Research Article

H_∞ Control for Multirate Networked Control Systems with Deadband Scheduling

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This paper is concerned with the problem of H_∞ control for multirate networked control systems with deadband scheduling policy and transmission delay. The transmission deadbands are set at the sensor nodes and the controller nodes to reduce the network traffic, and then the systems are modeled as discrete-time switch system with uncertain parameters. Using the Lyapunov theory, a robust H_∞ controller is designed to maintain the asymptotic stability of the closed-loop systems. Finally, a simulation example is presented to show the advantage of the deadband scheduling policy and the efficiency of the proposed theoretical results.

1. Introduction

Networked control systems (NCSs) are the complex systems in which the controlled plants, sensors, controllers, and actuators are distributed in different places and connected through a communication network [1]. Compared with the traditional control systems, NCSs have many advantages such as the flexibility of structure, high reliability, and simple installation. As the rapid development of information technology, NCSs have been applied to various applications including mobile robots, unmanned aerial vehicles, and remote surgeries [2–5]. However, the data transmission through a network brings many challenges, such as time delay, packet dropout, and data disorder, which will cause the system performance degradation and even, make the systems unstable. In recent years, the stability analysis and controller design of NCSs have been researched by many scholars [6–10].

When the bandwidth of NCSs is constrained, the packet collision becomes more severe. It is difficult to guarantee the stabilization and acceptable performance of systems only by means of controller design [11, 12]. In this case, the network scheduling becomes particularly important. The network nodes' unnecessary data traffic could be reduced by

introducing appropriate network scheduling strategy; thus, the network-induced delay and packet dropouts can be reduced remarkably [13–17]. In [13], the problem of integrated design of controller and communication sequences is addressed for a class of multiple inputs multiple outputs NCSs based on switch system theory, but not taking the network-induced delay into account, and the results are extended to NCSs with transmission delay in [14, 15]. In [16], a coordinate design scheme of the try-once-discard (TOD) dynamic scheduling strategy and robust H_∞ controller for a class of networked control systems with communication constraints and random time-delay is proposed. In [17], the wireless NCSs with communication delay and packet dropouts, which are based on the deadband scheduling policy, are modeled as continuous time-delay systems and an H_∞ controller is designed. Meanwhile, the event-triggering NCSs attract many researchers' attention [18–20]. Compared with the widely used time-triggering sampling scheme, the event-triggering sampling scheme, whose main property is that the signal is sampled only when some functions of the system state or output measurement exceeds a threshold, can also save network bandwidth for the NCSs.

It is worth pointing that all of the aforementioned results are under the assumption that NCSs work in a single rate

mode (either the actuators are event driven or the updating period of actuators is the same as the sampling period). The multirate NCSs, in which the updating rate of actuators is several times the sampling rate, have many advantages: (1) compared with the NCSs with event-driven actuators, the actuator updating moments are fixed in the multirate NCSs; thus, the analysis and synthesis of the multirate NCSs become more easier; (2) compared with the NCSs in which the actuator updating rate is the same as the sampling rate, the control input can be implemented to the plant more sooner in the multirate NCSs; therefore, better system performance can be ensured [21–23]. In [21], the multirate NCSs are models as the discrete switch systems, and an H_∞ controller is designed by using the average dwell time method. The model-predictive control problem for multirate NCSs is proposed in [22]. To the best of the authors' knowledge, H_∞ control problem of multirate NCSs with deadband scheduling has not been fully investigated to date, which motivates the present study.

In this paper, we focus on the problem of H_∞ control for multirate NCSs with deadband scheduling and transmission delay. By analyzing the influences of transmission deadband, a discrete-time switch system model with uncertain parameters is proposed to describe the multirate NCSs. The design method of a robust H_∞ controller that stabilizes this class of networked control systems is proposed. A simulation example is presented to show the advantage of the deadband scheduling policy and the efficiency of the proposed method.

This paper is organized as follows. In Section 2, the problem is stated and some useful definitions and lemmas are given, and then the main results of this paper are given in Section 3. Section 4 provides a simulation example to illustrate the effectiveness of our results. Finally, Section 5 gives some concluding remarks.

