Research Article

Synchronization of N Different Chaotic Systems Based on Antisymmetric Structure

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The problem of synchronization of N different chaotic systems is investigated. By using the direct design control method, the synchronization controller is designed to transform the error system into a nonlinear system with a special antisymmetric structure. The sufficient stability conditions are presented for such systems, and the complete synchronization of chaotic systems is realized. Finally, the corresponding numerical simulations demonstrate the effectiveness of the proposed schemes.

1. Introduction

Since Pecora and Carroll have synchronizes two identical chaotic systems with different initial conditions [1], chaotic synchronization has been intensively and extensively investigated due to its potential applications in many fields [2–4]. And most of research efforts have been done about some synchronization phenomena, such as phase synchronization [5], antisynchronization [6], and projective synchronization [7]. Many techniques have been developed to realize chaos synchronization which includes sliding mode control [8], direct design control method [9], and backstepping method [10]. However, most of researches mentioned above mainly concentrated on synchronizing two identical or different chaotic systems.

In fact, more and more applications of chaos synchronization in secure communication make it much more important to synchronize multiple chaotic systems. It can satisfy the synchronization of multiple chaotic communication systems with a lower cost, and it can make it possible to realize multiparty communications simultaneously. Therefore, the synchronization of multiple chaotic systems has become a hot topic. It is more essential and useful in real-life applications. For example, Yu and Zhang studied the global synchronization of three coupled chaotic systems with ring connection in [11]. The adaptive coupled synchronization among multi-Lorenz systems family is investigated in [12].

The synchronization of N different coupled chaotic systems with ring and chain connection was proposed in [13]. The synchronization of N -coupled fractional-order chaotic systems with ring connection was investigated using the stability criteria of fractional-order system in [14]. Zhang studied the synchronization of multi-chaotic systems based on the impulsive control theory in [15]. Grassi researched the propagation of projective synchronization in a series connection of N chaotic discrete-time drive systems and N response systems in [16]. Yang and Zhang studied the synchronization of three identical systems and its application for secure communication with noise perturbation in [17].

However, the realization of synchronization of N identical or nonidentical chaotic systems is much more difficult, so it is necessary to find an easy method to realize such synchronization of multiple chaotic systems. Motivated by the above discussions, in this paper, we consider the problem of synchronization of N different chaotic systems. With the active control method, the synchronization controller is designed to transform the error system into a nonlinear system with the special antisymmetric structure. The complete synchronization of multiple chaotic systems is realized.

This paper is organized as follows. In Section 2, the synchronization of N different chaotic systems is theoretically analyzed. A stability theorem for N different chaotic systems with a special antisymmetric structure is given. In Section 3, the proposed synchronization schemes with the direct design

control method are applied to three different chaotic systems, that is, New system, Lorenz system and Rössler system. The simulations demonstrate the effectiveness of proposed schemes. And finally some concluding remarks are given in Section 4.

2. Synchronization of N Different Chaotic Systems and Controllers Design

Consider the following chaotic systems:

$$\begin{aligned}\dot{x}_1 &= A_1 x_1 + g_1(x_1), \\ \dot{x}_2 &= A_2 x_2 + g_2(x_2), \\ &\vdots \\ \dot{x}_N &= A_N x_N + g_N(x_N),\end{aligned}\quad (1)$$

where $x_1, x_2, \dots, x_N \in \mathbb{R}^n$ ($N > 2$) represent state vectors of the chaotic systems; $g: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous nonlinear function; $A_1, A_2, \dots, A_N$ are constant matrices. For $i, j = 1, \dots, N$, and $i \neq j$, if

$$A_i \neq A_j, \quad g_i(\cdot) \neq g_j(\cdot), \quad (2)$$

then the systems (1) are the different chaotic systems.

