Research Article

Improving Delay-Range-Dependent Stability Condition for Systems with Interval Time-Varying Delay

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This paper discusses the delay-range-dependent stability for systems with interval time-varying delay. Through defining the new Lyapunov-Krasovskii functional and estimating the derivative of the LKF by introducing new vectors, using free matrices and reciprocally convex approach, the new delay-range-dependent stability conditions are obtained. Two well-known examples are given to illustrate the less conservatism of the proposed theoretical results.

1. Introduction

It is well known that time-varying delays are frequently encountered in many practical control systems, and they are usually regarded as a source of instability and poor performance. So the stability issue of time delay systems has received considerable attention [1–16]. In the last years, in order to further reduce conservatism of the stability results, some methods were developed, such as the delay-fraction approach in [1, 2], free weighting matrices method in [3, 4], the convex analysis method in [5, 6], the reciprocally convex approach in [7], LKF constructing method with matrices that depend on the time delays [8], LKF constructing method with triple-integral terms in [9], LKF constructing method with quadruple-integral terms in [10], and simple LKF having quadratic terms multiplied by a higher degree scalar function [11, 12]. These methods reduced the conservatism of the stability results. But when delay is interval time-varying, the information of the delay derivative is not full used, which causes the conservatism of the stability results.

Motivated by recent methods, in this paper, we further discuss the stability of linear systems with interval time-varying delay. Firstly, a novel LKF is introduced. Then, by introducing new vectors, using free matrices and reciprocally convex approach, the derivative of LKF is estimated less conservatively, and as a result, the stability criterion is

obtained in terms of LMI. Finally, two examples are given to illustrate the effectiveness of the proposed method.

Throughout the note, the used notations are standard. $\mathbf{R}^n$ denotes the n -dimensional Euclidean space, $\mathbf{R}^{n \times m}$ is a set of $n \times m$ real matrix, A^T is the transpose of A , $P > 0$ ($P < 0$) means symmetric positive (negative) definite matrix, and $*$ in the matrix denotes the symmetric element; I is the identity matrix of appropriate dimensions, $x_t = x(t + \theta)$, $\theta \in [-h, 0]$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulations

Consider the following time-delay system:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_1x(t-h(t)), \\ x(t) &= \phi(t), \quad t \in [-h, 0],\end{aligned}\tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, the initial condition $\phi(t)$ is a continuously differentiable vector-valued function, $A, A_1 \in \mathbf{R}^{n \times n}$ are known real constant matrices, and $h(t)$ is the time-varying delay satisfying

$$h_1 \leq h(t) \leq h_2, \quad \dot{h}(t) \leq \mu \leq \infty,\tag{2}$$

where h_1, h_2 , and μ are constants.

To obtain the main results, the following lemmas are needed.

Lemma 1 (see [9]). For any symmetric matrix $Z = Z^T \geq 0$, scalars $h_2 > h_1 > 0$ such that the integration is well defined; then

$$\begin{aligned} & - (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^t x^T(s) Z x(s) ds d\theta \\ & \leq - \int_{-h_2}^{-h_1} \int_{t+\theta}^t x^T(s) ds d\theta Z \int_{-h_2}^{-h_1} \int_{t+\theta}^t x(s) ds d\theta. \end{aligned} \quad (3)$$

Lemma 2 (see [7]). Let $f_1, f_2, \dots, f_N : \mathbf{R}^m \rightarrow \mathbf{R}$ have positive values in an open subset $\mathbf{D}$ of $\mathbf{R}^m$. Then, the reciprocally convex combination of f_i over $\mathbf{D}$ satisfies

$$\begin{aligned} & \min_{\{\alpha_i | \alpha_i > 0, \sum \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{ij}(t)} \sum_{i \neq j} g_{ij}(t) \\ & \text{subject to } g_{i,j} : \mathbf{R}^m \rightarrow \mathbf{R}, \quad g_{ij}(t) = g_{ji}(t), \quad (4) \\ & \begin{bmatrix} f_i(t) & g_{ij}(t) \\ g_{ij}(t) & f_j(t) \end{bmatrix} \geq 0. \end{aligned}$$

Lemma 3 (see [16]). The symmetric appropriately dimensional matrices $\Theta > 0, \Xi, Y$, if $\Xi - Y\Theta Y^T < 0$ hold; then there exists a matrix of appropriate dimension Π such that the following LMI holds:

$$\begin{bmatrix} \Xi + Y\Pi^T + \Pi Y^T & \Pi \\ * & -\Theta \end{bmatrix} < 0. \quad (5)$$

3. Main Results

In this section, the stability of system (1) is investigated. Through constructing a novel LKF and estimating the derivative of it, new stability condition is provided.

