Research Article

Delay-Dependent Robust H_∞ Filtering of the Takagi-Sugeno Fuzzy Stochastic Systems

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This paper is concerned with the problem of the robust H_∞ filtering for the Takagi-Sugeno (T-S) fuzzy stochastic systems with bounded parameter uncertainties. For a given T-S fuzzy stochastic system, this paper focuses on the stochastically mean-square stability of the filtering error system and the H_∞ performance level of the output error and the disturbance input. The design method for delay-dependent filter is developed based on linear matrix inequalities. Finally, the effectiveness of the proposed methods is substantiated with an illustrative example.

1. Introduction

As an efficient technique to linearize the nonlinear differential equations, the T-S fuzzy model [1] has been an active research area over the past three decades. This model is capable of representing linear input-output relations of nonlinear systems by appropriate fuzzy sets, such as the stirred tank reactor system [2] and the truck trailer system [3]. And it has been proved that this model can accurate to a compact set by a family of IF-THEN rules. This way, the stability analysis and synthesis of the nonlinear system turn into the analysis of linear systems, where the linear system theory can be conveniently applied. In recent years, the researches of T-S fuzzy have grown into a great number. To mention a few, the stability and control problem of T-S fuzzy systems have been investigated in [4–14] and the references therein.

Moreover, as a branch of state estimation theory, the filtering problem has become an important research field for many kinds of systems. The H_∞ filtering problems for the T-S fuzzy systems have been addressed in [15–19]; references [20, 21] have considered the $L_2 - L_\infty$ filtering problem for delayed T-S fuzzy systems with different methods.

On the other hand, stochastic system has received considerable attention because uncertain factors are unavoidable in most of the physical systems, for example, signal processing, engineering, finance, economics; biological movement

systems, and so forth. Stochastic modeling has become important in many branches of engineering applications [22]. And many results on the study of stochastic systems can be found in the literature. References [23, 24] study the problem of designing delay-dependent controllers and H_∞ output feedback controller for nonlinear stochastic time-delay system with method, respectively. The sliding mode control for the time-delay nonlinear Itô stochastic systems was proposed in [25, 26]. References [26, 27] have investigated the stability of the time-delay stochastic neutral networks. The problem of filtering is considered in [29–31]. Fault detection problem of the stochastic system has been addressed in [32, 33]. And the problem of H_∞ model reduction in stochastic framework is investigated in [34].

Through the above analysis, T-S fuzzy model could be used to represent the nonlinear stochastic system with several subsystems that could easily be analyzed. There have been a few works in this manner; [35] deals with the robust fault detection problem for T-S fuzzy stochastic systems. And [36] considers the stabilization for the stochastic fuzzy systems with delays. However, to the best of the authors knowledge, few results on filtering problem for TCS fuzzy stochastic systems are available which still remains challenging.

Inspired by the above discussions, this paper will focus on the robust fuzzy delay-dependent H_∞ filter design for a T-S fuzzy stochastic system with time-varying delays and

norm-bounded parameter uncertainties. The problem we consider here is to make sure that the fuzzy filters we design could ensure both the robust stochastic mean-square stability and a prescribed H_∞ performance level of the filtering error system. During the proof of the theorems, some useful free-weighting matrices are introduced to reduce the potential conservatism as much as we can. By using the Lyapunov-Krasovskii functional technique, a linear matrix inequality (LMI) approach is proposed to solve the problem.

The remainder of the paper is organized as follows. Section 2 formulates the filter design problem. Section 3 gives the delay-dependent conditions for the stochastic stability problem of the T-S fuzzy stochastic systems. And the solvability of the filtering design problem is obtained in terms of LMIs, which are presented in Section 4. In Section 5, a numerical example is shown to illustrate the effectiveness of the proposed methods. Finally, we conclude the paper in Section 6.

Notation. The notation used in this paper is fairly standard. The superscript “ T ” stands for matrix transposition. Throughout this paper, for real symmetric matrices X and Y , the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X - Y$ is positive semidefinite (resp., positive definite). $\mathbf{R}^n$ denotes the n -dimensional Euclidean space, and $\mathbf{R}^{m \times n}$ denotes the set of all $m \times n$ real matrices. I stands for an identity matrix of appropriate dimension, while $\mathbf{1}_n \in \mathbf{R}^n$ denotes a vector of ones. The notation $*$ is used as an ellipsis for terms that are induced by symmetry. $\text{diag}(\cdots)$ stands for a block-diagonal matrix. $|\cdot|$ denotes the Euclidean norm for vectors, and $\|\cdot\|$ denotes the spectral norm for matrices. $\mathbf{L}_2[0, \infty)$ represents the space of square-integrable vector functions over $[0, \infty)$. $\mathbf{E}(\cdot)$ stands for the mathematical expectation operator. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation and Preliminaries

Consider a T-S fuzzy stochastic time-delay system with time-varying parameter uncertainties.

Plant rule i : IF $s_1(t)$ is μ_{i1} , and $s_2(t)$ is μ_{i2} , and $\cdots$ and $s_g(t)$ is μ_{ig} , THEN

$$\begin{aligned} dx(t) = & [(A_i + \Delta A_i(t))x(t) \\ & + (A_{d1i} + \Delta A_{d1i}(t))x(t - \tau_1(t)) \\ & + (A_{d2i} + \Delta A_{d2i}(t))x(t - \tau_2(t)) + B_i v(t)] dt \\ & + [(H_i + \Delta H_i(t))x(t) \\ & + (H_{d1i} + \Delta H_{d1i}(t))x(t - \tau_1(t)) \\ & + (H_{d2i} + \Delta H_{d2i}(t))x(t - \tau_2(t))] d\omega(t), \end{aligned} \quad (1)$$

$$\begin{aligned} dy(t) = & [C_i x(t) + C_{d1i} x(t - \tau_1(t)) \\ & + C_{d2i} x(t - \tau_2(t)) + D_i v(t)] dt, \end{aligned} \quad (2)$$

$$z(t) = L_i x(t) + L_{d1i} x(t - \tau_1(t)) + L_{d2i} x(t - \tau_2(t)), \quad (3)$$

$$x(t) = \varphi(t), \quad t \in [-\max(h_1, h_2), 0], \quad i = 1, 2, \dots, r, \quad (4)$$

