Research Article

Composite Disturbance Observer-Based Control and H_∞ Output Tracking Control for Discrete-Time Switched Systems with Time-Varying Delay

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Received 28 January 2013; Accepted 3 April 2013

Academic Editor: Yang Yi

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This paper considers the problem of H_∞ output tracking control for discrete-time switched systems with time-varying delay and external disturbances. The control scheme combining disturbance observer-based control (DOBC) and H_∞ control is proposed. The disturbances are assumed to include two parts. One part is generated by an exogenous system, which imposes on system with control inputs in the same channel. The other part is supposed to have the bounded H_2 norm. A new disturbance observer is developed to estimate and reject the first case disturbances for switched system with time-varying delay, and the second case disturbances are attenuated by H_∞ control scheme. The stability analysis of the closed-loop system is developed by switched Lyapunov function, and a solvable delay-dependent sufficient condition is presented in terms of linear matrix inequalities (LMIs) and cone complement linearization (CCL) methods. A numerical example is given to demonstrate the effectiveness of the proposed composite control scheme.

1. Introduction

Switched systems are a special class of hybrid control systems, which are composed of a family of subsystems described by continuous or discrete time dynamics and a switched rule among them [1, 2]. Switched systems can be employed to represent many practical systems, for example, power electronics, embedded systems, chemical processes, computer-controlled systems, automotive industries, and so on. Hence, switched systems have attracted considerable attention during the last decade, and many meaningful results have been reported on stability analysis and controller design for switched systems (see, e.g., [3–14]). By far, there are a number of methodologies on dealing with the stability analysis and control synthesis of switched systems, such as common Lyapunov function [1, 11], multiple Lyapunov function [3, 12], dwell time and average dwell time method [7, 13], and switched Lyapunov function [4, 14].

On the one hand, time delay always encounters in various engineering systems, such as long transmission lines in pneumatic, hydraulic, and chemical processes, economic and rolling mill systems, to name a few, which usually leads to instability and poor performance of the closed-loop system. Recently, many researches have paid more and more attention to stabilization and performance analysis of systems or switched systems with delays [15–20]. On the other hand, disturbances widely exist in systems, which may degrade the closed-loop system performance. Usually, some performance indexes, that is, H_∞ and $L_2 - L_\infty$ ($l_2 - l_\infty$), are employed to deal with external disturbances. Many results have been developed with these performance indexes [10, 13, 18, 19, 21–25]. However, disturbances need to satisfy the bounded H_2 norm. In reality, many disturbances do not satisfy this condition, for example, constant disturbances or harmonics disturbances. To improve the ability of disturbance attenuation and rejection, disturbance observer-based

control (DOBC) scheme has been studied from 1980s and applied in many control fields [26–37], which can be regarded as an active disturbance rejection method. In this method, disturbances are estimated by disturbance observer (DOB), and the disturbance estimation is employed to feedforward compensation.

The problem of output tracking control is important and numerous results have been developed for kinds of systems in the literature [38–40]. Along with the development of switched system theory, increased attention has been paid to studying output tracking controller design for switched systems. In [41–43], the tracking control problems have been considered for kinds of continuous switched systems. Exponential l_2 – l_∞ output tracking control problem has been solved in [13] for a class of discrete-time switched systems. However, in [13], disturbances are assumed to be the bounded H_2 norm.

In this paper, a composite control scheme is developed to solve the problem of H_∞ output tracking control for discrete-time switched systems with time-varying delay and external disturbances. Here, external disturbances can be divided into two parts. One part is generated by exogenous system, which imposes on system in the same channel with control inputs. The other part satisfies bounded H_2 norm. A new disturbance observer is designed to estimate and reject the first case disturbances for switched system with time-varying delay. H_∞ control is employed to analyze attenuation performance with respect to the second case disturbances. Hence, a composite control method, consisting of DOBC and H_∞ control, is proposed. By resorting to the switched Lyapunov function approach and inspired by [44, 45], some delay-dependent conditions for the problem of H_∞ output tracking control are presented. In order to obtain the desired controller and observer gains, a cone complement linearization (CCL) method is used to transform the nonconvex feasibility problem to some sequential optimization problem subject to linear matrix inequalities (LMIs) constraints. Finally, a numerical example is provided to demonstrate the effectiveness of the main result.

2. Problem Formulation and Preliminaries

Consider the following discrete-time switched systems with time-varying delays described by

$$\begin{aligned} \Sigma_0: \quad & x(k+1) = A_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-\tau(k)) \\ & + B_{\sigma(k)}(u(k) + d_1(k)) + B_{1\sigma(k)}d_2(k), \\ & z(k) = C_{\sigma(k)}x(k) + C_{d\sigma(k)}x(k-\tau(k)) + D_{\sigma(k)}u(k), \\ & x(s) = \phi(s), \quad s = -\bar{\tau}, \dots, -1, 0, \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the states, $z(k) \in \mathbb{R}^q$ is the signal to be estimated, and $u(k)$ is the control input. $\sigma(k) : \mathbb{Z}^+ \rightarrow \mathcal{N} = \{1, 2, \dots, N\}$ is the switching signal, which specifies which subsystem will be activated at a certain discrete time

instant and $A_i, A_{di}, B_i, B_{1i}, C_i, C_{di}, D_i, i \in \mathcal{N}$, are constant matrices with appropriate dimensions. $\tau(k)$ is a time-varying delay of the system and satisfies

$$\underline{\tau} \leq \tau(k) \leq \bar{\tau}, \quad \forall k \in \mathbb{Z}^+, \quad (2)$$

where $\underline{\tau}, \bar{\tau}, \tau(k)$ are nonnegative integer numbers. $\phi(k)$ is the initial condition. $d_2(k) \in \mathbb{R}^p$ is the external disturbances, which is assumed to belong to $l_2[0, \infty)$. $d_1(k)$ is also the external disturbances, which is generated by the exogenous system

$$\omega(k+1) = W\omega(k), \quad d_1(k) = V\omega(k), \quad (3)$$

where (W, B_iV) is observable.

Remark 1. It is clear that $\tau(k)$ is an interval-like time-varying delay. When $\underline{\tau} = 0$, it is reduced to $0 \leq \tau(k) \leq \bar{\tau}$, for all $k \in \mathbb{Z}^+$, which was recently discussed in [23]. Therefore, this note can be viewed as some extensions of their results.

Remark 2. System Σ_0 contains two kinds of external disturbances: matched external disturbances and mismatched external disturbances. Two different methods are employed to deal with these disturbances. First, a disturbance observer is introduced to estimate matched disturbances; then the estimation values are used to feedforward compensation. Then, a performance index, that is, H_∞ performance index, is presented to reject and attenuate mismatched disturbances.

