Research Article

Analytical Solution of Flow and Heat Transfer over a Permeable Stretching Wall in a Porous Medium

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Boundary layer flow through a porous medium over a stretching porous wall has been solved with analytical solution. It has been considered two wall boundary conditions which are power-law distribution of either wall temperature or heat flux. These are general enough to cover the isothermal and isoflux cases. In addition to momentum, both first and second laws of thermodynamics analyses of the problem are investigated. The governing equations are transformed into a system of ordinary differential equations. The transformed ordinary equations are solved analytically using homotopy analysis method. A comprehensive parametric study is presented, and it is shown that the rate of heat transfer increases with Reynolds number, Prandtl number, and suction to the surface.

1. Introduction

Boundary layer flows over a stretching surface have application in engineering processes such as liquid composite molding, extrusion of plastic sheets, paper production, glass blowing, metal spinning, wire drawing, and hot rolling [1–3]. More importantly, the quality of the products, in the abovementioned processes, depends on the kinematics of stretching and the simultaneous heat and mass transfer rates during the fabrication process. Sakiadis [4, 5] and Crane [6] were the pioneers in the investigations of boundary layer flow over continuously moving surfaces that are quite different from the free stream flow over stationary flat plates. Pop and Na [7] studied free convection heat transfer of non-Newtonian fluids along a vertical wavy surface in a porous medium.

The flow field of a stretching surface with a power-law velocity variation was discussed by Banks [8]. Elbashbeshy [9] investigated heat transfer over a stretching surface with variable surface heat flux. Elbashbeshy and Bazid [10] analyzed heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection. This work was extended by Cortell [11] to include power-law temperature distribution. Steady flow and heat transfer of

a viscous incompressible fluid flow through porous medium over a stretching sheet were studied by Sriramalu et al. [12]. Ali [13] investigated thermal boundary layer on a power-law stretched surface with suction or injection for uniform and variable surface temperatures. Recently, Pantokratoras [14] published analytical solution for velocity distribution inside a Darcy-Brinkman porous medium with a stretching boundary. Elbashbeshy [15] included thermal radiation effects in a stretching surface problem. Tamayol and Bahrani [16] understood that porous materials can be used to enhance the heat transfer rate from stretching surfaces to improve processes such as hot rolling and composite fabrication.

In view of the above, an analytical solution is developed in the present study to solve fluid flow, heat transfer in a porous medium over a porous plate with linear velocity, subjected to different power-law thermal boundary conditions. The analytical solution is successfully validated in comparison with numerical analysis. There are many effective methods for obtaining the solutions of nonlinear equation such as variational iteration method [17], Adomian method [18], and homotopy perturbation method [19, 20], and we use one of a powerful technique, namely, the homotopy analysis method (HAM) which was expressed by Liao [21–24]. This method has been successfully applied to solve many types

of nonlinear problems [25–29] and provides us with great freedom to express solutions of a given nonlinear problem by means of different base functions. Secondly, unlike all previous analytic techniques, the homotopy analysis method always provides us with a family of solution expressions in the auxiliary parameter $\hbar$, even if a nonlinear problem has a unique solution. Thirdly, unlike perturbation techniques, the homotopy analysis method is independent of any small or large quantities. So, the homotopy analysis method can be applied no matter if governing equations and boundary/initial conditions of a given nonlinear problem contain small or large quantities or not.

Above all, there are no rigorous theories to direct us to choose the initial approximations, auxiliary linear operators, auxiliary functions, and auxiliary parameter $\hbar$. From the practical viewpoints, there are some fundamental rules such as the rule of solution expression, the rule of coefficient ergodicity, and the rule of solution existence, which play important roles within the homotopy analysis method. Unfortunately, the rule of solution expression implies such an assumption that we should have, more or less, some knowledge about a given nonlinear problem a priori. So, theoretically, this assumption impairs the homotopy analysis method, although we can always attempt some base functions even if a given nonlinear problem is completely new for us.