where $s(t) = [s_1(t) \ s_2(t) \ \cdots \ s_g(t)]$ are the premise variables, μ_{ij} is the fuzzy set, and r is the number of IF-THEN rules; $x(t) \in \mathbf{R}^n$ is the system state; $\varphi(t)$ is a given differential initial function on $[-u_2, 0]$, and $y(t) \in \mathbf{R}^m$ is the measured output; $z(t) \in \mathbf{R}^l$ is a signal to be estimated; $v(t) \in \mathbf{R}^s$ is the noise signal which belongs to $\mathcal{L}_2[0, \infty)$; $\tau_1(t)$ and $\tau_2(t)$ are continuous differentiable functions representing the time-varying delays, which are assumed to satisfy for all $t \geq 0$,

$$0 \leq \tau_1(t) < h_1, \quad 0 \leq \tau_2(t) < h_2. \quad (5)$$

In the considered system, $A_i, A_{d1i}, A_{d2i}, B_i, H_i, H_{d1i}, H_{d2i}, C_i, C_{d1i}, C_{d2i}, D_i, L_i, L_{d1i}$, and L_{d2i} are known constant matrices with appropriate dimensions. $\Delta A_i(t), \Delta A_{d1i}(t), \Delta A_{d2i}(t), \Delta H_i(t), \Delta H_{d1i}(t)$, and $\Delta H_{d2i}(t)$ represent the unknown time-varying parameter uncertainties and are assumed to satisfy

$$\begin{aligned} & \begin{bmatrix} \Delta A_i(t) & \Delta A_{d1i}(t) & \Delta A_{d2i}(t) \\ \Delta H_i(t) & \Delta H_{d1i}(t) & \Delta H_{d2i}(t) \end{bmatrix} \\ & = \begin{bmatrix} K_{1i} \\ K_{2i} \end{bmatrix} F_i(t) [T_{1i} \ T_{2i} \ T_{3i}], \end{aligned} \quad (6)$$

where $K_{1i}, K_{2i}, T_{1i}, T_{2i}$, and T_{3i} are known real constant matrices and the unknown time-varying matrix function satisfying

$$F_i(t)^T F_i(t) \leq I \quad \forall t. \quad (7)$$

Now, the defuzzified output of the dynamic fuzzy stochastic model in (1)–(4) can be represented as follows:

$$\begin{aligned} (\Sigma): dx(t) = & \sum_{i=1}^r \rho_i(s(t)) \\ & \times [(A_i + \Delta A_i(t))x(t) \\ & + (A_{d1i} + \Delta A_{d1i}(t))x(t - \tau_1(t)) \\ & + (A_{d2i} + \Delta A_{d2i}(t))x(t - \tau_2(t)) \\ & + B_i v(t)] dt \\ & + [(H_i + \Delta H_i(t))x(t) \\ & + (H_{d1i} + \Delta H_{d1i}(t))x(t - \tau_1(t)) \\ & + (H_{d2i} + \Delta H_{d2i}(t))x(t - \tau_2(t))] d\omega(t), \end{aligned}$$

$$\begin{aligned}
dy(t) &= \sum_{i=1}^r \rho_i(s(t)) \\
&\quad \times [C_i x(t) + C_{d1i} x(t - \tau_1(t)) \\
&\quad + C_{d2i} x(t - \tau_2(t)) + D_i v(t)] dt, \\
z(t) &= \sum_{i=1}^r \rho_i(s(t)) \\
&\quad \times [L_i x(t) + L_{d1i} x(t - \tau_1(t)) \\
&\quad + L_{d2i} x(t - \tau_2(t))], \\
x(t) &= \varphi(t), \quad t \in [-\max(h_1, h_2), 0],
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\rho_i(s(t)) &= \frac{\bar{\omega}_i(s(t))}{\sum_{j=1}^r \bar{\omega}_j(s(t))}, \\
\bar{\omega}_i(s(t)) &= \prod_{j=1}^g \mu_{ij}(s_j(t));
\end{aligned} \tag{9}$$

using the fuzzy theory, it is easy to see that, for all t ,

$$\begin{aligned}
\rho_i(s(t)) &\geq 0, \quad i = 1, 2, \dots, r, \\
\sum_{i=1}^r h_i(s(t)) &= 1.
\end{aligned} \tag{10}$$

Then, we consider the following fuzzy filters:

$$\begin{aligned}
d\hat{x}(t) &= \sum_{i=1}^r \rho_i(s(t)) [A_{fi} \hat{x}(t) dt + B_{fi} dy(t)], \\
\hat{z}(t) &= \sum_{i=1}^r \rho_i(s(t)) [L_{fi} \hat{x}(t)],
\end{aligned} \tag{11}$$

in which, the fuzzy rule has the same representation as in (1)–(4). Now we consider $\hat{x}(t) \in \mathbf{R}^n$ and $\hat{z}(t) \in \mathbf{R}^l$. The matrixes A_{fi} , B_{fi} , and L_{fi} are the filters need to be determined.

Let $\xi(t) = [x(t)^T \quad \hat{x}(t)^T]^T$, $\tilde{z}(t) = z(t) - \hat{z}(t)$.

For convenience, the filtering error dynamic system can be written as

$$\begin{aligned}
(\tilde{\Sigma}): d\xi(t) &= [(\bar{A} + \Delta \bar{A}(t)) \xi(t) \\
&\quad + (\bar{A}_{d1} + \Delta \bar{A}_{d1}(t)) G \xi(t - \tau_1(t)) \\
&\quad + (\bar{A}_{d2} + \Delta \bar{A}_{d2}(t)) G \xi(t - \tau_2(t)) + \bar{B} v(t)] dt \\
&\quad + [(\bar{H} + \Delta \bar{H}(t)) \xi(t) + (\bar{H}_{d1} + \Delta \bar{H}_{d1}(t))
\end{aligned}$$

