Research Article

Passive Control and ε -Bound Estimation of Singularly Perturbed Systems with Nonlinear Nonlinearities

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This paper considers the problems of passivity analysis and synthesis of singularly perturbed systems with nonlinear uncertainties. By a novel storage function depending on the singular perturbation parameter ε , a new method is proposed to estimate the ε -bound, such that the system is passive when the singular perturbation parameter is lower than the ε -bound. Furthermore, a controller design method is proposed to achieve a predefined ε -bound. The proposed results are shown to be less conservative than the existing ones because the adopted storage function is more general. Finally, an RLC circuit is presented to illustrate the advantages and effectiveness of the proposed methods.

1. Introduction

Singularly perturbed systems (SPSs), with a small singular perturbation parameter ε determining the degree of separation between the “slow” and “fast” modes of the systems, have attracted much attention due to their widespread applications in chemical process, robot, aerospace engineering, power systems, and so forth [1, 2]. A key problem for investigations of SPSs is to estimate the upper bound $\bar{\varepsilon}$ of the singular perturbation parameter ε such that the system preserves its stability or other desired performance for all $\varepsilon \in (0, \bar{\varepsilon}]$, which is referred to as the ε -bound problem [3–7]. The ε -bound problem on stability of SPSs has been widely considered and many frequency- and time-domain methods have been proposed [3–5]. H_∞ performance of SPSs was considered in [6, 7], and some methods for evaluating the upper bound $\bar{\varepsilon}$ to meet a prescribed H_∞ performance requirement were given. However, the ε -bound problem on passivity of SPSs is still open.

Passivity, which was introduced by Willems [8, 9], relates nicely to stability of systems. On one hand, storage functions induced by passivity are usually related with system energy and thus provide natural candidates for the Lyapunov functions; on the other hand, passivity is expected to be preserved under feedback interconnection, which provides a useful tool for stability analysis of feedback systems [10]. In addition,

the passivity theory which is intimately related to circuit analysis methods has played an important role in robust and nonlinear stabilization problems [11, 12]. It has been shown that passivity provides a suitable design approach in power systems, neural networks, signal processing, chaos control, and synchronization [11–16].

Recently, the problem of passivity analysis and synthesis of SPSs has attracted more attention. Applying the routine methods for normal systems to SPSs usually leads to ill-conditioned numerical problems [1]. The conventional approaches to avoiding the ill-conditioned problem are based on decomposing the original SPSs into fast and slow subsystems. In [17], passive control of T-S fuzzy SPSs was studied. It was shown that the passivity of SPSs is equivalent to the passivity of both the boundary system and the slow subsystem. The proposed method is based on the decomposition of the original systems, which leads to difficulties for computing the ε -bound for the SPSs to preserve passivity. An alternative approach to alleviating the ill-conditioned problem is based on the so-called descriptor system method and independent of the system decomposition [18–20]. Passivity analysis and passivity-based controller design were studied for uncertain singularly perturbed Markovian jump systems with time-varying delay in [18]; but it did not consider the ε -bound estimation problem. In [19], sufficient conditions in

terms of linear matrix inequalities (LMIs) were derived, and the estimation of ε -bound can be obtained by solving a generalized eigenvalue problem (GEVP). Following the line of [19], passivity-based integral sliding mode control was studied for uncertain singularly perturbed systems in [20]. All in all, passive control of SPSs with the consideration of ε -bound estimation is still an open problem.

This paper considers passive control and ε -bound estimation of singularly perturbed systems with nonlinear uncertainties. A storage function which gives full consideration of the singular perturbation structure is first constructed, by which a sufficient condition for the system to be passive for any $\varepsilon \in (0, \bar{\varepsilon}]$ is proposed, where $\bar{\varepsilon}$ is an upper bound of ε . Then, based on the proposed sufficient condition, a bisectional search algorithm is formulated to compute the best estimate of ε -bound. Furthermore, a controller design method is proposed to achieve a predefined ε -bound. Finally, an RLC circuit is presented to demonstrate the proposed results. The main contributions of the paper are as follows: (1) the proposed method implicitly employs the singular perturbation structure of the SPSs rather than depends on decomposing the original systems into reduced-order subsystems, which provides convenience for solving the ε -bound problem; (2) the adopted storage function is more general than the existing one, which leads to a less conservative method for passivity analysis of SPSs and provides a tighter estimate of the ε -bound; (3) a constructive algorithm for computing the best estimate of the ε -bound is proposed; (4) a predefined ε -bound is one of the controller design objectives.