2. Governing Equation

Consider a steady, constant property, two-dimensional flow through a homogenous porous medium of permeability K , over a stretching surface with linear velocity distribution; that is, $u_w = u_0 x/L$ (Figure 1). The transport properties of the medium can be assumed independent of temperature when the difference between wall and ambient temperatures is not significant [4]. The origin is kept fixed, while the wall is stretching, and the y -axis is perpendicular to the surface. The governing equations are [4, 5]

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\ \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= \mu_{\text{eff}} \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K'} u, \\ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha_{\text{eff}} \frac{\partial^2 T}{\partial y^2}, \end{aligned} \quad (1)$$

where u and v are velocity components in the x and y directions, respectively. μ_{eff} is the effective viscosity which for simplicity in the present study is considered to be identical to the dynamic viscosity, μ . The transport properties of the porous medium such as permeability depend on their microstructure and can be calculated either using existing correlations in the literature or through experimental measurements. α_{eff} is the effective thermal diffusivity of the medium. The hydrodynamic boundary conditions are

$$u(x^*, 0) = u_0 x^*, \quad v(x^*, 0) = v_w, \quad u(x^*, \infty) = 0, \quad (2)$$

FIGURE 1: Schematic diagram of problem.

where $x^* = x/L$ is the nondimensional x -coordinate and L is the length of the porous plate. The following thermal boundary conditions are considered:

$$\begin{aligned} T(x^*, 0) &= T_\infty + T_0(x^*)^n, & T(x^*, \infty) &= T_\infty, \\ -\kappa \frac{\partial T}{\partial y} \Big|_{(x^*, 0)} &= q_0(x^*)^n, & T(x^*, \infty) &= T_\infty, \end{aligned} \quad (3)$$

where κ is the effective thermal conductivity of the medium and is a function of thermal conductivities of the fluid and solid phases and the porous medium microstructure.

Using dimensionless parameters,

$$\begin{aligned} \eta &= \frac{y}{\sqrt{K}}, \\ u &= u_0 x^* f'(\eta), \\ v &= -\frac{u_0}{L} \sqrt{K} f(\eta), \\ \psi &= u_0 x^* \sqrt{K} f(\eta), \end{aligned} \quad (4)$$

where f' is $df/d\eta$.

The transformed nonlinear ordinary differential equations are

$$\begin{aligned} f''' + \text{Re}(ff'' - f'^2) - f' &= 0, \\ \theta'' + \text{Re} \cdot \text{Pr}(f\theta') - n \cdot \text{Re} \cdot \text{Pr}(f'\theta) &= 0, \end{aligned} \quad (5)$$

where $\text{Re} = \rho u_0 K / L \mu$ is the Reynolds number. Equation (5) should be solved subject to the following boundary conditions:

$$f'(0) = 1, \quad f(0) = -\frac{v_w L}{u_0 \sqrt{K}} = f_w, \quad f'(\infty) = 0, \quad (6)$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \quad \text{Power-law temperature}, \quad (7a)$$

$$\theta'(0) = -1, \quad \theta'(\infty) = 0 \quad \text{Power-law heat flux}, \quad (7b)$$

where f_w is the injection parameter. Positive/negative values of f_w show suction/injection into/from the porous surface, respectively. The wall shear stress term can be calculated as

$$\tau_L = \frac{-\mu u_0 L f''(0)}{2\sqrt{K}}. \quad (8)$$

For power-law fluid with constant temperature and heat flux boundary conditions, respectively. Employing the definition of convective heat transfer coefficient, the local Nusselt numbers become

$$\text{Nu}_x = \frac{hx}{k} = \begin{cases} -\frac{\theta'(0)x}{\sqrt{K}}, & \text{Power-law temperature,} \\ \frac{q_w x}{k(T_w - T_\infty)} = \frac{x}{\theta(0)\sqrt{K}}, & \text{Power-law heat flux.} \end{cases} \quad (9)$$

3. Solution of Problem by Homotopy Analysis Method (HAM)

As mentioned by Liao, a solution may be expressed with different base functions, among which some converge to the exact solution of the problem faster than others. Such base functions are obviously better suited for the final solution to be expressed in terms of. Noting these facts, we have decided to express $g(\eta)$ by a set of base functions of the following form:

$$\begin{aligned} f(\eta) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} b_{m,n}^k \eta^k \exp(-n\eta), \\ \theta(\eta) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} b_{m,n}^k \eta^k \exp(-n\eta). \end{aligned} \quad (10)$$

The rule of solution expression provides us with a starting point. It is under the rule of solution expression that initial approximations, auxiliary linear operators, and the auxiliary functions are determined. So, according to the rule of solution expression, we choose the initial guess and auxiliary linear operator in the following form:

$$f_0(\eta) = f_w + 1 - \exp(-\eta), \quad \theta_0(\eta) = \exp(-\eta). \quad (11)$$

We choose linear operator as follows:

$$\begin{aligned} \mathcal{L}_1(f) &= f''' + f'', & \mathcal{L}_2(\theta) &= \theta'' + \theta', \\ \mathcal{L}_1(c_1 + c_2\eta + c_3 \exp(-\eta)) &= 0, \\ \mathcal{L}_2(c_4 + c_5 \exp(-\eta)) &= 0, \end{aligned} \quad (12)$$

where c_i ($i = 1-5$) are constants. Let $P \in [0, 1]$ denote the embedding parameter, and let $\hbar$ indicate nonzero auxiliary parameters. Then, we construct the following equations.

