Research Article

Delay-Dependent Fuzzy Control of Networked Control Systems and Its Application

Hongbo Li, Fuchun Sun, and Zengqi Sun

Department of Computer Science and Technology, State Key Laboratory of Intelligent Technology and Systems, Tsinghua University, Beijing 100084, China

Correspondence should be addressed to Hongbo Li; hbli@mail.tsinghua.edu.cn

Received 11 January 2013; Accepted 9 March 2013

Academic Editor: Yang Tang

Copyright © 2013 Hongbo Li et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper is concerned with the state feedback stabilization problem for a class of Takagi-Sugeno (T-S) fuzzy networked control systems (NCSs) with random time delays. A delay-dependent fuzzy networked controller is constructed, where the control parameters are independent on both sensor-to-controller delay and controller-to-actuator delay simultaneously. The resulting NCS is transformed into a discrete-time fuzzy switched system, and under this framework, the stability conditions of the closed-loop NCS are derived by defining a multiple delay-dependent Lyapunov function. Based on the derived stability conditions, the stabilizing fuzzy networked controller design method is also provided. Finally, simulation results are given to illustrate the effectiveness of the obtained results.

1. Introduction

During the past decades, Fuzzy control technique has been widely developed and used in many scientific applications and engineering systems. Especially, the so-called Takagi-Sugeno (T-S) fuzzy model has been well recognized as an effective method in approximating complex nonlinear system and has been widely used in many real-world physical systems. In T-S fuzzy model, local dynamics in different state space regions are represented by different linear models, and the overall model of the system is achieved by fuzzy “blending” of these fuzzy models. Under this framework, the controller design of nonlinear system can be carried out by utilizing the well-known parallel distributed compensation (PDC) scheme. As a result, the fruitful linear system theory can be readily extended to the analysis and controller synthesis of nonlinear systems. Therefore, the last decades have witnessed a rapidly growing interest in T-S fuzzy systems, with many important results reported in the literature. For more details on this topic, we refer the readers to [1–3] and the reference therein.

However, it is worth noting that in traditional T-S fuzzy control systems, system components are located in the same place and connected by point-to-point wiring, where an

implicit assumption is that the plant measurements and the control signals transmitted between the physical plant and the controller do not exhibit time delays. However, in many modern control systems, it is difficult to do so, and thus the plant measurements and control signals might be transmitted from one place to another. In this situation, communication networks such as Internet are used to connect the spatially distributed system components, which gives rise to the so-called networked control systems (NCSs) [4]. Using NCSs has many advantages, such as low cost, reduced weight and power requirements, simple installation and maintenance, and resource sharing. Therefore, NCSs have emerged as a hot topic in research communities during the past decade. Many interesting and practical issues such as NCSs architecture [5], network protocol [6], time delay [7], and packet loss [8] have been investigated with many important results reported in the literature [9–17]. Moreover, NCSs have been finding applications in a broad range of areas such as networked DC motors, networked robots, and networked process control.

Among the aforementioned problems, time delay is one of the most important ones, since time delay is usually the major cause for NCSs performance deterioration and potential system instability. Therefore, the analysis and synthesis of NCSs with time delays have been the focus of some research

studies in recent years, with many interesting results reported in the literature; see [4, 7, 9, 18–22] and the references therein. It has been shown in [23, 24] that, in order to reduce the conservatism of the obtained results, it is of great significance to design two-mode-dependent networked controller for NCSs, where the control parameter depend on sensor-to-controller (S-C) delay and controller-to-actuator (C-A) delay simultaneously. Therefore, two-mode-dependent networked control has received increasing attention during the past few years. For example, for NCSs with Markov delays, [7] presents a delay-dependent state feedback controller with control gains dependent on the current S-C delay τ_k and the previous C-A delay d_{k-1} . Reference [24] proposes an output feedback networked controller for NCSs, where the control parameters depends on the current S-C delay τ_k and the most recent C-A delay $d_{k-\tau_k-1}$. In our earlier work [23], a more desirable networked control methodology with control parameter dependent on the current S-C delay τ_k and the current C-A delay d_k has been investigated. In this way, most recent delay information is effectively utilized, and therefore the control performance of NCSs should be improved. It is worth noting that most of the aforementioned results are for linear NCSs. However, there exist many complex nonlinear systems in practical situations, and therefore it is desirable to investigate two-model-dependent control for nonlinear NCSs. To the best of the authors' knowledge, the problem of two-model-dependent control for nonlinear NCSs, especially for the one with control parameters dependent on τ_k and d_k simultaneously, has not been investigated and still remains challenging, which motivates the present study.

Therefore the intention of this paper is to investigate the two-mode-dependent for a class of nonlinear NCSs with time delays, where the remote controlled plant is described by T-S fuzzy model. A τ_k - d_k -dependent fuzzy networked controller is constructed for the NCSs under study. The resulting NCS is transformed into a discrete-time fuzzy switched system, and under this framework, the stability conditions of the closed-loop NCS are derived by employing multiple delay-dependent Lyapunov approach. Based on the derived stability conditions, the stabilizing fuzzy controller design method is also provided. Simulation results are given to illustrate the effectiveness of the obtained results.

Notation. Throughout this paper, $\mathbb{R}^n$ denotes the n -dimensional Euclidean space, and the notation $\mathbf{P} > 0$ (≥ 0) means that $\mathbf{P}$ is real symmetric and positive definite (semidefinite). The superscript “ T ” denotes matrix transposition, and I is the identity matrix with appropriate dimensions. The notation $\mathbb{Z}^+$ stands for the set of nonnegative integers. In symmetric block matrices, we use “ $*$ ” as an ellipsis for the terms introduced by symmetry.

