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Research Article

Convergence of Sample Autocorrelation of Long-Range Dependent Traffic

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We depict our work on a fundamental issue in the theory of long-range dependent traffic in the aspect of the convergence of sample autocorrelation function (ACF) of real traffic. The present results suggest that the sample ACF of traffic is convergent. In addition, we show that the sample size has considerable effects in estimating the sample ACF of traffic. More precisely, a sample ACF of traffic tends to be smoother when the sample size increases.

1. Introduction

Start with the meaning of the convergence of a sample autocorrelation function (ACF) of a stationary random function $x(t)$ for $t \in (0, \infty)$. Denote by $x_T(t)$ a sample record of $x(t)$ with length T, that is, $x_T(t) = x(t)$ for $t \in (0, T)$. Let $r_T(\tau)$ be the ACF of $x_T(t)$, meaning $r_T(\tau) = E[x_T(t)x_T(t+\tau)]$ for $t, t+\tau \in (0, T)$. Then, we say that $r_T(\tau)$ is convergent if $\lim_{T\to} r_T(\tau) = r(\tau)$ exists, where $r(\tau)$ is the ACF of $x(t)$. Otherwise, $r_T(\tau)$ is divergent or does not exist. In the discrete case, $x(t)$ is replaced by $x(i)$ ($i = 0, \ldots, \infty$), $x_T(t)$ is substituted by $x_I(i)$ for $i = 0, \ldots, I-1$, and $r_T(\tau)$ is replaced with $r_I(k)$ ($k = 0, \ldots, I-1$). When $\lim_{I\to\infty} r_I(k) = r(k)$ exists, where $r(k)$ is the ACF of $x(i)$, we say that $r_I(k)$ is convergent. That is the meaning of the convergence or existence of a sample ACF of a random function.

The issue described above may be unnecessary to treat in the field of conventional second-order random functions because an ACF of a conventional 2-order random function generally exists [1–6]. However, the issue whether a sample ACF of teletraffic (traffic for short) time series with long-range dependence is convergent or not is worth discussing.

The research by Resnick et al. [7] stated that the sample ACF of stable random functions with infinite variance may be random when the sample size approaches infinity instead of being convergent. The example of such a type of random functions includes α-stable processes with infinite variance [7, Page 798]. Recall that traffic modeling using α-stable processes was described by reports, such as Karasaridis and Hatzinakos [8], Barbe and McCormick [9]. On the other side, the reports by Field et al. [10, 11] discussed the traffic modeling using the standard Cauchy distribution, which implied that traffic is of infinite variance because variance of a random function obeying the Cauchy distribution is infinite [2–4]. Therefore, a sample ACF of traffic may be divergent if it is of infinite variance. Nonetheless, correlations of traffic play a role in practice; see, for example [12–22], just to cite a few. Consequently, the answer to the question whether a sample ACF of traffic is convergent or not is desired from the point of view of the theory of traffic. This paper aims at exhibiting that sample ACFs of real-traffic data are convergent.

The rest of the paper is organized as follows. Problem statement is described in Section 2. Results are explained in Section 3. Discussions are given in Section 4, which is followed by conclusions.

2. Problem Statement

In what follows, we assume that traffic $x(i)$ is causal. By causal, we mean that $x(i)$ is defined for $i \geq 0$ and $x(i) = 0$ if $i < 0$. In the interval $[0, I]$, the sample ACF of $x_I(i)$ is given by

$$r_I(k) = \frac{1}{I}\sum_{i=0}^{I-1} x(i)\, x(i+k). \tag{1}$$

For $I_l \neq I_m$, assuming $I_l < I_m$, we have,

$$r_{I_l}(k) \neq r_{I_m}(k), \quad k = 0, \ldots, I_l - 1. \tag{2}$$

What is previously mentioned may be expressed by

$$r_{I_l}(k) + \Delta_{I_l}(k) = r_{I_m}(k), \quad k = 0, \ldots, I_l - 1, \tag{3}$$

where $\Delta_{I_l}(k)$ is a fluctuation noise. Further, For $I_l \neq I_m \neq I_n$, assuming $I_l < I_m < I_n$, we have

$$r_{I_m}(k) + \Delta_{I_m}(k) = r_{I_n}(k), \quad k = 0, \ldots, I_l - 1. \tag{4}$$

In general, if $r_I(k)$ is convergent, we have

$$\sum_{k=0}^{I_l-1} \left|\Delta_{I_m}(k)\right| \leq \sum_{k=0}^{I_l-1} \left|\Delta_{I_l}(k)\right|. \tag{5}$$

More precisely, in the case of $r_I(k)$ being convergent, we have

$$\lim_{I_l \to \infty} r_{I_l}(k) = \lim_{I_m \to \infty} r_{I_m}(k) = r(k). \tag{6}$$

Consequently, the above means that

$$\lim_{I_l \to \infty} \Delta_{I_l}(k) = 0. \tag{7}$$

Since a real-traffic data series is always of finite length and because the true ACF of $x(i)$ is never achieved, the issue we treat in this research is to investigate whether $r_I(k)$ tends smoother when I becomes larger. If that is so, we infer that $r_I(k)$ is convergent. If not, it is divergent.