Firstly, define the following vector as

$$\begin{aligned} \zeta^T(t) &= \begin{bmatrix} x^T(t) & \int_{t-h_1}^t x^T(s) ds \end{bmatrix}, \\ \xi^T(t) &= \begin{bmatrix} \dot{x}^T(t) & x^T(t) \end{bmatrix}, \end{aligned} \quad (6)$$

and then construct the L-K functional as follows:

$$V(x_t) = \sum_{i=1}^4 V_i(x_t), \quad (7)$$

where

$$\begin{aligned} V_1(x_t) &= \zeta^T(t) P \zeta(t), \\ V_2(x_t) &= \int_{t-h_1}^t x^T(s) Q_1 x(s) ds + \int_{t-h(t)}^{t-h_1} x^T(s) Q_2 x(s) ds \\ & \quad + \int_{t-h_2}^{t-h(t)} x^T(s) Q_3 x(s) ds, \\ V_3(x_t) &= h_1 \int_{-h_1}^0 \int_{t+\theta}^t \xi^T(s) \begin{bmatrix} R_1 & 0 \\ * & R_2 \end{bmatrix} \xi(s) ds d\theta \\ & \quad + (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^t \xi^T(s) \begin{bmatrix} X_1 & 0 \\ * & X_2 \end{bmatrix} \xi(s) ds d\theta, \\ V_4(x_t) &= \frac{h_1^2}{2} \int_{-h_1}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) U_1 \dot{x}(s) ds d\lambda d\theta \\ & \quad + \frac{h_2^2 - h_1^2}{2} \int_{-h_1}^{-h_2} \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) U_2 \dot{x}(s) ds d\lambda d\theta \\ & \quad + \frac{1}{2} \int_{t-h_2}^{t-h_1} \int_{\theta}^{t-h_1} \int_{\lambda}^t \dot{x}^T(s) U_3 \dot{x}(s) ds d\lambda d\theta, \end{aligned} \quad (8)$$

and matrices $Q_1, Q_2, Q_3, R_1, R_2, X_1, X_2, U_1, U_2, U_3 \in \mathbf{R}^{n \times n} > 0, P = [P_{ij}] (1 \leq i \leq j \leq 2) \in \mathbf{R}^{2n \times 2n} > 0$.

In order to make the proof process clear, the following notations are used:

$$\alpha(t) = h(t) - h_1, \quad \beta(t) = h_2 - h(t), \quad \varphi(t) = \frac{1}{\alpha(t)} \int_{t-h(t)}^{t-h_1} x^T(s) ds, \quad \phi(t) = \frac{1}{\beta(t)} \int_{t-h_2}^{t-h(t)} x^T(s) ds, \quad (9)$$

$$\chi^T(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & x^T(t-h_1) & x^T(t-h_2) & \int_{t-h_1}^t x^T(s) & \varphi^T(t) & \phi^T(t) & \dot{x}^T(t) \end{bmatrix}$$

and block entry matrices $e_i (i = 1, 2, \dots, 5)$ (e.g., $e_2^T = [0 \ I \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$).

Then taking the time derivatives of $V_i(x_t)$ along the trajectory of system (1) yields

$$\dot{V}_1(x_t) = 2\zeta^T(t) P \dot{\zeta}(t) = 2\chi^T(t) [e_1 \ e_5] P [e_8 \ e_1 - e_3]^T \chi(t),$$

$$\dot{V}_2(x_t) = x^T(t) Q_1 x(t) - x^T(t-h_1) (Q_1 - Q_2) x(t-h_1)$$

$$\begin{aligned} & - (1 - \dot{h}(t)) x^T(t-h(t)) (Q_2 - Q_3) x(t-h(t)) \\ & \quad - x^T(t-h_2) Q_3 x(t-h_2) \\ & \leq \chi^T(t) [e_1 Q_1 e_1^T - e_3 (Q_1 - Q_2) e_3^T \\ & \quad - (1 - \mu) e_2 (Q_2 - Q_3) e_2^T - e_4 Q_3 e_4^T] \chi(t), \end{aligned} \quad (10)$$

where $Q_2 - Q_3 > 0$.