We consider N different chaotic systems. The drive systems and the controlled response systems are described as follows:

$$\begin{aligned}\dot{x}_1 &= A_1 x_1 + g_1(x_1), \\ \dot{x}_2 &= A_2 x_2 + g_2(x_2) + u(x_1, x_2), \\ &\vdots \\ \dot{x}_N &= A_N x_N + g_N(x_N) + u(x_1, x_N).\end{aligned}\quad (3)$$

Let us define the errors systems between response systems and drive systems as $\dot{e}_i = \dot{x}_i - \dot{x}_1$, ($i = 2, \dots, N$), then the dynamics of the synchronization errors can be expressed as

$$\begin{aligned}\dot{e} &= \begin{bmatrix} \dot{e}_2 \\ \dot{e}_3 \\ \vdots \\ \dot{e}_N \end{bmatrix} \\ &= \begin{bmatrix} A_2 e_2 + (A_2 - A_1)x_1 + g_2(x_2) - g_1(x_1) + u(x_2, x_1) \\ A_3 e_3 + (A_3 - A_1)x_1 + g_3(x_3) - g_1(x_1) + u(x_3, x_1) \\ \vdots \\ A_N e_N + (A_N - A_1)x_1 + g_N(x_N) - g_1(x_1) + u(x_N, x_1) \end{bmatrix}.\end{aligned}\quad (4)$$

Our purpose is to design the appropriate controllers $u(x_i, x_1)$ for the response systems such that the error systems (4) are asymptotically stable, which implies that the complete synchronization of N different chaotic systems (3) is realized; that is,

$$\lim_{t \rightarrow \infty} \|e_i\| = \lim_{t \rightarrow \infty} \|x_i - x_1\| = 0. \quad (5)$$

Here, the direct design control method is used to achieve the objective. In accordance to the active control design strategy, we choose the control input $u(x_i, x_1)$ to eliminate all known items that cannot be shown in the form of the error system e_i . The controller can be given by

$$u(x_i, x_1) = u_i - (A_i - A_1)x_1 - g_i(x_i) + g_1(x_1), \quad (6)$$

$i = 2, \dots, N.$

The error systems (4) can be rewritten as

$$\dot{e} = \begin{bmatrix} \dot{e}_2 \\ \dot{e}_3 \\ \vdots \\ \dot{e}_N \end{bmatrix} = \begin{bmatrix} A_2 & 0 & \cdots & 0 \\ 0 & A_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_N \end{bmatrix} \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_N \end{bmatrix} + \begin{bmatrix} u_2 \\ u_3 \\ \vdots \\ u_N \end{bmatrix}. \quad (7)$$

Defining $u_i = p_i e_i$, we can get the error systems (8) as follows:

$$\dot{e} = \begin{bmatrix} \dot{e}_2 \\ \dot{e}_3 \\ \vdots \\ \dot{e}_N \end{bmatrix} = \begin{bmatrix} A_2 + p_2 & 0 & \cdots & 0 \\ 0 & A_3 + p_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_N + p_N \end{bmatrix} \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_N \end{bmatrix}. \quad (8)$$

There are many possible choices for p_i , as long as it assures that the error dynamic system (8) is asymptotically stable at the origin. Without loss of generality, let us define p_i as a state-dependent coefficient matrix, then the error systems (8) can be rewritten as

$$\dot{e} = \begin{bmatrix} B_2(e_2) & 0 & \cdots & 0 \\ 0 & B_3(e_3) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_N(e_N) \end{bmatrix} \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_N \end{bmatrix}. \quad (9)$$

The sufficient stability conditions of the systems (9) will be given by transforming it into a stable system with a special antisymmetric structure. The main result is described as follows.

Theorem 1. Consider the systems (9) with the state-dependent coefficient matrices $B_i(e_i) = B_{i1}(e_i) + B_{i2}$. If the matrices $B_{i1}(e_i)$ and B_{i2} satisfy the assumptions that

$$\begin{aligned}B_{i1}^T(e_i) &= -B_{i1}(e_i), \quad (i = 2, \dots, N), \\ B_{i2} &= \text{diag}(b_{i1}, \dots, b_{in}), \quad b_{ij} < 0, \quad (j = 1, \dots, n),\end{aligned}\quad (10)$$

and the invariant set of the system (9) only includes the origin, then the system (9) is asymptotically stable.

