Research Article

Adaptive Control of MEMS Gyroscope Based on Global Terminal Sliding Mode Controller

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An adaptive global fast terminal sliding mode control (GFTSM) is proposed for tracking control of Micro-Electro-Mechanical Systems (MEMS) vibratory gyroscopes under unknown model uncertainties and external disturbances. To improve the convergence rate of reaching the sliding surface, a global fast terminal sliding surface is employed which can integrate the advantages of traditional sliding mode control and terminal sliding mode control. It can be guaranteed that sliding surface and equilibrium point can be reached in a shorter finite time from any initial state. In the presence of unknown upper bound of system nonlinearities, an adaptive global fast terminal sliding mode controller is derived to estimate this unknown upper bound. Simulation results demonstrate that the tracking error can be attenuated efficiently and robustness of the control system can be improved with the proposed adaptive global fast terminal sliding mode control.

1. Introduction

MEMS gyroscopes are the basic measuring elements of inertial navigation and inertial guidance system, which are commonly used for measuring angular velocity. Because of their enormous advantages in size and cost, MEMS gyroscopes have a wide range of applications in the area of navigation, platform stabilization, control, and traffic. But, due to the manufacturing error and the temperature disturbance, which can cause deviations from the desired qualities and degrade the performance of a gyroscope. Besides, MEMS gyroscopes belong to MIMO systems, which have parameter uncertainties and are vulnerable to external environment influence. Therefore, it is necessary to solve these challenges by utilizing advanced control methods. In the last few years, many advanced control approaches have been proposed to enhance performance and robustness of MEMS gyroscope. Batur et al. [1] developed a sliding mode control for MEMS gyroscope system. Leland [2] presented an adaptive controller for tuning the natural frequency of the drive axis of a vibratory gyroscope. Park et al. [3, 4] presented an adaptive controller for a MEMS gyroscope which drives both axes of vibration and controls the entire operation

of the gyroscope. Adaptive controller has been developed using sliding mode control to control the vibration of MEMS gyroscope [5]. Raman et al. [6] proposed a closed-loop digitally controlled MEMS gyroscope using unconstrained sigma-delta force balanced feedback control. Tsai and Sue [7] developed integrated model reference adaptive control and time-varying angular rate estimator for microgyroscopes.

As we know, sliding mode control has many attractive features such as robustness to parameter variations and insensitivity to disturbance. The basic idea of the sliding mode control is to drive and maintain the system trajectory on a sliding surface designed a priori in the state space. However, one of the representative characteristics of conventional sliding mode control is that the convergence of the system states to the equilibrium point is usually asymptotical but not in finite time. Recently, a new type of sliding mode control technique called terminal sliding mode control is developed. Instead of using linear hyperplanes as the sliding surfaces, the terminal sliding mode control adopts nonlinear sliding surfaces. With the nonlinear sliding surfaces, the terminal sliding mode controller can make the system states reach the equilibrium point in a finite time

period. Saif et al. [8] presented a novel methodology for approximation of the unknown time-varying rotation rate by using a sliding mode observer along with a robust control system based on terminal sliding mode control for improving the performance of the MEMS gyroscope. Zhu and Chai [9] presented a global finite-time tracking controller based on terminal sliding mode control method for the trajectory tracking control problem of biped multilinked robots with uncertain disturbances. In order to solve the optimal problem of convergence time and estimate the unknown upper bound of system uncertainties, much research has been done to apply intelligent control approaches such as the adaptive global fast terminal sliding mode control. Feng et al. [10] and Keleher and Stonier [11] proposed an adaptive fast terminal sliding mode tracking control scheme for robotic manipulators to estimate the upper bound of uncertainties. Tao et al. [12] developed a new adaptive fuzzy terminal sliding mode controller for linear systems with mismatched time varying uncertainties. Zhang et al. [13] proposed a new approach of adaptive fuzzy global identical terminal sliding mode control for cross-beam system. Neila and Tarak [14] designed adaptive terminal sliding mode control for rigid robotic manipulators. Feng et al. [15] proposed a nonsingular terminal sliding mode controller for rigid manipulators. In this paper, robust adaptive global fast terminal sliding mode control strategy for trajectory tracking control of MEMS vibratory gyroscopes is proposed. The main motivations in the paper are highlighted as follows.

- (1) A global fast terminal sliding mode surface is designed to improve the convergence rate of reaching the sliding surface. It integrates the advantages of traditional sliding and terminal sliding control, and guarantees that the tracking error can converge to zero in a shorter finite time. Adaptive control and terminal sliding control are successfully integrated with MEMS gyroscope.
- (2) An adaptive global fast terminal sliding mode control is incorporated into the MEMS gyroscope to improve the tracking performance and strengthen the robustness of the control system. With this control, the upper bound of model uncertainties and external disturbances are not needed because the proposed controller can estimate the upper bound of these uncertainties using an adaptive mechanism. Also, the proposed controller eliminates the chattering effect without losing the robustness property and the precision.

This paper is organized as follows. In Section 2, the dynamics of MEMS gyroscope are described. In Section 3, global fast terminal sliding mode control is introduced. In Section 4, the proposed adaptive global fast terminal sliding mode controller is derived and the stability of this control system is proved by Lyapunov's stability theory. Simulation results are presented in Section 5. Finally, conclusions are provided in Section 6.

FIGURE 1: A simplified model of a z-axis MEMS Gyroscope.