3. Results

Before showing the results, we brief the real-traffic data used in this section. Let $x(t(i))$ be a sample record of traffic, where $t(i)$ is the series of timestamps, indicating the timestamp of the ith packet arriving at a server. Thus, $x(t(i))$ represents the packet size of the ith packet recorded at time $t(i)$. Note that the pattern of $x(t(i))$ is consistent with that of $x(i)$ that represents the size of the ith packet on a packet-by-packet basis.

We use two real-traffic traces. One is named BC-pOct89 that contains 1,000,000 packets. It was recorded on an Ethernet at the Bellcore Morristown Research and Engineering Facility [23], which is now Telcordia Technologies (http://en.wikipedia.org/wiki/Telcordia_Technologies), made by Will Leland and Dan Wilson. The other is DEC-PKT-1 with 3.3 million packets. It was made by Jeff Mogul of Digital's Western Research Lab [24]. The former was used in the pioneering research of the fractal statistics of traffic in [25, 26]. The latter was utilized by Paxson and Floyd in [27]. We cite relatively recent articles [28–30] for the nice description of the basic statistics of traffic.

Figure 1 is the plot of the first 2048 data points of traffic trace BC-pOct89. Figure 2 indicates the plots of sample ACFs of BC-pOct89 with different sample sizes as follows.

(i) Figure 2(a): sample ACF of BC-pOct89 with the sample size $I = 2^{11} = 2048$.

(ii) Figure 2(b): sample ACF of BC-pOct89 with the sample size $I = 2^{12} = 4096$.

(iii) Figure 2(c): sample ACF of BC-pOct89 with the sample size $I = 2^{13} = 8192$.

(iv) Figure 2(d): sample ACF of BC-pOct89 with the sample size $I = 2^{14} = 16384$.

(v) Figure 2(e): sample ACF of BC-pOct89 with the sample size $I = 2^{15} = 32768$.

(vi) Figure 2(f): sample ACF of BC-pOct89 with the sample size $I = 2^{16} = 65536$.

(vii) Figure 2(g): sample ACF of BC-pOct89 with the sample size $I = 2^{17} = 131027$.

From Figure 2, we obtain the following observations.

Observation 1. The sample ACF of BC-pOct89 with different sample sizes obeys a certain deterministic function.

Observation 2. The fluctuation of a sample ACF of BC-pOct89 decreases as the sample size increases.

In order to refine Observation 2, we demonstrate the first 1024 points of the sample ACFs in the sense of zoom as shown in Figure 3. Thus, comes the following consequence.

Consequence 1. The sample ACF of BC-pOct89 is convergent.

The demonstrations of the sample ACF of DEC-PKT-1 are given in the appendix.

4. Discussions

The modeling of the sample ACF of BC-pOct89 may be fractional Gaussian noise (fGn), which is appropriate when the time scaling is large [31, 32], or the generalized fGn [33]. Both BC-pOct89 and DEC-PKT-1 may be well described by the generalized Cauchy (GC) correlation structure [34]. The theme of this research is the convergence of sample ACF of traffic instead of correlation modeling of traffic. From the present results, nevertheless, one may infer that (5), (6), and (7) hold.

The data used on this research were measured in the last century, more precisely, BC-pOct89 in 1989 and DEC-PKT-1 in 1995. However, the research by Borgnat et al. [35] suggested that statistics of traffic remain identical from the early age of the Internet to today. Therefore, we infer that sample ACF of traffic today may be convergent.

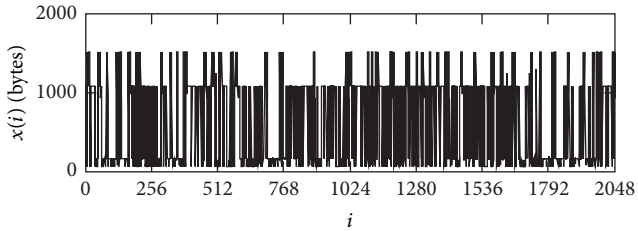

Figure 1: Plot of packet-size series of traffic trace BC-pOct89 on a packet-by-packet basis.