2. Preliminaries

This paper will consider the following singularly perturbed system:

$$\begin{aligned} E(\varepsilon) \dot{x}(t) &= Ax(t) + B_f f(t, x) + B_u u(t) + B_\omega \omega(t), \\ z(t) &= Cx(t) + D_g g(t, x) + D_\omega \omega(t), \end{aligned} \quad (1)$$

where $x(t) = [x_1^T \ x_2^T]^T \in R^n$ denotes the system state, $x_1 \in R^{n_1}$ and $x_2 \in R^{n_2}$, ($n_1 + n_2 = n$) represent the slow modes and the fast modes of the state, respectively, $u(t) \in R^l$ is the control input, $z(t) \in R^p$ is the controlled output, and $\omega(t) \in L_2^q[0, \infty)$ is the external disturbance. And $E(\varepsilon) = \begin{bmatrix} I_{n_1} & 0_{n_1 \times n_2} \\ 0_{n_2 \times n_1} & \varepsilon I_{n_2} \end{bmatrix} \in R^{n \times n}$, $A \in R^{n \times n}$, $B_f \in R^{n \times r}$, $B_u \in R^{n \times l}$, $B_\omega \in R^{n \times q}$, $C \in R^{m \times n}$, $D_g \in R^{n \times s}$, and $D_\omega \in R^{m \times q}$ are known constant matrices. $f(t, x) \in R^r$ and $g(t, x) \in R^s$ are vector-valued time-varying nonlinear uncertainties, which are bounded by

$$\|f(t, x)\| \leq \alpha \|Mx(t)\|, \quad (2)$$

$$\|g(t, x)\| \leq \beta \|Nx(t)\|, \quad (3)$$

where $\alpha > 0$, $\beta > 0$ are given constants and M, N known constant matrices of approximate dimensions.

Definition 1 (see [13, 21]). System (1) is said to be passive if

$$\int_0^T \omega^T(t) z(t) dt \geq 0, \quad \forall T > 0 \quad (4)$$

holds for all trajectories with zero initial condition $x(0) = 0$. It is said to have dissipation η if

$$\int_0^T \omega^T(t) z(t) - \eta \omega^T(t) \omega(t) dt \geq 0, \quad \forall T > 0 \quad (5)$$

holds for all trajectories with zero initial condition $x(0) = 0$.

The largest dissipation of the system, that is, the largest value of η such that (5) holds, is usually called its dissipativity [13, 21]. It is known that a system with larger dissipativity is able to tolerate larger uncertainties and disturbances [13]. When $\eta = 0$, it is the case of passivity that was discussed in [19]. In this paper, we will consider the more general case $\eta \geq 0$.

The following lemmas will be used in the sequel.

Lemma 2 (see [7]). Given $\bar{\varepsilon} > 0$ and symmetric matrices S_1, S_2 , and S_3 , if

$$\begin{aligned} S_1 &\geq 0, \\ S_1 + \bar{\varepsilon} S_2 &> 0, \\ S_1 + \bar{\varepsilon} S_2 + \bar{\varepsilon}^2 S_3 &> 0, \end{aligned} \quad (6)$$

then, it holds that

$$S_1 + \varepsilon S_2 + \varepsilon^2 S_3 > 0, \quad \forall \varepsilon \in (0, \bar{\varepsilon}]. \quad (7)$$

Lemma 3 (see [7]). If there exist matrices Z_i ($i = 1, 2, \dots, 5$) of appropriate dimensions with $Z_i = Z_i^T$ ($i = 1, 2, 3, 4$) satisfying the following LMIs:

$$\begin{aligned} Z_1 &> 0, \\ \begin{bmatrix} Z_1 + \bar{\varepsilon} Z_3 & \bar{\varepsilon} Z_5^T \\ \bar{\varepsilon} Z_5 & \bar{\varepsilon} Z_2 \end{bmatrix} &> 0, \\ \begin{bmatrix} Z_1 + \bar{\varepsilon} Z_3 & \bar{\varepsilon} Z_5^T \\ \bar{\varepsilon} Z_5 & \bar{\varepsilon} Z_2 + \bar{\varepsilon}^2 Z_4 \end{bmatrix} &> 0, \end{aligned} \quad (8)$$

then, it holds that

$$E(\varepsilon) Z(\varepsilon) = (E(\varepsilon) Z(\varepsilon))^T = Z^T(\varepsilon) E(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \bar{\varepsilon}], \quad (9)$$

where

$$Z(\varepsilon) = \begin{bmatrix} Z_1 + \varepsilon Z_3 & \varepsilon Z_5^T \\ Z_5 & Z_2 + \varepsilon Z_4 \end{bmatrix}, \quad E(\varepsilon) = \begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}. \quad (10)$$

3. Main Results

3.1. Passivity Analysis and ε -Bound Estimation. In this subsection, using a novel storage function depending on the singular perturbation parameter, a sufficient condition for the system to be passive is proposed. Furthermore, bisectional search algorithms are formulated to get the best estimate of the ε -bound and the optimal dissipation η , respectively.

