Research Article

Observer-Based Robust Control of Uncertain Switched Fuzzy Systems with Combined Switching Controller

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The observer-based robust control for a class of switched fuzzy (SF) time-delay systems involving uncertainties and external disturbances is investigated in this paper. A switched fuzzy system, which differs from existing ones, is firstly employed to describe a nonlinear system. Next, a combined switching controller is proposed. The designed controller based on the observer instead of the state information integrates the advantages of both the switching controllers and the supplementary controllers but eliminates their disadvantages. The proposed controller provides good performance during the transient period, and the chattering effect is removed when the system state approaches the origin. Sufficient condition for the solvability of the robust control problem is given for the case that the state of system is not available. Since convex combination techniques are used to derive the delay-independent criteria, some subsystems are allowed to be unstable. Finally, various comparisons of the elaborated examples are conducted to demonstrate the effectiveness of the proposed control design approach.

1. Introduction

In practical control systems, the nonlinear property, uncertainty, and time delay exist widely, because of the complexity of control procedure, disturbances, and the constraint of system. Therefore, the stabilization and control issues for nonlinear time-delay systems with uncertainties are very important issues, and they are challenging and important in both theory and practical applications [1–5].

Also recently, the switched systems, which are an important class of hybrid systems and have wide background and technological applications, have also been one of the main research focuses. In turn, a considerable number of fruitful results in analysis and design of switching systems have been derived too; for example, see [6–13]. On the other hand, the research activities on the synergy of fuzzy-system-based controls, as an important intelligent control approach, combined with some of math-analytical control theories have attracted great attention. In particular, the class of Takagi-Sugeno (T-S) fuzzy models has been found to be most effective as system model in various fuzzy-system-based

methods. Based on T-S fuzzy model representations and the feedback control strategy, stability and robust analysis and design as well as handling parameter uncertainties for fuzzy systems have acquired a considerable number of fruitful results; for example, see [14–26].

Recently, the stabilizability conditions and smoothness conditions for fuzzy switching control systems were reported. Based on the T-S fuzzy systems, Tanaka et al. [16–19] introduced new switching fuzzy systems for more complicated real systems such as multiple nonlinear systems, switched nonlinear hybrid systems, and second-order nonholonomic systems. This class of the model has two levels of structures. It will switch between the second-level region rules according to the first-level region rules. In fact, it is a type of switching for the same premise variable.

To effectively achieve nonlinear control, we propose an SF model in this paper [27]. Here, by assumption, Takagi-Sugeno fuzzy systems, which are also denoted as T-S fuzzy systems, are considered as terms of plant representation models. These fuzzy systems belong to the category of data-driven fuzzy systems, in contrast to the more traditional

category of semantic-driven fuzzy models. The proposed corresponding model of SF systems, presented below, differs from the existing ones cited in the literature in the fact that each subsystem is a T-S fuzzy system, hence defining a class of SF systems. It should be noted that this class inherits some essential features of hybrid systems while retaining all of the information and knowledge representation capacity of fuzzy systems. Furthermore, this model is used for the extra design of the switching law based on the previous works [16–19].

Comparing with the previous works [16–19], this model of SF systems does not have two levels of structures, but it is switching between each of the sub fuzzy systems, not depending on region fuzzy rules. In this case, stability analysis is often facilitated by the fact that properties of each individual sub fuzzy system are of concern only in the regions where this subsystem is active, and the behavior of the sub fuzzy systems in other parts of the state space has no influence on the SF system. It is possible to find a switching law that renders the SF systems stable for unstable sub fuzzy systems. Particularly, when the parameters suddenly change or discontinuously change, the switching rules can be designed as any combinational function of variables, which make up the insufficiency of the switching depending on the single variable of the switching fuzzy model [16–19].

Control of SF systems is a challenging task because no systematic mathematical tools exist to help find necessary and sufficient conditions to guarantee the stability and performance. The problem becomes more complex if some of the system's parameters are uncertain, especially observer-based control. The study of observer-based control, which has been well addressed for switched systems with delay [28–30] and fuzzy systems with delay [31–33], has been rarely investigated for SF systems. In [34], although the observer-based control of SF systems has been investigated, the external disturbance is not considered. However, to our knowledge, there are no papers considering observer-based control of SF time-delay systems with the external disturbance. Observer-based control for the SF system which is based on designed switching method is still an open and interesting issue.

It is well known that parallel distributed compensation (PDC) control has been found to be a particular means for fuzzy systems due to its universal property [16–19, 34]. However, an undesired chattering effect will occur. This paper presents a combined switching controller, which has nonfuzzy-rules-based combination of switching controller and supplementary controllers, for SF systems subject to parameter uncertainties. The switching controller consists of several linear controllers. One of the switching controllers is employed at each moment according to a switching scheme, which is derived based on the Lyapunov stability theory. The supplementary controller is employed to handle the external disturbance. The smooth transition acted on the switching controller and the supplementary controllers is governed. As a result, the combined switching controller provides good performance during the transient period, and the chattering effect is removed when the system state approaches the origin.

In this paper, we investigate the problem of observer-based robust control for SF systems involving uncertainties and external disturbances with time delay. Sufficient

condition for the solvability of the robust control problem is given for the case that the state of a system is unmeasurable. The design of a switching control law based on observer state instead of the original state information is considered. We use the multiple Lyapunov function technique to design a combined switching controller and a switching law such that the observer performance is satisfied. The feasibility of the problem can be realized by convex combination techniques and linear matrix inequalities. Finally, the simulation examples show the validity of the proposed design method.

This paper is organized as follows. Section 2 introduces a switching fuzzy system and an SF system. In Section 3, we describe the model of an SF system of time-delay case involving uncertainties and external disturbances. In Section 4, state observer and the combined switching controller are derived. Two compared example simulations will be presented in Section 5. Finally, a conclusion will be drawn in Section 6.

2. Preliminaries

We compare a switching fuzzy control system and an SF control system.

2.1. Switching Fuzzy Control System. Consider a switching fuzzy model defined by Tanaka et al. as follows [16–19].

The Region Rule j :

If $z_1(t)$ is N_{j1} and $\dots$ and $z_p(t)$ is N_{jp} ,
then the following is given.

The Local Region Rule i :

If $z_1(t)$ is M_{ji1} and $\dots$ and $z_p(t)$ is M_{jip} ,
then

$$\begin{aligned} \dot{x}(t) &= A_{ji}x(t) + B_{ji}u(t), & y(t) &= C_{ji}x(t), \\ i &= 1, 2, \dots, r, & j &= 1, 2, \dots, s. \end{aligned} \quad (1)$$

$z_1(t), \dots, z_p(t)$ are the premise variables. $M_{ji1}, \dots, M_{jip}$ are the fuzzy sets. r is the number of the if-then rules. This switching fuzzy model in [16–19] has two levels of structures: the region rule level and the local fuzzy rule level. The region rule is crisply switched according to the premise variables. s is the number of regions partitioned on the premise variables space. Note that $N_{jk}(z(t))$ is a crisp set.