3.1. Consider Zeroth-Order Deformation Equations.

$$\begin{aligned} (1-P)\mathcal{L}_1[f(\eta; p) - f_0(\eta)] &= p\hbar_1 H(\eta) N_1[f(\eta; p)], \\ f(0; p) &= f_w, \\ f'(0; p) &= 1, \quad f'(\infty; p) = 0, \end{aligned}$$

$$\begin{aligned} (1-P)\mathcal{L}_2[\theta(\eta; p) - \theta_0(\eta)] &= p\hbar_2 H(\eta) N_2[\theta(\eta; p)], \\ \theta(0; p) &= 1, \quad \theta(\infty; p) = 0 \quad \text{Power-law temperature,} \\ \theta'(0; p) &= -1, \quad \theta(\infty; p) = 0 \quad \text{Power-law heat flux,} \\ N_1[f(\eta; p), \theta(\eta; p)] &= \frac{\partial^3 f(\eta; p)}{\partial \eta^3} \\ &+ \text{Re} \left(f(\eta; p) \frac{\partial^2 f(\eta; p)}{\partial \eta^2} - \left(\frac{\partial f(\eta; p)}{\partial \eta} \right)^2 \right) \\ &- \frac{\partial f(\eta; p)}{\partial \eta}, \\ N_2[f(\eta; p), \theta(\eta; p)] &= \frac{\partial^2 \theta(\eta; p)}{\partial \eta^2} + \text{Re} \cdot \text{Pr} \left(f(\eta; p) \frac{\partial \theta(\eta; p)}{\partial \eta} \right) \\ &- n \cdot \text{Re} \cdot \text{Pr} \left(\theta(\eta; p) \frac{\partial f(\eta; p)}{\partial \eta} \right). \end{aligned} \quad (13)$$

For $p = 0$ and $p = 1$, we have

$$\begin{aligned} f(\eta; 0) &= f_0(\eta), & f(\eta; 1) &= f(\eta), \\ \theta(\eta; 0) &= \theta_0(\eta), & \theta(\eta; 1) &= \theta(\eta). \end{aligned} \quad (14)$$

When p increases from 0 to 1, then $f(\eta; p)$ and $\theta(\eta; p)$ vary from $f_0(\eta)$ and $\theta_0(\eta)$ to $f(\eta)$ and $\theta(\eta)$. By Taylor's theorem and using (14), we can write the following:

$$\begin{aligned} f(\eta; p) &= f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m, \\ f_m(\eta) &= \frac{1}{m!} \frac{\partial^m (f(\eta; p))}{\partial p^m}, \\ \theta(\eta; p) &= \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m, \\ \theta_m(\eta) &= \frac{1}{m!} \frac{\partial^m (\theta(\eta; p))}{\partial p^m}. \end{aligned} \quad (15)$$

In which $\hbar_1$ and $\hbar_2$ are chosen in such a way that these two series are convergent at $p = 1$; therefore we have the following through (15):

$$\begin{aligned} f(\eta) &= f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \\ \theta(\eta) &= \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta). \end{aligned} \quad (16)$$

3.2. Consider Mth-Order Deformation Equations.

$$\mathcal{L}_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 H(\eta) R_m^f(\eta), \quad (17)$$

$$f_m(0) = 0, \quad f'_m(0) = 0, \quad f'_m(\infty) = 0, \quad (18)$$

$$R_m^f(\eta) = f_{m-1}''' + \sum_{k=0}^{m-1} f_k f_{m-1-k}'' - \sum_{k=0}^{m-1} f'_k f'_{m-1-k} - K f'_{m-1}, \quad (19)$$

$$\mathcal{L}_2[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_2 H(\eta) R_m^\theta(\eta), \quad (20)$$

$$\theta_m(0) = 0, \quad \theta_m(\infty) = 0, \quad (21)$$

$$\theta'_m(0) = 0, \quad \theta'_m(\infty) = 0, \quad (22)$$

$$R_m^\theta(\eta) = \theta_{m-1}'' + \text{Re} \cdot \text{Pr} \left(\sum_{k=0}^{m-1} f_k \theta'_{m-1-k} \right) - n \cdot \text{Re} \cdot \text{Pr} \left(\sum_{k=0}^{m-1} \theta_k f'_{m-1-k} \right), \quad (23)$$