Assume that the reference signal $z_r(k)$ is generated by the following system:

$$\begin{aligned} x_r(k+1) &= A_r x_r(k) + B_r r(k), \\ z_r(k) &= C_r x_r(k), \end{aligned} \quad (4)$$

where $x_r(k)$ is reference states, $r(k) \in l_2[0, \infty)$ is reference input, and A_r is Hurwitz matrix with an appropriate dimension. Here we are interested in designing a state feedback controller by the following formula:

$$u(k) = K_{1\sigma(k)}x(k) - \hat{d}_1(k) + K_{2\sigma(k)}x_r(k), \quad (5)$$

where $\hat{d}_1(k)$ is the estimation of the disturbances $d_1(k)$, which is obtained by the following disturbance observer:

$$\begin{aligned} \hat{\omega}(k) &= v(k) - Lx(k), \\ v(k+1) &= W(v(k) - Lx(k)) \\ &+ L(A_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-\tau(k)) \\ &+ B_{\sigma(k)}(u(k) + \hat{d}_1(k))), \\ \hat{d}_1(k) &= V\hat{\omega}(k). \end{aligned} \quad (6)$$

$$(7)$$

Remark 3. In this paper, we assume that the switching signal $\sigma(k)$ is not known a priori but its instantaneous value is available in real time [4]. Here we only consider the case of synchronous switching; that is, the controller switches just as the system Σ_0 does.

Defining $e_\omega(k) = \omega(k) - \hat{\omega}(k)$ yields

$$e_\omega(k+1) = (W + LB_{\sigma(k)}V)e_\omega(k) + \bar{B}_{1\sigma(k)}d_2(k), \quad (8)$$

where $\bar{B}_{1\sigma(k)} = LB_{1\sigma(k)}$.

Applying controller (5) to system Σ_0 and combining (3) and (7), we obtain

$$\begin{aligned} x(k+1) &= \bar{A}_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-\tau(k)) \\ &\quad + B_{\sigma(k)}(d_1(k) - \hat{d}_1(k)) + B_{1\sigma(k)}d_2(k) \\ &\quad + \bar{B}_{\sigma(k)}x_r(k) \\ &= \bar{A}_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-\tau(k)) + \hat{B}_{\sigma(k)}e_\omega(k) \\ &\quad + B_{1\sigma(k)}d_2(k) + \bar{B}_{\sigma(k)}x_r(k), \end{aligned} \quad (9)$$

where

$$\begin{aligned} \bar{A}_\sigma(k) &= A_{\sigma(k)} + B_{\sigma(k)}K_{1\sigma(k)}, & \hat{B}_{\sigma(k)} &= B_{\sigma(k)}V, \\ \bar{B}_{\sigma(k)} &= B_{\sigma(k)}K_{2\sigma(k)}, & e_\omega(k) &= \omega(k) - \hat{\omega}(k). \end{aligned} \quad (10)$$

Let

$$e(k) = z(k) - z_r(k), \quad (11)$$

then we have the following augmented switched system:

$$\begin{aligned} \Sigma_e: \zeta(k+1) &= \mathcal{A}_{\sigma(k)}\zeta(k) + \mathcal{A}_{d\sigma(k)}\zeta(k-\tau(k)) \\ &\quad + \mathcal{B}_{\sigma(k)}e_\omega(k) + \mathcal{B}_{1\sigma(k)}v(k), \\ e_\omega(k+1) &= (W + LB_{\sigma(k)}V)e_\omega(k) + \bar{\mathcal{B}}_{1\sigma(k)}v(k), \\ e(k) &= \mathcal{C}_{\sigma(k)}\zeta(k) + \mathcal{C}_{d\sigma(k)}\zeta(k-\tau(k)), \end{aligned} \quad (12)$$

where

$$\begin{aligned} \mathcal{A}_{\sigma(k)} &= \begin{bmatrix} \bar{A}_{\sigma(k)} & \bar{B}_{\sigma(k)} \\ 0 & A_r \end{bmatrix}, & \mathcal{A}_{d\sigma(k)} &= \begin{bmatrix} A_{d\sigma(k)} & 0 \\ 0 & 0 \end{bmatrix}, \\ \mathcal{B}_{\sigma(k)} &= \begin{bmatrix} \hat{B}_{\sigma(k)} \\ 0 \end{bmatrix}, & \mathcal{B}_{1\sigma(k)} &= \begin{bmatrix} B_{1\sigma(k)} & 0 \\ 0 & B_r \end{bmatrix}, \\ \zeta(k) &= \begin{bmatrix} x(k) \\ x_r(k) \end{bmatrix} \in \mathbb{R}^m, & v(k) &= \begin{bmatrix} d_2(k) \\ r(k) \end{bmatrix}, \\ \mathcal{C}_{\sigma(k)} &= [C_{\sigma(k)} + D_{\sigma(k)}K_{1\sigma(k)} \quad D_{\sigma(k)}K_{2\sigma(k)} - C_r], \\ \mathcal{C}_{d\sigma(k)} &= [C_{d\sigma(k)} \quad 0], & \bar{\mathcal{B}}_{1\sigma(k)} &= [\bar{B}_{1\sigma(k)} \quad 0]. \end{aligned} \quad (13)$$

By defining

$$\begin{aligned} \widehat{\mathcal{A}}_{\sigma(k)} &= \begin{bmatrix} A_{\sigma(k)} & 0 \\ 0 & A_r \end{bmatrix}, & \widehat{\mathcal{B}}_{\sigma(k)} &= \begin{bmatrix} B_{\sigma(k)} \\ 0 \end{bmatrix}, \\ K_{\sigma(k)} &= [K_{1\sigma(k)} \quad K_{2\sigma(k)}], \\ \widehat{\mathcal{C}}_{\sigma(k)} &= [C_{\sigma(k)} \quad -C_r], \end{aligned} \quad (14)$$

then we have

$$\begin{aligned} \mathcal{A}_{\sigma(k)} &= \widehat{\mathcal{A}}_{\sigma(k)} + \widehat{\mathcal{B}}_{\sigma(k)}K_{\sigma(k)}, \\ \mathcal{C}_{\sigma(k)} &= \widehat{\mathcal{C}}_{\sigma(k)} + D_{\sigma(k)}K_{\sigma(k)}, \end{aligned} \quad (15)$$

and the controller can be rewritten as

$$u(k) = K_{\sigma(k)}\zeta(k) - \hat{d}_1(k). \quad (16)$$

In order to prepare for a precise formulation, we introduce the following definition.

Definition 4 (H_∞ output tracking control problem). Consider system Σ_e . Given a prescribed $\gamma > 0$, if there exists composite controller (5) such that the following two conditions are satisfied:

(R1) system Σ_e is asymptotically stable when $v(k) = 0$, for all $k \geq 0$;

(R2) under the zero-initial condition, the following inequality holds:

$$\|e(k)\|_2 < \gamma\|v(k)\|_2, \quad (17)$$

for any nonzero $v(k) \in l_2[0, \infty)$, then system Σ_e is asymptotically stable with an H_∞ performance index γ .

3. Main Results

3.1. H_∞ Output Tracking Performance Analysis. In this subsection, we focus on developing delay-dependent condition to solve the H_∞ output tracking control problem formulated in previous section.