To reduce the conservatism of the main result, by using the Jensen inequality and Lemma 2 with introducing appropriate matrices $Y_1, Y_2 \in \mathbf{R}^{n \times n}$, we can calculate $\dot{V}_3(x_t)$ as

$$\begin{aligned}
 \dot{V}_3(x_t) &= h_1^2 \xi^T(t) \begin{bmatrix} R_1 & 0 \\ * & R_2 \end{bmatrix} \xi(t) \\
 &\quad + (h_2 - h_1)^2 \xi^T(t) \begin{bmatrix} X_1 & 0 \\ * & X_2 \end{bmatrix} \xi(t) \\
 &\quad - h_1 \int_{t-h_1}^t \xi^T(s) \begin{bmatrix} R_1 & 0 \\ * & R_2 \end{bmatrix} \xi(s) ds \\
 &\quad - (h_2 - h_1) \int_{t-h_2}^{t-h_1} \xi^T(s) \begin{bmatrix} X_1 & 0 \\ * & X_2 \end{bmatrix} \xi(s) ds \\
 &\leq \chi^T(t) \{e_8 [h_1^2 R_1 + (h_2 - h_1)^2 X_1] e_8^T \\
 &\quad + e_1 [h_1^2 R_2 + (h_2 - h_1)^2 X_2] e_1^T \\
 &\quad - (e_1 - e_3) R_1 (e_1 - e_3)^T - e_5 R_2 e_5^T\} \chi(t) \\
 &\quad - \chi^T(t) [e_2 - e_3 \quad e_4 - e_2] \\
 &\quad \times \begin{bmatrix} X_1 & Y_1 \\ * & X_1 \end{bmatrix} [e_2 - e_3 \quad e_4 - e_2]^T \chi(t) - \Gamma_1,
 \end{aligned} \tag{11}$$

where by using Lemma 2 with matrix Y_1 , one can get

$$\begin{aligned}
 &- (h_2 - h_1) \int_{t-h_2}^{t-h_1} \dot{x}^T(s) X_1 \dot{x}(s) ds \\
 &= - (h_2 - h_1) \int_{t-h(t)}^{t-h_1} \dot{x}^T(s) X_1 \dot{x}(s) ds \\
 &- (h_2 - h_1) \int_{t-h_2}^{t-h(t)} \dot{x}^T(s) X_1 \dot{x}(s) ds \\
 &\leq - \begin{bmatrix} x(t-h(t)) - x(t-h_1) \\ x(t-h_2) - x(t-h(t)) \end{bmatrix}^T \\
 &\quad \times \begin{bmatrix} X_1 & Y_1 \\ * & X_1 \end{bmatrix} \begin{bmatrix} x(t-h(t)) - x(t-h_1) \\ x(t-h_2) - x(t-h(t)) \end{bmatrix} \\
 &= - \chi^T(t) [e_2 - e_3 \quad e_4 - e_2] \begin{bmatrix} X_1 & Y_1 \\ * & X_1 \end{bmatrix} \\
 &\quad \times [e_2 - e_3 \quad e_4 - e_2]^T \chi(t),
 \end{aligned} \tag{12}$$

and by using Lemma 2 with matrix Y_2 , one can get

$$\begin{aligned}
 &- (h_2 - h_1) \int_{t-h_2}^{t-h_1} x^T(s) X_2 x(s) ds \\
 &= - (h_2 - h_1) \int_{t-h(t)}^{t-h_1} x^T(s) X_2 x(s) ds \\
 &- (h_2 - h_1) \int_{t-h_2}^{t-h(t)} x^T(s) X_2 x(s) ds
 \end{aligned}$$

$$\begin{aligned}
 &\leq - \begin{bmatrix} \int_{t-h(t)}^{t-h_1} x(s) ds \\ \int_{t-h_2}^{t-h(t)} x(s) ds \end{bmatrix}^T \\
 &\quad \times \begin{bmatrix} X_2 & Y_2 \\ * & X_2 \end{bmatrix} \begin{bmatrix} \int_{t-h(t)}^{t-h_1} x(s) ds \\ \int_{t-h_2}^{t-h(t)} x(s) ds \end{bmatrix} = \Gamma_1.
 \end{aligned} \tag{13}$$