Notation. $\mathbb{R}^n$ denotes the n -dimensional Euclidean space, and I is identity matrix. A^T stands for the transpose of the corresponding matrix A . The notation $A \geq 0$ ($A > 0$) means that the matrix A is a positive semidefinite (positive definite) matrix. For an arbitrary matrix Y and two symmetric matrices X and Z , $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes a block matrix entry implied by symmetry.

2. Problem Formulation and Preliminaries

Consider NCSs whose setup is depicted in Figure 1. The controlled plant, sensor, and controller are connected with a communication network. The transmission deadbands are set at the sensor node and the controller node.

The plant is described by

$$\begin{aligned}\dot{x}(t) &= A_p x(t) + B_p u(t) + B_{pw} w(t) \\ y(t) &= C_p x(t),\end{aligned}\quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^l$ is the output, and $w(t) \in \mathbb{R}^p$ is the exogenous input. A_p , B_p , B_{pw} , and C_p are known real constant matrices with appropriate dimensions.

The sensors are periodically sampled with the sampling interval T_s . The sensor-to-controller transmission delay is described as τ^{sc} , and the controller-to-actuator transmission delay is described as τ^{ca} . We make the following assumptions about NCSs.

Assumption 1. $\tau = \tau^{sc} + \tau^{ca}$, $0 < \tau < T_s$.

Assumption 2. The controller is event driven; the sensors and actuators are time driven.

Assumption 3. The updating period of actuators is T_a , $T_s = NT_a$, N is positive integer, and $N > 1$.

First, the discrete-time model of multirate NCSs without transmission deadband is proposed.

The controller output $u(kT)$ arrives at the actuator at the instants $kT + \tau_k$; if $\tau_k \in ((n-1)T_a, nT_a]$, ($n \in \{1, 2, \dots, N\}$), then the actuator will update at the instances $kT_s + nT_a$. Therefore, in the sample interval $[kT_s, (k+1)T_s]$, system (1) is described as

$$\begin{aligned}\dot{x}(t) &= A_p x(t) + B_p K x((k-1)T_s) + B_{pw} w(t), \\ t &\in [kT_s, kT_s + nT_a],\end{aligned}\quad (2)$$

$$\begin{aligned}\dot{x}(t) &= A_p x(t) + B_p K x(kT_s) + B_{pw} w(t), \\ t &\in [kT_s + nT_a, (k+1)T_s], \\ y(t) &= C_p x(t).\end{aligned}\quad (3)$$

Let $A = e^{A_p T_s}$, system (1) with sampling period T_s is discretized to

$$\begin{aligned}x(k+1) &= Ax(k) + \int_0^{nT_a} e^{A_p(T_s-s)} B_p ds \cdot u(k-1) \\ &\quad + \int_{nT_a}^{T_s} e^{A_p(T_s-s)} B_p ds \cdot u(k) + B_{pw} \omega(k) \\ &= Ax(k) + \int_0^{nT_a} e^{A_p(NT_a-s)} B_p ds \cdot u(k-1) \\ &\quad + \int_{nT_a}^{NT_a} e^{A_p(NT_a-s)} B_p ds \cdot u(k) + B_{pw} \omega(k).\end{aligned}\quad (4)$$

Let

$$\begin{aligned}B_{1n} &= \begin{cases} \sum_{\alpha=n}^{N-1} \int_{\alpha T_a}^{(\alpha+1)T_a} e^{A_p(T_s-s)} B_p ds, & n < N \\ 0 & n = N, \end{cases} \\ B_{2n} &= \sum_{\alpha=0}^{n-1} \int_{\alpha T_a}^{(\alpha+1)T_a} e^{A_p(T_s-s)} B_p ds.\end{aligned}\quad (5)$$

Then

$$x(k+1) = Ax(k) + B_{1n} u(k) + B_{2n} u(k-1) + B_{pw} \omega(k).\quad (6)$$

FIGURE 1: Illustration of NCSs over communication network.