Theorem 5 (consider system Σ_e). For scalars $\underline{\tau} > 0$, $\bar{\tau} > 0$, $\gamma > 0$, the error system Σ_e is asymptotically stable with an H_∞ performance index, if there exist matrices $M_1, M_2, N_1, N_2, T_1, T_2, K_{1i}, K_{2i}, P_i > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, R_1 > 0, R_2 > 0, S > 0, i \in \mathcal{N}$, such that the following inequality holds:

$$\Phi_i = \begin{bmatrix} \Phi_{11i} & \Phi_{12} & 0 & 0 & M_1^T & -T_1^T & \mathcal{A}_i^T & \Phi_{18i} & \Phi_{19i} & \Phi_{1,10} & \Phi_{1,11} & \Phi_{1,12} & 0 \\ * & \Phi_{22i} & 0 & 0 & M_2^T & -T_2^T & \mathcal{A}_{di}^T & \mathcal{A}_{di}^T & \mathcal{A}_{di}^T & \Phi_{2,10} & \Phi_{2,11} & \Phi_{2,12} & 0 \\ * & * & -S & 0 & 0 & 0 & \mathcal{B}_i^T & \mathcal{B}_i^T & \mathcal{B}_i^T & 0 & 0 & 0 & \Phi_{3,13i} \\ * & * & * & -\gamma^2 I & 0 & 0 & \mathcal{B}_{1i}^T & \mathcal{B}_{1i}^T & \mathcal{B}_{1i}^T & 0 & 0 & 0 & \check{\mathcal{B}}_{1i}^T \\ * & * & * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -P_l^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Phi_{88} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Phi_{99} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Phi_{10,10} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Phi_{11,11} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Phi_{12,12} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -S^{-1} \end{bmatrix} < 0, \quad (18)$$

where

$$\begin{aligned} \Phi_{11i} = & -P_i + Q_1 + Q_2 + (\bar{\tau} - \underline{\tau} + 1)Q_3 \\ & + N_1 + N_1^T + \mathcal{C}_i^T \mathcal{C}_i + \mathcal{C}_i^T \mathcal{C}_{di}, \end{aligned}$$

$$\Phi_{12} = -N_1^T + N_2 + T_1^T - M_1^T, \quad \Phi_{18i} = \Phi_{19i} = (\mathcal{A}_i - I)^T,$$

$$\Phi_{1,10} = \bar{\tau} N_1^T, \quad \Phi_{1,11} = (\bar{\tau} - \underline{\tau}) T_1^T,$$

$$\Phi_{1,13} = (\bar{\tau} - \underline{\tau}) M_1^T,$$

$$\Phi_{22i} = -Q_3 - N_2 - N_2^T + T_2 + T_2^T - M_2 - M_2^T + \mathcal{C}_{di}^T \mathcal{C}_{di},$$

$$\Phi_{2,10} = \bar{\tau} N_2^T, \quad \Phi_{2,11} = (\bar{\tau} - \underline{\tau}) T_2^T,$$

$$\Phi_{2,12} = (\bar{\tau} - \underline{\tau}) M_2^T, \quad \Phi_{3,13i} = (W + LB_i V)^T,$$

$$\Phi_{88} = -(\bar{\tau} R_1)^{-1}, \quad \Phi_{99} = -((\bar{\tau} - \underline{\tau}) R_2)^{-1},$$

$$\Phi_{10,10} = -\bar{\tau} R_1,$$

$$\Phi_{11,11} = -(\bar{\tau} - \underline{\tau})(R_1 + R_2), \quad \Phi_{12,12} = -(\bar{\tau} - \underline{\tau}) R_2. \quad (19)$$

Proof. Choose a Lyapunov-Krasovskii functional candidate as

$$V(k) = V_1(k) + V_2(k) + V_3(k), \quad (20)$$

where

$$\begin{aligned} V_1(k) = & \zeta^T(k) P_{\sigma(k)} \zeta(k) + e_{\omega}^T(k) S e_{\omega}(k), \\ V_2(k) = & \sum_{\theta=k-\underline{\tau}}^{k-1} \zeta^T(\theta) Q_1 \zeta(\theta) + \sum_{\theta=k-\bar{\tau}}^{k-1} \zeta^T(\theta) Q_2 \zeta(\theta) \\ & + \sum_{\theta=k-\tau(k)}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta), \end{aligned}$$

$$\begin{aligned} V_3(k) = & \sum_{i=-\bar{\tau}}^{-1} \sum_{\theta=k+i}^{k-1} \rho^T(\theta) R_1 \rho(\theta) \\ & + \sum_{i=-\bar{\tau}+1}^{-\underline{\tau}} \sum_{\theta=k+i-1}^{k-1} \rho^T(\theta) R_2 \rho(\theta) \\ & + \sum_{i=-\bar{\tau}+2}^{-\underline{\tau}+1} \sum_{\theta=k+i-1}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta), \end{aligned} \quad (21)$$

and $\rho(\theta) = \zeta(\theta+1) - \zeta(\theta)$.

Without loss of generality, we assume that $\sigma(k+1) = l$, $\sigma(k) = i$, for all $i, l \in \mathcal{N}$. Then taking the forward difference yields

$$\begin{aligned} \Delta V_1(k) = & V_1(k+1) - V_1(k) \\ = & \zeta^T(k+1) P_l \zeta(k+1) - \zeta^T(k) P_i \zeta(k) \\ & + e_{\omega}^T(k+1) S e_{\omega}(k+1) - e_{\omega}^T(k) S e_{\omega}(k) \quad (22) \\ = & \xi^T(k) \mathcal{A}_i^T P_l \mathcal{A}_i \xi(k) - \zeta^T(k) P_i \zeta(k) \\ & + \xi^T(k) \mathcal{B}_i^T S \mathcal{B}_i \xi(k) - e_{\omega}^T(k) S e_{\omega}(k), \end{aligned}$$

where

$$\begin{aligned} \xi^T(k) = & [\zeta^T(k) \quad \zeta^T(k - \tau(k)) \quad e_{\omega}^T(k) \quad v^T(k)], \\ \mathcal{A}_i = & [\mathcal{A}_i \quad \mathcal{A}_{di} \quad \mathcal{B}_i \quad \mathcal{B}_{1i}], \quad (23) \\ \mathcal{B}_i = & [0 \quad 0 \quad (W + LB_i V) \quad \check{\mathcal{B}}_{1i}]. \end{aligned}$$

Direct computation gives

$$\begin{aligned}
\Delta V_2(k) &= \sum_{\theta=k+1-\underline{\tau}}^k \zeta^T(\theta) Q_1 \zeta(\theta) - \sum_{\theta=k-\underline{\tau}}^{k-1} \zeta^T(\theta) Q_1 \zeta(\theta) \\
&+ \sum_{\theta=k+1-\bar{\tau}}^k \zeta^T(\theta) Q_2 \zeta(\theta) - \sum_{\theta=k-\bar{\tau}}^{k-1} \zeta^T(\theta) Q_2 \zeta(\theta) \\
&+ \sum_{\theta=k+1-\tau(k+1)}^k \zeta^T(\theta) Q_3 \zeta(\theta) \\
&- \sum_{\theta=k-\tau(k)}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta) \\
&= \zeta^T(k) (Q_1 + Q_2 + Q_3) \zeta(k) \\
&- \zeta^T(k-\underline{\tau}) Q_1 \zeta(k-\underline{\tau}) \\
&- \zeta^T(k-\bar{\tau}) Q_2 \zeta(k-\bar{\tau}) \\
&- \zeta^T(k-\tau(k)) Q_3 \zeta(k-\tau(k)) \\
&+ \sum_{\theta=k+1-\tau(k+1)}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta) \\
&- \sum_{\theta=k+1-\tau(k)}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta).
\end{aligned} \tag{24}$$