Proof. Choose Lyapunov function as follows:

$$V = \frac{1}{2} e^T e. \quad (11)$$

Then

$$\begin{aligned}
\dot{V} &= \frac{1}{2} (\dot{e}^T e + e^T \dot{e}) \\
&= \frac{1}{2} e^T \left(\begin{bmatrix} B_2(e_2) & 0 & \cdots & 0 \\ 0 & B_3(e_3) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_N(e_N) \end{bmatrix}^T + \begin{bmatrix} B_2(e_2) & 0 & \cdots & 0 \\ 0 & B_3(e_3) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_N(e_N) \end{bmatrix} \right) e \\
&= \frac{1}{2} e^T \left(\begin{bmatrix} B_{21}(e_2) + B_{22}(e_2) & 0 & \cdots & 0 \\ 0 & B_{31}(e_3) + B_{32}(e_3) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_{N1}(e_N) + B_{N2}(e_N) \end{bmatrix}^T \right. \\
&\quad \left. + \begin{bmatrix} B_{21}(e_2) + B_{22}(e_2) & 0 & \cdots & 0 \\ 0 & B_{31}(e_3) + B_{32}(e_3) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_{N1}(e_N) + B_{N2}(e_N) \end{bmatrix} \right) e,
\end{aligned} \tag{12}$$

where $B_{i1}^T(e_i) = -B_{i1}(e_i)$, $(i = 2, \dots, N)$, and $B_{i2}^T(e_i) = B_{i2}(e_i)$. So we get that

$$\dot{V} = e^T \begin{bmatrix} B_{22}(e_2) & 0 & \cdots & 0 \\ 0 & B_{32}(e_3) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_{N2}(e_N) \end{bmatrix} e, \tag{13}$$

where, for $j = 1, \dots, n$ and $b_{ij} < 0$, $B_{i2} = \text{diag}(b_{i1}, \dots, b_{in})$ is negative definite. Therefore, $\dot{V}$ is negative definite. According to Lyapunov asymptotical stability theory, the complete synchronization of the different chaotic systems is achieved. $\square$

Remark 2. The system (9) is transformed into the system $\dot{e}_i = B_i(e_i)e_i$ under the control law $u(x_i, x_1)$, where $B_i(e_i)$ possesses the antisymmetric structure; then the error system (4) is asymptotically stable at the origin according to the Theorem 1.

Remark 3. The antisymmetric structures in Theorem 1 are the generalization of the tridiagonal structures. The error system constructed with the antisymmetric structure is more convenient than the one with tridiagonal structure, if the original system has some zero elements at the tridiagonal position and nonzero elements at other positions.

The selecting of the coefficient matrices with antisymmetric structure is an important and difficult technique, which relates to the coefficient matrices and the states of the original system. In the following section, we will demonstrate the proposed approaches for the special structure with an example.

3. Applications of Synchronization Control Schemes

In this section, we employ a simulation example to illustrate the effectiveness of the proposed schemes. The synchronization is simulated for three different chaotic systems. We choose New system as drive system, and we consider Lorenz system and Rössler system as the response systems. They are described as follows:

$$\begin{aligned}
\dot{x}_{11} &= 5x_{11} - x_{12}x_{13}, \\
\dot{x}_{12} &= -10x_{12} + x_{11}x_{13},
\end{aligned} \tag{14}$$

$$\begin{aligned}
\dot{x}_{13} &= -3.8x_{13} + \frac{1}{3}x_{11}x_{12}, \\
\dot{x}_{21} &= -10x_{21} + 10x_{22} + u_{21}, \\
\dot{x}_{22} &= 28x_{21} - x_{22} - x_{21}x_{23} + u_{22},
\end{aligned} \tag{15}$$