$$\begin{aligned}
&\quad \times G \xi(t - \tau_1(t)) \\
&\quad + (\bar{H}_{d2} + \Delta \bar{H}_{d2}(t)) G \xi(t - \tau_2(t))] d\omega(t), \\
\tilde{z}(t) &= \bar{L} \xi(t) + \bar{L}_{d1} G \xi(t - \tau_1(t)) \\
&\quad + \bar{L}_{d2} G \xi(t - \tau_2(t)),
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
\bar{A} &= \begin{bmatrix} \bar{A} & 0 \\ \bar{B}_f \bar{C} & \bar{A}_f \end{bmatrix}, & \bar{A}_{d1} &= \begin{bmatrix} \bar{A}_{d1} \\ \bar{B}_f \bar{C}_{d1} \end{bmatrix}, \\
\bar{A}_{d2} &= \begin{bmatrix} \bar{A}_{d2} \\ \bar{B}_f \bar{C}_{d2} \end{bmatrix}, & \bar{H} &= \begin{bmatrix} \bar{H} & 0 \\ 0 & 0 \end{bmatrix}, \\
\bar{H}_{d1} &= \begin{bmatrix} \bar{H}_{d1} \\ 0 \end{bmatrix}, & \bar{H}_{d2} &= \begin{bmatrix} \bar{H}_{d2} \\ 0 \end{bmatrix}, \\
\bar{B} &= \begin{bmatrix} \bar{B} \\ \bar{B}_f \bar{D} \end{bmatrix}, & \Delta \bar{A}(t) &= \begin{bmatrix} \Delta \bar{A}(t) & 0 \\ 0 & 0 \end{bmatrix}, \\
\Delta \bar{A}_{d1}(t) &= \begin{bmatrix} \Delta \bar{A}_{d1}(t) \\ 0 \end{bmatrix}, & \Delta \bar{A}_{d2}(t) &= \begin{bmatrix} \Delta \bar{A}_{d2}(t) \\ 0 \end{bmatrix}, \\
\Delta \bar{H}(t) &= \begin{bmatrix} \Delta \bar{H}(t) & 0 \\ 0 & 0 \end{bmatrix}, & \Delta \bar{H}_{d1}(t) &= \begin{bmatrix} \Delta \bar{H}_{d1}(t) \\ 0 \end{bmatrix}, \\
\Delta \bar{H}_{d2}(t) &= \begin{bmatrix} \Delta \bar{H}_{d2}(t) \\ 0 \end{bmatrix}, & \bar{A} &= \sum_{i=1}^r \rho_i(s(t)) A_i, \\
\bar{A}_{d1} &= \sum_{i=1}^r \rho_i(s(t)) A_{d1i}, & \bar{A}_{d2} &= \sum_{i=1}^r \rho_i(s(t)) A_{d2i}, \\
\bar{H} &= \sum_{i=1}^r \rho_i(s(t)) H_i, & \bar{H}_{d1} &= \sum_{i=1}^r \rho_i(s(t)) H_{d1i}, \\
\bar{H}_{d2} &= \sum_{i=1}^r \rho_i(s(t)) H_{d2i}, & \bar{C} &= \sum_{i=1}^r \rho_i(s(t)) C_i, \\
\bar{C}_{d1} &= \sum_{i=1}^r \rho_i(s(t)) C_{d1i}, & \bar{C}_{d2} &= \sum_{i=1}^r \rho_i(s(t)) C_{d2i}, \\
\bar{B} &= \sum_{i=1}^r \rho_i(s(t)) B_i, & \bar{D} &= \sum_{i=1}^r \rho_i(s(t)) D_i, \\
\bar{L} &= [\bar{L} \quad -\bar{L}_f], & G &= [I \quad 0], \\
\bar{A}_f &= \sum_{i=1}^r \rho_i(s(t)) A_{fi}, & \bar{B}_f &= \sum_{i=1}^r \rho_i(s(t)) B_{fi}, \\
\bar{L}_f &= \sum_{i=1}^r \rho_i(s(t)) L_{fi}, \\
\bar{L} &= \sum_{i=1}^r \rho_i(s(t)) L_i, & \bar{L}_{d1} &= \sum_{i=1}^r \rho_i(s(t)) L_{d1i},
\end{aligned}$$

$$\begin{aligned}
\Psi_{12} &= [\tilde{N}_1 \quad \tilde{N}_2 \quad h_1 \tilde{N}_1 \quad h_1 \tilde{N}_2 \quad \tilde{M}_1 \quad \tilde{M}_2 \quad h_2 \tilde{M}_1 \quad h_2 \tilde{M}_2], \\
\Psi_{22} &= \text{diag} \{-Z_3, -Z_3, -h_1 Z_1, -h_1 Z_1, -Z_4, -Z_4, -h_2 Z_2, -h_2 Z_2\}, \\
\Psi_{13} &= [\check{A} G^T U_1 \quad \check{H} G^T U_2 \quad \check{H} P \quad \check{L}], \quad \Psi_{33} = \text{diag} \{-U_1, -U_2, -P, -I\}, \\
\Omega_{11} &= P(\tilde{A} + \Delta \tilde{A}(t)) + (\tilde{A} + \Delta \tilde{A}(t))^T P + G^T (Q_1 + Q_2 + h_1 R_1 + h_2 R_2) G, \\
\Omega_{14} &= P(\tilde{A}_{d1} + \Delta \tilde{A}_{d1}(t)) + G^T N_1^T, \quad \Omega_{15} = P(\tilde{A}_{d2} + \Delta \tilde{A}_{d2}(t)) + G^T M_1^T, \\
\Omega_{22} &= -Q_2 - N_2 - N_2^T, \quad \Omega_{33} = -Q_2 - M_2 - M_2^T, \quad \Omega_{44} = -N_1 - N_1^T, \quad \Omega_{55} = -M_1 - M_1^T, \\
\tilde{N}_1 &= [0 \quad 0 \quad 0 \quad N_1^T \quad 0 \quad 0 \quad 0 \quad 0]^T, \quad \tilde{M}_1 = [0 \quad 0 \quad 0 \quad 0 \quad M_1^T \quad 0 \quad 0 \quad 0]^T, \\
\tilde{N}_2 &= [0 \quad N_2^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T, \quad \tilde{M}_2 = [0 \quad 0 \quad M_2^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T, \\
\check{A} &= [\tilde{A}^T + \Delta \tilde{A}^T(t) \quad 0 \quad 0 \quad \tilde{A}_{d1}^T + \Delta \tilde{A}_{d1}^T(t) \quad \tilde{A}_{d2}^T + \Delta \tilde{A}_{d2}^T(t) \quad 0 \quad 0 \quad \tilde{B}^T]^T, \\
\check{H} &= [\tilde{H}^T + \Delta \tilde{H}^T(t) \quad 0 \quad 0 \quad \tilde{H}_{d1}^T + \Delta \tilde{H}_{d1}^T(t) \quad \tilde{H}_{d2}^T + \Delta \tilde{H}_{d2}^T(t) \quad 0 \quad 0 \quad 0]^T, \\
\check{L} &= [\tilde{L}^T \quad 0 \quad 0 \quad \tilde{L}_{d1}^T \quad \tilde{L}_{d2}^T \quad 0 \quad 0 \quad 0]^T, \quad U_1 = h_1 Z_1 + h_2 Z_2, \quad U_2 = h_1 Z_3 + h_2 Z_4.
\end{aligned} \tag{21}$$