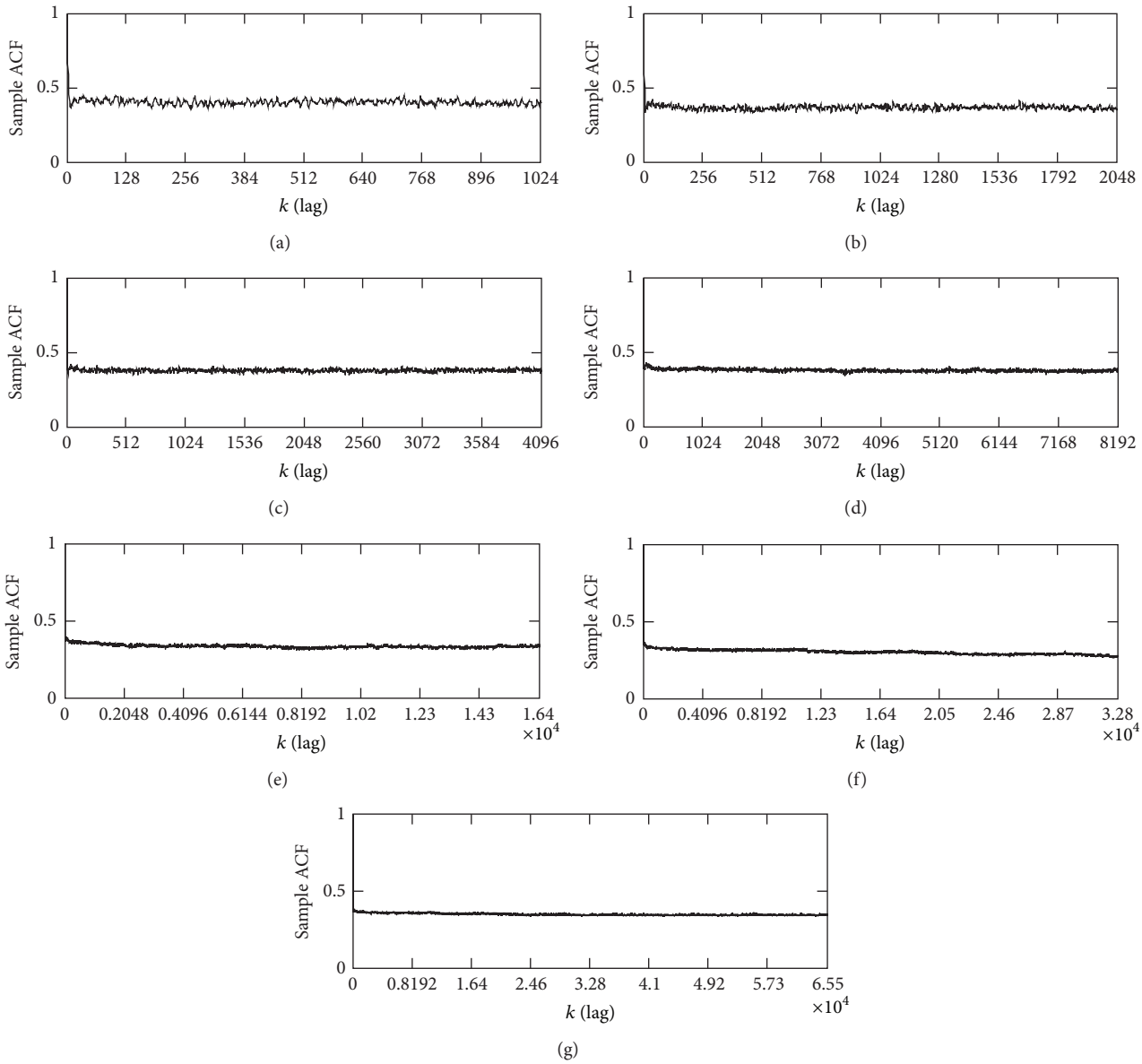

Figure 2: Sample ACFs of BC-pOct89 with the different sample sizes. (a) Sample ACF of BC-pOct89 with the sample size $I = 2048$. (b) Sample ACF of BC-pOct89 with $I = 4096$. (c) Sample ACF of BC-pOct89 with $I = 8192$. (d) Sample ACF of BC-pOct89 with $I = 16384$. (e) Sample ACF of BC-pOct89 with $I = 32768$. (f) Sample ACF of BC-pOct89 with $I = 65536$. (g) Sample ACF of BC-pOct89 with $I = 131027$.