Theorem 4. Given η and $\bar{\varepsilon} > 0$, system (1) with $u(t) = 0$ is passive with dissipation η for all $\varepsilon \in (0, \bar{\varepsilon}]$, if there exist scalars $\mu_1 > 0$, $\mu_2 > 0$, and matrices Z_i ($i = 1, \dots, 5$) with $Z_i = Z_i^T$ ($i = 1, 2, 3, 4$) satisfying the following LMIs:

$$Z_1 > 0,$$

$$\begin{bmatrix} Z_1 + \bar{\varepsilon}Z_3 & \bar{\varepsilon}Z_5^T \\ \bar{\varepsilon}Z_5 & \bar{\varepsilon}Z_2 \end{bmatrix} > 0, \quad (11)$$

$$\begin{bmatrix} Z_1 + \bar{\varepsilon}Z_3 & \bar{\varepsilon}Z_5^T \\ \bar{\varepsilon}Z_5 & \bar{\varepsilon}Z_2 + \bar{\varepsilon}^2Z_4 \end{bmatrix} > 0,$$

$$\begin{bmatrix} \mu_1\alpha^2M^TM + \mu_2\beta^2N^TN + \Delta(0) & Z^T(0)B_\omega - C^T & Z^T(0)B_f & 0 \\ * & 2\eta I - D_\omega^T - D_\omega & 0 & -D_g \\ * & * & -\mu_1 I & 0 \\ * & * & * & -\mu_2 I \end{bmatrix} < 0,$$

$$\begin{bmatrix} \mu_1\alpha^2M^TM + \mu_2\beta^2N^TN + \Delta(\bar{\varepsilon}) & Z^T(\bar{\varepsilon})B_\omega - C^T & Z^T(\bar{\varepsilon})B_f & 0 \\ * & 2\eta I - D_\omega^T - D_\omega & 0 & -D_g \\ * & * & -\mu_1 I & 0 \\ * & * & * & -\mu_2 I \end{bmatrix} < 0, \quad (12)$$

where $Z(\varepsilon) = \begin{bmatrix} Z_1 + \varepsilon Z_3 & \varepsilon Z_5^T \\ Z_5 & Z_2 + \varepsilon Z_4 \end{bmatrix}$, $\Delta(\varepsilon) = A^T Z(\varepsilon) + Z^T(\varepsilon)A$.

Proof. Define a storage function as follows:

$$V(x) = x^T(t) E(\varepsilon) Z(\varepsilon) x(t). \quad (13)$$

By Lemma 3, LMIs (11) imply that

$$E(\varepsilon) Z(\varepsilon) = Z^T(\varepsilon) E(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \bar{\varepsilon}]. \quad (14)$$

Thus, $V(x)$ defined by (13) is positive definite for any $\varepsilon \in (0, \bar{\varepsilon}]$.

By Lemma 2, it follows from (12) that, for any $\varepsilon \in (0, \bar{\varepsilon}]$,

$$\begin{bmatrix} A^T Z(\varepsilon) + Z^T(\varepsilon) A + \Delta(\varepsilon) & Z^T(\varepsilon) B_\omega - C^T & Z^T(\varepsilon) B_f & 0 \\ * & 2\eta I - D^T - D & 0 & -D_g \\ * & * & -\mu_1 I & 0 \\ * & * & * & -\mu_2 I \end{bmatrix} < 0. \quad (15)$$

Taking the derivative of $V(x)$ along the trajectories of system (1) with $u(t) = 0$ and using constraints (2) and (3), for any scalars $\mu_1 > 0$, $\mu_2 > 0$, and $\varepsilon \in (0, \bar{\varepsilon}]$, we have

$$\begin{aligned} \dot{V}(x)|_{(1)} &= \dot{x}^T(t) E(\varepsilon) Z(\varepsilon) x(t) \\ &\quad + x^T(t) E(\varepsilon) Z(\varepsilon) \dot{x}(t) \\ &= (Ax(t) + B_f f(t, x) + B_\omega \omega(t))^T \\ &\quad \times Z(\varepsilon) x(t) \\ &\quad + x^T(t) Z^T(\varepsilon) \\ &\quad \times (Ax(t) + B_f f(t, x) + B_\omega \omega(t)) \\ &= x^T(t) (A^T Z(\varepsilon) + Z^T(\varepsilon) A) x(t) \\ &\quad + 2x^T(t) Z^T(\varepsilon) B_f f(t, x) \\ &\quad + 2x^T(t) Z^T(\varepsilon) B_\omega \omega(t) \\ &\leq x^T(t) (A^T Z(\varepsilon) + Z^T(\varepsilon) A) x(t) \\ &\quad + 2x^T(t) Z^T(\varepsilon) B_f f(t, x) \\ &\quad + 2x^T(t) Z^T(\varepsilon) B_\omega \omega(t) \\ &\quad + \mu_1 (\alpha^2 x^T(t) M^T M x(t) - f^T f) \\ &\quad + \mu_2 (\beta^2 x^T(t) N^T N x(t) - g^T g). \end{aligned} \quad (16)$$