Proof. Define the following Lyapunov-Krasovskii candidate for system (2):

$$\begin{aligned}
V(\xi(t), t) &= \xi^T(t) P \xi(t) + \int_{t-h_1}^t \xi^T(s) G^T Q_1 G \xi(s) ds \\
&+ \int_{t-h_2}^t \xi^T(s) G^T Q_2 G \xi(s) ds \\
&+ \int_{-h_1}^0 \int_{t+\theta}^t \xi^T(s) G^T R_1 G \xi(s) ds d\theta \\
&+ \int_{-h_2}^0 \int_{t+\theta}^t \xi^T(s) G^T R_2 G \xi(s) ds d\theta \\
&+ \int_{-h_1}^0 \int_{t+\beta}^t \Phi^T(s) G^T Z_1 G \Phi(s) ds d\beta \\
&+ \int_{-h_2}^0 \int_{t+\beta}^t \Phi^T(s) G^T Z_2 G \Phi(s) ds d\beta \\
&+ \int_{-h_1}^0 \int_{t+\beta}^t g^T(s) G^T Z_3 G g(s) ds d\beta \\
&+ \int_{-h_2}^0 \int_{t+\beta}^t g^T(s) G^T Z_4 G g(s) ds d\beta.
\end{aligned} \tag{22}$$

When $v(t) = 0$,

$$dV(\xi(t), t) = LV(\xi(t), t) + 2\xi^T(t) P g(t) d\omega(t). \tag{23}$$

By the Newton-Leibnitz formula, we can get the following equations for any matrices $N_i, M_i, i = 1, 2$ with any appropriate dimensions:

$$\begin{aligned}
2\eta^T(t) \bar{N}_1 G \left[\xi(t) - \xi(t - \tau_1(t)) - \int_{t-\tau_1(t)}^t \Phi(s) ds \right. \\
\left. - \int_{t-\tau_1(t)}^t g(s) d\omega(s) \right] &= 0, \\
2\eta^T(t) \bar{N}_2 G \left[\xi(t - \tau_1(t)) - \xi(t - h_1) \right. \\
\left. - \int_{t-h_1}^{t-\tau_1(t)} \Phi(s) ds - \int_{t-h_1}^{t-\tau_1(t)} g(s) d\omega(s) \right] &= 0, \\
2\eta^T(t) \bar{M}_1 G \left[\xi(t) - \xi(t - \tau_2(t)) - \int_{t-\tau_2(t)}^t \Phi(s) ds \right. \\
\left. - \int_{t-\tau_2(t)}^t g(s) d\omega(s) \right] &= 0,
\end{aligned}$$

$$2\eta^T(t) \overline{M}_2 G \left[\xi(t - \tau_2(t)) - \xi(t - h_2) - \int_{t-h_2}^{t-\tau_2(t)} \Phi(s) ds - \int_{t-h_2}^{t-\tau_2(t)} g(s) d\omega(s) \right] = 0,$$

$$\begin{aligned} & \tau_1(t) \eta^T(t) \overline{N}_1 Z_1^{-1} \overline{N}_1^T \eta(t) \\ & - \int_{t-\tau_1(t)}^t \eta^T(s) \overline{N}_1 Z_1^{-1} \overline{N}_1^T \eta(s) ds = 0, \\ & \tau_2(t) \eta^T(t) \overline{M}_1 Z_2^{-1} \overline{M}_1^T \eta(t) \\ & - \int_{t-\tau_2(t)}^t \eta^T(s) \overline{M}_1 Z_2^{-1} \overline{M}_1^T \eta(s) ds = 0, \\ & (h_1 - \tau_1(t)) \eta^T(t) \overline{N}_2 Z_1^{-1} \overline{N}_1^T \eta(t) \\ & - \int_{t-h_1}^{t-\tau_1(t)} \eta^T(s) \overline{N}_1 Z_1^{-1} \overline{N}_1^T \eta(s) ds = 0, \end{aligned}$$

$$\begin{aligned} & (h_2 - \tau_2(t)) \eta^T(t) \overline{M}_2 Z_2^{-1} \overline{M}_1^T \eta(t) \\ & - \int_{t-h_2}^{t-\tau_2(t)} \eta^T(s) \overline{M}_2 Z_2^{-1} \overline{M}_1^T \eta(s) ds = 0, \end{aligned} \quad (24)$$

where

$$\begin{aligned} \overline{N}_1 &= [0 \ 0 \ 0 \ N_1^T \ 0 \ 0 \ 0]^T, \\ \overline{N}_2 &= [0 \ N_2^T \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\ \overline{M}_1 &= [0 \ 0 \ 0 \ 0 \ M_1^T \ 0 \ 0]^T, \\ \overline{M}_2 &= [0 \ 0 \ M_2^T \ 0 \ 0 \ 0 \ 0]^T, \end{aligned} \quad (25)$$

and $\eta(t)$ is a new vector defined as follows:

$$\eta^T(t) = \left[\xi^T(t) \ \xi^T(t - h_1) G^T \ \xi^T(t - h_2) G^T \ \xi^T(t - \tau_1(t)) G^T \ \xi^T(t - \tau_2(t)) G^T \ \left(\int_{t-h_1}^t \xi(s) ds \right)^T G^T \ \left(\int_{t-h_2}^t \xi(s) ds \right)^T G^T \right]. \quad (26)$$

Using the above formulas (24) and Lemma 3, it can be seen that

LV $(\xi(t), t)$

$$\begin{aligned} & = 2\xi^T(t) P \Phi(t) + g^T(t) P g(t) \\ & + \xi^T(t) G^T Q_1 G \xi(t) + \xi^T(t) G^T Q_2 G \xi(t) \\ & - \xi^T(t - h_1) G^T Q_1 G \xi(t - h_1) \\ & - \xi^T(t - h_2) G^T Q_2 G \xi(t - h_2) + h_1 \Phi^T(t) G^T Z_1 G \Phi(t) \\ & + h_1 \xi^T(t) G^T R_1 G \xi(t) + h_2 \xi^T(t) G^T R_2 G \xi(t) \\ & + h_2 \Phi^T(t) G^T Z_2 G \Phi(t) - \int_{t-h_1}^t \xi^T(s) G^T R_1 G \xi(s) ds \\ & - \int_{t-h_2}^t \xi^T(s) G^T R_2 G \xi(s) ds + h_1 g^T(t) G^T Z_3 G g(t) \\ & - \int_{t-h_1}^t \Phi^T(s) G^T Z_1 G \Phi(s) ds \\ & - \int_{t-h_2}^t \Phi^T(s) G^T Z_2 G \Phi(s) ds + h_2 g^T(t) G^T Z_4 G g(t) \\ & - \int_{t-h_1}^t g^T(s) G^T Z_3 G g(s) ds \\ & - \int_{t-h_2}^t g^T(s) G^T Z_4 G g(s) ds \end{aligned}$$