2. Problem Formulation

In this paper, we consider the state feedback stabilization problem for a class of discrete-time nonlinear NCSs, where the corresponding system framework is depicted in Figure 1. It can be seen that the NCS under study consists of four

FIGURE 1: The structure of the considered NCSs.

components: (i) the controlled plant with sensor; (ii) the networked controller; (iii) the communication network; (iv) the actuator. Each component is described in the following sections.

2.1. The Controlled Plant with the Sensor and State Observer.

In the NCSs under study, the dynamics of the controlled plant are described by the T-S fuzzy model and can be represented by the following form:

Plant rule i :

IF $\theta_1(k)$ is μ_{i1} , and $\dots, \theta_g(k)$ is μ_{ig} ,

THEN $\mathbf{x}(k+1) = \mathbf{F}_i \mathbf{x}(k) + \mathbf{G}_i \mathbf{u}(k)$ (1)

$\mathbf{y}_i(k) = \mathbf{C}_i \mathbf{x}(k)$,

(for $i = 1, 2, \dots, r$),

where $\mu_{i\omega}$ ($\omega = 1, 2, \dots, g$) are the fuzzy sets, $\mathbf{x}(k) \in \mathbb{R}^n$ is the plant state, $\mathbf{u}(k) \in \mathbb{R}^m$ is the control input, $\mathbf{y}(k) \in \mathbb{R}^p$ is the plant output, $\mathbf{F}_i$, $\mathbf{G}_i$, and $\mathbf{C}_i$ are matrices of compatible dimensions, r is the number of IF-THEN rules, and $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_g]$ are the premise variables. It is assumed that the premise variables do not depend on the input $\mathbf{u}(k)$.

By using the fuzzy inference method with a center-average defuzzifier, product inference, and singleton fuzzifier, the controlled plant in (1) can be expressed as

$$\begin{aligned} \mathbf{x}(k+1) &= \sum_{i=1}^r \mu_i(k) [\mathbf{F}_i \mathbf{x}(k) + \mathbf{G}_i \mathbf{u}(k)], \\ \mathbf{y}(k) &= \sum_{i=1}^r \mu_i(k) [\mathbf{C}_i \mathbf{x}(k)], \end{aligned} \quad (2)$$

where

$$\mu_i(k) = \frac{w_i(k)}{\sum_{i=1}^r w_i(k)}, \quad w_i(k) = \prod_{j=1}^g \mu_{ij}[\theta_j(k)]. \quad (3)$$

It is assumed that $w_i(\theta(k)) \geq 0$ for $i = 1, 2, \dots, r$ and $\sum_{i=1}^r w_i(\theta(k)) > 0$ for k . Therefore, we can conclude that $\sum_{i=1}^r \mu_i(\theta(k)) \geq 0$ for $i = 1, 2, \dots, r$ and $\sum_{i=1}^r \mu_i(\theta(k)) = 1$ for all k .

It is worth mentioning that the sensor in NCSs is time-driven, and it is assumed that the full state variables are available. At each sampling period, the sampled plant state

and its timestamp (i.e., the time the plant state is sampled) are encapsulated into a packet and sent to the controller via the network.

2.2. The Network. Networks exist in both channels from the sensor to the controller and from the controller to the actuator. The sensor packet will suffer a sensor-to-controller (S-C) delay during its transmission from the sensor to the controller, while the control packet will suffer a controller-to-actuator (C-A) delay during its transmission from the controller to the actuator. For notation simplicity, let τ_k and d_k denote S-C delay and C-A delay at time k , respectively. Then, a natural assumption can be made as follows:

$$\check{\tau} \leq \tau_k \leq \hat{\tau}, \quad \check{d} \leq d_k \leq \hat{d}, \quad (4)$$

where $\check{\tau} \geq 0$ and $\hat{\tau} \geq 0$ are the lower and the upper bounds of τ_k and $\check{d} \geq 0$ and $\hat{d} \geq 0$ are the lower and the upper bounds of d_k . Let $\mathcal{M} \triangleq \{\check{\tau}, \check{\tau} + 1, \dots, \hat{\tau}\}$ and $\mathcal{N} \triangleq \{\check{d}, \check{d} + 1, \dots, \hat{d}\}$.

2.3. The Networked Controller. Please note that the control signal in NCSs suffers the S-C delay τ_k and the C-A delay d_k , and therefore, the control signal for the plant at the time step k will be the one based on the state $\mathbf{x}(k - \tau_k - d_k)$. In view of this, it is more appealing from a delay-dependent point of view to construct the following fuzzy networked controller:

Observer rule i :

$$\begin{aligned} \text{IF } \theta_1(k) \text{ is } \mu_{i1}, \text{ and } \dots, \theta_g(k) \text{ is } \mu_{ig}, \\ \text{THEN } \mathbf{u} = \mathbf{L}_i(\tau_k, d_k) \mathbf{x}, \\ (\text{for } i = 1, 2, \dots, r), \end{aligned} \quad (5)$$

where $\mathbf{K}_i(m, n)$, ($m \in \mathcal{M}, n \in \mathcal{N}$) are the feedback gains to be designed. Then the final output of the networked fuzzy controller is

$$\mathbf{u} = \sum_{i=1}^r \mu_i(k) \mathbf{L}_i(\tau_k, d_k) \mathbf{x}. \quad (6)$$

In such a way, the control signal for the plant at the time step k can be expressed by

$$\mathbf{u}(k) = \sum_{i=1}^r \mu_i(k) \mathbf{L}_i(\tau_k, d_k) \mathbf{x}(k - \tau_k - d_k). \quad (7)$$

It can be seen from (7) that most recent delay information is effectively utilized in the controller, and therefore the control performance of NCSs should be improved.