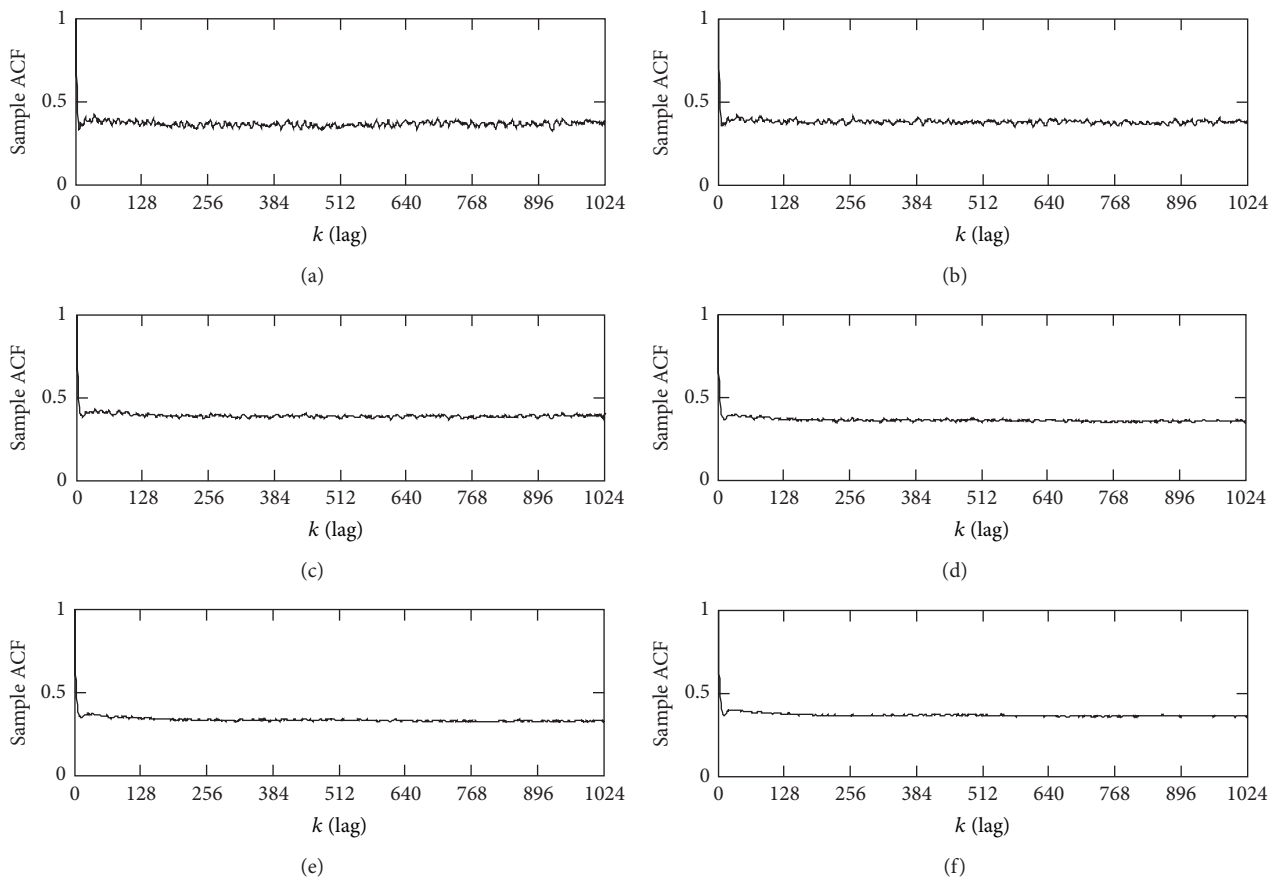

Figure 3: First 1024 points of sample ACFs of BC-pOct89 with the different sample sizes. (a) First 1024 points of sample ACF of BC-pOct89 with the sample size $I = 4096$. (b) First 1024 points of sample ACF of BC-pOct89 with $I = 8192$. (c) First 1024 points of sample ACF of BC-pOct89 with $I = 16384$. (d) First 1024 points of sample ACF of BC-pOct89 with $I = 32768$. (e) First 1024 sample ACF of BC-pOct89 with $I = 65536$. (f). First 1024 sample ACF of BC-pOct89 with $I = 131027$.

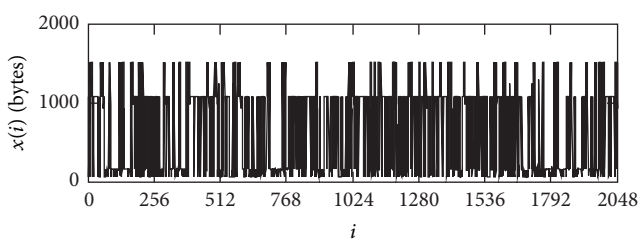

Figure 4: First 2048 points of packet-size series of traffic trace DEC-PKT-1 on a packet-by-packet basis.

Though we utilized traffic data in this research, the methodology described in this research might be possible for investigating the convergence of sample ACFs of other data series, for example, those in [36–43]. Finally, we note that this research does not imply something that might deviate from the point of view in traffic modeling using Levy stable structure, such as stable Levy motion [44], Levy flights [45], α-stable models [8] or the standard Cauchy distribution [10, 11], when those correspond to random functions with infinite variance such that they may yield divergent sample ACF. Rather, we suppose that it seems still faraway to thoroughly understand the statistics of traffic, without contradictions, with its commonly used models that may not be enough as Mandelbrot stated for modeling fractal random functions in general [46], but things may need developing; see, for example, Cohen and Lindner [47], Denby et al. [48], and Lazarou et al. [49].

5. Conclusions

We have exhibited that the sample ACFs of real-traffic data are convergent. Consequently, the sample ACFs of traffic exist. We have shown that the sample ACFs of traffic become smoother when the sample size increases.

Appendix

One More Case Study

The purpose of this appendix is to provide one more case study to demonstrate the convergence of the sample ACF of traffic using the real-traffic trace DEC-PKT-1.