$$\begin{aligned} & \leq \eta^T(t) \left[\overline{\Omega} + \overline{N}_1 Z_3^{-1} \overline{N}_1^T \right. \\ & + \widehat{H} (G^T h_1 Z_3 G + G^T h_2 Z_4 G + P) \widehat{H} \\ & + \widehat{A} G^T (h_1 Z_1 + h_2 Z_2) G \widehat{A}^T + \overline{N}_2 Z_3^{-1} \overline{N}_2^T \\ & + h_1 \overline{N}_1 Z_1^{-1} \overline{N}_1^T + h_1 \overline{N}_2 Z_1^{-1} \overline{N}_2^T + h_2 \overline{M}_1 Z_2^{-1} \overline{M}_1^T \\ & + h_2 \overline{M}_2 Z_2^{-1} \overline{M}_2^T + \overline{M}_1 Z_4^{-1} \overline{M}_1^T + \overline{M}_2 Z_4^{-1} \overline{M}_2^T \left. \right] \eta(t) \\ & - \int_{t-\tau_1(t)}^t \left[\eta^T(s) \overline{N}_1 + \Phi^T(s) G^T Z_1 \right] Z_1^{-1} \\ & \quad \times \left[Z_1 G \Phi(s) + \overline{N}_1^T \eta(s) \right] ds \\ & - \int_{t-h_1}^{t-\tau_1(t)} \left[\eta^T(s) \overline{N}_2 + \Phi^T(s) G^T Z_1 \right] Z_1^{-1} \\ & \quad \times \left[Z_1 G \Phi(s) + \overline{N}_2^T \eta(s) \right] ds \\ & - \int_{t-\tau_2(t)}^t \left[\eta^T(s) \overline{M}_1 + \Phi^T(s) G^T Z_2 \right] Z_2^{-1} \\ & \quad \times \left[Z_2 G \Phi(s) + \overline{M}_1^T \eta(s) \right] ds \\ & - \int_{t-h_2}^{t-\tau_2(t)} \left[\eta^T(s) \overline{M}_2 + \Phi^T(s) G^T Z_2 \right] Z_2^{-1} \\ & \quad \times \left[Z_2 G \Phi(s) + \overline{M}_2^T \eta(s) \right] ds \end{aligned}$$

$$\begin{aligned}
& + \left(\int_{t-\tau_1(t)}^t g(s) d\omega(s) \right)^T G^T Z_3 G \left(\int_{t-\tau_1(t)}^t g(s) d\omega(s) \right) - \int_{t-\tau_2(t)}^t g^T(s) G^T Z_4 G g(s) ds \\
& - \int_{t-\tau_1(t)}^t g^T(s) G^T Z_3 G g(s) ds \\
& + \left(\int_{t-h_1}^{t-\tau_1(t)} g(s) d\omega(s) \right)^T G^T Z_3 G \left(\int_{t-h_1}^{t-\tau_1(t)} g(s) d\omega(s) \right) \\
& - \int_{t-h_1}^{t-\tau_1(t)} g^T(s) G^T Z_3 G g(s) ds \\
& + \left(\int_{t-\tau_2(t)}^t g(s) d\omega(s) \right)^T G^T Z_4 G \left(\int_{t-\tau_2(t)}^t g(s) d\omega(s) \right) \\
& - \int_{t-h_2}^{t-\tau_2(t)} g^T(s) G^T Z_4 G g(s) ds,
\end{aligned} \tag{27}$$

where

$$\bar{\Omega} = \begin{bmatrix} \Omega_{11} & 0 & 0 & \Omega_{14} & \Omega_{15} & 0 & 0 \\ * & \Omega_{22} & 0 & 0 & N_2 & M_2 & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 \\ * & * & * & * & * & -\frac{R_1}{h_1} & 0 \\ * & * & * & * & * & * & -\frac{R_2}{h_2} \end{bmatrix}, \tag{28}$$

$$\hat{A} = \begin{bmatrix} (\bar{A} + \Delta \bar{A}(t))^T & 0 & 0 & (\bar{A}_{d1} + \Delta \bar{A}_{d1}(t))^T & (\bar{A}_{d2} + \Delta \bar{A}_{d2}(t))^T & 0 & 0 \end{bmatrix}^T,$$

$$\hat{H} = \begin{bmatrix} (\bar{H} + \Delta \bar{H}(t))^T & 0 & 0 & (\bar{H}_{d1} + \Delta \bar{H}_{d1}(t))^T & (\bar{H}_{d2} + \Delta \bar{H}_{d2}(t))^T & 0 & 0 \end{bmatrix}^T.$$

Note that

$$\begin{aligned}
& \mathbb{E} \left(\int_{t-\tau_1(t)}^t g^T(s) G^T Z_3 G g(s) ds \right) \\
& = \mathbb{E} \left(\left(\int_{t-\tau_1(t)}^t g^T(s) d\omega(s) \right)^T \right. \\
& \quad \left. \times G^T Z_3 G \left(\int_{t-\tau_1(t)}^t g^T(s) d\omega(s) \right) \right), \\
& \mathbb{E} \left(\int_{t-h_1}^{t-\tau_1(t)} g^T(s) G^T Z_3 G g(s) ds \right) \\
& = \mathbb{E} \left(\left(\int_{t-h_1}^{t-\tau_1(t)} g^T(s) d\omega(s) \right)^T \right. \\
& \quad \left. \times G^T Z_3 G \left(\int_{t-h_1}^{t-\tau_1(t)} g^T(s) d\omega(s) \right) \right), \\
& \mathbb{E} \left(\int_{t-\tau_2(t)}^t g^T(s) G^T Z_4 G g(s) ds \right) \\
& = \mathbb{E} \left(\left(\int_{t-\tau_2(t)}^t g^T(s) d\omega(s) \right)^T \right. \\
& \quad \left. \times G^T Z_4 G \left(\int_{t-\tau_2(t)}^t g^T(s) d\omega(s) \right) \right), \\
& \mathbb{E} \left(\int_{t-h_2}^{t-\tau_2(t)} g^T(s) G^T Z_4 G g(s) ds \right) \\
& = \mathbb{E} \left(\left(\int_{t-h_2}^{t-\tau_2(t)} g^T(s) d\omega(s) \right)^T \right. \\
& \quad \left. \times G^T Z_4 G \left(\int_{t-h_2}^{t-\tau_2(t)} g^T(s) d\omega(s) \right) \right),
\end{aligned} \tag{29}$$