The networked controller is time-driven. At each sampling period, it calculates the control signals with the most recent sensor packet available. Immediately after the calculation, the new control signals and the timestamp of the used plant state are encapsulated into a packet and sent to the actuator via the network. The timestamp will ensure that the actuator selects the appropriate control signal to control the plant.

2.4. The Actuator. The actuator in NCS is time-driven. The actuator and the sensor have the same sampling period h , and they are synchronized. It is worth noting that the actuator and the sensor are both located at the plant side, and therefore the synchronization between them can be easily achieved by hardware synchronization, for instance, by using special wiring to distribute a global clock signal to the sensor and the actuator. The actuator has a buffer size of 1, which means that the latest control packet is used to control the plant.

It is worth noting that when the networked controller (6) calculates the control signal, it does not know the value of d_k because it does not happen yet. To circumvent this problem, in our earlier work [23], we propose the strategy that sends a control sequence in a packet and uses an actuator with selection logic to choose the appropriate control signal based on d_k to overcome the aforementioned problem. Generally speaking, the proposed strategy works in the following way. When a sensor packet arrives at the controller node, the networked controller will calculate a set of control signals using the control parameter set $\{\mathbf{L}_i(\tau_k, \check{d}), \mathbf{L}_i(\tau_k, \check{d} + 1), \dots, \mathbf{L}_i(\tau_k, \hat{d})\}$ ($i = 1, 2, \dots, r$), then the obtained control signal set will be sent to the actuator via the network; when the control packet arrives at the actuator node, the actuator will select the appropriate control signal from the control signal set based on d_k and then uses it to control the plant. In this paper, we also employ this strategy to deal with the aforementioned issue. For more details on the aforementioned strategy, we refer the reader to [23].

The objective of this paper is to design the fuzzy networked controller (6), such that the resulting closed-loop system with random delays is stable.

3. Main Results

3.1. Modeling of NCSs. For the convenience of notation, we let $\mu_i = \mu_i(\theta(k))$ in the following. By substituting (7) into (2), we have

$$\begin{aligned} \mathbf{x}(k+1) \\ = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j [\mathbf{F}_i \mathbf{x}(k) + \mathbf{G}_j \mathbf{L}_j(\tau_k, d_k) \mathbf{x}(k - \tau_k - d_k)]. \end{aligned} \quad (8)$$

One can readily infer from $\tau_k \leq \hat{\tau}$ and $d_k \leq \hat{d}$ that, at time step k , the control signal no older than $k - \hat{\tau} - \hat{d}$ can be used to control the plant. Introduce the following augmented state

$$\mathbf{z}(k) = [\mathbf{x}(k)^T \quad \mathbf{x}(k-1)^T \quad \dots \quad \mathbf{x}(k - \hat{\tau} - \hat{d})^T]^T, \quad (9)$$

into (8), then the closed-loop NCS can be expressed with the following fuzzy switched model:

$$\mathbf{z}(k+1) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j [\tilde{\mathbf{F}}_i + \tilde{\mathbf{G}}_j \mathbf{L}_j(\tau_k, d_k) \tilde{\mathbf{E}}(\tau_k, d_k)] \mathbf{z}(k), \quad (10)$$

with

$$\tilde{\mathbf{F}}_i = \begin{bmatrix} \mathbf{F}_i & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \quad \tilde{\mathbf{G}}_i = \begin{bmatrix} \mathbf{G}_i \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (11)$$

$$\tilde{\mathbf{E}}(\tau_k, d_k) = [0 \quad \cdots \quad I \quad \cdots \quad 0],$$

where $\tilde{\mathbf{E}}(\tau_k, d_k)$ has all elements being zeros except for the $(\tau_k + d_k + 1)$ th block being identity. Apparently, the closed-loop system (10) is a discrete-time fuzzy switched system, where the control parameter $\mathbf{L}_i(\tau_k, d_k)$ depends on τ_k and d_k simultaneously.

For notation convenience, we define the following matrix variable:

$$\Pi_{ij}(\tau_k, d_k) = \tilde{\mathbf{F}}_i + \tilde{\mathbf{G}}_i \mathbf{L}_j(\tau_k, d_k) \tilde{\mathbf{E}}(\tau_k, d_k). \quad (12)$$

Then closed-loop NCS in (10) can be rewritten as the following compact form:

$$\mathbf{z}(k+1) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Pi_{ij}(\tau_k, d_k) \mathbf{z}(k). \quad (13)$$

Remark 1. Apparently, the most appealing advantage of the proposed networked controller (5) is efficiently utilizing the τ_k - d_k -dependent control gains, in such a way that most recent delay information is used in the networked controller, and therefore better control performance could be obtained.

3.2. Stability Analysis and Controller Synthesis. Before proceeding further, we introduce the following definition, and it will be used throughout this paper.

Definition 2. The delays in NCSs are called arbitrary bounded delays, if $\{\tau_k : k \in \mathbb{Z}^+\}$ and $\{d_k : k \in \mathbb{Z}^+\}$ take values arbitrarily in $\mathcal{M}$ and $\mathcal{N}$, respectively.

In the following theorem, the stability conditions are derived for NCS (13) via a multiple delay-dependent Lyapunov approach.