Note that

$$\begin{aligned}
&\sum_{\theta=k+1-\tau(k+1)}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta) \\
&= \sum_{\theta=k+1-\underline{\tau}}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta) + \sum_{\theta=k+1-\tau(k+1)}^{k-\underline{\tau}} \zeta^T(\theta) Q_3 \zeta(\theta) \\
&\leq \sum_{\theta=k+1-\tau(k)}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta) + \sum_{\theta=k+1-\bar{\tau}}^{k-\bar{\tau}} \zeta^T(\theta) Q_3 \zeta(\theta).
\end{aligned} \tag{25}$$

Hence we obtain

$$\begin{aligned}
\Delta V_2(k) &\leq \zeta^T(k) (Q_1 + Q_2 + Q_3) \zeta(k) \\
&- \zeta^T(k-\underline{\tau}) Q_1 \zeta(k-\underline{\tau}) \\
&- \zeta^T(k-\bar{\tau}) Q_2 \zeta(k-\bar{\tau}) \\
&- \zeta^T(k-\tau(k)) Q_3 \zeta(k-\tau(k)) \\
&+ \sum_{\theta=k+1-\bar{\tau}}^{k-\bar{\tau}} \zeta^T(\theta) Q_3 \zeta(\theta).
\end{aligned} \tag{26}$$

After some manipulations, the following inequality is satisfied:

$$\begin{aligned}
\Delta V_3(k) &= \sum_{i=-\bar{\tau}}^{-1} \sum_{\theta=k+1+i}^k \rho^T(\theta) R_1 \rho(\theta) \\
&- \sum_{i=-\bar{\tau}}^{-1} \sum_{\theta=k+i}^{k-1} \rho^T(\theta) R_1 \rho(\theta) \\
&+ \sum_{i=-\bar{\tau}+1}^{-\bar{\tau}} \sum_{\theta=k+i}^k \rho^T(\theta) R_2 \rho(\theta) \\
&- \sum_{i=-\bar{\tau}+1}^{-\bar{\tau}} \sum_{\theta=k+i-1}^{k-1} \rho^T(\theta) R_2 \rho(\theta) \\
&+ \sum_{i=-\bar{\tau}+2}^{-\bar{\tau}+1} \sum_{\theta=k+i}^k \zeta^T(\theta) Q_3 \zeta(\theta) \\
&- \sum_{i=-\bar{\tau}+2}^{-\bar{\tau}+1} \sum_{\theta=k+i-1}^{k-1} \zeta^T(\theta) Q_3 \zeta(\theta) \\
&= \bar{\tau} \rho^T(k) R_1 \rho(k) - \sum_{\theta=k-\bar{\tau}}^{k-1} \rho^T(\theta) R_1 \rho(\theta) \\
&+ (\bar{\tau} - \underline{\tau}) \rho^T(k) R_2 \rho(k) - \sum_{\theta=k-\bar{\tau}}^{k-\underline{\tau}-1} \rho^T(\theta) R_2 \rho(\theta) \\
&+ (\bar{\tau} - \underline{\tau}) \zeta^T(k) Q_3 \zeta(k) - \sum_{\theta=k+1-\bar{\tau}}^{k-\bar{\tau}} \zeta^T(\theta) Q_3 \zeta(\theta).
\end{aligned} \tag{27}$$

Observe that the following equalities hold naturally:

$$\begin{aligned}
&2 \left(\zeta^T(k) N_1^T + \zeta^T(k-\tau(k)) N_2^T \right) \\
&\times \left(\zeta(k) - \zeta(k-\tau(k)) - \sum_{\theta=k-\tau(k)}^{k-1} \rho(\theta) \right) = 0, \\
&2 \left(\zeta^T(k) T_1^T + \zeta^T(k-d_j(k)) T_2^T \right) \\
&\times \left(\zeta(k-\tau(k)) - \zeta(k-\bar{\tau}) - \sum_{\theta=k-\bar{\tau}}^{k-1-\tau(k)} \rho(\theta) \right) = 0, \\
&2 \left(\zeta^T(k) M_1^T + \zeta^T(k-\tau(k)) M_2^T \right) \\
&\times \left(\zeta(k-\underline{\tau}) - \zeta(k-\tau(k)) - \sum_{\theta=k-\tau(k)}^{k-1-\underline{\tau}} \rho(\theta) \right) = 0.
\end{aligned} \tag{28}$$

Then

$$\begin{aligned}
& - \sum_{\theta=k-\bar{\tau}}^{k-1} \rho^T(\theta) R_1 \rho(\theta) - \sum_{\theta=k-\bar{\tau}}^{k-\bar{\tau}-1} \rho^T(\theta) R_2 \rho(\theta) \\
& = - \sum_{\theta=k-\bar{\tau}}^{k-\tau(k)-1} \rho^T(\theta) (R_1 + R_2) \rho(\theta) \\
& \quad - \sum_{\theta=k-\tau(k)}^{k-1} \rho^T(\theta) R_1 \rho(\theta) - \sum_{\theta=k-\tau(k)}^{k-\bar{\tau}-1} \rho^T(\theta) R_2 \rho(\theta) \\
& \quad + 2 \left(\zeta^T(k) N_1^T + \zeta^T(k - \tau(k)) N_2^T \right) \\
& \quad \times \left(\zeta(k) - \zeta(k - \tau(k)) - \sum_{\theta=k-\tau(k)}^{k-1} \rho(\theta) \right) \\
& \quad + 2 \left(\zeta^T(k) T_1^T + \zeta^T(k - \tau(k)) T_2^T \right) \\
& \quad \times \left(\zeta(k - \tau(k)) - \zeta(k - \bar{\tau}) - \sum_{\theta=k-\bar{\tau}}^{k-1-\tau(k)} \rho(\theta) \right) \\
& \quad + 2 \left(\zeta^T(k) M_1^T + \zeta^T(k - \tau(k)) M_2^T \right) \\
& \quad \times \left(\zeta(k - \underline{\tau}) - \zeta(k - \tau(k)) - \sum_{\theta=k-\tau(k)}^{k-1-\bar{\tau}} \rho(\theta) \right) \\
& = 2 \left(\zeta^T(k) N_1^T + \zeta^T(k - \tau(k)) N_2^T \right) \\
& \quad \times (\zeta(k) - \zeta(k - \tau(k))) \\
& \quad + 2 \left(\zeta^T(k) T_1^T + \zeta^T(k - \tau(k)) T_2^T \right) \\
& \quad \times (\zeta(k - \tau(k)) - \zeta(k - \bar{\tau})) \\
& \quad + 2 \left(\zeta^T(k) M_1^T + \zeta^T(k - \tau(k)) M_2^T \right) \\
& \quad \times (\zeta(k - \underline{\tau}) - \zeta(k - \tau(k))) \\
& \quad - 2 \left(\zeta^T(k) T_1^T + \zeta^T(k - \tau(k)) T_2^T \right) \\
& \quad \times \sum_{\theta=k-\bar{\tau}}^{k-1-\tau(k)} \rho(\theta) - \sum_{\theta=k-\bar{\tau}}^{k-\tau(k)-1} \rho^T(\theta) (R_1 + R_2) \rho(\theta) \\
& \quad - 2 \left(\zeta^T(k) N_1^T + \zeta^T(k - \tau(k)) N_2^T \right) \\
& \quad \times \sum_{\theta=k-\tau(k)}^{k-1} \rho(\theta) - \sum_{\theta=k-\tau(k)}^{k-1} \rho^T(\theta) R_1 \rho(\theta) \\
& \quad - 2 \left(\zeta^T(k) M_1^T + \zeta^T(k - \tau(k)) M_2^T \right) \\
& \quad \times \sum_{\theta=k-\tau(k)}^{k-1-\bar{\tau}} \rho(\theta) - \sum_{\theta=k-\tau(k)}^{k-\bar{\tau}-1} \rho^T(\theta) R_2 \rho(\theta)
\end{aligned}$$