Where

$$N_{jk}(z(t)) = \begin{cases} 1, & z(t) \in N_{jk}, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

When the variables $z_1(t), \dots, z_p(t)$ satisfy the condition of the region rule, the fuzzy model which belongs to the local region rule below the region rule is active.

2.2. SF Control System. Comparing with the previous works [16–19], the SF model breaks through two levels of structures, but it is switching between each of the sub fuzzy models

FIGURE 1: Sketch map of the switched fuzzy systems.

directly. The SF system is a more general system than the model of [16–19]. Now, we introduce the SF systems.

When subsystems of the switched systems are T-S fuzzy systems, the systems are switched fuzzy systems. Now, we define SF model including $N_{\sigma(t)}$ pieces of rules as follows:

$R_{\sigma(t)}^l$: if $\bar{x}_{\sigma(t)1}(t)$ is $M_{\sigma(t)1}^l, \dots$, and $\bar{x}_{\sigma(t)p}(t)$ is $M_{\sigma(t)p}^l$,

then

$$\dot{x}(t) = A_{\sigma(t)l}x(t) + B_{\sigma(t)l}u_{\sigma(t)}(t), \quad l = 1, 2, \dots, N_{\sigma(t)}, \quad (3)$$

with

$$\sigma(t) : M = \{1, 2, \dots, m\}, \quad (4)$$

which is a piecewise constant function, called a switching signal.

$R_{\sigma(t)}^l$ denotes the l th fuzzy inference rule, $N_{\sigma(t)}$ is the number of inference rules, $u_{\sigma(t)}(t)$ is the input variable, $x(t)$ is the state variable vector, $A_{\sigma(t)l}$ and $B_{\sigma(t)l}$ are matrices of appropriate dimensions and $\bar{x}_{\sigma(t)1}, \bar{x}_{\sigma(t)2}, \dots, \bar{x}_{\sigma(t)p}$ are the vectors of premise variables.

Sketch map of the switched fuzzy systems is depicted in Figure 1; Ω_i denotes the state area of the i th switched subsystem; Ω_{il} denote the l th sub fuzzy area in Ω_i . In fact, the switched fuzzy systems again partition the Ω_i subarea into l sub fuzzy areas $\Omega_{i1}, \dots, \Omega_{il}, \dots, \Omega_{in}$. There is local linear model in every sub fuzzy area; namely, local linear model in Ω_{il} is $\dot{x}(t) = A_{il}x(t) + B_{il}u_i(t)$. The model of every switched subarea $\Omega_1, \dots, \Omega_m$ is composed of local linear model which is linked by fuzzy membership function. We design the switching law for sub fuzzy area model to ensure stability of the switched fuzzy system. When local model in sub fuzzy area satisfies the switching law, we switch to the Ω_i th subsystem to ensure stability of the switched fuzzy system.

Also, switching law can be designed by arbitrary form, not only depending on fuzzy rules as in [16–19]. That is, the SF systems are extended further for the switching fuzzy systems (1) and (2).

3. Novel Models of Uncertain Switched Fuzzy Time-Delay Systems

So in here, we will introduce an innovated representation modeling of uncertain SF time-delay systems. In this model, each subsystem is an uncertain fuzzy time-delay system, namely, uncertain sub fuzzy time-delay system.

Consider the uncertain SF time-delay model including $N_{\sigma(t)}$ pieces of rules as follows:

$$\begin{aligned} R_{\sigma(t)}^l &: \text{if } \bar{x}_{\sigma(t)1}(t) \text{ is } M_{\sigma(t)1}^l, \dots, \text{and } \bar{x}_{\sigma(t)p}(t) \text{ is } M_{\sigma(t)p}^l, \\ &\text{then} \\ \dot{x}(t) &= (A_{\sigma(t)l} + \Delta A_{\sigma(t)l})x(t) + D_{\sigma(t)l}x(t - \tau) \\ &\quad + B_{\sigma(t)l}u_{\sigma(t)l}(t) + \bar{D}_{\sigma(t)l}\omega(t), \\ y(t) &= C_{\sigma(t)l}x(t), \quad t \in [0, \infty), \quad x(0) = 0, \\ \varphi_{\sigma(t)}(\theta) &= x(t + \theta), \quad \theta \in [t_j - \tau, t_j], \\ j &= 0, 1, \dots, \quad l = 1, 2, \dots, N_{\sigma(t)}, \end{aligned} \quad (5)$$

with

$$\sigma(t) : M = \{1, 2, \dots, m\}, \quad (6)$$

which is a piecewise constant function, called a switching signal. It can be characterized by the switching sequence $\Sigma = \{x_0; (i_0, t_0), (i_1, t_1), \dots, (i_j, t_j) \mid i_j \in M, j = 0, 1, \dots\}$. Moreover, $\sigma(t) = i$ implies that the i th subsystem (A_i, D_i, B_i, C_i) is active.

$R_{\sigma(t)}^l$ denotes the l th fuzzy inference rule, $N_{\sigma(t)}$ is the number of inference rules, $u_{\sigma(t)l}(t) \in \mathbb{R}^q$ is the control input, $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^s$ denote the state vector and the output vector, respectively, $\omega(t) \in \mathbb{R}^n$ is the bounded exogenous disturbance, $\bar{D}_{\sigma(t)l}$ are known constant matrices of appropriate dimensions, $A_{\sigma(t)l}$ and $D_{\sigma(t)l} \in \mathbb{R}^{n \times n}$ are known system matrices, and $B_{\sigma(t)l} \in \mathbb{R}^{n \times q}$ is the input matrix. Moreover, $C_{\sigma(t)l} \in \mathbb{R}^{s \times n}$ is the output matrix, τ is the constant-bounded time delay in the state, and $\varphi_{\sigma(t)}(t)$ is the continuous vector-valued function.

It is readily seen that the i th sub fuzzy system can be represented as follows:

$$\begin{aligned} R_i^l &: \text{if } \bar{x}_{i1}(t) \text{ is } M_{i1}^l, \dots, \text{and } \bar{x}_{ip}(t) \text{ is } M_{ip}^l, \\ &\text{then} \\ \dot{x}(t) &= (A_{il} + \Delta A_{il})x(t) + D_{il}x(t - \tau) \\ &\quad + B_{il}u_{il}(t) + \bar{D}_i\omega(t), \\ y(t) &= C_{il}x(t), \quad t \in [0, \infty), \quad x(0) = 0, \\ l &= 1, 2, \dots, N_i, \quad i = 1, 2, \dots, m. \end{aligned} \quad (7)$$

Therefore, the global model of the i th sub fuzzy system is described by means of the following equation:

$$\begin{aligned} \dot{x}(t) &= \sum_{l=1}^{N_i} \eta_{il}(\bar{x}(t)) [(A_{il} + \Delta A_{il})x(t) \\ &\quad + D_{il}x(t - \tau) + B_{il}u_{il}(t) + \bar{D}_i\omega(t)], \\ y(t) &= \sum_{l=1}^{N_i} \eta_{il}C_{il}x(t), \end{aligned} \quad (8)$$

along with

$$0 \leq \eta_{il}(\bar{x}(t)) \leq 1, \quad \sum_{l=1}^{N_i} \eta_{il}(\bar{x}(t)) = 1,$$

$$w_{il}(\bar{x}(t)) = \prod_{\rho=1}^p M_{i\rho}^l(\bar{x}_\rho(t)), \quad \eta_{il}(\bar{x}(t)) = \frac{w_{il}(\bar{x}(t))}{\sum_{l=1}^{N_i} w_{il}(\bar{x}(t))}, \quad (9)$$

where $M_{i\rho}^l(\bar{x}_\rho(t))$ denotes the membership function in which $\bar{x}_\rho(t)$ belongs to the fuzzy set $M_{i\rho}^l$.