Next, we introduce the deadband scheduling policy. The transmission deadbands located at the sensor node and controller node are based on such a deadband scheduling policy: if the difference between the present signal and the last transmitted signal exceeds the threshold of deadband, the present signal will be transmitted, otherwise it will not be transmitted for the purpose of reducing the network traffic and saving the network bandwidth.

Let $x(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T$, $x_i(k)$ is the component of $x(k)$ and sampled separately by each sensor at the sampling instant. If $x_i(k)$ enters into the deadband, according to the proposed scheduling policy, $x_i(k)$ will not be transmitted and the controller will hold the previous transmitted one, which is equivalent to transmitting the previous transmitted one again. Therefore, the deadband 1 is described as

$$\begin{aligned} \bar{x}_i(k) &= f_i(x_i(k)) \\ &= \begin{cases} x_i(k) & |\bar{x}_i(k-1) - x_i(k)| > \delta_{1i} |x_i(k)| \\ \bar{x}_i(k-1) & |\bar{x}_i(k-1) - x_i(k)| \leq \delta_{1i} |x_i(k)|, \end{cases} \end{aligned} \quad (7)$$

where δ_{1i} is the deadband threshold coefficient, $\delta_{1i} \in (0, 1)$.

According to (7),

$$x_i(k) - \delta_{1i} |x_i(k)| \leq \bar{x}_i(k) \leq x_i(k) + \delta_{1i} |x_i(k)|. \quad (8)$$

Define $\bar{\Delta}_1 = \text{diag}\{\bar{\Delta}_{11}, \bar{\Delta}_{12}, \dots, \bar{\Delta}_{1n}\}$, $\bar{\Delta}_{1j} \in [-\delta_{1j}, \delta_{1j}]$, ($j = 1, 2, \dots, n$) and $\delta_1 = \text{diag}\{\delta_{11}, \delta_{12}, \dots, \delta_{1n}\}$; therefore, one can obtain

$$\bar{x}(k) = (I + \bar{\Delta}_1) x(k). \quad (9)$$

Similarly, deadband 2 is described as

$$\begin{aligned} \bar{u}_i(k) &= g_i(u_i(k)) \\ &= \begin{cases} u_i(k) & |\bar{u}_i(k-1) - u_i(k)| > \delta_{2i} |u_i(k)| \\ \bar{u}_i(k-1) & |\bar{u}_i(k-1) - u_i(k)| \leq \delta_{2i} |u_i(k)|, \end{cases} \end{aligned} \quad (10)$$

where δ_{2i} is the deadband threshold coefficient, $\delta_{2i} \in (0, 1)$.

Define $\bar{\Delta}_2 = \text{diag}\{\bar{\Delta}_{21}, \bar{\Delta}_{22}, \dots, \bar{\Delta}_{2m}\}$, $\bar{\Delta}_{2j} \in [-\delta_{2j}, \delta_{2j}]$, ($j = 1, 2, \dots, m$) and $\delta_2 = \text{diag}\{\delta_{21}, \delta_{22}, \dots, \delta_{2m}\}$; therefore, one can obtain

$$\bar{u}(k) = (I + \bar{\Delta}_2) u(k). \quad (11)$$

Combining (6), (9), and (11), the discrete-time model of multirate NCSs is described as

$$\begin{aligned} x(k+1) &= Ax(k) + B_{1n} (I + \Delta_1) K (I + \Delta_2) x(k) \\ &\quad + B_{2n} (I + \Delta_3) K (I + \Delta_4) x(k-1) + B_w \omega(k) \\ y(k) &= Cx(k). \end{aligned} \quad (12)$$

The following definition and lemmas will be essential for the proofs in the next section.

Definition 1. System (12) with a given matrix K is said to be asymptotically stable with an H_∞ norm bound γ if the following conditions hold:

- (1) system (12) is asymptotically stable with $\omega(k) = 0$;
- (2) under the zero initial condition, the controlled output $y(k)$ satisfies $\|y(k)\|_2 < \gamma \|\omega(k)\|_2$, where $\omega(t) \in L_2[0, \infty)$.