Therefore,

$$\begin{aligned} \dot{V}(x)|_{(1)} &- 2\omega^T(t) z(t) + 2\eta \omega^T(t) \omega(t) \\ &= \begin{bmatrix} x \\ \omega \\ f \\ g \end{bmatrix}^T \begin{bmatrix} A^T Z(\varepsilon) + Z^T(\varepsilon) A + \Delta(\varepsilon) & Z^T(\varepsilon) B_\omega - C^T & Z^T(\varepsilon) B_f & 0 \\ * & 2\eta I - D^T - D & 0 & -D_g \\ * & * & -\mu_1 I & 0 \\ * & * & * & -\mu_2 I \end{bmatrix} \begin{bmatrix} x \\ \omega \\ f \\ g \end{bmatrix}. \end{aligned} \quad (17)$$

Taking into account (15) and (17), we have

Taking the integral on the two sides of (18) from 0 to T , we obtain

$$\dot{V}(x)|_{(1)} - 2\omega^T(t) z(t) + 2\eta \omega^T(t) \omega(t) \leq 0. \quad (18)$$

$$\int_0^T \dot{V}(x) - 2\omega^T(t) z(t) + 2\eta \omega^T(t) \omega(t) \leq 0, \quad (19)$$

which implies

$$\int_0^T 2\omega^T(t) z(t) - 2\eta\omega^T(t) \omega(t) \geq V(x(T)) - V(x(0)). \quad (20)$$

Therefore,

$$\int_0^T \omega^T(t) z(t) - \eta\omega^T(t) \omega(t) dt \geq 0, \quad \forall T > 0 \quad (21)$$

holds for all trajectories with zero initial condition $x(0) = 0$.

Hence, system (1) is passive with dissipation η for all $\varepsilon \in (0, \bar{\varepsilon}]$. $\square$

Remark 5. In [17], passivity of fuzzy SPSs was investigated based on decomposing the original system into fast and slow subsystems. Since the system decomposition requires the singular perturbation to be small enough, the proposed results are only sufficient conditions for the existence of an ε -bound for the SPSs to preserve passivity but cannot produce an estimate of the ε -bound. In this paper, the newly developed method implicitly employs the singular perturbation structure of the SPSs rather than depends on system decomposition, which provides convenience for estimating the value of the ε -bound.

Remark 6. In [19], a sufficient condition for system (1) to be passive with dissipation $\eta = 0$ was proposed by using a storage function in the form of

$$V(x) = \begin{bmatrix} x_1^T & x_2^T \end{bmatrix} \begin{bmatrix} Z_1 & \varepsilon Z_5^T \\ \varepsilon Z_5 & \varepsilon Z_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad (22)$$

which is a special case of (13) with $Z_3 = 0$ and $Z_4 = 0$. Thus, Theorem 4 is more general and less conservative than the existing results in [19], which will be illustrated by the example in the next section.

The upper bound $\bar{\varepsilon}$ given by Theorem 4 is guessed. We now propose a bisectional search algorithm to get the best estimate of $\bar{\varepsilon}$.

Algorithm 7. Bisectional search algorithm optimizing $\bar{\varepsilon}$ for given dissipation η .

Step 1. Given positive scalars $\rho, \sigma, \gamma, \delta$, where ρ, δ are sufficiently small, γ is sufficiently large, and $\rho < \sigma < \gamma$. Set $\underline{\tau} = \bar{\tau} = \sigma$.

Step 2. Check LMIs (11)-(12) with $\bar{\varepsilon} = \sigma$. If they are feasible, set $\underline{\tau} = \sigma$ and $\sigma := 2\sigma$; otherwise, set $\bar{\tau} = \sigma$ and $\sigma := 0.5\sigma$.

Step 3. If $\bar{\tau} = \rho$ or $\underline{\tau} > \gamma$, go to Step 7. Else if $\underline{\tau} \geq \bar{\tau}$, go to Step 2.

Step 4. Set $\varepsilon^* = 0.5(\underline{\tau} + \bar{\tau})$.

Step 5. Check LMIs (11)-(12) with $\bar{\varepsilon} = \varepsilon^*$. If they are feasible, set $\underline{\tau} = \varepsilon^*$; otherwise, set $\bar{\tau} = \varepsilon^*$.

Step 6. Go to Step 4 if $|\underline{\tau} - \bar{\tau}| > \delta$; otherwise, go to Step 7.

Step 7. If $\bar{\tau} < \rho$, the proposed method cannot give an answer. If $\underline{\tau} > \gamma$, the optimal upper bound $\bar{\varepsilon}$ is larger than γ . Otherwise, the optimal upper bound produced by the proposed method is the value of $\bar{\varepsilon}$. End.

Remark 8. In Algorithm 7, Step 1 presents the initial and terminal conditions. Steps 2-3 will show that the proposed method does not work or the optimal upper bound $\bar{\varepsilon}$ is larger than the given γ or determine a search interval $[\underline{\tau}, \bar{\tau}]$ for Steps 4-6 such that LMIs (11)-(12) are feasible with $\bar{\varepsilon} = \underline{\tau}$ but not with $\bar{\varepsilon} = \bar{\tau}$. Steps 4-6 are used to search the best estimate of the upper bound $\bar{\varepsilon}$ in $[\underline{\tau}, \bar{\tau}]$.