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases} \quad (24)$$

The general solutions of (17)–(23) are

$$\begin{aligned} f_m(\eta) - \chi_m f_{m-1}(\eta) &= f_m^*(\eta) + C_1^m + C_2^m \eta + C_3^m \exp(-\eta), \\ \theta_m(\eta) - \chi_m \theta_{m-1}(\eta) &= \theta_m^*(\eta) + C_4^m + C_5^m \exp(-\eta), \end{aligned} \quad (25)$$

where C_1^m to C_5^m are constants that can be obtained by applying the boundary condition in (18), (21), and (22).

As discussed by Liao, the rule of coefficient ergodicity and the rule of solution existence play important roles in determining the auxiliary function and ensuring that the high-order deformation equations are closed and have solutions. In many cases, by means of the rule of solution expression and the rule of coefficient ergodicity, auxiliary functions can be uniquely determined. So we define the auxiliary function $H(\eta)$ in the following form:

$$H(\eta) = \exp(-\eta). \quad (26)$$

4. Convergence of the HAM Solution

As was mentioned in introduction, the convergence and the rate of approximation of the HAM solution strongly depend on the values of auxiliary parameter $\hbar$. By means of the so-called $\hbar$ curves, it is easy to find out the so-called valid regions of $\hbar$ to gain a convergent solution series. According to Figures 2 and 3, the acceptable range of auxiliary parameter for $\text{Pr} = 1, n = 0, \text{Re} = 1$, and $f_w = 0$ is $-0.3 < \hbar_1 < -0.1$ and $-1.8 < \hbar_2 < -0.3$.

Figures 4 and 5 show how auxiliary parameters $\hbar_1$ and $\hbar_2$ vary with changing f_w . If f_w increases, the range of convergency of solution is restricted and then decreased.

FIGURE 2: The $\hbar_1$ validity for $\text{Pr} = 1, n = 0, \text{Re} = 1$, and $f_w = 0$.

FIGURE 3: The $\hbar_2$ validity for $\text{Pr} = 1, n = 0, \text{Re} = 1$, and $f_w = 0$.

5. Results and Discussion

In the present study, the Homotopy analysis method has been used to solve the nonlinear equations of heat transfer and fluid flow over a permeable stretching wall in a porous medium. The nondimensional numbers introduced in the present analysis are Reynolds number (Re), Prandtl number

TABLE 1: The results of HAM and NS for $f(\eta)$, $f'(\eta)$, and $\theta(\eta)$ when $f_w = 0$, $Pr = 1$, $n = 0$, and $Re = 1$.

η	$f(\eta)$		$f'(\eta)$		$\theta(\eta)$ (temperature)		$\theta(\eta)$ (heat flux)	
	HAM	NS	HAM	NS	HAM	NS	HAM	NS
0.0	0.00000	0.00000	0.99999	1.00000	0.999999	1.00000	1.9565	1.9615
0.2	0.17042	0.17420	0.75376	0.75363	0.89633	0.89996	1.7677	1.7627
0.4	0.30564	0.30549	0.56813	0.75363	0.80205	0.80327	1.5658	1.5706
0.6	0.40571	0.40443	0.42820	0.42804	0.71301	0.71225	1.3856	1.3898
0.8	0.47505	0.47900	0.32272	0.32259	0.62792	0.62826	1.2210	1.2229
1.0	0.53802	0.53519	0.24322	0.24311	0.55226	0.55191	1.0690	1.0712
1.2	0.58010	0.57754	0.18330	0.18322	0.48134	0.48329	0.9313	0.9340
1.4	0.61161	0.60946	0.13814	0.13808	0.42056	0.42214	0.8094	0.8134
1.6	0.64096	0.63352	0.10410	0.10406	0.36595	0.36800	0.6943	0.7058
1.8	0.65374	0.65165	0.07845	0.07842	0.31817	0.32029	0.6095	0.6110
2.0	0.66952	0.66531	0.05912	0.05910	0.27657	0.27841	0.5193	0.5278
2.2	0.67812	0.67560	0.04455	0.04454	0.23934	0.24175	0.4480	0.4549
2.4	0.68567	0.68336	0.03357	0.03357	0.20813	0.20971	0.3861	0.3913
2.6	0.69054	0.68921	0.02530	0.02529	0.17972	0.18177	0.3323	0.3358
2.8	0.69675	0.69362	0.01907	0.01906	0.15519	0.15743	0.2858	0.2874
3.0	0.69842	0.69694	0.01437	0.01436	0.13407	0.13623	0.2456	0.2453

FIGURE 4: The h_1 validity for various f_w , $Pr = 1$, $n = 0$, and $Re = 1$.