Theorem 3. *The closed-loop NCS (13) with arbitrary bounded delays is asymptotically stable, if there exist $n \times n$ matrices $\mathbf{P}_i(m, s) > 0$ and $\mathcal{M}_{ij}$, satisfying*

$$\begin{bmatrix} -\mathbf{P}_l(n, t) & \mathbf{P}_l(n, t) \Pi_{li}(m, s) \\ * & -\mathbf{P}_i(m, s) - \mathcal{M}_{ii} \end{bmatrix} < 0, \quad (14)$$

$$(i, l \in \{1, 2, \dots, r\}, m, n \in \mathcal{M}, s, t \in \mathcal{N}),$$

$$\begin{bmatrix} -2\mathbf{P}_l(n, t) & \mathbf{P}_l(n, t) [\Pi_{lj}(m, s) + \Pi_{ji}(m, s)] \\ * & -\mathbf{P}_i(m, s) - \mathbf{P}_j(m, s) - \mathcal{M}_{ij} - \mathcal{M}_{ij}^T \end{bmatrix} < 0, \quad (15)$$

$$(1 \leq i < j \leq r, m, n \in \mathcal{M}, s, t \in \mathcal{N}),$$

$$\Omega = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \cdots & \mathcal{M}_{1r} \\ \mathcal{M}_{12}^T & \mathcal{M}_{22} & \cdots & \mathcal{M}_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{M}_{1r}^T & \mathcal{M}_{2r}^T & \cdots & \mathcal{M}_{rr} \end{bmatrix} < 0. \quad (16)$$

Proof. For NCS (13), we define the Lyapunov function as

$$V(\mathbf{z}(k), \mu(k)) = \mathbf{z}^T(k) \bar{\mathbf{P}}(\tau_k, d_k) \mathbf{z}(k), \quad (17)$$

$$\bar{\mathbf{P}}(\tau_k, d_k) = \sum_{i=1}^r \mu_i \mathbf{P}_i(\tau_k, d_k),$$

where $\mathbf{P}_i(\tau_k, d_k)$ are matrices dependent on time delays τ_k and d_k simultaneously.

Let $\tau_k = m$, $\tau_{k+1} = n$, $d_k = s$, and $d_{k+1} = t$, where $m, n \in \mathcal{M}$, $s, t \in \mathcal{N}$. The difference of $V(\mathbf{z}(k), \mu(k))$ can be given by

$$\Delta V = V(\mathbf{z}(k+1), \mu(k+1)) - V(\mathbf{z}(k), \mu(k))$$

$$= \mathbf{z}^T(k) \bar{\mathbf{P}}_+(n, t) \mathbf{z}(k) - \mathbf{z}^T(k) \bar{\mathbf{P}}_-(m, s) \mathbf{z}(k), \quad (18)$$

where

$$\bar{\mathbf{P}}_+(n, t) = \sum_{l=1}^r \mu_l(k+1) \mathbf{P}_l(n, t), \quad (19)$$

$$\bar{\mathbf{P}}_-(m, s) = \sum_{i=1}^r \mu_i(k) \mathbf{P}_i(m, s).$$

Then, along the trajectory of NCS (13), we have

$$\Delta V = \mathbf{z}^T(k) \left[\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Pi_{ij}(m, s) \right]^T$$

$$\times \bar{\mathbf{P}}_+(n, t) \left[\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Pi_{ij}(m, s) \right] \mathbf{z}(k)$$

$$- \mathbf{z}^T(k) \bar{\mathbf{P}}_-(m, s) \mathbf{z}(k)$$

$$= \mathbf{z}^T(k) \left[\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left[\frac{\Pi_{ij}(m, s) + \Pi_{ji}(m, s)}{2} \right] \right]^T$$

$$\times \bar{\mathbf{P}}_+(n, t) \left[\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left[\frac{\Pi_{ij}(m, s) + \Pi_{ji}(m, s)}{2} \right] \right]$$

$$\times \mathbf{z}(k) - \mathbf{z}^T(k) \bar{\mathbf{P}}_-(m, s) \mathbf{z}(k)$$

$$= \mathbf{z}^T(k)$$

$$\begin{aligned}
& \times \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left\{ \left[\frac{\Pi_{ij}(m, s) + \Pi_{ji}(m, s)}{2} \right]^T \right. \\
& \quad \times \bar{\mathbf{P}}_+(n, t) \left[\frac{\Pi_{ij}(m, s) + \Pi_{ji}(m, s)}{2} \right] \\
& \quad \left. - \mathbf{P}_i(m, s) \right\} \times \mathbf{z}(k) \\
& = \mathbf{z}^T(k) \sum_{i=1}^r \mu_i^2 \left[\Pi_{ii}^T(m, s) \bar{\mathbf{P}}_+(n, t) \Pi_{ii}(m, s) \right. \\
& \quad \left. - \mathbf{P}_i(m, s) \right] \mathbf{z}(k) + \mathbf{z}^T(k) \\
& \quad \times \sum_{i=1}^{r-1} \sum_{j=i+1}^r \mu_i \mu_j \\
& \quad \times \left[\frac{1}{2} (\Pi_{ij}(m, s) + \Pi_{ji}(m, s))^T \times \bar{\mathbf{P}}_+(n, t) \right. \\
& \quad \times (\Pi_{ij}(m, s) + \Pi_{ji}(m, s)) \\
& \quad \left. - \mathbf{P}_i(m, s) - \mathbf{P}_j(m, s) \right] \mathbf{z}(k). \tag{20}
\end{aligned}$$

On the other hand, by applying Schur complement to (14) and (15), we readily have

$$\begin{aligned}
& \Pi_{ii}^T(m, s) \mathbf{P}_l(n, t) \Pi_{ii}(m, s) - \mathbf{P}_i(m, s) < 0, \\
& \frac{1}{2} \Xi_{ij}^T(m, s) \mathbf{P}_l(n, t) \Xi_{ij}(m, s) \\
& \quad - \mathbf{P}_i(m, s) - \mathbf{P}_j(m, s) - \mathcal{M}_{ij} - \mathcal{M}_{ij}^T < 0, \tag{21}
\end{aligned}$$

where $\Xi_{ij}(m, s) = \Pi_{ij}(m, s) + \Pi_{ji}(m, s)$.