$$\begin{aligned}
\dot{x}_{23} &= -\frac{8}{3}x_{23} + x_{21}x_{22} + u_{23}, \\
\dot{x}_{31} &= -x_{32} - x_{33} + u_{31}, \\
\dot{x}_{32} &= x_{31} + 0.2x_{32} + u_{32},
\end{aligned} \tag{16}$$

$$\dot{x}_{33} = -5.7x_{33} + 0.2 + x_{31}x_{33} + u_{33},$$

where

$$A_1 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -3.8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & -5.7 \end{bmatrix}, \quad g_1(x_1) = \begin{bmatrix} -x_{12}x_{13} \\ x_{11}x_{13} \\ \frac{1}{3}x_{11}x_{12} \end{bmatrix},$$

$$g_2(x_2) = \begin{bmatrix} 0 \\ -x_{21}x_{23} \\ x_{21}x_{22} \end{bmatrix}, \quad g_3(x_3) = \begin{bmatrix} 0 \\ 0 \\ 0.2 + x_{31}x_{33} \end{bmatrix}. \quad (17)$$

Let the synchronization error state be $\dot{e}_i = \dot{x}_i - \dot{x}_1$, ($i = 2, 3$), then the error dynamical states can be written as

$$\begin{aligned} \dot{e}_{21} &= -10e_{21} + 10e_{22} + u_{21} + x_{12}x_{13} - 15x_{11} + 10x_{12}, \\ \dot{e}_{22} &= 28e_{21} - e_{22} + u_{22} - x_{21}x_{23} - x_{11}x_{13} + 28x_{11} + 9x_{12}, \\ \dot{e}_{23} &= -\frac{8}{3}e_{23} + u_{23} + x_{21}x_{22} - \frac{1}{3}x_{11}x_{12} + \left(3.8 - \frac{8}{3}\right)x_{13}, \end{aligned} \quad (18)$$

$$\begin{aligned} \dot{e}_{31} &= -e_{32} - e_{33} + u_{31} + x_{12}x_{13} - 5x_{11} - x_{12} - x_{13}, \\ \dot{e}_{32} &= e_{31} + 0.2e_{32} + u_{32} + x_{11} + 10.2x_{12} - x_{11}x_{13}, \end{aligned} \quad (19)$$

$$\dot{e}_{33} = -5.7e_{33} + u_{33} - 1.9x_{13} + 0.2 + x_{31}x_{33} - \frac{1}{3}x_{11}x_{12}.$$

Then the controllers are designed as follows,

$$\begin{aligned} u(x_2, x_1) &= \begin{bmatrix} u_{21} \\ u_{22} \\ u_{23} \end{bmatrix} \\ &= p_1 \begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} - \begin{bmatrix} 15 & -10 & 0 \\ 28 & 9 & 0 \\ 0 & 0 & \left(3.8 - \frac{8}{3}\right) \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \\ &\quad - g_2(x_2) + g_1(x_1), \\ u(x_3, x_1) &= \begin{bmatrix} u_{31} \\ u_{32} \\ u_{33} \end{bmatrix} \\ &= p_2 \begin{bmatrix} e_{31} \\ e_{32} \\ e_{33} \end{bmatrix} - \begin{bmatrix} -5 & -1 & -1 \\ 1 & 10.2 & 0 \\ 0 & 0 & -1.9 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \\ &\quad - g_3(x_3) + g_1(x_1), \end{aligned} \quad (20)$$

where

$$\begin{aligned} p_1 &= \begin{bmatrix} 0 & 0 & -x_{12} \\ -18 & 0 & -(x_{11} + e_{21}) \\ x_{12} & (x_{11} + e_{21}) & 0 \end{bmatrix}, \\ p_2 &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1.2 & -(x_{11} + e_{31}) \\ 1 & (x_{11} + e_{31}) & 0 \end{bmatrix}. \end{aligned} \quad (21)$$