(Pr), and the injection number (f_w). Another important parameter is the power of the surface temperature/heat flux distribution (n), which is considered in the present study.

In order to ensure the convergence of the solution series, the variation of f has been plotted at different orders of approximation in Figure 6. The comparison between results of HAM and numerical solution (Runge-Kutta) has been displayed in Table 1. It can be observed that there is a good agreement between HAM method and numerical solution. Figures 7, 8, and 9 illustrate effect of variation of wall injection/suction parameter (f_w) on velocity and

FIGURE 5: The h_2 validity for various f_w , $Pr = 1$, $n = 0$, $Re = 1$, and $f_w = 0$.

temperature distribution. It can be observed that all boundary layer thickness decreased by increasing f_w from negative to positive (from injection to suction). Figures 10, 11, and 12 show the effect of Reynolds number on the velocity profile when $Pr = 1$, $n = 0$, and $f_w = 0$. This figure shows that the boundary layer thickness and thermal boundary layer thickness are quite opposite to that of Reynolds number. The influence of n on temperature field for both types of the thermal boundary conditions considered has been studied in Figures 13 and 14. We notice that increasing n reduces

FIGURE 6: The variation of f at different orders of approximations, $Pr = 1$, $n = 0$, $Re = 1$, and $f_w = 0$.

FIGURE 8: Temperature profile (isothermal) θ for various f_w when $Pr = 1$, $n = 0$, and $Re = 1$.

FIGURE 7: Velocity profile f' for various f_w when $Pr = 1$, $n = 0$, and $Re = 1$.

FIGURE 9: Temperature profile (isoflux) θ for various f_w when $Pr = 1$, $n = 0$, and $Re = 1$.

the thermal boundary layer thickness regardless of the boundary condition type leading to a heat transfer augmentation. Effect of Prandtl number (isothermal) on the temperature field is plotted in Figure 15. This figure shows that thermal boundary layer thickness directly depends on Prandtl number.

Tables 2 and 3 compare the results of HAM and numerical solution when the Reynolds number varies for $f_w = 0$,

$Pr = 1$, and $n = 0$ for isothermal and heat flux boundary conditions, respectively. It can be seen that for isothermal boundary condition with increasing Reynolds number, the wall shear stress for both boundary conditions considered here (isothermal and heat flux) and Nusselt number increase with Reynolds number for isothermal state and independent of Reynolds number for heat flux boundary condition.

FIGURE 10: Velocity profile f' for various Re when $Pr = 1$, $n = 0$, and $f_w = 0$.

FIGURE 12: Temperature profile (isoflux) θ for various Re when $Pr = 1$, $n = 0$, and $f_w = 0$.

FIGURE 11: Temperature profile (isothermal) θ for various Re when $Pr = 1$, $n = 0$, and $f_w = 0$.

FIGURE 13: Temperature profile (isothermal) θ for various n when $Pr = 1$, $n = 0$, and $f_w = 0$.

Comparison of the results of HAM and numerical solution has been shown in Tables 4 and 5 for various Prandtl numbers when $Re = 1$, $n = 0$ and $f_w = -0.3$, $f_w = 0.3$, respectively. It can be observed that the Prandtl number has no effect on the wall shear stress for both boundary conditions and Nusselt number for heat flux boundary condition.

6. Summary and Conclusions

Homotopy analysis method (HAM) is applied to compute wall driven flow through a porous medium over a stretching permeable surface subjected to power-law temperature and heat flux boundary conditions. The proper range of

FIGURE 14: Temperature profile (isoflux) θ for various n when $Pr = 1$, $n = 0$, and $f_w = 0$.

TABLE 2: The results of HAM and NS for $f''(0)$, $\theta'(0)$ when $f_w = 0$, $Pr = 1$, and $n = 0$ for temperature.

Re	$\tilde{h}_1$	$\tilde{h}_2$	$-f''(0)$		$-\theta'(0)$	
			HAM	NS	HAM	NS
1	-0.2	-0.4	1.4198	1.4242	0.5030	0.5033
1.5	-0.25	-0.4	1.5799	1.5811	0.6456	0.6422
2	-0.2	-0.2	1.7234	1.7320	0.7518	0.7592
5	-0.1	-0.1	2.4394	2.4494	1.2636	1.2576

TABLE 3: The results of HAM and NS for $f''(0)$, $\theta'(0)$ when $f_w = 0$, $Pr = 1$, and $n = 0$ for heat flux.