4. New Stabilization Based on State Observer and Combined Switching Controller

Suppose that the state observer is given by the following:

$$R_{\sigma(t)}^l: \text{if } \bar{x}_{\sigma(t)1}(t) \text{ is } M_{\sigma(t)1}^l, \dots, \text{ and } \bar{x}_{\sigma(t)p}(t) \text{ is } M_{\sigma(t)p}^l, \\ \text{then}$$

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{\sigma(t)l} \hat{x}(t) + D_{\sigma(t)l} \hat{x}(t - \tau) + B_{\sigma(t)l} u_{\sigma(t)l}(t) \\ &\quad + \bar{D}_{\sigma(t)l} \omega(t) + L_{\sigma(t)l} (y(t) - \hat{y}(t)), \\ \hat{y}(t) &= C_{\sigma(t)l} \hat{x}(t) \end{aligned} \quad (10)$$

in which $y(t)$ and $\sigma(t)$ are measurable output and switching signal of systems (5), respectively. The matrices $L_{\sigma(t)l} \in \mathbb{R}^{n \times s}$ are to be determined later.

Then, the global model of the i th sub fuzzy observer is described as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{l=1}^{N_i} \eta_{il}(\bar{x}(t)) [A_{il} \hat{x}(t) + D_{il} \hat{x}(t - \tau) \\ &\quad + B_{il} u_{il}(t) + \bar{D}_i \omega(t) \\ &\quad + L_{il} (y(t) - \hat{y}(t))], \\ \hat{y}(t) &= \sum_{l=1}^{N_i} \eta_{il}(\bar{x}(t)) C_{il} \hat{x}(t). \end{aligned} \quad (11)$$

Define the difference between the real state and the observer state as follows:

$$e(t) = x(t) - \hat{x}(t). \quad (12)$$

Subtracting (11) from (8) gives the following error switched system:

$$\begin{aligned} \dot{e}(t) &= \sum_{l=1}^{N_i} \eta_{il} \sum_{h=1}^{N_i} \eta_{ih} [(A_{il} + \Delta A_{il} - L_{il} C_{ih}) e(t) \\ &\quad + \Delta A_{il} \hat{x} + D_{il} e(t - \tau)]. \end{aligned} \quad (13)$$

Our purpose is to design a controller $u = u(t)$ and a switching law $\sigma = \sigma(t)$ such that (5) is asymptotically stable for the observer-based robust control.

Now, a combined switching controller is employed to control the SF time-delay model of (5). The combined switching controller, which has nonfuzzy rules, integrates the advantages of both switching controllers and supplementary controllers but eliminates their disadvantages. The switching controllers consist of some simple subcontrollers. These subcontrollers will switch among each other to control the system of (5) according to an appropriate switching scheme. The supplementary controller is employed to handle the external disturbance. It is shown in the sequel how to design controllers to achieve asymptotical stability in the closed loop and under the switching law.

The combined switching controller for the sub fuzzy time-delay system is described by

$$u_{il} = u_{il}^1 + u_{il}^2, \quad (14)$$

where u_{il}^1 is the switching controller and u_{il}^2 is the supplementary controller. Consider the following:

$$u_{il}^1 = - \sum_{a=1}^{N_i} \rho_{ila}(\hat{x}(t)) R B_{ia}^T P_i \hat{x}(t), \quad (15)$$

$$u_{il}^2 = \begin{cases} 0, & \text{if } \|B_{il}^T P_i \hat{x}(t)\| = 0, \\ \Theta_{il}, & \text{otherwise,} \end{cases}$$

$$\begin{aligned} \Theta_{il} &= - \frac{\theta(t) \cdot B_{il}^T P_i \hat{x}(t)}{\|B_{il}^T P_i \hat{x}(t)\|} \\ &\quad - \sum_{\beta=1}^{N_i} \frac{\gamma_\beta \theta(t) \cdot B_{il}^T P_i \hat{x}(t)}{\|B_{il}^T P_i \hat{x}(t)\|^2} \|B_{i\beta}^T P_i \hat{x}(t)\|, \end{aligned} \quad (16)$$

where R is symmetric positive definite.

Now, we will make the following standard assumptions for the uncertain SF system (5).

Assumption 1. The admissible parameter uncertainties are of the norm-bounded form

$$\Delta A_{il} = M_{il} F_{il}(t) N_{il}, \quad (17)$$

where M_{il} and N_{il} are known real constant matrices of proper dimensions which represent the structure of the uncertainties and $F_{il}(t)$ is unknown time-varying matrix satisfying

$$F_{il}^T(t) F_{il}(t) \leq I. \quad (18)$$

Assumption 2. There exists constant γ_l ($l = 1, 2, \dots, N_i$), such that

$$\bar{D}_i = \sum_{l=1}^{N_i} \gamma_l B_{il}. \quad (19)$$

Assumption 3. There exists known continuous nonnegative function $\theta(t)$, such that

$$\max \|\omega(t)\| \leq \theta(t). \quad (20)$$

The following theorem provides a condition for asymptotic stability of the system (5) and gives the design of state-dependent switching laws.