Then the error systems (18) can be rewritten as

$$\begin{aligned} \dot{e} &= \begin{bmatrix} \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = [\dot{e}_{21} \quad \dot{e}_{22} \quad \dot{e}_{23} \quad \dot{e}_{31} \quad \dot{e}_{32} \quad \dot{e}_{33}]^T \\ &= \begin{bmatrix} -10 & -10 & -x_{12} & 0 & 0 & 0 \\ 10 & -1 & -(x_{11} + e_{21}) & 0 & 0 & 0 \\ x_{12} & (x_{11} + e_{21}) & -\frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & -1 & -(x_{11} + e_{31}) \\ 0 & 0 & 0 & 1 & (x_{11} + e_{31}) & -5.7 \end{bmatrix} \begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \\ e_{31} \\ e_{32} \\ e_{33} \end{bmatrix}. \end{aligned} \quad (22)$$

Let us define $B_2(e_2)$ and $B_3(e_3)$ as follows:

$$\begin{aligned} B_{21}(e_2) &= \begin{bmatrix} 0 & -10 & -x_{12} \\ 10 & 0 & -(x_{11} + e_{21}) \\ x_{12} & (x_{11} + e_{21}) & 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} -10 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} B_3(e_3) &= \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -(x_{11} + e_{31}) \\ 1 & (x_{11} + e_{31}) & 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -5.7 \end{bmatrix}. \end{aligned} \quad (23)$$

Then we declare that the error systems (18) and (19) are asymptotically stable at the origin under the controllers (20)

FIGURE 1: Dynamics of the variables e_{21} , e_{22} , and e_{23} of the error system (18) with time t .

FIGURE 2: Dynamics of the variables e_{31} , e_{32} , and e_{33} of the error system (19) with time t .

according to Theorem 1. Then the synchronization between the response systems and the drive system is realized.

The fourth-order Runge-Kutta integration method is used to numerical simulation with time step size 0.001s. Let the initial conditions of the drive system and the response systems be $(x_{11}(0), x_{12}(0), x_{13}(0))^T = (11, 15, 26)^T$, $(x_{21}(0), x_{22}(0), x_{23}(0))^T = (1, 6, 8)^T$, and $(x_{31}(0), x_{32}(0), x_{33}(0))^T = (4.8, 0.8, 0.2)^T$, respectively. Then the initial values of the error states are $(e_{21}(0), e_{22}(0), e_{23}(0))^T = (-10, -9, -18)^T$ and $(e_{31}(0), e_{32}(0), e_{33}(0))^T = (-6.2, -14.2, -25.8)^T$. The state trajectories of the error systems are shown in Figures 1, 2, and 3, and the state trajectories of 3 different chaotic systems are shown in Figures 4, 5, and 6. We can make out that, from Figures 1–6, the state trajectories asymptotically converge to zero near 2.5 s, 4.5 s, and 5 s under the controllers, and the state vectors of the different chaotic systems achieve the complete synchronization. The numerical simulations demonstrate that the proposed design method is feasible and effective to realize the complete synchronization of N different chaotic systems which satisfy Theorem 1.

FIGURE 3: Dynamics of the variables of the error system between the response systems (15) and (16) with time t .

FIGURE 4: The state trajectories x_{11} , x_{21} , and x_{31} of the chaotic systems with time t .

4. Conclusions

In this paper, the synchronization problem of N different chaotic systems is investigated. The direct design method is adopted to realize the complete synchronization of N different chaotic systems according to the proposed theorems. And then the stability theorems about the error systems with the special antisymmetric structure are presented. Numerical simulations of the synchronization about the different chaotic systems, respectively, illustrate the validity of the proposed schemes. How to realize other types of synchronization of N -coupled chaotic systems is our further work. Inspired by the studies [18–20], how to extend the current results to chaotic systems with discontinuous functions is also our future research directions.

FIGURE 5: The state trajectories x_{12} , x_{22} , and x_{32} of the chaotic systems with time t .

FIGURE 6: The state trajectories x_{13} , x_{23} , and x_{33} of the chaotic systems with time t .

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