Similarly, we have the following algorithm to get the largest dissipation η for given $\bar{\varepsilon}$.

Algorithm 9. Bisectional search algorithm optimizing η for given $\bar{\varepsilon}$.

Step 1. Given positive scalars $\rho, \sigma, \gamma, \delta$, where ρ, δ are sufficiently small, γ is sufficiently large, and $\rho < \sigma < \gamma$. Set $\underline{\tau} = \bar{\tau} = \sigma$.

Step 2. Check LMIs (11)-(12) with $\eta = \sigma$. If they are feasible, set $\underline{\tau} = \sigma$ and $\sigma := 2\sigma$; otherwise, set $\bar{\tau} = \sigma$ and $\sigma := 0.5\sigma$.

Step 3. If $\bar{\tau} = \rho$ or $\underline{\tau} > \gamma$, go to Step 7. Else if $\underline{\tau} \geq \bar{\tau}$, go to Step 2.

Step 4. Set $\eta^* = 0.5(\underline{\tau} + \bar{\tau})$.

Step 5. Check LMIs (11)-(12) with $\eta = \eta^*$. If they are feasible, set $\underline{\tau} = \eta^*$; otherwise, set $\bar{\tau} = \eta^*$.

Step 6. Go to Step 4 if $|\underline{\tau} - \bar{\tau}| > \delta$; otherwise, go to Step 7.

Step 7. If $\bar{\tau} < \rho$, the proposed method cannot give an answer. If $\underline{\tau} > \gamma$, the optimal value of η is larger than γ . Otherwise, the optimal value produced by the proposed method is the value of η . End.

Remark 10. Since the conditions proposed in Theorem 4 are sufficient but not necessary for the system to be passive, thus the best estimates of ε -bound and dissipation η obtained by Algorithms 7 and 9 are less than or equal to their true values. Thus, the conservatism of the algorithms results from Theorem 4. Although Theorem 4 is less conservative than the existing results in [19], there may be some room to further reduce the conservatism, as will be considered in our future work.

3.2. Passive Controller Design. In this subsection, we will design a state feedback controller for system (1) to achieve a given ε -bound. The controller under consideration is in the form of

$$u(t) = F(\varepsilon) x(t), \quad (23)$$

where $F(\varepsilon) \in R^{l \times n}$ is the controller gain to be designed. Then,

the closed-loop system is as follows:

$$E(\varepsilon) \dot{x}(t) = (A + B_u F(\varepsilon)) x(t) + B_f f(t, x) + B_\omega \omega(t),$$

$$z(t) = Cx(t) + D_g g(t, x) + D_\omega \omega(t).$$

(24)

Theorem 11. Given η and $\bar{\varepsilon} > 0$, if there exist scalars $\mu_1 > 0$, $\mu_2 > 0$, $\bar{\mu}_1 > 0$, $\bar{\mu}_2 > 0$, and matrices Z_i ($i = 1, \dots, 5$) with $Z_i = Z_i^T$ ($i = 1, 2, 3, 4$) and K satisfying

$$Z_1 > 0,$$

$$\begin{bmatrix} Z_1 + \bar{\varepsilon} Z_3 & \bar{\varepsilon} Z_5^T \\ \bar{\varepsilon} Z_5 & \bar{\varepsilon} Z_2 \end{bmatrix} > 0, \quad (25)$$

$$\begin{bmatrix} Z_1 + \bar{\varepsilon} Z_3 & \bar{\varepsilon} Z_5^T \\ \bar{\varepsilon} Z_5 & \bar{\varepsilon} Z_2 + \bar{\varepsilon}^2 Z_4 \end{bmatrix} > 0, \quad (26)$$

$$\begin{bmatrix} \Theta(0) & B_\omega - Z^T(0)C^T & B_f & 0 & \alpha Z^T(0)M^T & \beta Z^T(0)N^T \\ * & 2\eta I - D_\omega^T - D_\omega & 0 & -D_g & 0 & 0 \\ * & * & -\mu_1 I & 0 & 0 & 0 \\ * & * & * & -\mu_2 I & 0 & 0 \\ * & * & * & * & -\bar{\mu}_1 I & 0 \\ * & * & * & * & * & -\bar{\mu}_2 I \end{bmatrix} < 0, \quad (27)$$

$$\begin{bmatrix} \Theta(\bar{\varepsilon}) & B_\omega - Z^T(\bar{\varepsilon})C^T & B_f & 0 & \alpha Z^T(\bar{\varepsilon})M^T & \beta Z^T(\bar{\varepsilon})N^T \\ * & 2\eta I - D_\omega^T - D_\omega & 0 & -D_g & 0 & 0 \\ * & * & -\mu_1 I & 0 & 0 & 0 \\ * & * & * & -\mu_2 I & 0 & 0 \\ * & * & * & * & -\bar{\mu}_1 I & 0 \\ * & * & * & * & * & -\bar{\mu}_2 I \end{bmatrix} < 0, \quad (28)$$

$$\mu_1 \bar{\mu}_1 = 1, \quad \mu_2 \bar{\mu}_2 = 1, \quad (29)$$

where $Z(\varepsilon) = \begin{bmatrix} Z_1 + \varepsilon Z_3 & \varepsilon Z_5^T \\ Z_5 & Z_2 + \varepsilon Z_4 \end{bmatrix}$, $\Theta(\varepsilon) = AZ(\varepsilon) + Z^T(\varepsilon)A^T + BK + K^T B^T$, then, the closed-loop system (24) is passive with dissipation η for all $\varepsilon \in (0, \bar{\varepsilon}]$, and the gain matrix can be designed as $F(\varepsilon) = KZ^{-1}(\varepsilon)$.