Theorem 4. Suppose that there exist constants β_{ij} ($i, j \in \overline{M}$) (either all nonnegative or all nonpositive) and positive definite symmetric matrices P_i , Q_i such that the following matrix inequalities are satisfied. Then, the system (5) along with fuzzy observer (10) is asymptotical stable via the combined switching controller (14):

$$\begin{bmatrix} Y_{ial} + \sum_{j=1}^m \beta_{ij} (P_i - P_j) & P_i D_{il} & P_i L_{il} C_{ih} & 0 \\ D_{il}^T P_i & -P_i & 0 & 0 \\ C_{ih}^T L_{il}^T P_i & 0 & \Psi_{ilh} & Q_i D_{il} \\ 0 & 0 & D_{il}^T Q_i & -Q_i \end{bmatrix} < 0, \quad (21)$$

$$i = 1, 2, \dots, m, \quad l, a, h = 1, 2, \dots, N_i,$$

where $Y_{ial} = -P_i B_{ia} R B_{il}^T P_i - P_i B_{il} R B_{ia}^T P_i + A_{il}^T P_i + P_i A_{il} + N_{il}^T N_{il} + P_i$,

$$\begin{aligned} \Psi_{ilh} &= Q_i + (A_{il}^T - C_{ih}^T L_{il}^T) Q_i + Q_i (A_{il} - L_{il} C_{ih}) \\ &+ 2Q_i M_{il} M_{il}^T Q_i + N_{il}^T N_{il}. \end{aligned} \quad (22)$$

And, the switching law is designed as follows:

$$\sigma = \sigma(\hat{x}(t)) = i : [0, +\infty) \longrightarrow \overline{M} = \{1, 2, \dots, m\}. \quad (23)$$

Proof. Without loss of generality, suppose that $\beta_{ij} \geq 0$. For any $i \in \overline{M}$, if $\hat{x}^T(t)(P_i - P_j)\hat{x}(t) \geq 0$, for all $j \in \overline{M}$, then from (21), we have

$$\begin{bmatrix} Y_{ial} & P_i D_{il} & P_i L_{il} C_{ih} & 0 \\ D_{il}^T P_i & -P_i & 0 & 0 \\ C_{ih}^T L_{il}^T P_i & 0 & \Psi_{ilh} & Q_i D_{il} \\ 0 & 0 & D_{il}^T Q_i & -Q_i \end{bmatrix} < 0. \quad (24)$$

Obviously, for all $\hat{x}(t) \in \mathbb{R}^n \setminus \{0\}$, there is certainly an $i \in \overline{M}$ such that

$$\hat{x}^T(t)(P_i - P_j)\hat{x}(t) \geq 0, \quad \forall j \in \overline{M}. \quad (25)$$

For arbitrary $i \in \overline{M}$, let

$$\Omega_i = \{\hat{x}(t) \in \mathbb{R}^n \setminus \{0\} \mid \hat{x}^T(t)(P_i - P_j)\hat{x}(t) \geq 0, \quad \forall j \in \overline{M}\}. \quad (26)$$

Then, $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n \setminus \{0\}$. Furthermore, let us construct the sets $\overline{\Omega}_1 = \Omega_1, \dots, \overline{\Omega}_i = \Omega_i - \bigcup_{j=1}^{i-1} \overline{\Omega}_j, \dots, \overline{\Omega}_m = \Omega_m - \bigcup_{j=1}^{m-1} \overline{\Omega}_j$. Obviously, we have $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n \setminus \{0\}$ and $\overline{\Omega}_i \cap \overline{\Omega}_j = \Phi, i \neq j$.

Note that (24) holds; then there exists

$$\Xi^T(t) \begin{bmatrix} Y_{ial} & P_i D_{il} & P_i L_{il} C_{ih} & 0 \\ D_{il}^T P_i & -P_i & 0 & 0 \\ C_{ih}^T L_{il}^T P_i & 0 & \Psi_{ilh} & Q_i D_{il} \\ 0 & 0 & D_{il}^T Q_i & -Q_i \end{bmatrix} \Xi(t) < 0, \quad (27)$$

where $\Xi(t) \triangleq [\hat{x}^T(t) \quad \hat{x}^T(t-\tau) \quad e^T(t) \quad e^T(t-\tau)]^T$.

When $\hat{x}(t) \in \overline{\Omega}_i, \sigma(t) = i$, then choose the multiple Lyapunov functional as follows:

$$\begin{aligned} V_i(t) &= \hat{x}^T(t) P_i \hat{x}(t) + e^T(t) Q_i e(t) \\ &+ \int_{t-\tau}^t \hat{x}^T(\theta) P_i \hat{x}(\theta) d\theta + \int_{t-\tau}^t e^T(\theta) Q_i e(\theta) d\theta, \end{aligned} \quad (28)$$

which is positive definite since P_i and Q_i are positive definite matrices.

The time derivative of $V_i(t)$ is given as follows:

$$\begin{aligned} \dot{V}_i(t) &= \hat{x}^T(t) P_i \dot{\hat{x}}(t) + \hat{x}^T(t) P_i \dot{\hat{x}}(t) + \dot{e}^T(t) Q_i e(t) \\ &+ e^T(t) Q_i \dot{e}(t) + \hat{x}^T(t) P_i \hat{x}(t) - \hat{x}^T(t-\tau) P_i \hat{x}(t-\tau) \\ &+ e^T(t) Q_i e(t) - e^T(t-\tau) Q_i e(t-\tau) \\ &= \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} [A_{il} \hat{x}(t) + D_{il} \hat{x}(t-\tau) + B_{il} u_{il}(t) \\ &+ \overline{D}_i \omega + L_{il} C_{ih} e(t)]^T P_i \hat{x}(t) \\ &+ \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} \hat{x}^T(t) P_i [A_{il} \hat{x}(t) + D_{il} \hat{x}(t-\tau) \\ &+ B_{il} u_{il}(t) + \overline{D}_i \omega + L_{il} C_{ih} e(t)] \\ &+ \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} [(A_{il} + \Delta A_{il} - L_{il} C_{ih}) e(t) \\ &+ \Delta A_{il} \hat{x}(t) + D_{il} e(t-\tau)]^T Q_i e(t) \\ &+ \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} e^T(t) Q_i [(A_{il} + \Delta A_{il} - L_{il} C_{ih}) e(t) \\ &+ \Delta A_{il} \hat{x}(t) + D_{il} e(t-\tau)] \\ &+ \hat{x}^T(t) P_i \hat{x}(t) - \hat{x}^T(t-\tau) P_i \hat{x}(t-\tau) \\ &+ e^T(t) Q_i e(t) - e^T(t-\tau) Q_i e(t-\tau) \\ &= \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} [\hat{x}^T(t) A_{il}^T + \hat{x}^T(t-\tau) D_{il}^T \\ &+ u_{il}^T(t) B_{il}^T + \omega^T \overline{D}_i^T + e^T(t) C_{ih}^T L_{il}^T] P_i \hat{x}(t) \end{aligned}$$