Lemma 2 (see [24] Schur complement). For matrices $A, Q = Q^T, P = P^T > 0$, the matrix inequality $A^T P A + Q < 0$ holds, if and only if the following matrix inequalities

$$\begin{bmatrix} Q & A^T \\ A & -P^{-1} \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -P^{-1} & A \\ A^T & Q \end{bmatrix} < 0 \quad (13)$$

hold.

Lemma 3 (see [25]). *For some given matrices W , M , and N of appropriate dimensions and with W being symmetric, then*

$$W + N^T F^T(t) M^T + M F(t) N < 0. \quad (14)$$

holds, where $F(t)$ satisfying $F^T(t)F(t) \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$W + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0. \quad (15)$$

3. Main Results

In this section, we will find a state feedback control matrix K such that the system is asymptotically stable with an H_∞ norm bound γ .

Theorem 4. *For given state feedback control matrix K , system (12) is asymptotically stable with an H_∞ norm bound γ , if there exist symmetric positive definite matrices P and Q , such that the following condition holds:*

$$\begin{bmatrix} Q - P & 0 & 0 & C^T (A + B_{1n}(I + \Delta_1)K(I + \Delta_2))^T \\ * & -Q & 0 & 0 & (B_{2n}(I + \Delta_3)K(I + \Delta_4))^T \\ * & * & -\gamma^2 I & 0 & B_\omega \\ * & * & * & -I & * \\ * & * & * & * & -P^{-1} \end{bmatrix} < 0. \quad (16)$$

Proof. Consider the following Lyapunov candidate functional:

$$V(k) = x^T(k) P x(k) + x^T(k-1) Q x(k-1). \quad (17)$$

First, we proof that system (12) is asymptotically stable with $\omega(k) = 0$ under (16).

Let

$$\begin{aligned} \bar{A} &= A + B_{1n}(I + \Delta_1)K(I + \Delta_2), \\ \bar{B} &= B_{2n}(I + \Delta_3)K(I + \Delta_4), \end{aligned} \quad (18)$$

along the trajectory of the system (12), we have

$$\begin{aligned} \Delta V(k) &= x^T(k+1) P x(k+1) + x^T(k) Q x(k) \\ &\quad - x^T(k) P x(k) - x^T(k-1) Q x(k-1) \\ &= [x^T(k+1) \ x^T(k)] \begin{bmatrix} Q - P + \bar{A}^T P \bar{A} & * \\ \bar{B}^T P \bar{A} & \bar{B} P \bar{B} - Q \end{bmatrix} \\ &\quad \times [x^T(k+1) \ x^T(k)]^T, \end{aligned} \quad (19)$$

using Lemma 2 (Schur complement), we have $\Delta V(k) < 0$ if and only if

$$\begin{bmatrix} Q - P & 0 & \bar{A}^T \\ * & -Q & \bar{B}^T \\ * & * & -P^{-1} \end{bmatrix} < 0. \quad (20)$$

By Lemma 2 (Schur complement), we know that (16) implies (20); Therefore, system (12) is asymptotically stable with $\omega(k) = 0$ being proved.

Now, we will consider the H_∞ performance of system (12).

When $\omega(k) \neq 0$, define

$$\begin{aligned} J &= \|y(k)\|_2^2 - \gamma^2 \|\omega(k)\|_2^2 \\ &= \sum_{k=0}^{\infty} [y^T(k) y(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(k)] \\ &\quad - V(\infty) + V(0); \end{aligned} \quad (21)$$

under zero condition and $V(\infty) \geq 0$, we have

$$J \leq \sum_{k=0}^{\infty} [y^T(k) y(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(k)]. \quad (22)$$

Let $\eta(k) = [x^T(k) \ x^T(k-1) \ \omega^T(k)]^T$, we obtain

$$J \leq \sum_{k=0}^{\infty} \eta^T(k) H \eta(k), \quad (23)$$

where

$$H = \begin{bmatrix} Q - P + \bar{A}^T P \bar{A} + C^T C & * & * \\ \bar{B}^T P \bar{A} & \bar{B} P \bar{B} - Q & * \\ B_\omega^T P \bar{A} & B_\omega^T P \bar{B} & B_\omega^T P B_\omega - \gamma^2 \end{bmatrix}. \quad (24)$$

By Lemma 2 (Schur complement), (16) is equivalent to $H < 0$, which implies that $J < 0$; thus, we have $\|y(k)\|_2 < \gamma \|\omega(k)\|_2$. This completes the proof. $\square$

Next, the design method of the state feedback controller is proposed.