To the time-derivative of $V_4(x_t)$, it can be calculated as

$$\begin{aligned}
 \dot{V}_4(x_t) &= \frac{h_1^4}{4} \dot{x}^T(t) U_1 \dot{x}(t) + \frac{(h_2^2 - h_1^2)^2}{4} \dot{x}^T(t) U_2 \dot{x}(t) \\
 &\quad + \frac{(h_2 - h_1)^2}{4} \dot{x}^T(t) U_3 \dot{x}(t) \\
 &\quad - \frac{h_1^2}{2} \int_{-h_1}^0 \int_{t+\theta}^t \dot{x}^T(s) U_1 \dot{x}(s) ds d\theta \\
 &\quad - \frac{h_2^2 - h_1^2}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^t \dot{x}^T(s) U_2 \dot{x}(s) ds d\theta \\
 &\quad - \frac{1}{2} \int_{t-h_2}^{t-h_1} \int_{\theta}^{t-h_1} \dot{x}^T(s) U_3 \dot{x}(s) ds d\theta \\
 &= \chi^T(t) e_8 \left[\frac{h_1^2}{2} U_1 + \frac{(h_2^2 - h_1^2)^2}{4} U_2 \right. \\
 &\quad \left. + \frac{(h_2 - h_1)^2}{4} U_3 \right] e_8^T \chi(t) \\
 &\quad - \chi^T(t) (h_1 e_1 - e_5) U_1 (h_1 e_1 - e_5)^T \chi(t) \\
 &\quad - \chi^T(t) [(e_3 - e_6)^T U_3 (e_3 - e_6) \\
 &\quad - (e_2 - e_7)^T U_3 (e_2 - e_7)] \chi(t) - \Gamma_2,
 \end{aligned} \tag{14}$$

where by using Lemma 1, we can get

$$\begin{aligned}
 &- \frac{h_1^2}{2} \int_{-h_1}^0 \int_{t+\theta}^t \dot{x}^T(s) U_1 \dot{x}(s) ds d\theta \\
 &\leq - \left[h_1 x(t) - \int_{t-h_1}^t x(s) ds \right]^T \\
 &\quad \times U_1 \left[h_1 x(t) - \int_{t-h_1}^t x(s) ds \right] \\
 &= - \chi^T(t) (h_1 e_1 - e_5) U_1 (h_1 e_1 - e_5)^T \chi(t), \\
 &- \frac{1}{2} \int_{t-h_2}^{t-h_1} \int_{\theta}^{t-h_1} \dot{x}^T(s) U_3 \dot{x}(s) ds d\theta
 \end{aligned}$$

$$\begin{aligned}
&\leq -\frac{1}{2} \int_{t-h(t)}^{t-h_1} \int_{\theta}^{t-h_1} \dot{x}^T(s) U_3 \dot{x}(s) ds d\theta \\
&\quad - \frac{1}{2} \int_{t-h_2}^{t-h(t)} \int_{\theta}^{t-h(t)} \dot{x}^T(s) U_3 \dot{x}(s) ds d\theta \\
&\leq -\left[(h(t) - h_1) x(t - h_1) - \int_{t-h(t)}^{t-h_1} x(s) ds \right]^T \\
&\quad \times \frac{U_3}{\alpha^2(t)} \left[(h(t) - h_1) x(t - h_1) - \int_{t-h(t)}^{t-h_1} x(s) ds \right] \\
&\quad - \left[(h_2 - h(t)) x(t - h(t)) - \int_{t-h_2}^{t-h(t)} x(s) ds \right]^T \\
&\quad \times \frac{U_3}{\beta^2(t)} \left[(h_2 - h(t)) x(t - h(t)) - \int_{t-h_2}^{t-h(t)} x(s) ds \right] \\
&= -\chi^T(t) \left[(e_3 - e_6)^T U_3 (e_3 - e_6) \right. \\
&\quad \left. - (e_2 - e_7)^T U_3 (e_2 - e_7) \right] \chi(t).
\end{aligned} \tag{15}$$