$$\begin{aligned}
& + \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} \hat{x}^T(t) P_i [A_{il} \hat{x}(t) + D_{il} \hat{x}(t - \tau) \\
& \quad + B_{il} u_{il}(t) + \bar{D}_i \omega + L_{il} C_{ih} e(t)] \\
& + \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} [e^T(t) (A_{il}^T + \Delta A_{il}^T - C_{ih}^T L_{il}^T) \\
& \quad + \hat{x}^T(t) \Delta A_{il}^T + e^T(t - \tau) D_{il}^T] Q_i e(t) \\
& + \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} e^T(t) Q_i [(A_{il} + \Delta A_{il} - L_{il} C_{ih}) e(t) \\
& \quad + \Delta A_{il} \hat{x}(t) + D_{il} e(t - \tau)] \\
& + \hat{x}^T(t) P_i \hat{x}(t) - \hat{x}^T(t - \tau) P_i \hat{x}(t - \tau) \\
& + e^T(t) Q_i e(t) - e^T(t - \tau) Q_i e(t - \tau) \\
& = \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} [\hat{x}^T(t) (A_{il}^T P_i + P_i A_{il}) \hat{x}(t) \\
& \quad + u_{il}^T(t) B_{il}^T P_i \hat{x}(t) \\
& \quad + \hat{x}^T(t) P_i B_{il} u_{il}(t) + \hat{x}^T(t - \tau) D_{il}^T P_i \hat{x}(t) \\
& \quad + \hat{x}^T(t) P_i D_{il} \hat{x}(t - \tau) + \omega^T \bar{D}_i^T P_i \hat{x}(t) \\
& \quad + \hat{x}^T(t) P_i \bar{D}_i \omega + e^T(t) C_{ih}^T L_{il}^T P_i \hat{x}(t) \\
& \quad + \hat{x}^T(t) P_i L_{il} C_{ih} e(t)] \\
& + \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \eta_{il} \eta_{ih} [\hat{x}^T(t) \Delta A_{il}^T Q_i e(t) + e^T(t) Q_i \Delta A_{il} \hat{x}(t) \\
& \quad + e^T(t) (A_{il}^T + \Delta A_{il}^T - C_{ih}^T L_{il}^T) Q_i e(t) \\
& \quad + e^T(t) Q_i (A_{il} + \Delta A_{il} - L_{il} C_{ih}) e(t) \\
& \quad + e^T(t - \tau) D_{il}^T Q_i e(t) \\
& \quad + e^T(t) Q_i D_{il} e(t - \tau)] \\
& + \hat{x}^T(t) P_i \hat{x}(t) - \hat{x}^T(t - \tau) P_i \hat{x}(t - \tau) \\
& + e^T(t) Q_i e(t) - e^T(t - \tau) Q_i e(t - \tau).
\end{aligned} \tag{29}$$

□

(I) When $B_{il}^T P_i \hat{x}(t) \neq 0$, then we have

$$\begin{aligned}
& \hat{x}^T(t) (A_{il}^T P_i + P_i A_{il}) \hat{x}(t) + (u_{il}^1)^T(t) B_{il}^T P_i \hat{x}(t) \\
& \quad + \hat{x}^T(t) P_i B_{il} u_{il}^1(t) \\
& = \hat{x}^T(t) \left[\left(-\sum_{a=1}^{N_i} \rho_{ila} R B_{ia}^T P_i \right)^T B_{il}^T P_i \right.
\end{aligned}$$

$$\begin{aligned}
& \left. + P_i B_{il} \left(-\sum_{a=1}^{N_i} \rho_{ila} R B_{ia}^T P_i \right) \right. \\
& \quad \left. + A_{il}^T P_i + P_i A_{il} \right] \hat{x}(t) \\
& = \hat{x}^T(t) \left\{ \left[\left(-\sum_{a=1}^{N_i} \rho_{ila} R B_{ia}^T P_i \right)^T B_{il}^T P_i \right. \right. \\
& \quad \left. - \sum_{a=1}^{N_i} \eta_{ia} (B_{il} R B_{ia}^T P_i)^T P_i \right. \\
& \quad \left. + \sum_{a=1}^{N_i} \eta_{ia} (B_{il} R B_{ia}^T P_i)^T P_i \right] \\
& \quad \left. + P_i B_{il} \left(-\sum_{a=1}^{N_i} \rho_{ila} R B_{ia}^T P_i \right) \right. \\
& \quad \left. - \sum_{a=1}^{N_i} \eta_{ia} P_i (B_{il} R B_{ia}^T P_i) \right. \\
& \quad \left. + \sum_{a=1}^{N_i} \eta_{ia} P_i (B_{il} R B_{ia}^T P_i) + A_{il}^T P_i + P_i A_{il} \right\} \hat{x}(t) \\
& = \hat{x}^T(t) \left\{ -\sum_{a=1}^{N_i} \eta_{ia} (B_{il} R B_{ia}^T P_i)^T P_i - \sum_{a=1}^{N_i} (\rho_{ila} - \eta_{ia}) \right. \\
& \quad \left. \times (B_{il} R B_{ia}^T P_i)^T P_i \right. \\
& \quad \left. - \sum_{a=1}^{N_i} \eta_{ia} P_i (B_{il} R B_{ia}^T P_i) - \sum_{a=1}^{N_i} (\rho_{ila} - \eta_{ia}) \right. \\
& \quad \left. \times P_i (B_{il} R B_{ia}^T P_i) \right. \\
& \quad \left. + A_{il}^T P_i + P_i A_{il} \right\} \hat{x}(t).
\end{aligned} \tag{30}$$

Let

$$\rho_{ila} = \frac{1 + \operatorname{sgn}(\hat{x}^T(t) P_i B_{il} R B_{ia}^T P_i \hat{x}(t))}{2}, \tag{31}$$

$$\operatorname{sgn}(z) = \begin{cases} 1, & \text{if } z > 0, \\ -1, & \text{otherwise.} \end{cases} \tag{32}$$

From (30), we obtain

$$\begin{aligned}
& = \hat{x}^T(t) \left\{ \sum_{a=1}^{N_i} \eta_{ia} (-P_i B_{ia} R B_{il}^T P_i - P_i B_{il} R B_{ia}^T P_i \right. \\
& \quad \left. + A_{il}^T P_i + P_i A_{il}) \right. \\
& \quad \left. - 2 \sum_{a=1}^{N_i} \left(\frac{\operatorname{sgn}(e_r^T(t) P_i B_{il} R B_{ia}^T P_i e_r(t))}{2} \right. \right. \\
& \quad \left. \left. - \left(\eta_{ia} - \frac{1}{2} \right) \right) (B_{il} R B_{ia}^T P_i)^T P_i \right\} \hat{x}(t)
\end{aligned}$$

$$\begin{aligned}
&\leq \sum_{a=1}^{N_i} \eta_{ia} \hat{x}^T(t) \left(-P_i B_{ia} R B_{il}^T P_i - P_i B_{il} R B_{ia}^T P_i \right. \\
&\quad \left. + A_{il}^T P_i + P_i A_{il} \right) \hat{x}(t) \\
&\quad - 2 \sum_{a=1}^{N_i} \left(\frac{1}{2} - \left(\eta_{ia} - \frac{1}{2} \right) \right) \left| \hat{x}^T(t) P_i B_{il} R B_{ia}^T P_i \hat{x}(t) \right|.
\end{aligned} \tag{33}$$