Proof. By Lemma 3, LMIs (25) and (26) imply that

$$E(\varepsilon) Z(\varepsilon) = Z^T(\varepsilon) E(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \bar{\varepsilon}], \quad (30)$$

which shows that

$$Z^{-T}(\varepsilon) E(\varepsilon) = E(\varepsilon) Z^{-1}(\varepsilon) > 0, \quad \forall \varepsilon \in (0, \bar{\varepsilon}]. \quad (31)$$

Define a storage function as follows:

$$V(x) = x^T(t) Z^{-T}(\varepsilon) E(\varepsilon) x(t). \quad (32)$$

Thus, $V(x)$ defined by (32) is positive definite for any $\varepsilon \in (0, \bar{\varepsilon}]$.

By Lemma 2, it follows from (27), (28), and (29) that, for any $\varepsilon \in (0, \bar{\varepsilon}]$,

$$\begin{bmatrix} \Theta(\varepsilon) & B_\omega - Z^T(\varepsilon)C^T & B_f & 0 & \alpha Z^T(\varepsilon)M^T & \beta Z^T(\varepsilon)N^T \\ * & 2\eta I - D_\omega^T - D_\omega & 0 & -D_g & 0 & 0 \\ * & * & -\mu_1 I & 0 & 0 & 0 \\ * & * & * & -\mu_2 I & 0 & 0 \\ * & * & * & * & -\bar{\mu}_1 I & 0 \\ * & * & * & * & * & -\bar{\mu}_2 I \end{bmatrix} < 0, \quad (33)$$

which is equivalent to

$$\begin{bmatrix} \Theta(\varepsilon) + \mu_1 \alpha^2 Z^T(\varepsilon) M^T M Z(\varepsilon) & B_\omega - Z^T(\varepsilon) C^T & B_f & 0 \\ + \mu_2 \beta^2 Z^T(\varepsilon) N^T N Z(\varepsilon) & 2\eta I - D_\omega^T - D_\omega & 0 & -D_g \\ * & * & -\mu_1 I & 0 \\ * & * & * & -\mu_2 I \end{bmatrix} < 0. \quad (34)$$

Pre- and postmultiplying (34) by $\text{diag}\{Z^{-T}, I, I, I\}$ and its transpose gives

$$\begin{bmatrix} \bar{\Theta}(\varepsilon) + \mu_1 \alpha^2 M^T M & Z^{-T}(\varepsilon) B_\omega - C^T & Z^{-T}(\varepsilon) B_f & 0 \\ + \mu_2 \beta^2 N^T N & 2\eta I - D_\omega^T - D_\omega & 0 & -D_g \\ * & * & -\mu_1 I & 0 \\ * & * & * & -\mu_2 I \end{bmatrix} < 0, \quad \varepsilon \in (0, \bar{\varepsilon}], \quad (35)$$

where $\bar{\Theta}(\varepsilon) = Z^{-T}(\varepsilon)(A + BF(\varepsilon)) + (A + BF(\varepsilon))^T Z^{-1}(\varepsilon)$, $F(\varepsilon) = KZ^{-1}(\varepsilon)$.

Taking the derivative of $V(x)$ along the trajectories of system (24) and using constraints (2) and (3), for any scalars $\mu_1 > 0$, $\mu_2 > 0$, and $\varepsilon \in (0, \bar{\varepsilon}]$, we have

$$\begin{aligned} \dot{V}(x)|_{(22)} &= 2x^T(t) Z^{-T}(\varepsilon) E(\varepsilon) \dot{x}(t) \\ &= 2x^T(t) Z^{-T}(\varepsilon) \\ &\quad \times ((A + BF(\varepsilon))x(t) + B_f f(t, x) + B_\omega \omega(t)) \\ &= x^T(t) (Z(\varepsilon)^{-T} (A + BF(\varepsilon)) \\ &\quad + (A + BF(\varepsilon))^T Z^{-1}(\varepsilon)) x(t) \\ &\quad + 2x^T(t) Z^{-T}(\varepsilon) B_f f(t, x) \end{aligned}$$

$$\begin{aligned} &+ 2x^T(t) Z^{-T}(\varepsilon) B_\omega \omega(t) \\ &\leq x^T(t) (Z(\varepsilon)^{-T} (A + BF(\varepsilon)) \\ &\quad + (A + BF(\varepsilon))^T Z^{-1}(\varepsilon)) x(t) \\ &\quad + 2x^T(t) Z^{-T}(\varepsilon) B_f f(t, x) \\ &\quad + 2x^T(t) Z^{-T}(\varepsilon) B_\omega \omega(t) \\ &\quad + \mu_1 (\alpha^2 x^T(t) M^T M x(t) - f^T f) \\ &\quad + \mu_2 (\beta^2 x^T(t) N^T N x(t) - g^T g). \end{aligned} \quad (36)$$