Theorem 5. *For given diagonal matrices δ_1, δ_2 and a scalar λ , if there exist positive definite matrices $\bar{Q}$, X , and W , scalars $\varepsilon_i > 0$ ($i = 1, 2, \dots, 6$) and a matrix Y , such that the following linear matrix inequalities*

$$\begin{bmatrix} -W & Y^T \\ * & -\lambda I \end{bmatrix} < 0, \quad (25)$$

$$W < 2X - I, \quad (26)$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ * & \Gamma_{22} \end{bmatrix} < 0, \quad (27)$$

hold, then the system is asymptotically stable with an H_∞ norm bound γ ; the desired controllers are given as follows:

$$K = YX^{-1}, \quad (28)$$

where

$$\Gamma_{11} = \begin{bmatrix} \bar{Q} - X & 0 & 0 & (CX)^T & (AX + B_{1n}Y)^T \\ * & -\bar{Q} & 0 & 0 & (B_{2n}Y)^T \\ * & * & -\gamma^2 I & 0 & B_\omega^T \\ * & * & * & -I & 0 \\ * & * & * & * & -X \end{bmatrix},$$

$$\Gamma_{22} = \text{diag} \{-\varepsilon_1, -\varepsilon_1, -\varepsilon_2, -\lambda\varepsilon_2, -\varepsilon_3, -\lambda\varepsilon_3, -\varepsilon_4, -\varepsilon_4, -\varepsilon_5, -\lambda\varepsilon_5, -\varepsilon_6, -\lambda\varepsilon_6\}, \quad (29)$$

$$\Gamma_{12} = [\varepsilon_1 L_1 \delta_1 \quad L_2 Y^T \quad \varepsilon_2 L_1 \quad \lambda L_2 X \delta_2 \quad \varepsilon_3 L_1 \delta_1 \quad \lambda L_2 X \delta_2 \quad \varepsilon_4 L_3 \delta_1 \quad L_4 Y^T \quad \varepsilon_5 L_3 \quad \lambda L_4 X \delta_2 \quad \varepsilon_6 L_3 \delta_1 \quad \lambda L_4 X \delta_2],$$

$$L_1 = [0 \quad 0 \quad 0 \quad 0 \quad B_{1n}^T]^T, \quad L_2 = [I \quad 0 \quad 0 \quad 0 \quad 0]^T,$$

$$L_3 = [0 \quad 0 \quad 0 \quad 0 \quad B_{2n}^T]^T, \quad L_4 = [0 \quad I \quad 0 \quad 0 \quad 0]^T.$$

Proof. Rewrite (16) as

$$F = F_1 + F_2 < 0, \quad (30)$$

where

$$F_1 = \begin{bmatrix} Q - P & 0 & 0 & C^T & (A + B_{1n}K)^T \\ * & -Q & 0 & 0 & (B_{2n}K)^T \\ * & * & -\gamma^2 I & 0 & B_\omega^T \\ * & * & * & -I & * \\ * & * & * & * & -P^{-1} \end{bmatrix},$$

$$F_2 = L_1 \Delta_1 K L_2^T + (L_1 \Delta_1 K L_2^T)^T + L_1 K \Delta_2 L_2^T + (L_1 K \Delta_2 L_2^T)^T \\ + L_1 \Delta_1 K \Delta_2 L_2^T + (L_1 \Delta_1 K \Delta_2 L_2^T)^T + L_3 \Delta_3 K L_4^T \\ + (L_3 \Delta_3 K L_4^T)^T + L_3 K \Delta_4 L_4^T + (L_3 K \Delta_4 L_4^T)^T \\ + L_3 \Delta_3 K \Delta_4 L_4^T + (L_3 \Delta_3 K \Delta_4 L_4^T)^T, \quad (31)$$

L_1, L_2, L_3 , and L_4 are defined above in Theorem 5.