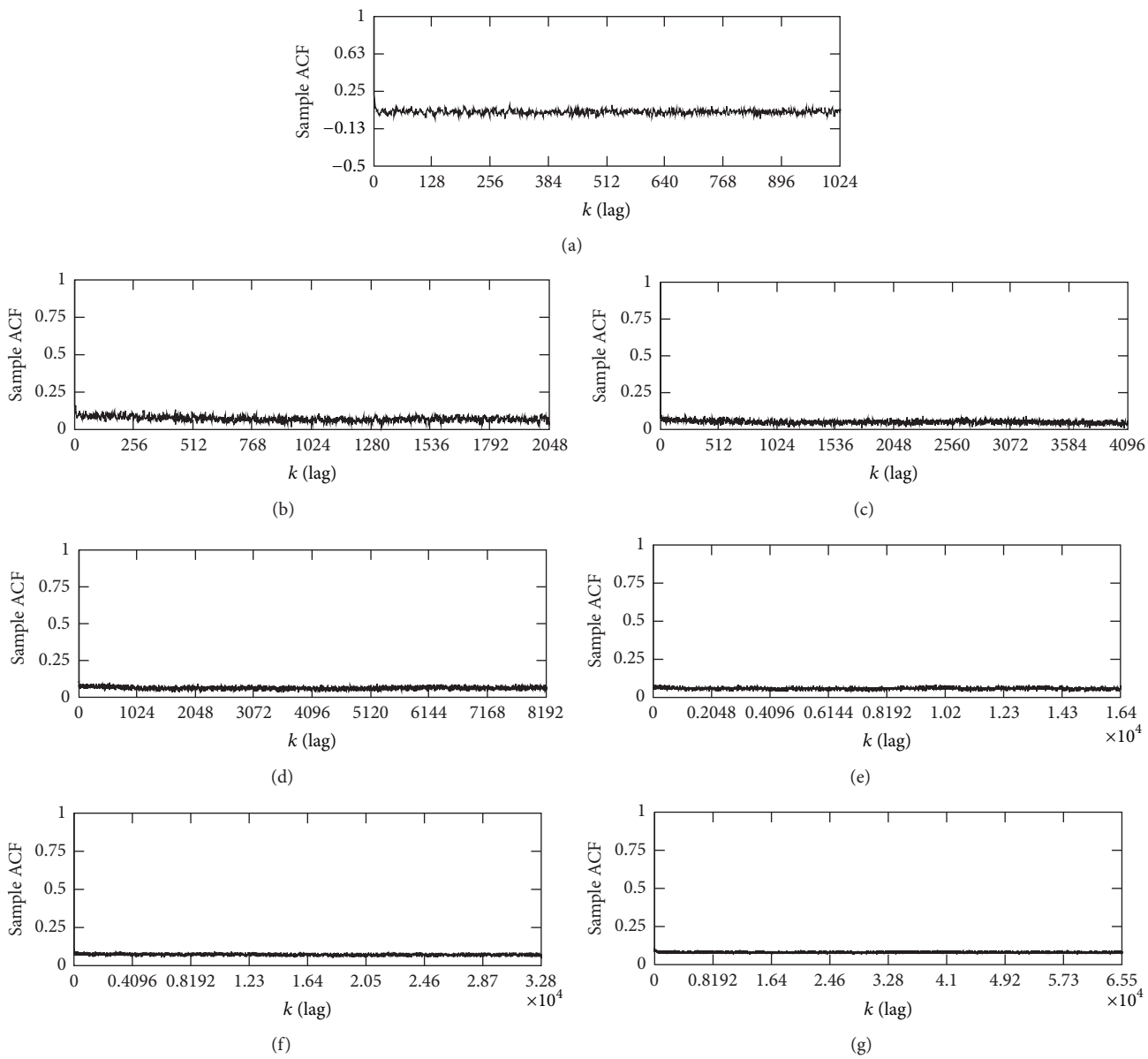

Figure 5: Plots of sample ACFs of DEC-PKT-1 with the different sample sizes. (a) Sample ACF of DEC-PKT-1 with the sample size $I = 2048$. (b) Sample ACF of DEC-PKT-1 with $I = 4096$. (c) Sample ACF of DEC-PKT-1 with $I = 8192$. (d) Sample ACF of DEC-PKT-1 with $I = 16384$. (e) Sample ACF of DEC-PKT-1 with $I = 32768$. (f) Sample ACF of DEC-PKT-1 with $I = 65536$. (g) Sample ACF of DEC-PKT-1 with $I = 131027$.

The first 2048 data points of DEC-PKT-1 are plotted in Figure 4. Figure 5 shows its plots of sample ACFs with different sample sizes from $I = 2^{11} = 2048$ to $I = 2^{17} = 131027$.

Figure 5 implies that the sample ACF of DEC-PKT-1 with different sample sizes is a deterministic function. Besides, the fluctuation of its sample ACF decreases as the sample size increases. We refine the observation that the fluctuation of its sample ACF decreases as the sample size increases in Figure 6 by indicating the first 1024 points of each sample ACF, making the result that the sample ACF of DEC-PKT-1 is convergent clearer.