Similar to the method in (12)-(13), by introducing matrix $Y_3 \in \mathbf{R}^{n \times n}$ and using Lemma 2, we can estimate the following:

$$\begin{aligned}
&-\frac{h_2^2 - h_1^2}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^t \dot{x}^T(s) U_2 \dot{x}(s) ds d\theta \\
&= -\frac{h_2^2 - h_1^2}{2} \int_{-h(t)}^{-h_1} \int_{t+\theta}^t \dot{x}^T(s) U_2 \dot{x}(s) ds d\theta \\
&\quad - \frac{h_2^2 - h_1^2}{2} \int_{-h_2}^{-h(t)} \int_{t+\theta}^t \dot{x}^T(s) U_2 \dot{x}(s) ds d\theta \\
&\leq -\left[\begin{array}{c} (h(t) - h_1)x(t) - \int_{t-h(t)}^{t-h_1} x(s) ds \\ (h_2 - h(t))x(t) - \int_{t-h_2}^{t-h(t)} x(s) ds \end{array} \right]^T \\
&\quad \times \begin{bmatrix} U_2 & Y_3 \\ * & U_2 \end{bmatrix} \left[\begin{array}{c} (h(t) - h_1)x(t) - \int_{t-h(t)}^{t-h_1} x(s) ds \\ (h_2 - h(t))x(t) - \int_{t-h_2}^{t-h(t)} x(s) ds \end{array} \right] = \Gamma_2.
\end{aligned} \tag{16}$$

In the last, in order to obtain the stability result base on LMI, we can deal with Γ_1, Γ_2 as follows:

$$\begin{aligned}
\Gamma_1 + \Gamma_2 = &-\chi^T(t) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \alpha(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta(t) & 0 \end{bmatrix}^T \\
&\begin{bmatrix} X_2 & Y_2 \\ * & X_2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \alpha(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta(t) & 0 \end{bmatrix} \chi(t)
\end{aligned}$$

$$\begin{aligned}
&-\chi^T(t) \begin{bmatrix} \alpha(t) & 0 & 0 & 0 & 0 & -\alpha(t) & 0 & 0 \\ \beta(t) & 0 & 0 & 0 & 0 & 0 & -\beta(t) & 0 \end{bmatrix}^T \\
&\times \begin{bmatrix} U_2 & Y_3 \\ * & U_2 \end{bmatrix} \\
&\times \begin{bmatrix} \alpha(t) & 0 & 0 & 0 & 0 & -\alpha(t) & 0 & 0 \\ \beta(t) & 0 & 0 & 0 & 0 & 0 & -\beta(t) & 0 \end{bmatrix} \chi(t) \\
&= -\chi^T(t) Y(t) \Theta Y^T(t) \chi(t),
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
Y^T(t) &= \begin{bmatrix} \alpha(t) & 0 & 0 & 0 & 0 & -\alpha(t) & 0 & 0 \\ \beta(t) & 0 & 0 & 0 & 0 & 0 & -\beta(t) & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta(t) & 0 \end{bmatrix}, \\
\Theta &= \begin{bmatrix} U_2 & Y_3 & 0 & 0 \\ * & U_2 & 0 & 0 \\ 0 & 0 & X_2 & Y_2 \\ 0 & 0 & * & X_2 \end{bmatrix}.
\end{aligned} \tag{18}$$

Also, for appropriate matrices N_1, N_2 , one can have

$$\begin{aligned}
&2 \left[x^T(t) N_1 + \dot{x}^T(t) N_2 \right] \\
&\times [Ax(t) + A_1 x(t - h(t)) - \dot{x}(t)] = 0.
\end{aligned} \tag{19}$$