By Lemma 3, $F < 0$ is equivalent to

$$\bar{F} = F_1 + \bar{F}_2 < 0, \quad (32)$$

where

$$\bar{F}_2 = \varepsilon_1 L_1 \Delta_1 \Delta_1^T L_1^T + \varepsilon_1^{-1} L_2 K^T K L_2^T + \varepsilon_2 L_1 L_1^T \\ + \varepsilon_2^{-1} L_2 \Delta_2^T K^T K \Delta_2 L_2^T + \varepsilon_3 L_1 \Delta_1 \Delta_1^T L_1^T \\ + \varepsilon_3^{-1} L_2 \Delta_2^T K^T K \Delta_2 L_2^T + \varepsilon_4 L_3 \Delta_3 \Delta_3^T L_3^T \\ + \varepsilon_4^{-1} L_4 K^T K L_4^T + \varepsilon_5 L_3 L_3^T + \varepsilon_5^{-1} L_4 \Delta_4^T K^T K \Delta_4 L_4^T \\ + \varepsilon_6 L_3 \Delta_3 \Delta_3^T L_3^T + \varepsilon_6^{-1} L_4 \Delta_4^T K^T K \Delta_4 L_4^T. \quad (33)$$

By the definitions of $\Delta_1, \Delta_2, \Delta_3$, and Δ_4 , we have

$$\Delta_1 \Delta_1^T = \Delta_1^T \Delta_1 \leq \delta_1^T \delta_1, \quad \Delta_3 \Delta_3^T = \Delta_3^T \Delta_3 \leq \delta_1^T \delta_1, \\ \Delta_2 \Delta_2^T = \Delta_2^T \Delta_2 \leq \delta_2^T \delta_2, \quad \Delta_4 \Delta_4^T = \Delta_4^T \Delta_4 \leq \delta_1^T \delta_1, \quad (34)$$

using Lemma 2 (Schur complement), (25) is equivalent to $Y^T Y \leq \lambda W$. Define $K = YX^{-1}$, (26) implies that

$$K^T K \leq \lambda I. \quad (35)$$

Combining (33), (34), and (35), we have

$$\bar{F}_2 \leq \tilde{F}_2 \\ = \varepsilon_1 L_1 \delta_1 \delta_1^T L_1^T + \varepsilon_1^{-1} L_2 K^T K L_2^T + \varepsilon_2 L_1 L_1^T \\ + \lambda \varepsilon_2^{-1} L_2 \delta_2^T \delta_2 L_2^T + \varepsilon_3 L_1 \delta_1 \delta_1^T L_1^T + \lambda \varepsilon_3^{-1} L_2 \delta_2^T \delta_2 L_2^T \\ + \varepsilon_4 L_3 \delta_3 \delta_3^T L_3^T + \varepsilon_4^{-1} L_4 K^T K L_4^T + \varepsilon_5 L_3 L_3^T \\ + \lambda \varepsilon_5^{-1} L_4 \delta_4^T \delta_4 L_4^T + \varepsilon_6 L_3 \delta_3 \delta_3^T L_3^T + \lambda \varepsilon_6^{-1} L_4 \delta_4^T \delta_4 L_4^T. \quad (36)$$

Therefore,

$$\tilde{F} = \bar{F}_1 + \tilde{F}_2 < 0 \quad (37)$$

Guarantees (16).

By Lemma 2 (Schur complement), (37) is equivalent to

$$\hat{F} = \begin{bmatrix} F_1 & \hat{F}_{12} \\ * & \hat{F}_{22} \end{bmatrix} < 0, \quad (38)$$

where

$$\begin{aligned} \hat{F}_{12} &= [\varepsilon_1 L_1 \delta_1 \quad L_2 K^T \quad \varepsilon_2 L_1 \quad \lambda L_2 \delta_2 \quad \varepsilon_3 L_1 \delta_1 \quad \lambda L_2 \delta_2 \quad \varepsilon_4 L_3 \delta_1 \quad L_4 K^T \quad \varepsilon_5 L_3 \quad \lambda L_4 \delta_2 \quad \varepsilon_6 L_3 \delta_1 \quad \lambda L_4 \delta_2], \\ \hat{F}_{22} &= \text{diag}\{-\varepsilon_1, -\varepsilon_1, -\varepsilon_2, -\lambda \varepsilon_2, -\varepsilon_3, -\lambda \varepsilon_3, -\varepsilon_4, -\varepsilon_4, -\varepsilon_5, -\lambda \varepsilon_5, -\varepsilon_6, -\lambda \varepsilon_6\}, \end{aligned} \quad (39)$$