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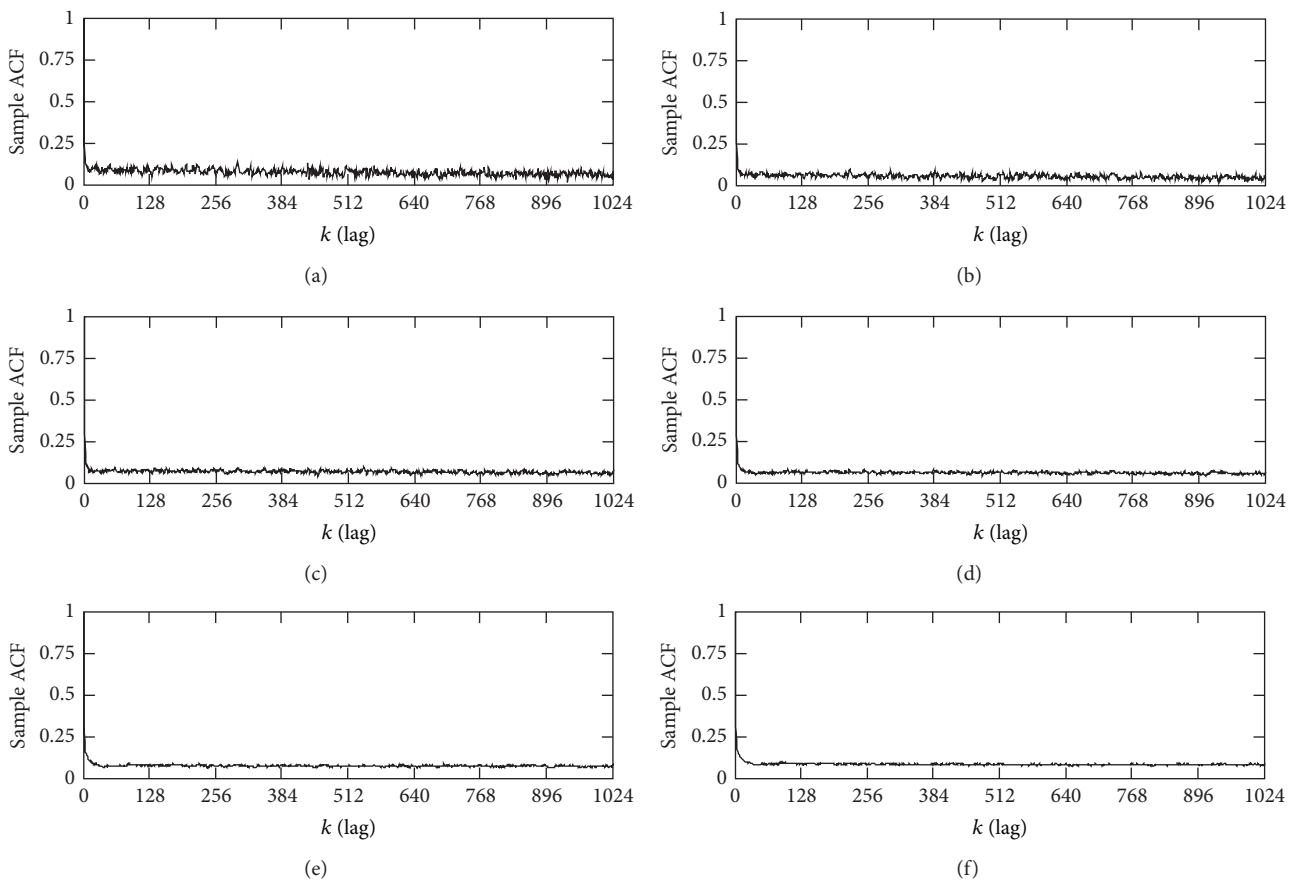

Figure 6: First 1024 points of sample ACFs of DEC-PKT-1 with the different sample sizes. (a) First 1024 points of sample ACF of DEC-PKT-1 with the sample size $I = 4096$. (b) First 1024 points of sample ACF of DEC-PKT-1 with $I = 8192$. (c) Sample ACF of DEC-PKT-1 with $I = 16384$. (d) First 1024 points of sample ACF of DEC-PKT-1 with $I = 32768$. (e) First 1024 sample ACF of DEC-PKT-1 with $I = 65536$. (f) First 1024 sample ACF of DEC-PKT-1 with $I = 131027$.

(dvw@bellcore.com), and Jeff Mogul (mogul@pa.dec.com), who provided them with their data in this research.

References

[1] A. M. Yaglom, *Correlation Theory of Stationary and Related Random Functions. Vol. I*, Springer, New York, NY, USA, 1987.

[2] B. W. Lindgren and G. W. McElrath, *Introduction to Probability and Statistics*, The Macmillan Company, New York, NY, USA, 1959.