Therefore, combining (10)–(19), we can obtain

$$\dot{V}(x_t) = \sum_{i=1}^4 \dot{V}_i(x_t) \leq \chi^T(t) [\Xi - Y(t) \Theta Y^T(t)] \chi(t), \tag{20}$$

where

$$\begin{aligned}
\Xi = &[e_1 \ e_5] P [e_8 \ e_1 - e_3]^T + [e_8 \ e_1 - e_3] P [e_1 \ e_5]^T \\
&- e_3 (Q_1 - Q_2) e_3^T - (1 - \mu) e_2 (Q_2 - Q_3) e_2^T - e_4 Q_3 e_4^T \\
&+ e_8 \left[h_1^2 R_1 + (h_2 - h_1)^2 X_1 + \frac{h_1^2}{2} U_1 \right. \\
&\quad \left. + \frac{(h_2^2 - h_1^2)^2}{4} U_2 + \frac{(h_2 - h_1)^2}{4} U_3 \right] e_8^T \\
&+ e_1 [Q_1 + h_1^2 R_2 + (h_2 - h_1)^2 X_2] e_1^T \\
&- (e_1 - e_3) R_1 (e_1 - e_3)^T - e_5 R_2 e_5^T \\
&- [e_2 - e_3 \ e_4 - e_2] \begin{bmatrix} X_1 & Y_1 \\ * & X_1 \end{bmatrix} [e_2 - e_3 \ e_4 - e_2]^T \\
&- (h_1 e_1 - e_5) U_1 (h_1 e_1 - e_5)^T \\
&- (e_3 - e_6)^T U_3 (e_3 - e_6) - (e_2 - e_7)^T U_3 (e_2 - e_7) \\
&+ (e_1 N_1 + e_8 N_2) (e_1^T A + e_2^T A_1 - e_8)^T \\
&+ (e_1^T A + e_2^T A_1 - e_8) (e_1 N_1 + e_8 N_2)^T.
\end{aligned} \tag{21}$$

As we know, if $\Xi - \Upsilon(t)\Theta Y^T(t) < 0$ holds, then $\dot{V}(x_t) < 0$, which means that system (1) is asymptotically stable. So, by Lemma 3, there exists a matrix of appropriate dimension Π such that the following LMI holds:

$$\begin{bmatrix} \Xi + \Upsilon \Pi^T + \Pi Y^T & \Pi \\ * & -\Theta \end{bmatrix} < 0. \quad (22)$$

So, we give the main theorem of this paper as follows.

Theorem 4. *The time-delayed system (1) having constrains (2) is asymptotically stable, if there exist matrices $Q_1, Q_2, Q_3, R_1, R_2, X_1, X_2, U_1, U_2, U_3 \in \mathbf{R}^{n \times n} > 0$, matrix $P \in \mathbf{R}^{2n \times 2n} > 0$, and appropriate matrices $Y_1, Y_2, Y_3, N_1, N_2, \Pi$ such that the following LMIs hold:*

$$\begin{aligned} Q_2 - Q_3 &> 0, \\ \begin{bmatrix} \Xi + Y_1 \Pi^T + \Pi Y_1^T & \Pi \\ * & -\Theta \end{bmatrix} &< 0, \\ \begin{bmatrix} \Xi + Y_2 \Pi^T + \Pi Y_2^T & \Pi \\ * & -\Theta \end{bmatrix} &< 0, \end{aligned} \quad (23)$$

where Ξ, Y, Π , and Θ are defined in (18) and (21).

Proof. From above, one can see that $\Xi + \Upsilon \Pi^T + \Pi Y^T$ is a convex combination of matrix Π as

$$\begin{aligned} \Xi + \Upsilon \Pi^T + \Pi Y^T &= \Xi + (h(t) - h_1)(Y_1 \Pi^T + \Pi Y_1^T) \\ &\quad + (h_2 - h(t))(Y_2 \Pi^T + \Pi Y_2^T) < 0. \end{aligned} \quad (24)$$

So (22) can be handled nonconservatively by the two corresponding boundary LMIs:

$$\begin{aligned} \begin{bmatrix} \Xi + Y_1 \Pi^T + \Pi Y_1^T & \Pi \\ * & -\Theta \end{bmatrix} &< 0, \\ \begin{bmatrix} \Xi + Y_2 \Pi^T + \Pi Y_2^T & \Pi \\ * & -\Theta \end{bmatrix} &< 0, \end{aligned} \quad (25)$$

where $Y_1 = Y(h(t) - h_1 = 0)$, $Y_2 = Y(h_2 - h(t) = 0)$. This completes the proof. $\square$

Remark 5. From (17) and (22), it can be seen that, in order to obtain the less conservative stability condition in terms of LMI, $\alpha(t), \beta(t)$ are used, but free matrices $\Pi = [\Pi_{ij}]$ ($1 \leq i \leq j \leq 8$) are also introduced, which increases the computational burden; this is a disadvantage of the proposed method.