$$\begin{aligned}
& = 2 \left(\zeta^T(k) N_1^T + \zeta^T(k - \tau(k)) N_2^T \right) \\
& \quad \times (\zeta(k) - \zeta(k - \tau(k))) \\
& \quad + 2 \left(\zeta^T(k) T_1^T + \zeta^T(k - \tau(k)) T_2^T \right) \\
& \quad \times (\zeta(k - \tau(k)) - \zeta(k - \bar{\tau})) \\
& \quad + 2 \left(\zeta^T(k) M_1^T + \zeta^T(k - \tau(k)) M_2^T \right) \\
& \quad \times (\zeta(k - \underline{\tau}) - \zeta(k - \tau(k))) \\
& \quad + \tau(k) \left(\zeta^T(k) N_1^T + \zeta^T(k - \tau(k)) N_2^T \right) R_1^{-1} \\
& \quad \times (N_1 \zeta(k) + N_2 \zeta(k - \tau(k))) \\
& \quad + (\bar{\tau} - \tau(k)) \left(\zeta^T(k) T_1^T + \zeta^T(k - \tau(k)) T_2^T \right) \\
& \quad \times (R_1 + R_2)^{-1} (T_1 \zeta(k) + T_2 \zeta(k - \tau(k))) \\
& \quad + (\tau(k) - \underline{\tau}) \left(\zeta^T(k) M_1^T + \zeta^T(k - \tau(k)) M_2^T \right) R_2^{-1} \\
& \quad \times (M_1 \zeta(k) + M_2 \zeta(k - \tau(k))) \\
& \quad - \sum_{\theta=k-\tau(k)}^{k-1} \eta^T(k) N^T R_1^{-1} N \eta(k) \\
& \quad - \sum_{\theta=k-\bar{\tau}}^{k-1-\tau(k)} \eta^T(k) T^T (R_1 + R_2)^{-1} T \eta(k) \\
& \quad - \sum_{\theta=k-\tau(k)}^{k-\bar{\tau}-1} \eta^T(k) M^T R_2^{-1} M \eta(k) \\
& \leq 2 \left(\zeta^T(k) N_1^T + \zeta^T(k - \tau(k)) N_2^T \right) \\
& \quad \times (\zeta(k) - \zeta(k - \tau(k))) \\
& \quad + 2 \left(\zeta^T(k) T_1^T + \zeta^T(k - \tau(k)) T_2^T \right) \\
& \quad \times (\zeta(k - \tau(k)) - \zeta(k - \bar{\tau})) \\
& \quad + 2 \left(\zeta^T(k) M_1^T + \zeta^T(k - \tau(k)) M_2^T \right) \\
& \quad \times (\zeta(k - \underline{\tau}) - \zeta(k - \tau(k))) \\
& \quad + \bar{\tau} \left(\zeta^T(k) N_1^T + \zeta^T(k - \tau(k)) N_2^T \right) R_1^{-1} \\
& \quad \times (N_1 \zeta(k) + N_2 \zeta(k - \tau(k))) \\
& \quad + (\bar{\tau} - \underline{\tau}) \left(\zeta^T(k) T_1^T + \zeta^T(k - \tau(k)) T_2^T \right) \\
& \quad \times (R_1 + R_2)^{-1} (T_1 \zeta(k) + T_2 \zeta(k - \tau(k))) \\
& \quad + (\bar{\tau} - \underline{\tau}) \left(\zeta^T(k) M_1^T + \zeta^T(k - \tau(k)) M_2^T \right) R_2^{-1} \\
& \quad \times (M_1 \zeta(k) + M_2 \zeta(k - \tau(k))),
\end{aligned}$$

(29)

where

$$\begin{aligned}\eta^T(k) &= [\zeta^T(k) \quad \zeta^T(k - \tau(k)) \quad \rho^T(\theta)], \\ N &= [N_1 \quad N_2 \quad R_1], \\ M &= [M_1 \quad M_2 \quad R_2], \quad T = [T_1 \quad T_2 \quad R_1 + R_2].\end{aligned}\quad (30)$$

From (22)–(29), and by some manipulations, we obtain

$$\begin{aligned}\Delta V(k) &\leq \lambda^T(k) \Phi_{1i} \lambda(k) + \xi^T(k) \Phi_{2i} \xi(k) + \bar{\eta}^T(k) \Phi_3 \bar{\eta}(k) \\ &\quad - \bar{\eta}^T(k) \begin{bmatrix} \mathcal{C}_i^T \\ \mathcal{C}_{di}^T \end{bmatrix} [\mathcal{C}_i \quad \mathcal{C}_{di}] \bar{\eta}(k),\end{aligned}\quad (31)$$

where

$$\begin{aligned}\lambda^T(k) &= [\zeta^T(k) \quad \zeta^T(k - \tau(k)) \quad e_\omega^T(k) \quad v^T(k) \quad \zeta^T(k - \underline{\tau}) \quad \zeta^T(k - \bar{\tau})], \\ \bar{\eta}^T(k) &= [\zeta^T(k) \quad \zeta^T(k - \tau(k))], \\ \Phi_{2i} &= [\mathcal{A}_i^T P_i \mathcal{A}_i + \mathcal{B}_i^T S \mathcal{B}_i + \bar{\mathcal{A}}_i^T (\bar{\tau} R_1 + (\bar{\tau} - \underline{\tau}) R_2) \bar{\mathcal{A}}_i], \\ \bar{\mathcal{A}}_i &= [\mathcal{A}_i - \bar{I}], \quad \bar{I} = [I \quad 0 \quad 0 \quad 0], \\ \Phi_3 &= \bar{\tau} [N_1 \quad N_2]^T R_1^{-1} [N_1 \quad N_2] + (\bar{\tau} - \underline{\tau}) [T_1 \quad T_2]^T \\ &\quad \times (R_1 + R_2)^{-1} [T_1 \quad T_2] \\ &\quad + (\bar{\tau} - \underline{\tau}) [M_1 \quad M_2]^T R_2^{-1} [M_1 \quad M_2], \\ \Phi_{1i} &= \begin{bmatrix} \Phi_{11i} & \Phi_{12} & 0 & 0 & M_1^T & -T_1^T \\ * & \Phi_{22i} & 0 & 0 & M_2^T & -T_2^T \\ * & * & -S & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & -Q_1 & 0 \\ * & * & * & * & * & -Q_2 \end{bmatrix}.\end{aligned}\quad (32)$$