[3] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, New York, NY, USA, 3rd edition, 2002.

[4] W. A. Fuller, *Introduction to Statistical Time Series*, John Wiley & Sons, New York, NY, USA, 2nd edition, 1996.

[5] J. S. Bendat and A. G. Piersol, *Random Data: Analysis and Measurement Procedure*, John Wiley & Sons, New York, NY, USA, 3rd edition, 2000.

[6] J. Beran, *Statistics for Long-Memory Processes*, Chapman & Hall, New York, NY, USA, 1994.

[7] S. Resnick, G. Samorodnitsky, and F. Xue, "How misleading can sample ACFs of stable MAs be? (Very!)," *The Annals of Applied Probability*, vol. 9, no. 3, pp. 797–817, 1999.

[8] A. Karasaridis and D. Hatzinakos, "Network heavy traffic modeling using α-stable self-similar processes," *IEEE Transactions on Communications*, vol. 49, no. 7, pp. 1203–1214, 2001.

[9] Ph. Barbe and W. P. McCormick, "Heavy-traffic approximations for fractionally integrated random walks in the domain of attraction of a non-Gaussian stable distribution," *Stochastic Processes and their Applications*, vol. 122, no. 4, pp. 1276–1303, 2012.

[10] A. J. Field, U. Harder, and P. G. Harrison, "Measurement and modelling of self-similar traffic in computer networks," *IEE Proceedings-Communications*, vol. 151, no. 4, pp. 355–363, 2004.

[11] T. Field, U. Harder, and P. Harrison, "Network traffic behaviour in switched Ethernet systems," *Performance Evaluation*, vol. 58, no. 2-3, pp. 243–260, 2004.

[12] Y. Afek, A. Bremler-Barr, and Y. Koral, "Space efficient deep packet inspection of compressed web traffic," *Computer Communications*, vol. 35, no. 7, pp. 810–819, 2012.

[13] V. N. G. J. Soares, F. Farahmand, and J. J. P. C. Rodrigues, "Traffic differentiation support in vehicular delay-tolerant networks," *Telecommunication Systems*, vol. 48, no. 1-2, pp. 151–162, 2011.

[14] A. H. Taherinia and M. Jamzad, "A two-step watermarking attack using long-range correlation image restoration," *Security and Communication Networks*, vol. 5, no. 6, pp. 625–635, 2012.

[15] Y. Cai, P. P. C. Lee, W. Gong, and D. Towsley, "Analysis of traffic correlation attacks on router queues," *Computer Networks*, vol. 55, no. 3, pp. 734–747, 2011.

[16] Y. Zhu, X. Fu, B. Graham, R. Bettati, and W. Zhao, "Correlation-based traffic analysis attacks on anonymity networks," *IEEE Transactions on Parallel and Distributed Systems*, vol. 21, no. 7, pp. 954–967, 2010.

[17] S. Lee, B. Chung, H. Kim, Y. Lee, C. Park, and H. Yoon, "Real-time analysis of intrusion detection alerts via correlation," *Computers and Security*, vol. 25, no. 3, pp. 169–183, 2006.

[18] J. Zhou, M. Heckman, B. Reynolds, A. Carlson, and M. Bishop, "Modeling network intrusion detection alerts for correlation," *ACM Transactions on Information and System Security*, vol. 10, no. 1, article 4, 2007.

[19] G. Min and M. Ould-Khaoua, "Prediction of communication delay in torus networks under multiple time-scale correlated traffic," *Performance Evaluation*, vol. 60, no. 1–4, pp. 255–273, 2005.

[20] B. Balcıoglu, D. L. Jagerman, and T. Altıok, "Merging and splitting autocorrelated arrival processes and impact on queueing performance," *Performance Evaluation*, vol. 65, no. 9, pp. 653–669, 2008.

[21] L. B. Lim, L. Guan, A. Grigg, I. W. Phillips, X. G. Wang, and I. U. Awan, "Controlling mean queuing delay under multi-class bursty and correlated traffic," *Journal of Computer and System Sciences*, vol. 77, no. 5, pp. 898–916, 2011.

[22] M. Ashour and T. Le-Ngoc, "Priority queuing of long-range dependent traffic," *Computer Communications*, vol. 31, no. 17, pp. 3954–3963, 2008.

[23] http://ita.ee.lbl.gov/html/contrib/BC.html .

[24] http://ita.ee.lbl.gov/html/contrib/DEC-PKT.html .

[25] W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson, "On the self-similar nature of Ethernet traffic (extended version)," *IEEE/ACM Transactions on Networking*, vol. 2, no. 1, pp. 1–15, 1994.

[26] H. J. Fowler and W. E. Leland, "Local area network traffic characteristics, with implications for broadband network congestion management," *IEEE Journal on Selected Areas in Communications*, vol. 9, no. 7, pp. 1139–1149, 1991.