Let $X = P^{-1}$, we pre- and postmultiply both sides of (38) with $\text{diag}\{X, X, \overbrace{I, I, \dots, I}^{15}\}$ we have (27). This completes the proof. $\square$

4. Simulations

In this section, we will give an example to show the usefulness of the derived results and the effectiveness of the proposed methods.

The controlled plant parameters are given as follows:

$$\begin{aligned} A_p &= \begin{bmatrix} -3 & 0.5 \\ 0.6 & -2 \end{bmatrix}, & B_p &= \begin{bmatrix} 1 & 0.5 \\ 0.3 & 1 \end{bmatrix}, \\ B_{pw} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, & C_p &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned} \quad (40)$$

Given the sampling period $T_s = 50$ ms, and the actuator updating period $T_a = T_s/3$, following (6), we have

$$\begin{aligned} A &= \begin{bmatrix} 0.861 & 0.0221 \\ 0.0266 & 0.9052 \end{bmatrix}, & B_w &= \begin{bmatrix} 0.0046 & 0.0001 \\ 0.0001 & 0.0048 \end{bmatrix}, \\ B_{11} &= \begin{bmatrix} 0.0318 & 0.0161 \\ 0.0100 & 0.0324 \end{bmatrix}, & B_{21} &= \begin{bmatrix} 0.0148 & 0.0077 \\ 0.0050 & 0.0155 \end{bmatrix}, \\ B_{12} &= \begin{bmatrix} 0.0163 & 0.0082 \\ 0.0050 & 0.0164 \end{bmatrix}, & B_{22} &= \begin{bmatrix} 0.0303 & 0.0156 \\ 0.0100 & 0.0315 \end{bmatrix}, \\ B_{13} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, & B_{23} &= \begin{bmatrix} 0.0466 & 0.0238 \\ 0.0150 & 0.0479 \end{bmatrix}. \end{aligned} \quad (41)$$

The deadband threshold coefficients of deadbands 1 and 2 are given as $\delta_1 = \text{diag}\{0.3, 0.3\}$, $\delta_2 = \text{diag}\{0.2, 0.2\}$, and the disturbance attenuation $\gamma = 0.3$. According to Theorem 5, using the LMI toolbox of MATLAB, the controller parameters can be calculated as follows:

$$K = \begin{bmatrix} -2.726 & -0.8501 \\ -0.8501 & -3.897 \end{bmatrix}. \quad (42)$$

The exogenous input is chosen as $w(t) = 5 \sin(t)$, the states responses of the closed-loop multirate NCSs are shown

FIGURE 2: State trajectories of the closed-loop system.

in Figure 2, which confirms that the closed-loop multirate NCSs are robustly asymptotically stable under the exogenous input.

The packet dropout rates of the S-to-C channel and the C-to-A channel are shown in Figure 3. The packet dropout mentioned here is the data packets which are lost “actively” when the multirate NCSs enter into the transmission deadband; thus, Figure 3 confirms that the proposed deadband scheduling policy could reduce the network traffic remarkably.

5. Conclusions

In this paper, H_∞ control problem for multirate networked control systems with deadband scheduling policy and transmission delay is considered. The transmission deadbands are set at the sensor node and controller node to reduce the network traffic. The systems are modeled as the discrete switch system with uncertain parameters and the design method of H_∞ controller is established by using the Lyapunov theorem. Finally, the simulation example is presented to show the advantage of the deadband scheduling policy and the efficiency of the proposed theoretical results.

FIGURE 3: Packet dropout rate curve of the closed-loop system.

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