For (14) and (15), multiplying the corresponding $l = 1, \dots, r$ inequalities by $\mu_l(k+1)$, summing up the resulting inequalities, and noting the fact that $\sum_{l=1}^r \mu_l(k+1) = 1$, we have

$$\begin{aligned}
& \Pi_{ii}^T(m, s) \bar{\mathbf{P}}_+(n, t) \Pi_{ii}(m, s) - \mathbf{P}_i(m, s) - \mathcal{M}_{ii} < 0, \\
& \quad (i, l \in \{1, 2, \dots, r\}, m, n \in \mathcal{M}, s, t \in \mathcal{N}), \\
& \frac{1}{2} \Xi_{ij}^T(m, s) \bar{\mathbf{P}}_+(n, t) \Xi_{ij}(m, s) \\
& \quad - \mathbf{P}_i(m, s) - \mathbf{P}_j(m, s) - \mathcal{M}_{ij} - \mathcal{M}_{ij}^T < 0, \\
& \quad (1 \leq i < j \leq r, m, n \in \mathcal{M}, s, t \in \mathcal{N}). \tag{22}
\end{aligned}$$

Then, it follows from (23), (22) that

$$\begin{aligned}
\Delta V & \leq \mathbf{z}^T(k) \sum_{i=1}^{r-1} \sum_{j=i+1}^r \mu_i \mu_j \left[\mathcal{M}_{ij} + \mathcal{M}_{ij}^T \right] \mathbf{z}(k) \\
& \quad + \mathbf{z}^T(k) \sum_{i=1}^r \mu_i^2 \mathcal{M}_{ii} \mathbf{z}(k) \\
& = \begin{bmatrix} \mu_1 \mathbf{z}(k) \\ \mu_2 \mathbf{z}(k) \\ \vdots \\ \mu_r \mathbf{z}(k) \end{bmatrix}^T \Omega \begin{bmatrix} \mu_1 \mathbf{z}(k) \\ \mu_2 \mathbf{z}(k) \\ \vdots \\ \mu_r \mathbf{z}(k) \end{bmatrix}. \tag{23}
\end{aligned}$$

Therefore, if the conditions (14)–(16) hold, we can readily obtain $\Delta V(\mathbf{z}(k), \mu(k)) < 0$ for any $\mathbf{z}(k) \neq 0$. Then we have $\lim_{k \rightarrow \infty} V(\mathbf{z}(k)) = 0$ and $\lim_{k \rightarrow \infty} \mathbf{z}(k) = 0$, which imply that the closed-loop NCS (13) is asymptotically stable. This completes the proof. $\square$

Now, we are in a position to present the stabilizing controller design method. To this end, we proposed equivalent stability conditions for NCSs in the following theorem.

Theorem 4. *The closed-loop NCS (13) with arbitrary bounded delays is asymptotically stable, if there exist $n \times n$ matrices $\mathbf{P}_i(m, s) > 0$, $\mathbf{Q}_i(m, s) > 0$, and $\mathcal{M}_{ij}$, satisfying (16) and the following:*

$$\begin{bmatrix} -\mathbf{Q}_l(n, t) & \tilde{\mathbf{F}}_i + \tilde{\mathbf{G}}_i \mathbf{L}_i(m, s) \tilde{\mathbf{E}}(m, s) \\ * & -\mathbf{P}_i(m, s) - \mathcal{M}_{ii} \end{bmatrix} < 0, \tag{24}$$

($i, l \in \{1, 2, \dots, r\}, m, n \in \mathcal{M}, s, t \in \mathcal{N}$),

$$\begin{bmatrix} -2\mathbf{Q}_l(n, t) & \Pi_{ij}(m, s) + \Pi_{ji}(m, s) \\ * & -\mathbf{P}_i(m, s) - \mathbf{P}_j(m, s) - \mathcal{M}_{ij} - \mathcal{M}_{ij}^T \end{bmatrix} < 0, \tag{25}$$

($1 \leq i < j \leq r, m, n \in \mathcal{M}, s, t \in \mathcal{N}$),

$$\mathbf{P}_l(n, t) \mathbf{Q}_l(n, t) = I \quad (l \in \{1, 2, \dots, r\}, n \in \mathcal{M}, s \in \mathcal{N}), \tag{26}$$

where

$$\Pi_{ij}(m, s) = \tilde{\mathbf{F}}_i + \tilde{\mathbf{G}}_i \mathbf{L}_j(m, s) \tilde{\mathbf{E}}(m, s). \tag{27}$$

Proof. Condition (26) implies

$$\mathbf{Q}_l(n, t) = \mathbf{P}_l^{-1}(n, t). \tag{28}$$

Substituting (28) into (24) and (25) and then performing congruence transformations to the resulting inequalities by $\text{diag}\{\mathbf{P}_l(n, t), I\}$, respectively, lead to (14) and (15). Then from Theorem 3 we can conclude that if the conditions (16), (24), and (25) hold, the closed-loop system (5) is asymptotically stable. This completes the proof. $\square$

Note that the conditions stated in Theorem 4 are a set of LMIs with nonconvex constraints. In the literature, there are several approaches to solve such nonconvex problem, among

which *cone complementarity linearization* (CCL) approach is the most commonly used one [7, 24], since it is simple and very efficient in numerical implementation. Therefore, we employ CCL approach in this paper to deal with this problem. Note that the CCL-based controller design procedure is quite standard, and the one in our earlier work [22] can be easily adapted to solve the controller design problem in this paper. To save space and avoid repetition, the CCL-based controller design procedure is omitted here. For more details on this topic, please refer to [7, 22, 24] and the reference therein.