[27] V. Paxson and S. Floyd, "Wide area traffic: The failure of Poisson modeling," *IEEE/ACM Transactions on Networking*, vol. 3, no. 3, pp. 226–244, 1995.

[28] J. Cao, W. S. Cleveland, and D. X. Sun, "Bandwidth estimation for best-effort internet traffic," *Statistical Science*, vol. 19, no. 3, pp. 518–543, 2004.

[29] S. Resnick, "On the foundations of multivariate heavy-tail analysis," *Journal of Applied Probability*, vol. 41, pp. 191–212, 2004.

[30] B. D'Auria and S. I. Resnick, "The influence of dependence on data network models," *Advances in Applied Probability*, vol. 40, no. 1, pp. 60–94, 2008.

[31] A. Feldmann, A. C. Gilbert, W. Willinger, and T. G. Kurtz, "The changing nature of network traffic: scaling phenomena," *ACM SIGCOMM Computer Communication Review*, vol. 28, no. 2, pp. 5–29, 1998.

[32] M. Li, W. Zhao, W. Jia, D. Long, and C.-H. Chi, "Modeling autocorrelation functions of self-similar teletraffic in communication networks based on optimal approximation in Hilbert space," *Applied Mathematical Modelling*, vol. 27, no. 3, pp. 155–168, 2003.

[33] M. Li, "Modeling autocorrelation functions of long-range dependent teletraffic series based on optimal approximation in Hilbert space-A further study," *Applied Mathematical Modelling*, vol. 31, no. 3, pp. 625–631, 2007.

[34] M. Li and S. C. Lim, "Modeling network traffic using generalized Cauchy process," *Physica A*, vol. 387, no. 11, pp. 2584–2594, 2008.

[35] P. Borgnat, G. Dewaele, K. Fukuda, P. Abry, and K. Cho, "Seven years and one day: sketching the evolution of internet traffic," in *Proceedings of the 28th IEEE Conference on Computer Communications (INFOCOM '09)*, pp. 711–719, Rio de Janeiro, Brazil, April 2009.

[36] C. Cattani, E. Laserra, I. Bochicchio, and K. K. Nandi, "Correct light deflection in Weyl conformal gravity," *Physical Review D*, vol. 87, no. 4, Article ID 47503, 4 pages, 2013.

[37] C. Cattani, "Fractional calculus and Shannon wavelet," *Mathematical Problems in Engineering*, vol. 2012, Article ID 502812, 26 pages, 2012.

[38] C. Cattani, G. Pierro, and G. Altieri, "Entropy and multifractality for the myeloma multiple TET 2 gene," *Mathematical Problems in Engineering*, vol. 2012, Article ID 193761, 14 pages, 2012.

[39] C. Toma, "Advanced signal processing and command synthesis for memory-limited complex systems," *Mathematical Problems in Engineering*, vol. 2012, Article ID 927821, 13 pages, 2012.

[40] E. G. Bakhoum and M. H. M. Cheng, "Miniature carbon monoxide detector based on nanotechnology," , *IEEE Transactions on Instrumentation and Measurement*, vol. 62, no. 1, pp. 240–245, 2013.

[41] E. G. Bakhoum, "High-sensitivity miniature smoke detector," *IEEE Sensors Journal*, vol. 12, no. 10, pp. 3031–3035, 2012.

[42] E. G. Bakhoum and C. Toma, "Modeling transitions in complex systems by multiplicative effect of temporal patterns extracted from signal flows," *Mathematical Problems in Engineering*, vol. 2012, Article ID 409856, 11 pages, 2012.

[43] E. G. Bakhoum and C. Toma, "Specific mathematical aspects of dynamics generated by coherence functions," *Mathematical Problems in Engineering*, vol. 2011, Article ID 436198, 10 pages, 2011.

[44] T. Mikosch, S. Resnick, H. Rootzén, and A. Stegeman, "Is network traffic approximated by stable Lévy motion or fractional Brownian motion?" *The Annals of Applied Probability*, vol. 12, no. 1, pp. 23–68, 2002.

[45] G. Terdik and T. Gyires, "Lévy flights and fractal modeling of internet traffic," *IEEE/ACM Transactions on Networking*, vol. 17, no. 1, pp. 120–129, 2009.

[46] B. B. Mandelbrot, *Multifractals and 1/f Noise*, Springer, New York, NY, USA, 1999.

[47] S. Cohen and A. Lindner, "A central limit theorem for the sample autocorrelations of a Lévy driven continuous time moving average process," *Journal of Statistical Planning and Inference*, vol. 143, no. 8, pp. 1295–1306, 2013.

[48] L. Denby, J. M. Landwehr, C. L. Mallows et al., "Statistical aspects of the analysis of data networks," *Technometrics*, vol. 49, no. 3, pp. 318–334, 2007.

[49] G. Y. Lazarou, J. Baca, V. S. Frost, and J. B. Evans, "Describing network traffic using the index of variability," *IEEE/ACM Transactions on Networking*, vol. 17, no. 5, pp. 1672–1683, 2009.