Remark 6. When $h_1 \leq h(t) \leq h_2$, $\mu_1 \leq \dot{h}(t) \leq \mu_2$, let $\Xi_1 = \Xi(\mu = \mu_1)$, $\Xi_2 = \Xi(\mu = \mu_2)$, and using the convex combination again with $\begin{bmatrix} \Xi_1 + Y_1 \Pi^T + \Pi Y_1^T & \Pi \\ * & -\Theta \end{bmatrix} < 0$, $\begin{bmatrix} \Xi_1 + Y_2 \Pi^T + \Pi Y_2^T & \Pi \\ * & -\Theta \end{bmatrix} < 0$, $\begin{bmatrix} \Xi_2 + Y_1 \Pi^T + \Pi Y_1^T & \Pi \\ * & -\Theta \end{bmatrix} < 0$, $\begin{bmatrix} \Xi_2 + Y_2 \Pi^T + \Pi Y_2^T & \Pi \\ * & -\Theta \end{bmatrix} < 0$, the corresponding stability criterion can also be obtained.

4. Numerical Examples

In this section, the effectiveness of the obtained results in this paper is shown by the two well-known numerical examples.

TABLE 1: Admissible upper bound h_2 for various h_1 and μ in Example 1.

h_1	Methods	$\mu = 0.3$	$\mu = 0.5$	$\mu = 0.9$
2	Shao [6]	2.69	2.50	2.50
	Sun et al. [9]	2.91	2.50	2.50
	Theorem 4	3.00	2.64	2.64
3	Shao [6]	3.25	3.25	3.25
	Sun et al. [9]	3.34	3.34	3.34
	Theorem 4	3.35	3.35	3.35

TABLE 2: Admissible upper bound h_2 for various h_1 and $\mu = 0.3$ in Example 1.

Methods	h_1	0.3	0.5	0.8	1
Shao [6]	h_2	2.22	2.22	2.23	2.24
Sun et al. [9]	h_2	2.26	2.28	2.30	2.30
Theorem 4	h_2	2.39	2.41	2.42	2.43

Example 1. Consider the linear system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \quad (26)$$

The purpose is to compare the admissible upper bounds h_2 given lower bound h_1 and μ .

For $h_1 = 0$, $\mu = 0.5, 0.9$, in [14], the result is $h_2 = 2.04, 1.37$, in [17], the result is $h_2 = 2.33, 1.87$, and our results are $h_2 = 2.33, 1.87$.

For $h_1 = 1$, $\mu = 0.5, 0.9$, in [14], the result is $h_2 = 2.07, 1.74$, in [17], the result is $h_2 = 2.33, 2.07$, and our results are $h_2 = 2.43, 2.08$.

For $h_1 = 2$, $\mu = 0.5, 0.9$, in [14], the result is $h_2 = 2.43, 2.43$, in [17], the result is $h_2 = 2.61, 2.61$, and our results are $h_2 = 2.64, 2.64$.

For various h_1, μ , the maximum upper bounds on delay h_2 by different methods are also listed in Table 1. It can be seen that the result obtained in this paper is less conservative.

Example 2. Consider the linear system (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}. \quad (27)$$

The purpose is to compare the admissible upper bounds h_2 which guarantee the asymptotic stability of the above system for given lower bound h_1 and μ . This example is used in many recent papers, such as [6, 9], For $\mu = 0.3$, Table 2 lists the comparison of our results with some recent ones, and it is easy to see that the results obtained in this paper are less conservative.

5. Conclusions

In this note, the stability of interval time-varying delay systems has been discussed. Through constructing a novel LKF and using some new analysis methods, the delay-range-dependent stability criteria were derived. Compared with

some previous stability conditions, the obtained main results in this paper have less conservatism. In the end, numerical examples were given to show the superiority of the obtained criteria and their improvements over the existing results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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