Now, we develop the conclusion from two aspects. We first establish the asymptotic stability of system Σ_e under the condition of zero disturbances. In fact, when $v(k) = 0$, it is verified that

$$\begin{aligned}\Delta V(k) &\leq \tilde{\lambda}^T(k) \tilde{\Phi}_{1i} \tilde{\lambda}(k) + \xi^T(k) \tilde{\Phi}_{2i} \xi(k) \\ &\quad + \sum_{j=1}^m \bar{\eta}_j^T(k) \Phi_{3j} \bar{\eta}_j(k) - \zeta^T(k) \mathcal{G}_i^T \mathcal{G}_i \zeta(k),\end{aligned}\quad (33)$$

where

$$\begin{aligned}\tilde{\lambda}^T(k) &= [\zeta^T(k) \quad \zeta^T(k - \tau(k)) \quad e_\omega^T(k) \quad \zeta^T(k - \underline{\tau}) \quad \zeta^T(k - \bar{\tau})], \\ \tilde{\Phi}_{2i} &= [\tilde{\mathcal{A}}_i^T P_i \tilde{\mathcal{A}}_i + \tilde{\mathcal{B}}_i^T S \tilde{\mathcal{B}}_i + \tilde{\mathcal{A}}_i^T (\bar{\tau} R_1 + (\bar{\tau} - \underline{\tau}) R_2) \tilde{\mathcal{A}}_i], \\ \tilde{\mathcal{A}}_i &= [\mathcal{A}_i \quad \mathcal{A}_{di} \quad \mathcal{B}], \quad \tilde{\mathcal{B}}_i = [\mathcal{A}_i - \bar{I}], \quad \bar{I} = [I \quad 0 \quad 0], \\ \tilde{\mathcal{B}} &= [0 \quad 0 \quad W + LB_i V], \\ \tilde{\Phi}_{1i} &= \begin{bmatrix} \Phi_{11i} & \Phi_{12} & 0 & M_1^T & -T_1^T \\ * & \Phi_{22i} & 0 & M_2^T & -T_2^T \\ * & * & -S & 0 & 0 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_2 \end{bmatrix}.\end{aligned}\quad (34)$$

Applying Schur complement formula, we obtain $\Delta V(k) < 0$ if (18) is true. Therefore, it is easy to see that the error system Σ_e is asymptotically stable by the Lyapunov-Krasovskii stability theorem.

Next, we will present that under the zero-initial condition, the time-delay system Σ_e satisfies (17) for all nonzero $v(k) \in l_2[0, \infty)$. To this end, we introduce

$$J_N = \sum_{k=0}^N (e^T(k) e(k) - \gamma^2 v^T(k) v(k) + \Delta V(k)) - V(N+1). \quad (35)$$

Then, by the Schur complement formula, it easily follows from (18) and (31) that

$$J_N \leq \sum_{k=0}^{\infty} (e^T(k) e(k) - \gamma^2 v^T(k) v(k) + \Delta V(k)) < 0, \quad (36)$$

which implies that $\|e(k)\|_2 < \gamma \|v(k)\|_2$ holds under the zero-initial condition. This completes the proof. $\square$

Remark 6. It is obvious that the condition in (18) is not an LMI with respect to the parameters $P_1, \dots, P_N, R_1, R_2, S$. In order to solve the controller in the form of (5), we will cast the H_∞ output tracking control problem into an LMI framework.

3.2. H_∞ Output Tracking Controller Design. In this subsection, we try to obtain a solvable condition for the problem of H_∞ output tracking controller design using CCL method.

Define

$$\begin{aligned} X_i &= P_i^{-1}, \quad \bar{R}_1 = R_1^{-1}, \quad \bar{R}_2 = R_2^{-1}, \\ \bar{K}_i &= K_i X_i, \quad \bar{L} = SL, \\ \bar{Q}_{1i} &= X_i Q_1 X_i, \quad \bar{Q}_{2i} = X_i Q_2 X_i, \quad \bar{Q}_{3i} = X_i Q_3 X_i, \\ \bar{M}_1 &= X_i M_1 X_i, \quad \bar{M}_2 = X_i M_2 X_i, \quad \bar{N}_1 = X_i N_1 X_i, \end{aligned}$$

$$\bar{N}_2 = X_i N_2 X_i, \quad \bar{T}_1 = X_i T_1 X_i,$$

$$\bar{T}_2 = X_i T_2 X_i,$$

$$\Gamma = \text{diag} \{X_i, X_i, I, I, X_i, X_i, I, I, I, X_i, X_i, X_i, S\}.$$

(37)

Pre- and postmultiplying Γ and Γ^T on (18), and using Schur complement formula, we obtain

$$\bar{\Phi}_i = \begin{bmatrix} \bar{\Phi}_{11i} & \bar{\Phi}_{12} & 0 & 0 & \bar{M}_1^T & -\bar{T}_1^T & \bar{\Phi}_{17i} & \bar{\Phi}_{18i} & \bar{\Phi}_{19i} & \bar{\Phi}_{1,10} & \bar{\Phi}_{1,11} & \bar{\Phi}_{1,12} & 0 & \bar{\Phi}_{1,14i} \\ * & \bar{\Phi}_{22i} & 0 & 0 & \bar{M}_2^T & -\bar{T}_2^T & X_i \mathcal{A}_{di}^T & X_i \mathcal{A}_{di}^T & X_i \mathcal{A}_{di}^T & \bar{\Phi}_{2,10} & \bar{\Phi}_{2,11} & \bar{\Phi}_{2,12} & 0 & \bar{\Phi}_{2,14i} \\ * & * & -S & 0 & 0 & 0 & \mathcal{B}_i^T & \mathcal{B}_i^T & \mathcal{B}_i^T & 0 & 0 & 0 & \bar{\Phi}_{3,13i} & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & \mathcal{B}_{1i}^T & \mathcal{B}_{1i}^T & \mathcal{B}_{1i}^T & 0 & 0 & 0 & \bar{\Phi}_{4,13i} & 0 \\ * & * & * & * & -\bar{Q}_{1i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{Q}_{2i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -X_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \bar{\Phi}_{88} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \bar{\Phi}_{99} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \bar{\Phi}_{10,10i} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \bar{\Phi}_{11,11i} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \bar{\Phi}_{12,12i} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (38)$$

where

$$\bar{\Phi}_{11i} = -X_i + \bar{Q}_{1i} + \bar{Q}_{2i} + (\bar{\tau} - \underline{\tau} + 1) \bar{Q}_{3i} + \bar{N}_1 + \bar{N}_1^T,$$