So, the mathematical expectation of the last eight parts of (27) equals 0. And applying the Schur complement to (3), we can derive the following inequality with $v(t) = 0$:

$$\begin{aligned}
& \bar{\Omega} + \bar{N}_1 Z_3^{-1} \bar{N}_1^T + \hat{H} (G^T h_1 Z_3 G + G^T h_2 Z_4 G + P) \hat{H} \\
& + \hat{A} G^T (h_1 Z_1 + H_2 Z_2) G \hat{A}^T + \bar{N}_2 Z_3^{-1} \bar{N}_2^T \\
& + h_1 \bar{N}_1 Z_1^{-1} \bar{N}_1^T + h_1 \bar{N}_2 Z_1^{-1} \bar{N}_2^T + h_2 \bar{M}_1 Z_2^{-1} \bar{M}_1^T \\
& + h_2 \bar{M}_2 Z_2^{-1} \bar{M}_2^T + \bar{M}_1 Z_4^{-1} \bar{M}_1^T + \bar{M}_2 Z_4^{-1} \bar{M}_2^T < 0.
\end{aligned} \tag{30}$$

From (27), (29), and (30), we can see that

$$\mathbf{L}V(\xi(t), t) < 0, \quad (31)$$

which, by Definition 2.1 [37, 38], ensures that system $(\tilde{\Sigma})$ with $v(t) = 0$ is robustly stochastically stable. By Itô's formula, we can derive

$$\mathbb{E}(V(\xi(t), t)) = \mathbb{E}\left(\int_0^t \mathbf{L}V(\xi(s), s) ds\right). \quad (32)$$

Now, set a functional J_T as

$$J_T = \mathbb{E}\left(\int_0^T [z(t)^T z(t) - \gamma^2 v(t)^T v(t)] ds\right), \quad (33)$$

where $t > 0$. From (32) and (33), it is easy to show that

$$\begin{aligned} J_T &= \mathbb{E}\left(\int_0^T [z(t)^T z(t) - \gamma^2 v(t)^T v(t) + \mathbf{L}V(\xi(s), s)] ds\right) \\ &\quad - \mathbb{E}(V(\xi(t), t)) \\ &\leq \mathbb{E}\left(\int_0^T [z(t)^T z(t) - \gamma^2 v(t)^T v(t) + \mathbf{L}V(\xi(s), s)] ds\right), \end{aligned} \quad (34)$$

for all $t > 0$. By using the Schur complement to (3), there is

$$z(t)^T z(t) - \gamma^2 v(t)^T v(t) + \mathbf{L}V(\xi(s), s) \leq \bar{\eta}(t)^T \Psi \bar{\eta}(t) < 0, \quad (35)$$

where

$$\bar{\eta}^T(t) = \begin{bmatrix} \xi^T(t) & \xi^T(t-h_1)G^T & \xi^T(t-h_2)G^T & \xi^T(t-\tau_1(t))G^T & \xi^T(t-\tau_2(t))G^T & \left(\int_{t-h_1}^t \xi(s)ds\right)G^T & \left(\int_{t-h_2}^t \xi(s)ds\right)G^T & v(t) \end{bmatrix}. \quad (36)$$

It follows from (33) and (34) that

$$J(t) < 0, \quad (37)$$

for all $t > 0$, which implies that (10) is satisfied. This completes the proof. $\square$

Remark 5. The Lyapunov functional (22) contains the information of the uppers bound of the delays; by such a choice, delay-dependent results are obtained.

4. Robust H_∞ Filter Design

In this section, a sufficient condition for the solvability of robust H_∞ filter problem for uncertain T-S fuzzy stochastic time-delay system is investigated. The main result is given in the following theorem by LMI form.

Theorem 6. Consider the uncertain T-S fuzzy stochastic time-delay system (Σ) . The robust H_∞ filtering problem is solvable with disturbance attenuation $\gamma > 0$ if there exist scalars $\varepsilon_i > 0$ and matrices $W > 0$, $X > 0$, $Q_i > 0$, $R_i > 0$, $i = 1, 2$; $Z_j > 0$, $j = 1, 2, 3, 4$; N_{1i} , N_{2i} , M_{1i} , M_{2i} , $i = 1, 2, 3, 4$;

$\{\Gamma_i = \Gamma_i^T, 1 \leq i \leq r\}$, $\{\Pi_{ij}, 1 \leq i < j \leq r\}$, such that the following LMIs hold:

$$\begin{bmatrix} \Theta_{ii} - \Gamma_i + \varepsilon_i \Xi_i \Xi_i^T & \varrho_i \\ * & -\varepsilon_i \end{bmatrix} < 0, \quad (1 \leq i \leq r), \quad (38)$$

$$\begin{bmatrix} \Theta_{ij} + \Theta_{ji} - \Pi_{ij} - \Pi_{ij}^T + \varepsilon_i \Xi_i \Xi_i^T + \varepsilon_j \Xi_j \Xi_j^T & \varrho_i & \varrho_j \\ * & -\varepsilon_i & 0 \\ * & * & -\varepsilon_j \end{bmatrix} < 0, \quad (1 \leq i < j \leq r), \quad (39)$$

$$\begin{bmatrix} \Gamma_1 & \Pi_{12} & \cdots & \Pi_{1r} \\ * & \Gamma_2 & \cdots & \Pi_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Gamma_r \end{bmatrix} < 0, \quad (40)$$

$$X - W > 0, \quad (41)$$

where

$$\varrho_i^T = [K_{1i}^T W \quad K_{1i}^T X \quad \mathbf{0}_{1 \times 15} \quad K_{1i}^T U_1 \quad K_{2i}^T U_2 \quad K_{2i}^T X \quad K_{2i}^T],$$

$$\Xi_i^T = [T_{1i}^T \quad T_{1i}^T \quad 0 \quad 0 \quad T_{2i}^T \quad T_{3i}^T \quad \mathbf{0}_{15 \times 1}], \quad \Theta_{ij} = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \mathbf{0}_{8 \times 8} \\ * & * & \Theta_{33} \end{bmatrix},$$