Remark 5. It has been demonstrate that delay-dependent strategy is an effective way to improve the control performance and reduce the conservatism of NCSs. Therefore, the stabilization of NCSs with time delays and/or packet losses, either under sensor-to-controller (SCC) delay-dependent strategy or under two sides delay-dependent strategy (i.e., the control parameter depends on sensor-to-controller (S-C) delay and controller-to-actuator (C-A) delay simultaneously), has received a lot of attentions [7, 23, 24]. There are two main differences between this work and the aforementioned results. The first one is that the aforementioned results are for linear NCSs, while this work is for nonlinear NCSs. The second one is that this work employs most recent S-C and C-A delay information in the delay-dependent strategy.

It is not difficult to see that if we consider a fuzzy controller with delay-independent gains and define the following matrix variable:

$$\bar{\Pi}_{ij}(\tau_k, d_k) = \bar{\mathbf{F}}_i + \bar{\mathbf{G}}_i \mathbf{L}_j \bar{\mathbf{E}}(\tau_k, d_k), \quad (29)$$

the closed-loop NCS under delay-independent fuzzy controller can be expressed as

$$\mathbf{z}(k+1) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \bar{\Pi}_{ij}(\tau_k, d_k) \mathbf{z}(k). \quad (30)$$

Then by following similar lines in proof of Theorem 3, one can readily obtain the following corollary.

Corollary 6. *The closed-loop NCS (30) with delay-independent control parameters and arbitrary bounded delays is asymptotically stable, if there exist $n \times n$ matrices $\mathbf{P}_i > 0$ and $\mathcal{M}_{ij}$, satisfying*

$$\begin{bmatrix} -\mathbf{P}_l & \mathbf{P}_l \bar{\Pi}_{ii} \\ * & -\mathbf{P}_i - \mathcal{M}_{ii} \end{bmatrix} < 0, \quad (i, l \in \{1, 2, \dots, r\}),$$

$$\begin{bmatrix} -2\mathbf{P}_l & \mathbf{P}_l [\bar{\Pi}_{ij} + \bar{\Pi}_{ji}] \\ * & -\mathbf{P}_i - \mathbf{P}_j - \mathcal{M}_{ij} - \mathcal{M}_{ij}^T \end{bmatrix} < 0, \quad (1 \leq i < j \leq r), \quad (31)$$

$$\Omega = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \cdots & \mathcal{M}_{1r} \\ \mathcal{M}_{12}^T & \mathcal{M}_{22} & \cdots & \mathcal{M}_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{M}_{1r}^T & \mathcal{M}_{2r}^T & \cdots & \mathcal{M}_{rr} \end{bmatrix} < 0.$$

Remark 7. One can readily infer that, by remaining the control parameter constant (i.e., $\mathbf{L}_i = \mathbf{L}_i(\tau_k, d_k)$), Theorem 3 implies Corollary 6. This indicates that Theorem 3 is no more conservative than Corollary 6. In other words, from a theoretical point of view, using delay-dependent control parameter in NCSs obtains no more conservative results than using delay-independent control parameter. The previous theoretical analysis demonstrates the advantage of the proposed method.

Remark 8. To make our idea more lucid, in this paper, we only consider the stabilization case under a simple framework. However, it is worth mentioning that the previous derived results can be easily extended to the robust control case or H_∞ control case.

4. Illustrative Example

In this section, an illustrative example will be presented to demonstrate the effectiveness of the proposed approach. To this end, let us consider an NCS shown in Figure 1, where the controlled plant is a cart and inverted pendulum system and it is borrowed from our earlier work [25]. The dynamics of the cart and inverted pendulum system are described as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{1}{[(M+m)(J+ml^2) - m^2 l^2 x_1]} \\ &\quad \times [-f_1(M+m)x_2 - m^2 l^2 x_2^2 \sin x_1 \cos x_1 \\ &\quad + f_0 m l x_4 \cos x_1 \\ &\quad + (M+m)mgl \sin x_1 - ml \cos x_1 u], \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \frac{1}{[(M+m)(J+ml^2) - m^2 l^2 x_1]} \\ &\quad \times [f_1 m l x_2 \cos x_1 + (J+ml^2)m l x_2^2 \sin x_1 \\ &\quad - f_0(J+ml^2)x_4 - m^2 g l^2 \sin x_1 \cos x_1 \\ &\quad + (J+ml^2)u]. \end{aligned} \quad (32)$$

For more details on the physical meanings and parameters of each variables, please refer to our earlier work [25]. Let $\mathbf{x} = [x_1, x_2, x_3, x_4]$, where x_1 denotes the angle (rad) of the pendulum from the vertical, x_2 is the angular velocity (rad/s), x_3 is the displacement (m) of the cart, and x_4 is the velocity (m/s) of the cart. When the sampling period is set to $h = 0.005$ s, the considered cart and inverted pendulum system can be expressed by the following T-S fuzzy model:

Plant rule 1:

IF $\mathbf{x}_1(k)$ is about 0,

THEN $\mathbf{x}(k+1) = \mathbf{F}_1 \mathbf{x}(k) + \mathbf{G}_1 \mathbf{u}(k)$,

$\mathbf{y}_1(k) = \mathbf{C}_1 \mathbf{x}(k)$,

Plant rule 2:

IF $\mathbf{x}_1(k)$ is about $\pm \frac{\pi}{3}$,

THEN $\mathbf{x}(k+1) = \mathbf{F}_2 \mathbf{x}(k) + \mathbf{G}_2 \mathbf{u}(k)$,

$\mathbf{y}_2(k) = \mathbf{C}_2 \mathbf{x}(k)$,

(33)

where the corresponding parameters are given by

$$\begin{aligned} \mathbf{F}_1 &= \begin{bmatrix} 1.000364 & 0.004996 & 0 & 0.000536 \\ 0.145489 & 0.998798 & 0 & 0.211786 \\ -0.000015 & 0.0 & 1 & 0.004796 \\ -0.006057 & 0.000049 & 0 & 0.919852 \end{bmatrix}, \\ \mathbf{G}_1 &= \begin{bmatrix} -0.000023 \\ -0.009242 \\ 0.000008 \\ 0.003497 \end{bmatrix}, \quad \mathbf{C}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}^T, \\ \mathbf{F}_2 &= \begin{bmatrix} 1.000275 & 0.004996 & 0 & 0.000245 \\ 0.110107 & 0.998842 & 0 & 0.096910 \\ -0.000005 & 0.000000 & 1 & 0.004814 \\ -0.002292 & 0.000024 & 0 & 0.926650 \end{bmatrix}, \\ \mathbf{G}_2 &= \begin{bmatrix} -0.000010 \\ -0.004229 \\ 0.000008 \\ 0.003200 \end{bmatrix}, \quad \mathbf{C}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}^T \end{aligned} \quad (34)$$

and the membership functions for plant rule 1 and 2 are of the following form:

$$\begin{aligned} \mu_1[\mathbf{x}_1(k)] &= \left\{ 1 - \frac{1}{1 + e^{-7[\mathbf{x}_1(k) - \pi/6]}} \right\} \times \frac{1}{1 + e^{-7[\mathbf{x}_1(k) - \pi/6]}}, \\ \mu_2[\mathbf{x}_1(k)] &= 1 - \mu_1[\mathbf{x}_1(k)]. \end{aligned} \quad (35)$$

For more details on the controlled plant, we refer the reader to our earlier work [25].

In this scenario, the random delays are set to $\tau_k \in \{1, 2\}$ and $d_k \in \{1, 2\}$. By the proposed method, we obtain a stabilizing T-S fuzzy controller of the form (6), with the following parameters:

$$\begin{aligned} \mathbf{L}_1(1, 1) &= [55.2252 \quad 29.0898 \quad 10.8434 \quad 57.5585], \\ \mathbf{L}_1(1, 2) &= [52.0972 \quad 10.0643 \quad 8.3996 \quad 44.1605], \\ \mathbf{L}_1(2, 1) &= [51.3889 \quad 9.9974 \quad 8.8001 \quad 44.3360], \\ \mathbf{L}_1(2, 2) &= [48.1232 \quad 8.5432 \quad 4.1590 \quad 40.8940], \\ \mathbf{L}_2(1, 1) &= [140.5636 \quad 32.9458 \quad 7.7856 \quad 33.3936], \\ \mathbf{L}_2(1, 2) &= [127.3991 \quad 30.7697 \quad 7.2025 \quad 32.9037], \\ \mathbf{L}_2(2, 1) &= [126.7388 \quad 30.4958 \quad 7.3644 \quad 32.8196], \\ \mathbf{L}_2(2, 2) &= [100.7092 \quad 28.2467 \quad 6.9607 \quad 31.886]. \end{aligned} \quad (36)$$

FIGURE 2: Typical simulation results using the proposed networked controller.

(a) Sensor-controller random delays τ_k

(b) Controller-actuator random delays d_k

FIGURE 3: The corresponding network conditions.

With the initial state $\mathbf{x}_0 = [10, 0, -10, 0]^T$, typical simulation result of the previous networked inverted pendulum system is depicted in Figure 2, where the corresponding time delays are depicted in Figure 3. It can be seen that the previous networked system is asymptotically stable and shows satisfactory control performance, which illustrates the effectiveness of the proposed method.

Then to further illustrate the advantage of the proposed method, let us consider the networked system with the delay-independent controller. To this end, we applied Corollary 6 to the previous NCS and obtain a stabilizing T-S fuzzy controller with the following parameters:

$$\begin{aligned} \mathbf{L}_1 &= [50.8743 \quad 9.0431 \quad 6.8321 \quad 43.8548], \\ \mathbf{L}_2 &= [105.8937 \quad 29.7894 \quad 7.1743 \quad 32.4361]. \end{aligned} \quad (37)$$

FIGURE 4: Typical simulation results using the proposed networked controller.

Then with the same initial state $\mathbf{x} = [0.4, 0, 0, 0]^T$, the simulation result of the networked system with previous delay-independent controller is plotted in Figure 4. Apparently, the proposed delay-dependent controller shows better control performance than the delay-independent one, which illustrates the advantage of the proposed method.

5. Conclusions

This paper presents a delay-dependent state feedback stabilization method for a class of T-S fuzzy NCSs with random time delays. A two-mode-dependent fuzzy controller is constructed, and the resulting NCSs is transformed into discrete-time fuzzy switched system. Under this framework, the stability conditions are derived for the closed-loop NCS, and the corresponding stabilizing controller design method is also provided. The main advantage of the proposed method is that the control signal computation can effectively employ most recent delay information, and therefore better control performance of NCSs could be obtained. Simulation and experimental results are given to illustrate the effectiveness of the obtained results. In the future work, we will consider more performance requirements such as H_∞ specification during the controller design stage.