As $\eta_{ia} - (1/2) \in [-1/2, 1/2]$, due to the property of the switched fuzzy model with time delay, it can be shown that (33) satisfies the following inequality:

$$\begin{aligned}
&\hat{x}^T(t) \left(A_{il}^T P_i + P_i A_{il} \right) \hat{x}(t) + \left(u_{il}^1 \right)^T(t) B_{il}^T P_i \hat{x}(t) \\
&\quad + \hat{x}^T(t) P_i B_{il} u_{il}^1(t) \\
&\leq \sum_{a=1}^{N_i} \eta_{ia} \hat{x}^T(t) \left(-P_i B_{ia} R B_{il}^T P_i - P_i B_{il} R B_{ia}^T P_i \right. \\
&\quad \left. + A_{il}^T P_i + P_i A_{il} \right) \hat{x}(t).
\end{aligned} \tag{34}$$

Considering Assumptions 2 and 3, we have

$$\begin{aligned}
&\omega^T \bar{D}_i^T P \hat{x}(t) + \hat{x}^T(t) P \bar{D}_i \omega + \left(u_{il}^2 \right)^T(t) B_{il}^T P \hat{x}(t) \\
&\quad + \hat{x}^T(t) P B_{il} u_{il}^2(t) \\
&= 2 \left(\hat{x}^T(t) P_i B_{il} u_{il}^2 + \hat{x}^T(t) P_i \bar{D}_i \omega \right) \\
&= 2 \hat{x}^T(t) P_i B_{il} \left[-\frac{\theta(t) \left(B_{il}^T P_i \hat{x}(t) \right)}{\|B_{il}^T P_i \hat{x}(t)\|} \right. \\
&\quad \left. - \sum_{\beta=1}^{N_i} \frac{\gamma_\beta \theta(t) \left(B_{il}^T P_i \hat{x}(t) \right)}{\|B_{il}^T P_i \hat{x}(t)\|^2} \cdot \|B_{i\beta}^T P_i \hat{x}(t)\| \right] \\
&\quad + 2 \left[\hat{x}^T(t) P_i \left(\sum_{\beta=1}^{N_i} \gamma_\beta B_{i\beta} \right) \omega \right] \\
&\leq 2 \left(-\theta(t) \|B_{il}^T P_i \hat{x}(t)\| - \sum_{\beta=1}^{N_i} \gamma_\beta \theta(t) \|B_{i\beta}^T P_i \hat{x}(t)\| \right) \\
&\quad + 2 \sum_{\beta=1}^{N_i} \gamma_\beta \theta(t) \|B_{i\beta}^T P_i \hat{x}(t)\| \\
&= 2 \left(-\theta(t) \|B_{il}^T P_i \hat{x}(t)\| \right).
\end{aligned} \tag{35}$$

According to Assumption 1, we have

$$\begin{aligned}
&\hat{x}^T(t) \Delta A_{il}^T Q_i e(t) + e^T(t) Q_i \Delta A_{il} \hat{x}(t) + e^T(t) \Delta A_{il}^T Q_i e(t) \\
&\quad + e^T(t) Q_i \Delta A_{il} e(t) \leq \hat{x}^T(t) N_{il}^T N_{il} \hat{x}(t) \\
&\quad + 2e^T(t) Q_i M_{il} M_{il}^T Q_i e(t) \\
&\quad + e^T(t) N_{il}^T N_{il} e(t).
\end{aligned} \tag{36}$$

Then, from (34)–(36), it holds true that

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \sum_{a=1}^{N_i} \eta_{il} \eta_{ih} \eta_{ia} \\
&\quad \times \left[\hat{x}^T(t) \left(-P_i B_{ia} R B_{il}^T P_i - P_i B_{il} R B_{ia}^T P_i \right. \right. \\
&\quad \left. \left. + A_{il}^T P_i + P_i A_{il} \right) \hat{x}(t) \right. \\
&\quad + \hat{x}^T(t - \tau) D_{il}^T P_i \hat{x}(t) \\
&\quad + \hat{x}^T(t) P_i D_{il} \hat{x}(t - \tau) \\
&\quad + e^T(t) C_{ih}^T L_{il}^T P_i \hat{x}(t) \\
&\quad + \hat{x}^T(t) P_i L_{il} C_{ih} e(t) \\
&\quad + \hat{x}^T(t) N_{il}^T N_{il} \hat{x}(t) \\
&\quad + 2e^T(t) Q_i M_{il} M_{il}^T Q_i e(t) \\
&\quad + e^T(t) N_{il}^T N_{il} e(t) \\
&\quad + e^T(t) \left(A_{il}^T - C_{ih}^T L_{il}^T \right) Q_i e(t) \\
&\quad + e^T(t) Q_i \left(A_{il} - L_{il} C_{ih} \right) e(t) \\
&\quad + e^T(t - \tau) D_{il}^T Q_i e(t) \\
&\quad + e^T(t) Q_i D_{il} e(t - \tau) \\
&\quad + \hat{x}^T(t) P_i \hat{x}(t) - \hat{x}^T(t - \tau) P_i \hat{x}(t - \tau) \\
&\quad \left. + e^T(t) Q_i e(t) - e^T(t - \tau) Q_i e(t - \tau) \right] \\
&= \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \sum_{a=1}^{N_i} \eta_{il} \eta_{ih} \eta_{ia} \begin{bmatrix} \hat{x}(t) \\ \hat{x}(t - \tau) \\ e(t) \\ e(t - \tau) \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} Y_{ial} & P_i D_{il} & P_i L_{il} C_{ih} & 0 \\ D_{il}^T P_i & -P_i & 0 & 0 \\ C_{ih}^T L_{il}^T P_i & 0 & \Psi_{ilh} & Q_i D_{il} \\ 0 & 0 & D_{il}^T Q_i & -Q_i \end{bmatrix} \\
&\quad \times \begin{bmatrix} \hat{x}(t) \\ \hat{x}(t - \tau) \\ e(t) \\ e(t - \tau) \end{bmatrix},
\end{aligned} \tag{37}$$

where

$$\begin{aligned} Y_{ial} &= -P_i B_{ia} R B_{il}^T P_i - P_i B_{il} R B_{ia}^T P_i + A_{il}^T P_i \\ &\quad + P_i A_{il} + N_{il}^T N_{il} + P_i, \\ \Psi_{ilh} &= Q_i + (A_{il}^T - C_{ih}^T L_{il}^T) Q_i + Q_i (A_{il} - L_{il} C_{ih}) \\ &\quad + 2Q_i M_{il} M_{il}^T Q_i + N_{il}^T N_{il}, \end{aligned} \quad (38)$$

in which $i = \sigma(t)$ given by (23). Taking (9), (27), and (37) into account, we deduce that $(d/dt)V(t) < 0, \Xi(t) \neq 0$. Therefore, the system (5) is asymptotically stable under the switching law (23).