$$\bar{\Phi}_{12} = -\bar{N}_1^T + \bar{N}_2 + \bar{T}_1^T - \bar{M}_1^T,$$

$$\bar{\Phi}_{17i} = X_i \mathcal{A}_i^T + \bar{K}_i^T \mathcal{B}_i^T,$$

$$\bar{\Phi}_{18i} = \bar{\Phi}_{19i} = \bar{\Phi}_{17i} - X_i,$$

$$\bar{\Phi}_{1,10} = \bar{\tau} \bar{N}_1^T, \quad \bar{\Phi}_{1,11} = (\bar{\tau} - \underline{\tau}) \bar{T}_1^T,$$

$$\bar{\Phi}_{1,13} = (\bar{\tau} - \underline{\tau}) \bar{M}_1^T, \quad \bar{\Phi}_{1,14i} = X_i \mathcal{E}_i^T + \bar{K}_i^T D_i^T,$$

$$\bar{\Phi}_{22i} = -\bar{Q}_{3i} - \bar{N}_2 - \bar{N}_2^T + \bar{T}_2 + \bar{T}_2^T - \bar{M}_2 - \bar{M}_2^T,$$

$$\bar{\Phi}_{2,14i} = X_i \mathcal{E}_{di}^T, \quad \bar{\Phi}_{2,10} = \bar{\tau} \bar{N}_2^T,$$

$$\bar{\Phi}_{2,11} = (\bar{\tau} - \underline{\tau}) \bar{T}_2^T, \quad \bar{\Phi}_{2,12} = (\bar{\tau} - \underline{\tau}) \bar{M}_2^T,$$

$$\bar{\Phi}_{3,13i} = W^T S + (\bar{L} B_i V)^T,$$

$$\bar{\Phi}_{88} = -\frac{1}{\bar{\tau} \bar{R}_1}, \quad \bar{\Phi}_{99} = -\frac{1}{(\bar{\tau} - \underline{\tau}) \bar{R}_2},$$

$$\bar{\Phi}_{10,10i} = -\bar{\tau} X_i R_1 X_i, \quad \bar{\Phi}_{4,13i} = \check{\mathcal{B}}_{1i}^T S,$$

$$\bar{\Phi}_{11,11i} = -(\bar{\tau} - \underline{\tau}) X_i (R_1 + R_2) X_i,$$

$$\bar{\Phi}_{12,12i} = -(\bar{\tau} - \underline{\tau}) X_i R_2 X_i.$$

(39)

Theorem 7 (consider the system Σ_e). For scalars $\underline{\tau} > 0$, $\bar{\tau} > 0$, $\gamma > 0$, the error system Σ_e is asymptotically stable with an H_∞ performance index, if there exist matrices $\bar{M}_1, \bar{M}_2, \bar{N}_1, \bar{N}_2, \bar{T}_1, \bar{T}_2, \bar{K}_{1i}, \bar{K}_{2i}, X_i > 0, \bar{Q}_1 > 0, \bar{Q}_2 > 0, \bar{Q}_3 > 0, \bar{R}_1 > 0, \bar{R}_2 > 0, R_1 > 0, R_2 > 0, S > 0, i \in \mathcal{N}$, such that inequality (38) holds.

It is clear that (38) is a nonlinear matrix inequality due to the existence of terms $\bar{R}_1, \bar{R}_2, X_i R_1 X_i$ and $X_i R_2 X_i$. In the sequel, the CCL method is resorted to solve the desired controller gains and observer gain.

Introduce two new variables S_{1i} and S_{2i} such that $X_i R_1 X_i \geq S_{1i}$ and $X_i R_2 X_i \geq S_{2i}$, then we obtain the following results.

Theorem 8 (Consider system Σ_e). For scalars $\underline{\tau} > 0$, $\bar{\tau} > 0$, $\gamma > 0$, the error system Σ_e is asymptotically stable with an H_∞ performance index, if there exist matrices $\bar{M}_1, \bar{M}_2, \bar{N}_1, \bar{N}_2, \bar{T}_1, \bar{T}_2, \bar{K}_{1i}, \bar{K}_{2i}, X_i > 0, \bar{Q}_1 > 0, \bar{Q}_2 > 0, \bar{Q}_3 > 0, \bar{R}_1 > 0, \bar{R}_2 > 0, R_1 > 0, R_2 > 0, S > 0, S_{1i}, S_{2i}, i \in \mathcal{N}$, such that the following inequalities hold

$$\widehat{\Phi}_i = \begin{bmatrix} \overline{\Phi}_{11i} & \overline{\Phi}_{12} & 0 & 0 & \overline{M}_1^T & -\overline{T}_1^T & \overline{\Phi}_{17i} & \overline{\Phi}_{18i} & \overline{\Phi}_{19i} & \overline{\Phi}_{1,10} & \overline{\Phi}_{1,11} & \overline{\Phi}_{1,12} & 0 & \overline{\Phi}_{1,14i} \\ * & \overline{\Phi}_{22i} & 0 & 0 & \overline{M}_2^T & -\overline{T}_2^T & X_i \mathcal{A}_{di}^T & X_i \mathcal{A}_{di}^T & X_i \mathcal{A}_{di}^T & \overline{\Phi}_{2,10} & \overline{\Phi}_{2,11} & \overline{\Phi}_{2,12} & 0 & \overline{\Phi}_{2,14i} \\ * & * & -S & 0 & 0 & 0 & \mathcal{B}_i^T & \mathcal{B}_i^T & \mathcal{B}_i^T & 0 & 0 & 0 & \overline{\Phi}_{3,13i} & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & \mathcal{B}_{1i}^T & \mathcal{B}_{1i}^T & \mathcal{B}_{1i}^T & 0 & 0 & 0 & \overline{\Phi}_{4,13i} & 0 \\ * & * & * & * & -\overline{Q}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\overline{Q}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -X_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \overline{\Phi}_{88} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \overline{\Phi}_{99} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \widehat{\Phi}_{10,10i} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \widehat{\Phi}_{11,11i} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \widehat{\Phi}_{12,12i} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (40)$$

$$X_i R_1 X_i \geq S_{1i}, \quad (41)$$

$$X_i R_2 X_i \geq S_{2i}, \quad (42)$$

where $\widehat{\Phi}_{10,10i} = -\overline{\tau} S_{1i}$, $\widehat{\Phi}_{11,11i} = -(\overline{\tau} - \underline{\tau})(S_{1i} + S_{2i})$, $\widehat{\Phi}_{12,12i} = -(\overline{\tau} - \underline{\tau}) S_{2i}$.