Re	$\tilde{h}_1$	$\tilde{h}_2$	$-f''(0)$		$-\theta'(0)$	
			HAM	NS	HAM	NS
1	-0.25	-0.2	1.4191	1.4242	0.9997	1
1.5	-0.2	-0.4	1.5791	1.5811	1.0000	1
2	-0.15	-0.3	1.7298	1.7320	0.9985	1
2.5	-0.1	-0.1	2.4445	2.4494	0.9939	1

the auxiliary parameter $\tilde{h}$ to ensure the convergency of the solution series was obtained through the so-called $\tilde{h}$ curves. When compared with other analytical methods, it is clear that HAM provides highly accurate analytical solutions for non-linear problems. Moreover, second-law (of thermodynamics) aspects of the problem are investigated. The highlights of this study are the following.

- (i) The nondimensional viscous boundary layer thickness has a direct relationship with Reynolds number; thus Nusselt number rate increases with Re.

FIGURE 15: Temperature profile θ (isothermal) for various Pr when $Pr = 1$, $n = 0$, and $f_w = 0$.

TABLE 4: The results of HAM and NS for $f''(0)$, $\theta'(0)$ for various Pr when, $Re = 1$, $n = 0$, and $f_w = -0.3$.

Pr	$-f''(0)$				$-\theta'(0)$			
	$f_w = -0.3$				$f_w = -0.3$			
	Temperature		Heat flux		Temperature		Heat flux	
	HAM	NS	HAM	NS	HAM	NS	HAM	NS
0.5	1.2699	1.2721	1.2711	1.2721	0.216	0.219	1.000	0.999
1	1.2699	1.2721	1.2711	1.2721	0.3128	0.313	1.000	1.000
5	1.2699	1.2721	1.2711	1.2721	0.636	0.623	1.000	0.999

TABLE 5: The results of HAM and NS for $f''(0)$, $\theta'(0)$ for various Pr when, $Re = 1$, $n = 0$, and $f_w = 0.3$.

Pr	$-f''(0)$				$-\theta'(0)$			
	$f_w = 0.3$				$f_w = 0.3$			
	Temperature		Heat flux		Temperature		Heat flux	
	HAM	NS	HAM	NS	HAM	NS	HAM	NS
0.5	1.5699	1.5721	1.5695	1.5721	0.4145	0.4100	1.0002	1.0000
1	1.5699	1.5721	1.5695	1.5721	0.7258	0.7216	1.0002	1.0000
5	1.5699	1.5721	1.5695	1.5721	2.5821	2.5826	1.0002	1.0000

- (ii) Nusselt number, wall shear stress have a reverse relationship with and mass transfer from the wall, f_w .

- (iii) Increasing the Prandtl number results in reduction of thermal boundary layer thickness.

Consequently, Nusselt number increase with Pr .

Nomenclature

f :	Similarity function for velocity
f_w :	Injection parameter, $f_w = -v_w L/u_0 \sqrt{K}$
$\tilde{h}$:	Auxiliary parameter
HAM:	Homotopy analysis method
H :	Auxiliary function
$\mathcal{L}$:	Linear operator of HAM
K :	Permeability of the porous medium, m^2
N :	Non-linear operator
n :	Power of temperature/heat flux distribution
Nu :	Local Nusselt number
Nu_L :	Averaged Nusselt number
Pr :	Prandtl number, $Pr = \nu/\alpha_{eff}$
q_0 :	Wall heat flux coefficient, W/m^2
Re :	Reynolds number, $Re = \rho u_0 K/L$
T :	Temperature
T_0 :	Wall temperature coefficient, K
u :	Velocity in x direction
u_0 :	Wall velocity coefficient, m/s
v :	Velocity in y direction
v_w :	Injection velocity, m/s
B :	Positive constant
θ :	Similarity function for temperature
x :	Coordinate system, m
y :	Coordinate system, m
Q :	Volumetric rate of heat generation
λ :	Heat generation
N :	Radiation parameter
ρ :	Density of the fluid
Ψ :	Stream function
μ :	Dynamic viscosity
σ_1 :	Effective viscosity
κ_1 :	Absorption coefficient
η :	Dimensionless similarity variable
ν :	Kinematic viscosity.

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