in Figure 1 with the initial condition given by $\phi(t) = [-3 \ 2]^T (t \in [-h \ 0])$. Therefore, we aim to design a sliding mode controller $u(t)$ such that the closed-loop system is asymptotically stable. To check the stability of

FIGURE 1: States of the open-loop system.

FIGURE 2: States of the closed-loop system.

(9), we solve conditions (23)–(25) in Theorem 8, which gives that

$$C = 0.0384. \quad (38)$$

According to (27), we have

$$S(t) = [0.0384 \ 1] z(t). \quad (39)$$

The existence of a feasible solution shows that there exists a delay-dependent LPK functional for checking the stability of the sliding mode dynamics in (9); that is, we can find a desired sliding surface in (39) such that the corresponding sliding mode dynamics in (9) is asymptotically stable. The remaining task is to design a sliding mode controller such that the system trajectories can be driven onto the predefined sliding surface in (39) and maintained there for all subsequent time.

FIGURE 3: Sliding surface function.

FIGURE 4: Control input.

According to Theorem 9, the sliding mode controller in (31) can be designed as

$$u(t) = - \left[\begin{bmatrix} 1.4232 \\ -3.8464 \end{bmatrix} z(t) + \begin{bmatrix} 0.8346 \\ 0.7230 \end{bmatrix} z(t-d(t)) \right] - (\rho(t) + \varepsilon) \times \text{sgn}(s(t)). \quad (40)$$

Set $\varepsilon = 2$, $\rho(t) = 1$. To prevent the control signals from chattering, we replace $\text{sgn}(B_2^T S(t))$ with $B_2^T S(t) / (\|B_2^T S(t)\| + 0.01)$.

The simulation results of the closed-loop system with (40) are illustrated in Figures 2–4 with the initial condition $\phi(t) = [-3 \ 2]^T$ ($t \in [-h \ 0]$). Among them, Figure 2 shows the states responses of system (1) under the presented SMC law. The sliding surface $S(t)$ and control input signal $u(t)$ are given in Figures 3 and 4, respectively. From the simulation,

it is shown that the designed SMC scheme in this paper is feasible and effective since the resulting closed-loop system is stable.

5. Conclusion

The problem of sliding mode control for time-delay systems has been studied in this paper. The main contribution of this paper lies in that a novel Lyapunov-Krasovskii functional that reflects the delay-fractioning nature is constructed in order to reduce the possible conservatism introduced by the time delays. It should be pointed out that the delay partitioning technique of this paper is different from others in the literature, and one of the advantages of the method is that it is not necessary to represent the time delay model with two parts, the constant part and the time varying part. Firstly, by constructing a novel Lyapunov-Krasovskii functional based on the delay partitioning approach, a sufficient condition is given to guarantee the asymptotic stability of the sliding mode dynamics. Furthermore, a sliding mode control law is proposed to ensure the reachability of the system's trajectories to the predefined sliding surface. Finally, a numerical example has been provided to demonstrate the effectiveness of the proposed methods.

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