From (41) and (42), we know that the conditions in Theorem 8 are not strict linear matrix inequalities. By the assistance of the CCL method [46], the nonconvex feasibility problem formulated by (40)–(42) can be transformed into the following nonlinear minimization problem:

$$\text{Minimize} \quad \text{Tr} \left(\sum_{i=1}^N (X_i H_i + S_{1i} J_{1i} + S_{2i} J_{2i}) + \overline{R}_1 G_1 + \overline{R}_2 G_2 \right)$$

Subject to (40) and

$$\begin{aligned} \begin{bmatrix} G_1 & H_i \\ H_i & J_{1i} \end{bmatrix} &\geq I, & \begin{bmatrix} G_2 & H_i \\ H_i & J_{2i} \end{bmatrix} &\geq I, \\ \begin{bmatrix} S_{1i} & I \\ I & J_{1i} \end{bmatrix} &\geq 0, & \begin{bmatrix} S_{2i} & I \\ I & J_{2i} \end{bmatrix} &\geq 0, \\ \begin{bmatrix} \overline{R}_1 & I \\ I & G_1 \end{bmatrix} &\geq 0, & \begin{bmatrix} \overline{R}_2 & I \\ I & G_2 \end{bmatrix} &\geq 0, \\ \begin{bmatrix} X_i & I \\ I & H_i \end{bmatrix} &\geq 0. \end{aligned} \quad (43)$$

If the solution of the above minimization problem is $(6N + 4)m$, then system Σ_e is asymptotically stable with an H_∞ performance index via controller (5) and observer (6) with gains $K_i = \overline{K}_i X_i^{-1}$ and $L = S^{-1} \overline{L}$, respectively.

4. A Numerical Example

Now, we provide an example to show the effectiveness of the main result in this paper.

Consider discrete-time switched system with parameters as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.2 & 0 \\ 0 & -0.035 \end{bmatrix}, & A_{d1} &= \begin{bmatrix} 0.02 & 0 \\ 0.01 & -0.03 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 8 \\ 7 \end{bmatrix}, \\ B_{11} &= \begin{bmatrix} -0.03 \\ 0.05 \end{bmatrix}, & C_1 &= [1.2 \quad 0.8], \\ C_{d1} &= [1 \quad -0.6], & D_1 &= 16, \\ A_2 &= \begin{bmatrix} -0.3 & 0 \\ 0.01 & -0.015 \end{bmatrix}, & A_{d2} &= \begin{bmatrix} 0.01 & -0.01 \\ 0.02 & -0.05 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 10 \\ 10 \end{bmatrix}, & B_{12} &= \begin{bmatrix} -0.01 \\ 0.07 \end{bmatrix}, & C_2 &= [0.6 \quad 1.6], \\ C_{d2} &= [0.8 \quad -0.4], & D_2 &= 14. \end{aligned} \quad (44)$$

The reference model is given by the following parameters:

$$\begin{aligned} A_r &= \begin{bmatrix} 0.5 & -1 \\ 0.3 & -0.035 \end{bmatrix}, & B_r &= \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}, \\ C_r &= [0.5 \quad 0.5]. \end{aligned} \quad (45)$$

The disturbance model is presented by the following parameters:

$$W = \begin{bmatrix} 0.8776 & 0.4794 \\ -0.4794 & 0.8776 \end{bmatrix}, \quad V = [10 \ 0]. \quad (46)$$

Set $\tau = 1$ and $\bar{\tau} = 4$. Here, we suppose the disturbance attenuation level $\gamma = 0.8$. Then, using Matlab Control Toolbox to solve Theorem 8, we obtain controller gains and observer gain as follows:

$$\begin{aligned} K_{11} &= [-0.0477 \ -0.0308], & K_{21} &= [-0.0011 \ 0.0432], \\ L &= \begin{bmatrix} -0.0012 & -0.0004 \\ 0.0005 & 0.0002 \end{bmatrix}, \\ K_{12} &= [-0.0159 \ -0.0103], & K_{22} &= [-0.0004 \ 0.0144]. \end{aligned} \quad (47)$$

Suppose the switching sequence as 121212... The initial value of the states is chosen as $\phi(s) = [-0.2e^{-s} \ 0.1e^{-s}]^T$, and the reference model of initial condition is selected as $x_r = [0.1 \ -0.2]^T$. In the sequel, two kinds of reference inputs, that is, step reference input and sinusoidal reference input, are considered to demonstrate the effectiveness of the proposed method.

Case 1 (step reference input). Let

$$\begin{aligned} r(k) &= \begin{cases} 10, & 80 \leq k < 150, \\ 0, & \text{else } k, \end{cases} \\ d_2(k) &= \begin{cases} \frac{30}{k+1}, & 80 \leq k < 150, \\ 0, & \text{else } k. \end{cases} \end{aligned} \quad (48)$$

Curves of z_k and z_r are depicted in Figure 1 under input signal (48). From Figure 1, we can see that the system output can effectively track the reference model output in presence of matched disturbances and mismatched disturbances, which demonstrates the effectiveness of the proposed method. In order to evaluate the effectiveness of the DOB, curves of disturbances and disturbances estimation are shown in Figure 2. It illustrates that DOB can effectively estimate disturbances.

Case 2 (sinusoidal reference input). Let

$$\begin{aligned} r(k) &= \begin{cases} 10 \sin(k), & 80 \leq k < 150, \\ 0, & \text{else } k, \end{cases} \\ d_2(k) &= \begin{cases} \frac{30}{k+1}, & 80 \leq k < 150, \\ 0, & \text{else } k. \end{cases} \end{aligned} \quad (49)$$

Figure 3 presents the curves of z_k and z_r under input signal (49), which demonstrates that the proposed method obtains good tracking performance in spite of matched disturbances and mismatched disturbances. Figure 4 shows the disturbances estimation results, which depicts that the DOB can effectively estimate disturbances.

FIGURE 1: Curves of $z(k)$ and $z_r(k)$ under Case 1.

FIGURE 2: Curves of disturbances and disturbances estimation under Case 1.

Remark 9. In this paper, a composite tracking controller is designed for a class of discrete-time switched systems with time-varying delay. In reality, many physical systems can be modelled as switched system, for example, flight control systems [47, 48] and inverted-pendulum system [49]. Take flight control system; for example, the flight control system can be modelled as switched system corresponding to finite operating points within the flight envelope, where $d_2(t)$ is denoted as wind gust and $d_1(t)$ is regarded as unknown harmonic disturbances. In order to better serve engineering, we will pay attention to studying a tracking controller design for a practical system based on our method in future.

5. Conclusions

The problem of H_∞ output tracking control for discrete-time switched systems subject to time-varying delay and disturbances has been studied. H_∞ control has achieved the attenuation performance with respect to norm bounded disturbances. DOBC has been employed to reject the disturbances with some known information. In this paper, a

FIGURE 3: Curves of $z(k)$ and $z_r(k)$ under Case 2.

FIGURE 4: Curves of disturbances and disturbances estimation under Case 2.

composite control scheme, that is, consisting of H_∞ control method and DOBC technique, has been proposed, which can effectively attenuate and reject the external disturbances. A numerical example has been provided to show the effectiveness of the proposed algorithm.

Acknowledgments

The authors thank Beihang University Lei Guo Professor for helpful discussions and the anonymous associate editor and reviewers for constructive suggestions. This work was supported in part by Key Project of Chinese National Programs for Fundamental Research and Development (973 program) under Grant (2012CB720003) and the National Natural Science Foundation of China (nos. 61273123, 61203011, and 61203013).

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