(II) When $B_{il}^T P_i \hat{x}(t) = 0$, then we have

$$\begin{aligned} \dot{V}_i(t) &\leq \sum_{l=1}^{N_i} \sum_{h=1}^{N_i} \sum_{a=1}^{N_i} \eta_{il} \eta_{ih} \eta_{ia} \begin{bmatrix} \hat{x}(t) \\ \hat{x}(t-\tau) \\ e(t) \\ e(t-\tau) \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} Y_{ial} & P_i D_{il} & P_i L_{il} C_{ih} & 0 \\ D_{il}^T P_i & -P_i & 0 & 0 \\ C_{ih}^T L_{il}^T P_i & 0 & \Psi_{ilh} & Q_i D_{il} \\ 0 & 0 & D_{il}^T Q_i & -Q_i \end{bmatrix} \\ &\quad \times \begin{bmatrix} \hat{x}(t) \\ \hat{x}(t-\tau) \\ e(t) \\ e(t-\tau) \end{bmatrix}. \end{aligned} \quad (39)$$

Taking (9) and (21) into account, we deduce that $(d/dt)V_i(x(t)) < 0, x(t) \neq 0$. Therefore, asymptotic stability follows.

Remark 5. Although any individual subsystem of (5) alone usually cannot achieve the observer-based robust control, a suitable switching may solve the robust control problem, as shown in Theorem 4.

Remark 6. (i) This controller structure consists of two parts: the switching controller u_{il}^1 to stabilize the nominal system and the supplementary controller u_{il}^2 to handle the external disturbance. Hence, in practice, this controller is a hybrid switching-robust controller. (ii) It can be seen from (15) that the switching controller u_{il}^1 consists of 2^{N_i} controllers which are linear combinations of $-RB_{ia}^T P_i e_r(t), a = 1, 2, \dots, N_i$.

In other words, we formulate the finding of P_i and Q_i of (21) into LMI problem. Considering (21) and multiplying both sides of (21) by the matrix $\text{diag}\{P_i^{-1}, I, I, I\}$, we restate as follows.

Theorem 7. Suppose that there exist constants β_{ij} ($i, j \in \overline{M}$) (either all nonnegative or all nonpositive) and positive definite symmetric matrices P_i, Q_i and matrices G_{il} such that the following matrix inequalities are satisfied. Then, the system (5) along with the fuzzy observer (10) is asymptotically stable via

the combined switching controller (14), and the switching law is designed as (23):

$$\begin{aligned} &\begin{bmatrix} \bar{Y}_{ial} + \sum_{j=1}^m \beta_{ij} (P_i - P_j) & P_i^{-1} N_{il}^T & D_{il} & L_{il} C_{ih} & 0 & 0 \\ * & -I & 0 & 0 & 0 & 0 \\ * & * & -P_i & 0 & 0 & 0 \\ * & * & * & \bar{\Psi}_{ilh} & Q_i M_{il} & Q_i D_{il} \\ * & * & * & * & -2I & 0 \\ * & * & * & * & * & -Q_i \end{bmatrix} < 0, \\ &l, a, h = 1, 2, \dots, N_i, \quad i = 1, 2, \dots, m, \end{aligned} \quad (40)$$

where

$$\begin{aligned} \bar{Y}_{ial} &= -B_{ia} R B_{il}^T - B_{il} R B_{ia}^T + P_i^{-1} A_{il}^T + A_{il} P_i^{-1} + P_i^{-1}, \\ \bar{\Psi}_{ilh} &= Q_i + A_{il}^T Q_i - C_{ih}^T G_{il}^T + Q_i A_{il} - G_{il} C_{ih} + N_{il}^T N_{il}, \\ G_{il} &= Q_i L_{il}. \end{aligned} \quad (41)$$

Once one has P_i, Q_i from (40), then the controllers (14) can be constructed.

Remark 8. The design procedure can be summarized by the following steps.

Step 1. Design the combined switching controller (14) by choosing the symmetric matrix $R > 0$.

Step 2. Check the parameters of the combined switching controller (14) by solving the LMIs in Theorem 7.

Step 3. Design the law of (31) according to Theorem 7.

Remark 9. If (40) holds for some i , which is the condition for the observer-based robust control problem of nonswitched system to be solvable, then no switching is needed. Obviously, (40) is much weaker than the condition for nonswitched systems because (40) allows the case that the observer-based robust control problem is solvable for no subsystems.

5. Illustrative Examples and Results

In order to demonstrate the efficiency and feasible performance of the proposed control synthesis, now we consider the model as follows:

$$R_1^1: \text{ if } z_2(t) \text{ is } M_{11}^1,$$

then

$$\dot{x}(t) = A_{11}x(t) + D_{11}x(t-\tau) + B_{11}u_{11}(t) + \bar{D}_1\omega(t),$$

$$y(t) = C_{11}x(t);$$

$$R_1^2: \text{ if } z_2(t) \text{ is } M_{11}^2,$$

then

$$\dot{x}(t) = A_{12}x(t) + D_{12}x(t-\tau) + B_{12}u_{12}(t) + \bar{D}_1\omega(t),$$

$$y(t) = C_{12}x(t);$$

R_2^1 : if $z_2(t)$ is M_{21}^1 ,

then

$$\begin{aligned}\dot{x}(t) &= A_{21}x(t) + D_{21}x(t - \tau) + B_{21}u_{21}(t) + \bar{D}_2\omega(t), \\ y(t) &= C_{21}x(t); \end{aligned}$$

R_2^2 : if $z_2(t)$ is M_{21}^2 ,

then

$$\begin{aligned}\dot{x}(t) &= A_{22}x(t) + D_{22}x(t - \tau) + B_{22}u_{22}(t) + \bar{D}_2\omega(t), \\ y(t) &= C_{22}x(t). \end{aligned} \quad (42)$$

Also, this model is used for the extra design of the switching law (23) of the state-dependent form. For the purpose of comparing with SF system, Example 1 in [16] can be described for the following.

Region Rule 1. If $z_2(t) \geq 1$, then the following is given.

Local Region Rule 1. If $z_2(t)$ is $h_{11}(x_2(t))$,

then

$$\begin{aligned}\dot{x}(t) &= A_{11}x(t) + D_{11}x(t - \tau) + B_{11}u(t) + \bar{D}_1\omega(t), \\ y(t) &= C_{11}x(t). \end{aligned} \quad (43)$$

Local Region Rule 2. If $z_2(t)$ is $h_{12}(x_2(t))$,

then

$$\begin{aligned}\dot{x}(t) &= A_{12}x(t) + D_{12}x(t - \tau) + B_{12}u(t) + \bar{D}_1\omega(t), \\ y(t) &= C_{12}x(t). \end{aligned} \quad (44)$$

Region Rule 2. If $z_2(t) < 1$, then the following is given.

Local Region Rule 1. If $z_2(t)$ is $h_{21}(x_2(t))$,

then

$$\begin{aligned}\dot{x}(t) &= A_{21}x(t) + D_{21}x(t - \tau) + B_{21}u(t) + \bar{D}_2\omega(t), \\ y(t) &= C_{21}x(t). \end{aligned} \quad (45)$$

Local Region Rule 2. If $z_2(t)$ is $h_{22}(x_2(t))$,

then

$$\begin{aligned}\dot{x}(t) &= A_{22}x(t) + D_{22}x(t - \tau) + B_{22}u(t) + \bar{D}_2\omega(t), \\ y(t) &= C_{22}x(t). \end{aligned} \quad (46)$$

Obviously, switching fuzzy system of Example 1 in [16] is switching between each of the sub fuzzy systems and depending on region fuzzy rules.