Therefore,

$$\begin{aligned} \dot{V}(x)|_{(22)} - 2\omega^T(t) z(t) + 2\eta \omega^T(t) \omega(t) \\ = \begin{bmatrix} x \\ \omega \\ f \\ g \end{bmatrix}^T \begin{bmatrix} \bar{\Theta}(\varepsilon) + \mu_1 \alpha^2 M^T M & Z^{-T}(\varepsilon) B_\omega - C^T & Z^{-T}(\varepsilon) B_f & 0 \\ + \mu_2 \beta^2 N^T N & 2\eta I - D_\omega^T - D_\omega & 0 & -D_g \\ * & * & -\mu_1 I & 0 \\ * & * & * & -\mu_2 I \end{bmatrix} \begin{bmatrix} x \\ \omega \\ f \\ g \end{bmatrix}. \end{aligned} \quad (37)$$

Taking into account (35) and (37), we have

$$\int_0^T \dot{V}(x) - 2\omega^T(t) z(t) + 2\eta \omega^T(t) \omega(t) \leq 0, \quad (39)$$

$$\dot{V}(x)|_{(22)} - 2\omega^T(t) z(t) + 2\eta \omega^T(t) \omega(t) \leq 0. \quad (38) \quad \text{which implies}$$

Taking the integral on the two sides of (38) from 0 to T , we obtain

$$\int_0^T 2\omega^T(t) z(t) - 2\eta \omega^T(t) \omega(t) \geq V(x(T)) - V(x(0)). \quad (40)$$

Therefore,

$$\int_0^T \omega^T(t) z(t) - \eta \omega^T(t) \omega(t) dt \geq 0, \quad \forall T > 0 \quad (41)$$

holds for all trajectories with zero initial condition $x(0) = 0$.

Hence, system (24) is passive with dissipation η for all $\varepsilon \in (0, \bar{\varepsilon}]$. $\square$

Remark 12. It follows from LMIs (25) that Z_1 and Z_2 are nonsingular matrices, which guarantees that $Z(0)$ is nonsingular. Then, $F = KZ^{-1}(\varepsilon)$ is well defined for all $\varepsilon \in (0, \bar{\varepsilon}]$. Furthermore, it can be seen that the controller reduces to an ε -independent one, that is, $u = Fx = K \begin{bmatrix} Z_1 & 0 \\ Z_5 & Z_2 \end{bmatrix}^{-1} x$, when ε is sufficiently small.

Using Theorem 4, we have the following corollary to design an ε -independent controller.

Corollary 13. Given η and $\bar{\varepsilon} > 0$, if there exist scalars $\mu_1 > 0$, $\mu_2 > 0$, $\bar{\mu}_1 > 0$, $\bar{\mu}_2 > 0$, and matrices Z_i ($i = 1, 2, 5$) with $Z_i = Z_i^T > 0$ ($i = 1, 2$) and K satisfying (27) and (29), then the closed-loop system (24) is passive with dissipation η for all $\varepsilon \in (0, \bar{\varepsilon}]$, and the gain matrix can be designed as $F = K \begin{bmatrix} Z_1 & 0 \\ Z_5 & Z_2 \end{bmatrix}^{-1}$.

Remark 14. It is noted that the condition presented in Theorem 11 is not a convex set due to the equality constraints in (29). Several approaches have been proposed to solve such nonconvex feasibility problems, among which the cone complementarity linearization (CCL) method [22] is most commonly used. For computational simplicity, one can set $\mu_1 = \bar{\mu}_1 = \mu_2 = \bar{\mu}_2 = 1$; then, the LMI conditions in Theorem 11 are easy to be solved by LMI toolbox. However, this may lead to conservatism.

4. Example

In this section, the RLC circuit shown in Figure 1 will be used to demonstrate the utility of the proposed method and show the advantages over the existing results.

In Figure 1, R , L , and C denote resistor, inductance, and capacitor, respectively. I is the current, V is voltage of the capacitor, u is the input voltage, and R_δ is a nonlinear resistor with the voltage-current characteristics $V(R_\delta) = I + f(I)$, where $f(I)$ satisfies $\|f(I)\| \leq 0.5\|I\|$.

Let $L = 1H$, $R = 10\Omega$, and C be a small positive scalar with $C = \varepsilon$, $x_1 = I$, and $x_2 = V$. Then, the circuit shown in Figure 1 can be described by the following singularly perturbed system

$$\begin{aligned} \dot{x}_1 &= -x_1 - f(x_1) - x_2, \\ \varepsilon \dot{x}_2 &= x_1 + 0.1x_2 - 0.1\omega - 0.1u, \\ z &= x_1 + 2\omega, \end{aligned} \quad (42)$$

where $\omega(t) \in L_2^1[0, \infty)$ denotes the external disturbance.