$$\begin{aligned}
\Theta_{11} &= \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & 0 & 0 & \Lambda_{15} & \Lambda_{16} & 0 & 0 & \Lambda_{19} \\ * & \Lambda_{22} & 0 & 0 & \Lambda_{25} & \Lambda_{26} & 0 & 0 & \Lambda_{29} \\ * & * & \Lambda_{33} & 0 & \Lambda_{35} & \Lambda_{36} & 0 & 0 & 0 \\ * & * & * & \Lambda_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -N_1 - N_1^T & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -M_1 - M_1^T & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{R_1}{h_1} & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{R_2}{h_2} & 0 \\ * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}, \\
\Lambda_{11} &= WA_i + A_i^T W + Q_1 + Q_2 + h_1 R_1 + h_2 R_2, \\
\Lambda_{12} &= WA_i + A_i^T X + C_j^T \Phi_{1i}^T + \Phi_{2i}^T + Q_1 + Q_2 + h_1 R_1 + h_2 R_2, \\
\Lambda_{15} &= N_1^T + WA_{d1i}, \quad \Lambda_{16} = M_1^T + WA_{d2i}, \quad \Lambda_{19} = WB_i, \quad \Lambda_{29} = XB_i + \Phi_{1i} D_j, \\
\Lambda_{22} &= X^T A_i + \Phi_{1i} C_j + C_j^T \Phi_{1i}^T + A_i^T X + Q_1 + Q_2 + h_1 R_1 + h_2 R_2, \\
\Lambda_{25} &= N_1^T + X^T A_{d1i} + \Phi_{1i} C_{d1j}, \quad \Lambda_{26} = M_1^T + X^T A_{d2i} + \Phi_{1i} C_{d2j}, \\
\Lambda_{33} &= -Q_1 - N_2 - N_2^T, \quad \Lambda_{35} = N_2, \quad \Lambda_{36} = M_2, \quad \Lambda_{44} = -Q_2 - M_2 - M_2^T, \\
\Theta_{12} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_2 & 0 & h_2 N_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_2 & 0 & h_2 M_2 \\ N_1 & 0 & h_1 N_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_1 & 0 & h_2 M_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Theta_{22} &= \text{diag} \{-Z_3, -Z_3, -h_1 Z_1, -h_1 Z_1, -Z_4, -Z_4, -h_2 Z_2, -h_2 Z_2\}, \\
\Theta_{13} &= \begin{bmatrix} h_1 A_i^T Z_1 + H_2 A_i^T Z_2 & h_1 H_i Z_3 + h_2 H_i Z_4 & H_i^T X & H_i^T & L_i^T - \Phi_{3i}^T \\ h_1 A_i^T Z_1 + H_2 A_i^T Z_2 & h_1 H_i Z_3 + h_2 H_i Z_4 & H_i^T X & H_i^T & L_i^T \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ h_1 A_{d1i}^T Z_1 + H_2 A_{d1i}^T Z_2 & h_1 H_{d1i} Z_3 + h_2 H_{d1i} Z_4 & H_{d1i}^T X & H_{d1i}^T & 0 \\ h_1 A_{d2i}^T Z_1 + H_2 A_{d2i}^T Z_2 & h_1 H_{d2i} Z_3 + h_2 H_{d2i} Z_4 & H_{d2i}^T X & H_{d2i}^T & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Theta_{33} &= \text{diag} \left\{ -h_1 Z_1 - h_2 Z_2, -h_1 Z_3 - h_2 Z_4, \begin{bmatrix} -X & -I \\ * & -\Phi_4 \end{bmatrix}, -I \right\}.
\end{aligned} \tag{42}$$

When the LMIs (38)–(41) are feasible, there exist nonsingular matrices σ and β satisfying $\sigma\beta^T = I - XW^{-1}$. Under this condition, a desired time-dependent filter can be chosen as

$$L_{fi} = \Phi_{3i} W^{-1} \beta^{-T}, \quad i = 1, \dots, r. \tag{43}$$

Proof. From the demonstration in [21], we know that $I - XW^{-1}$ is nonsingular. Therefore, there always exist nonsingular matrices σ and β such that $\sigma\beta^T = I - XW^{-1}$

holds. Then, we define the nonsingular matrices χ_1 and χ_2 as follows:

$$\chi_1 = \begin{bmatrix} W^{-1} & I \\ \beta^T & o \end{bmatrix}; \quad \chi_2 = \begin{bmatrix} I & X \\ 0 & \sigma^T \end{bmatrix}. \tag{44}$$

Let $P = \chi_2 \chi_1^{-1}$; there is

$$P = \begin{bmatrix} X & \sigma \\ \sigma^T & \beta^{-1} W^{-1} (X_W) W^{-1} \beta^{-T} \end{bmatrix}, \tag{45}$$

which means that $P > 0$.

Now, using Lemma 3 and recalling (40), we can deduce that

$$\begin{aligned}
 \kappa &= \sum_{i=1}^r \rho_i^2(s(t)) \left[\Theta_{ii} + \varrho_i F_i(t) \Xi_i^T + \Xi_i F_i(t) \varrho_i^T \right] \\
 &\quad + \sum_{i=1}^r \sum_{j>i}^r \rho_i(s(t)) \rho_j(s(t)) \\
 &\quad \times \left[\Theta_{ij} + \varrho_i F_i(t) \Xi_i^T + \Xi_i F_i(t) \varrho_i^T \right. \\
 &\quad \left. + \Theta_{ji} + \varrho_j F_j(t) \Xi_j^T + \Xi_j F_j(t) \varrho_j^T \right] \\
 &< \sum_{i=1}^r \rho_i^2(s(t)) \left[\Theta_{ii} + \varepsilon_i^{-1} \varrho_i \varrho_i^T + \varepsilon_i \Xi_i \Xi_i^T \right] \\
 &\quad + \sum_{i=1}^r \sum_{j>i}^r \rho_i(s(t)) \rho_j(s(t)) \left[\Theta_{ij} + \varepsilon_i^{-1} \varrho_i \varrho_j^T + \varepsilon_i \Xi_i \Xi_j^T \right. \\
 &\quad \left. + \Theta_{ji} + \varepsilon_j^{-1} \varrho_j \varrho_i^T + \varepsilon_j \Xi_j \Xi_i^T \right] \\
 &< \sum_{i=1}^r \rho_i^2(s(t)) \Gamma_i + \sum_{i=1}^r \sum_{j>i}^r \rho_i(s(t)) \rho_j(s(t)) \left[\Pi_{ij} + \Pi_{ji}^T \right] \\
 &= \begin{bmatrix} h_1(s(t))I \\ h_2(s(t))I \\ \vdots \\ h_r(s(t))I \end{bmatrix}^T \begin{bmatrix} \Gamma_1 & \Pi_{12} & \cdots & \Pi_{1r} \\ * & \Gamma_2 & \cdots & \Pi_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Gamma_r \end{bmatrix} \begin{bmatrix} h_1(s(t))I \\ h_2(s(t))I \\ \vdots \\ h_r(s(t))I \end{bmatrix} < 0.
 \end{aligned} \tag{46}$$