We have

$$A_{11} = \begin{bmatrix} -11.5 & 4 \\ -0.843 & -1.0493 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} -10.01 \\ -0.4926 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} -4.8 & 1 \\ -0.232 & -1.4312 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} -10.2 \\ -0.1516 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -10 & 0.6 \\ -0.6 & -0.8 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} -3.4 \\ -1.7 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -12 & 1.2 \\ -2 & -4.529 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} -1.2 \\ -0.1765 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} -0.5 & 0.7 \\ -0.1 & -0.4 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0.2 & 0.3 \\ -0.2 & -0.2 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix},$$

$$C_{11} = [2 \ -1], \quad C_{12} = [-0.8 \ 0.1],$$

$$C_{21} = [5 \ -1], \quad C_{22} = [1.6 \ 4],$$

$$M_{11} = M_{12} = \begin{bmatrix} -0.1125 & 1 \\ 1 & 0 \end{bmatrix}, \quad M_{21} = M_{22} = \begin{bmatrix} 0.01 & 1 \\ 1 & 0 \end{bmatrix},$$

$$N_{11} = N_{12} = \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}, \quad N_{21} = N_{22} = \begin{bmatrix} 0.5 & 1 \\ 0 & 0 \end{bmatrix},$$

$$F_{11}(t) = F_{12}(t) = F_{21}(t) = F_{22}(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix},$$

$$\omega_1(t) = \omega_2(t) = \frac{\cos t^2}{\sin t},$$

$$\bar{D}_1 = \bar{D}_2 = (1 \ 1)^T, \quad \theta(t) = 1.$$

(48)

The fuzzy sets of " $M_{11}^1, M_{11}^2, M_{21}^1, M_{21}^2$ " are represented by the following membership functions, respectively:

$$M_{11}^1(z_2(t)) = h_{11}(z_2(t)) = 1 - \frac{1}{1 + e^{-2z_2(t)}},$$

$$M_{11}^2(z_2(t)) = h_{12}(z_2(t)) = \frac{1}{1 + e^{-2z_2(t)}},$$

$$M_{21}^1(z_2(t)) = h_{21}(z_2(t)) = 1 - \frac{1}{1 + e^{(-2(z_2(t)-0.3))}},$$

$$M_{21}^2(z_2(t)) = h_{22}(z_2(t)) = \frac{1}{1 + e^{(-2(z_2(t)-0.3))}}.$$

Consider the closed-loop systems (13). Choosing $l, a, h = 1, 2, \dots, N_i, i = 1, 2, R = 8.4317$, and $\tau = 1$ for (40), we can have

$$P_1 = \begin{bmatrix} 524.5761 & 342.5681 \\ 342.5681 & 578.1421 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 631.1245 & 412.7413 \\ 412.7413 & 514.2356 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 5.3152 & 0.2942 \\ 0.2942 & 2.0512 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 4.8514 & 0.3571 \\ 0.3571 & 3.6316 \end{bmatrix}. \quad (50)$$

FIGURE 2: (a) System state $x_1(t)$ (solid line) and observer state $\hat{x}_1(t)$ (dotted line) with the combined switching controller. (b) System state $x_2(t)$ (solid line) and observer state $\hat{x}_2(t)$ (dotted line) with the combined switching controller. (c) Control signal of the combined switching controller $u_{ii}(t)$.

And we have the candidate observer gains via arbitrary switching as

$$\begin{aligned} L_{11} &= \begin{bmatrix} 0.8921 \\ 0.2324 \end{bmatrix}, & L_{12} &= \begin{bmatrix} 5.0821 \\ 0.5973 \end{bmatrix}, \\ L_{21} &= \begin{bmatrix} 0.9075 \\ 0.3032 \end{bmatrix}, & L_{22} &= \begin{bmatrix} 0.8132 \\ 0.2019 \end{bmatrix}. \end{aligned} \quad (51)$$

Taking the initial condition as $x(0) = [-5, 13]^T$, then the simulation result for the combined switching controller (14) of SF time-delay system (5) is depicted in Figure 2.

To investigate the effectiveness of the proposed SF system here, we now compare the switching fuzzy system in [16–19]. The SF model is used for the extra design of the switching law (23), but switching fuzzy system switches depending on

region fuzzy rules. For the same data of Example 1 in [16] and the same initial condition $x(0) = [-5, 13]^T$, the simulation result is depicted in Figure 3. It can be seen that the proposed SF system provides the best performance in terms of faster transient responses and shorter settling time.

The traditional PDC fuzzy controller is employed to handle the same SF time-delay system in [34]. According to [34], the global control of the traditional PDC fuzzy controller is

$$u(t) = \sum_{i=1}^{N_i} \sum_{l=1}^{N_l} v_i h_{il} K_{il} \hat{x}(t), \quad (52)$$

where

$$v_j(z(t)) = \begin{cases} 1, & z(t) \in \text{Region } j, \\ 0, & \text{otherwise.} \end{cases} \quad (53)$$

FIGURE 3: (a) System state $x_1(t)$ (solid line) and observer state $\hat{x}_1(t)$ (dotted line) of switching fuzzy system with its switching controller in [16]. (b) System state $x_2(t)$ (solid line) and observer state $\hat{x}_2(t)$ (dotted line) of switching fuzzy system with its switching controller in [16]. (c) Control signal of the switching controller $u(t)$ in [16] of switching fuzzy system.

The state feedback gains of subsystems are obtained as follows:

$$\begin{aligned}
 K_{11} &= [-0.542 \quad -0.125], \\
 K_{12} &= [-0.6532 \quad -7.4367], \\
 K_{21} &= [-3.4 \quad -5.32], \\
 K_{22} &= [-3.12 \quad -4.21].
 \end{aligned} \tag{54}$$

Figure 4 shows the SF system responses and the control signals under the same initial condition $x(0) = [-5, 13]^T$. Referring to Figure 2, it can be seen that the performance

of the combined switching controller (14) is slightly better in terms of transient response time.

6. Conclusion

In this paper, the observer-based robust control for SF system with time delay is investigated. We used the multiple Lyapunov function technique and a switching law to design a control law such that the system along with fuzzy observer is asymptotically stable. The combined switching controller design problem can be solved efficiently by using linear matrix inequalities and convex optimization techniques.

FIGURE 4: (a) System state $x_1(t)$ (solid line) and observer state $\hat{x}_1(t)$ (dotted line) with the traditional PDC fuzzy controller of SF system. (b) System state $x_2(t)$ (solid line) and observer state $\hat{x}_2(t)$ (dotted line) with the traditional PDC fuzzy controller of SF system. (c) Control signal of the traditional PDC fuzzy controller $u(t)$ of SF system.

The simulation example shows the validity of the switching control law.

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