We first show that using Theorem 4, we can obtain a less conservative estimate of ε -bound than the existing methods in [19]. To do this, we will study the passivity of system (42)

FIGURE 1: An RLC circuit.

TABLE 1: Maximal $\bar{\varepsilon}$ for given η .

Given η	Maximal $\bar{\varepsilon}$
1.2	0.5849
1.3	0.5571
1.4	0.5198
1.5	0.4666
1.6	0.3817

with $u = 5x_1 + 6x_2$, which can be rewritten in the form of (1) with

$$\begin{aligned} E(\varepsilon) &= \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}, \quad A = \begin{bmatrix} -1 & -1 \\ 0.5 & -0.5 \end{bmatrix}, \\ B_u &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad B_f = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad B_\omega = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}, \\ C &= [1 \ 0], \quad D_g = 0, \quad D_\omega = 2. \end{aligned} \quad (43)$$

It is easy to verify that the nonlinearity $f(x_1)$ satisfies (2) with $\alpha = 0.5$ and $M = [1 \ 0]$.

By the existing method of [19], the system is passive with dissipation $\eta = 0$ for any $\varepsilon \in (0, 0.6335]$.

Setting $\eta = 0$ and using Algorithm 7, we have $\bar{\varepsilon} = 0.7015$ and the corresponding solution to LMIs (11)-(12) is as follows:

$$\begin{aligned} Z_1 &= 10.0359, \quad Z_2 = 25.4846, \quad Z_3 = 5.5166, \\ Z_4 &= 8.0593, \quad Z_5 = -17.4796, \\ \mu_1 &= 7.3139, \quad \mu_2 = 5.4073 \times 10^7. \end{aligned} \quad (44)$$

It can be seen that Algorithm 7 leads to a tighter ε -bound than the existing method in [19].

By Algorithm 7, the best estimate of the ε -bound can be obtained for different values of dissipation η . By Algorithm 9, the best estimate of dissipation η can be obtained for different value of $\bar{\varepsilon}$. The results are summarized in Tables 1 and 2, respectively, which shows that there is a competition between the dissipation η and the upper bound $\bar{\varepsilon}$.

TABLE 2: Maximal η for given $\bar{\varepsilon}$.

Given $\bar{\varepsilon}$	Maximal η
0.2	1.6666
0.3	1.6530
0.4	1.5803
0.5	1.4419
0.6	1.1328

Next, we demonstrate the utility of Theorem 11. System (42) can be rewritten in the form of (1) with

$$\begin{aligned}
 E(\varepsilon) &= \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}, & A &= \begin{bmatrix} -1 & -1 \\ 0.5 & -0.5 \end{bmatrix}, \\
 B_u &= \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}, & B_f &= \begin{bmatrix} -1 \\ 0 \end{bmatrix}, & B_\omega &= \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}, \\
 C &= [1 \ 0], & D_g &= 0, & D_\omega &= 2.
 \end{aligned} \quad (45)$$

Letting $\bar{\varepsilon} = 0.8$, $\eta = 1.8$, and solving the LMI conditions in Theorem 11 with $\mu_1 = \bar{\mu}_1 = \mu_2 = \bar{\mu}_2 = 1$, we have

$$\begin{aligned}
 Z_1 &= 0.7339, & Z_2 &= 10.5514, & Z_3 &= -0.2041, \\
 Z_4 &= -1.5881, & Z_5 &= 1.2496, \\
 K &= [85.5984 \ -47.7435].
 \end{aligned} \quad (46)$$

Then, by Theorem 11, the controller (23) with

$$\begin{aligned}
 F(\varepsilon) &= [85.5984 \ -47.7435] \\
 &\times \begin{bmatrix} 0.7339 - 0.2041\varepsilon & 1.2496\varepsilon \\ 1.2496 & 10.5514 - 1.5881\varepsilon \end{bmatrix}^{-1}
 \end{aligned} \quad (47)$$

renders the closed-loop system passive with dissipation $\eta = 1.8$ for all $\varepsilon \in (0, 0.8]$.

5. Conclusion

In this paper, the problems of passive control and ε -bound estimation were investigated for singularly perturbed systems subject to nonlinear uncertainties. A storage function which gives full consideration of the singular perturbation structure was constructed, by which an ε -independent LMI-based passivity analysis method was proposed. And a bisectional search algorithm was formulated to get the best estimate of the ε -bound. The proposed method was shown to be less conservative than the existing results. Furthermore, a controller design method was proposed to achieve a predefined ε -bound. The presented RLC circuit has demonstrated the utility of the proposed results and shown the advantages over the existing results. In our future work, we will try to propose less conservative results and generalize the proposed results of this paper to more general nonlinear singularly perturbed systems.

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