Acknowledgments

The authors would like to thank the editor and the anonymous reviewers for their valuable comments and suggestions to improve the quality of the paper. The work of H. Li was supported by National Basic Research Program of China (973 Program) under Grant 2012CB821206, the National Natural Science Foundation of China under Grant 61004021, and Beijing Natural Science Foundation under Grant 4122037. The work of Z. Sun was supported in part by the National Natural Science Foundation of China under Grants 61174069, 61174103, and 61004023.

References

- [1] G. Feng, "Stability analysis of discrete-time fuzzy dynamic systems based on piecewise Lyapunov functions," *IEEE Transactions on Fuzzy Systems*, vol. 12, no. 1, pp. 22–28, 2004.
- [2] R. E. Precup and H. Hellendoorn, "A survey on industrial applications of fuzzy control," *Computers in Industry*, vol. 62, no. 3, pp. 213–226, 2011.
- [3] Q. Gao, X. J. Zeng, G. Feng et al., "T-s-fuzzy-model-based approximation and controller design for general nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 42, no. 4, pp. 1143–1154, 2012.
- [4] T. C. Yang, "Networked control system: a brief survey," *IEE Proceedings*, vol. 153, no. 4, pp. 403–412, 2006.
- [5] C. G. Goodwin, D. E. Quevedo, and E. I. Silva, "Architectures and coder design for networked control systems," *Automatica*, vol. 44, no. 1, pp. 248–257, 2008.
- [6] I. G. Polushin, P. X. Liu, and C.-H. Lung, "On the model-based approach to nonlinear networked control systems," *Automatica*, vol. 44, no. 9, pp. 2409–2414, 2008.
- [7] G. P. Liu, J. X. Mu, D. Rees, and S. C. Chai, "Design and stability analysis of networked control systems with random communication time delay using the modified MPC," *International Journal of Control*, vol. 79, no. 4, pp. 288–297, 2006.
- [8] J. Xiong and J. Lama, "Stabilization of linear systems over networks with bounded packet loss," *Automatica*, vol. 43, no. 1, pp. 80–87, 2007.
- [9] H. Yang, Y. Xia, and P. Shi, "Stabilization of networked control systems with nonuniform random sampling periods," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 5, pp. 501–526, 2011.
- [10] D. Yue, Q.-L. Han, and J. Lam, "Network-based robust H_∞ control of systems with uncertainty," *Automatica*, vol. 41, no. 6, pp. 999–1007, 2005.
- [11] Y.-B. Zhao, J. Kim, and G.-P. Liu, "Error bounded sensing for packet-based networked control systems," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 5, pp. 1980–1989, 2011.
- [12] D. B. Đaćić and D. Nešić, "Quadratic stabilization of linear networked control systems via simultaneous protocol and controller design," *Automatica*, vol. 43, no. 7, pp. 1145–1155, 2007.
- [13] H. Gao, T. Chen, and J. Lam, "A new delay system approach to network-based control," *Automatica*, vol. 44, no. 1, pp. 39–52, 2008.
- [14] C. Hua, P.-X. Liu, and X. Guan, "Backstepping control for nonlinear systems with time delays and applications to chemical reactor systems," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 9, pp. 3723–3732, 2009.
- [15] C. Hua and P.-X. Liu, "Teleoperation over the onternet with/without velocity signal," *IEEE Transactions on Instrumentation and Measurement*, vol. 60, no. 1, pp. 4–3, 2011.
- [16] K. You and L. Xie, "Minimum data rate for mean square stabilizability of linear systems with Markovian packet losses," *IEEE Transactions on Automatic Control*, vol. 56, no. 4, pp. 772–785, 2011.
- [17] D. E. Quevedo and D. Nešić, "Robust stability of packetized predictive control of nonlinear systems with disturbances and Markovian packet losses," *Automatica*, vol. 48, no. 8, pp. 1803–1811, 2012.
- [18] N. van de Wouw, D. Nešić, W. P. M. H. Heemels et al., "A discrete-time framework for stability analysis of nonlinear networked control systems," *Automatica*, vol. 48, no. 6, pp. 1144–1153, 2012.

- [19] L. Zhang, Y. Shi, T. Chen, and B. Huang, "A new method for stabilization of networked control systems with random delays," *IEEE Transactions on Automatic Control*, vol. 50, no. 8, pp. 1177–1181, 2005.
- [20] D. Yue, Q.-L. Han, and C. Peng, "State feedback controller design of networked control systems," *IEEE Transactions on Circuits and Systems II*, vol. 51, no. 11, pp. 640–644, 2004.
- [21] F. W. Yang, Z. D. Wang, Y. S. Hung, and M. Gani, " H_∞ control for networked systems with random communication delays," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 511–518, 2006.
- [22] H. Li, M.-Y. Chow, and Z. Sun, "State feedback stabilisation of networked control systems," *IET Control Theory & Applications*, vol. 3, no. 7, pp. 929–940, 2009.
- [23] H. Li, Z. Sun, H. Liu, and F. Sun, "Stabilisation of networked control systems using delay-dependent control gains," *IET Control Theory & Applications*, vol. 6, no. 5, pp. 698–706, 2012.
- [24] Y. Shi and B. Yu, "Output feedback stabilization of networked control systems with random delays modeled by Markov chains," *IEEE Transactions on Automatic Control*, vol. 54, no. 7, pp. 1668–1674, 2009.
- [25] X. Ma, Z. Sun, and Y. He, "Analysis and design of fuzzy controller and fuzzy observer," *IEEE Transactions on Fuzzy Systems*, vol. 6, no. 1, pp. 41–51, 1998.