Then, it is easy to see that

$$\begin{aligned}
 &\left\{ \text{diag} \left(\chi_2^{-T} \begin{bmatrix} W^{-1} & 0 \\ 0 & I \end{bmatrix}, I, \dots, I \right) \right\} \kappa \left\{ \text{diag} \left(\chi_2^{-T} \begin{bmatrix} W^{-1} & 0 \\ 0 & I \end{bmatrix}, I, \dots, I \right) \right\} \\
 &= \begin{bmatrix} \Omega & \Psi_{12} & \Psi_{13} \\ * & \Psi_{22} & \mathbf{0} \\ * & * & \Psi_{33} \end{bmatrix} < 0,
 \end{aligned} \tag{47}$$

which is equivalent to (3). Therefore, the condition in Theorem 4 is satisfied when the LMIs in (38)–(41) hold. Finally, it can be concluded that the filtering error system $(\tilde{\Sigma})$ is stochastically stable with H_∞ performance level γ . $\square$

Remark 7. There is more than one time delay appear in the electric system, the network system, and so on. So, it makes sense to investigate the system containing two terms of time delays as shown in this paper. The model we consider here contains two terms of time-varying delay as $\tau_1(t)$ and $\tau_1(t)$ in the state $x(t)$. The method we use can be easily drawn back to the system that has only one time delay. Meanwhile, the result we get can be extended to the system that contains more delays at the same time by developing the Lyapunov function with $h_3, h_4, \dots$ integral terms in the same way.

5. Numerical Example

In this section, we provide a numerical example to show the effectiveness of the results obtained in the previous section.

FIGURE 1: State responses of $x(t)$ and $\hat{x}(t)$.

FIGURE 2: Responses of the signal $\tilde{z}(t) = z(t) - \hat{z}(t)$.

Example 1. Consider the T-S fuzzy stochastic system $(\tilde{\Sigma})$ with model parameters given as follows:

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -1.9 & 0.4 \\ 0.2 & -1.1 \end{bmatrix}, & A_{d11} &= \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \\
 A_{d21} &= \begin{bmatrix} -0.02 & 0.01 \\ 0.05 & -0.05 \end{bmatrix}, & H_1 &= \begin{bmatrix} -0.04 & 0.1 \\ 0.03 & -0.05 \end{bmatrix}, \\
 H_{d11} &= \begin{bmatrix} -0.01 & 0.02 \\ 0.01 & -0.05 \end{bmatrix}, & H_{d21} &= \begin{bmatrix} -0.04 & 0.01 \\ 0.03 & -0.05 \end{bmatrix},
 \end{aligned}$$

$$\begin{aligned}
C_1 &= [1 \ -0.4], & C_{d11} &= [-0.4 \ 0.1], \\
C_{d21} &= [-0.1 \ -0.4], & D_1 &= 0.3, \\
L_1 &= [1.0 \ -0.5], & L_{d11} &= [1 \ -0.5], \\
L_{d21} &= [1 \ -0.5], & A_2 &= \begin{bmatrix} -2.1 & 0.2 \\ 0.1 & -1.1 \end{bmatrix}, \\
A_{d12} &= \begin{bmatrix} -0.9 & 0.1 \\ -1.1 & -1.2 \end{bmatrix}, & A_{d22} &= \begin{bmatrix} -0.04 & 0.05 \\ 0.02 & -0.04 \end{bmatrix}, \\
H_2 &= \begin{bmatrix} -0.4 & 0.01 \\ 0.03 & -0.05 \end{bmatrix}, & H_{d12} &= \begin{bmatrix} -0.05 & 0.01 \\ 0.03 & -0.05 \end{bmatrix}, \\
H_{d22} &= \begin{bmatrix} -0.06 & 0.02 \\ 0.01 & -0.05 \end{bmatrix}, \\
C_2 &= [0.2 \ -0.5], & C_{d12} &= [-0.4 \ 0.5], \\
C_{d22} &= [-0.3 \ -0.4], & L_2 &= [-0.2 \ 0.3], \\
L_{d12} &= [-0.2 \ -0.5], & L_{d22} &= [-0.3 \ -0.5] \\
B_1 &= \begin{bmatrix} 1.0 \\ -0.2 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, & D_2 &= -0.3,
\end{aligned} \tag{48}$$

and the parameter uncertainties are shown as

$$\begin{aligned}
K_{11} &= \begin{bmatrix} 1.0 \\ -0.5 \end{bmatrix}, & K_{12} &= \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix}, \\
K_{21} &= \begin{bmatrix} 0.8 \\ -0.1 \end{bmatrix}, & K_{22} &= \begin{bmatrix} -0.1 \\ 0.4 \end{bmatrix}, \\
T_{11} &= [0 \ -0.3], & T_{21} &= [-0.2 \ 0.1] \\
T_{31} &= [-0.5 \ 0.1], & T_{12} &= [0 \ -0.3] \\
T_{22} &= [0 \ -0.1], & T_{32} &= [-0.3 \ -0.2].
\end{aligned} \tag{49}$$

The two membership functions are, respectively, described as

$$\begin{aligned}
h_1(x_1(t)) &= \left(1 - \frac{1}{1 + \exp(6x_1(t) + 2)}\right), \\
h_2(x_1(t)) &= 1 - h_1.
\end{aligned} \tag{50}$$

In this example, the noise attenuation level γ is set to $\gamma = 0.42$. From Theorem 6, it is to see that the robust H_∞ filtering problem is solvable for any $0 < h_1(t) \leq 0.8$, $0 < h_2(t) \leq 0.9$. Thus, a desired fuzzy H_∞ filter can be constructed as in the form of (11) with

$$\left[\begin{array}{c|c|c} A_{f1} & B_{f1} & L_{f1} \\ \hline A_{f2} & B_{f2} & L_{f2} \end{array} \right] = \left[\begin{array}{cc|cc} -69.057 & 76.641 & -0.514 & -24.7515 \\ 48.860 & 53.611 & 0.427 & -26.0768 \\ \hline 46.823 & 58.857 & -0.083 & -76.7602 \\ -131.781 & -161.087 & 0.116 & -93.953 \end{array} \right]. \tag{51}$$

The simulation results of the state response of the plant and the filter are given in Figure 1, where the initial condition is $x_0(t) = [0.4 \ 2.5]^T$, $\hat{x}_0(t) = [0 \ 0]^T$. Figure 2 shows the simulation results of the signal $z(t) - \hat{z}(t)$, and the exogenous disturbance input $v(t)$ is given by $v(t) = 12/(5 + 3.1t)$, $t \geq 0$, which belongs to $\mathcal{L}_2[0, \infty)$.

6. Conclusion

This paper has investigated the filter design problem for the uncertain T-S fuzzy stochastic system with time-varying delays. An LMI approach has been developed to design the fuzzy filter ensuring not only the robust stochastic mean-square stability but also a prescribed H_∞ performance level of the filtering error system for all admissible uncertainties. A numerical example has been provided to show the effectiveness of the proposed filter design methods.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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