2. Dynamics of MEMS Gyroscope

A z-axis MEMS gyroscope is depicted in Figure 1. Generally, a typical MEMS vibratory gyroscope includes a proof mass suspended by spring beams, electrostatic actuations, and sensing mechanisms for forcing an oscillatory motion and sensing the position and velocity of the proof mass. Dynamics of a MEMS gyroscope are derived from Newton's law in the rotating frame.

As we know, Newton's law in the rotating frame becomes

$$F_r = F_{\text{phy}} + F_{\text{centri}} + F_{\text{Coriolis}} + F_{\text{Euler}} = m a_r. \quad (1)$$

In (1), F_r is the total applied force to the proof mass in the gyro frame, F_{phy} is the total physical force to the proof mass in the inertial frame, F_{centri} is the centrifugal force, F_{Coriolis} is the Coriolis force, F_{Euler} is the Euler force, and a_r is the acceleration of the proof mass with respect to the gyro frame. F_{phy} , F_{Coriolis} , and F_{Euler} are inertial forces caused by the rotation of the gyro frame.

With the definition of r_r and v_r as the position and velocity vectors relative to the rotating gyroscope frame, and Ω as the angular velocity vector of the gyroscope frame, the expressions for the inertial forces are reduced to

$$\begin{aligned} F_{\text{Coriolis}} &= -2m\Omega \times v_r, & F_{\text{centri}} &= -m\Omega \times (\Omega \times r_r), \\ F_{\text{Euler}} &= -m \frac{d\Omega}{dt} \times r_r, \end{aligned} \quad (2)$$

then,

$$m a_r + m\Omega \times (\Omega \times r_r) + 2m\Omega \times v_r + m \dot{\Omega} \times r_r = F_{\text{phy}}, \quad (3)$$

where F_{phy} contains spring, damping, and control forces applied to the proof mass.

In a z-axis MEMS gyroscope, by supposing the stiffness of spring in z direction much larger than that in x and y directions, motion of proof mass is constrained to only along the x - y plane. Referring to [16], with assuming that the gyroscope is almost rotating at a constant angular velocity

Ω_z over a sufficiently long time interval, the dynamics of gyroscope based on (3) are simplified as follows:

$$\begin{aligned} m\ddot{x} + d_x\dot{x} + [k_x - m(\Omega_y^2 + \Omega_z^2)]x + m\Omega_x\Omega_y y \\ = u_x + 2m\Omega_z\dot{y}, \\ m\ddot{y} + d_y\dot{y} + [k_y - m(\Omega_x^2 + \Omega_z^2)]y + m\Omega_x\Omega_y x \\ = u_y - 2m\Omega_z\dot{x}, \end{aligned} \quad (4)$$

where x and y are the coordinates of the proof mass with respect to the gyro frame in a Cartesian coordinate system, $d_{x,y}$ and $k_{x,y}$ are damping and spring coefficients, $\Omega_{x,y,z}$ are the angular rate components along each axis of the gyro frame, and $u_{x,y}$ are the control forces in x and y directions. In this design, the control forces are the electrostatic forces in parallel plate actuator which can be expressed as the gradient of the potential energy stored on the capacitor. The last two terms in (4), $2m\Omega_z\dot{y}$ and $2m\Omega_z\dot{x}$ are the Coriolis forces used to reconstruct the unknown input angular rate Ω_z . Under typical assumptions $\Omega_x \approx \Omega_y \approx 0$, only the component of the angular rate Ω_z causes a dynamic coupling between x - and y -axes.

Taking fabrication imperfections into account, which cause extra coupling between x - and y -axes, the lumped parameter mathematical model for an actual z -axis MEMS gyroscope becomes

$$\begin{aligned} m\ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + k_{xx}x + k_{xy}y = u_x + 2m\Omega_z\dot{y}, \\ m\ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + k_{xy}x + k_{yy}y = u_y - 2m\Omega_z\dot{x}. \end{aligned} \quad (5)$$

In (5), d_{xx} and d_{yy} are damping coefficients, k_{xx} and k_{yy} are spring coefficients, d_{xy} and k_{xy} , called quadrature errors, are coupled damping and spring terms, respectively, mainly due to the asymmetries in suspension structure and misalignment of sensors and actuators. The coupled spring and damping terms are unknown, but can be assumed to be small. The nominal values of the x - and y -axes spring and damping terms are known, but there are small unknown variations. The proof mass can be determined accurately.

Dividing both sides of (5) by m , q_0 , and ω_0^2 , which are a reference mass, length, and natural resonance frequency, respectively, we get the form of the nondimensional equation of motion as

$$\begin{aligned} \ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + \omega_x^2 x + \omega_{xy}y = u_x + 2\Omega_z\dot{y}, \\ \ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + \omega_{xy}x + \omega_y^2 y = u_y - 2\Omega_z\dot{x}, \end{aligned} \quad (6)$$

where $d_{xx}/m\omega_0 \rightarrow d_{xx}$, $d_{xy}/m\omega_0 \rightarrow d_{xy}$, $d_{yy}/m\omega_0 \rightarrow d_{yy}$, $\sqrt{k_{xx}/m\omega_0^2} \rightarrow \omega_x$, $k_{xy}/m\omega_0^2 \rightarrow \omega_{xy}$, $\sqrt{k_{yy}/m\omega_0^2} \rightarrow \omega_y$, $\Omega_z/\omega_0 \rightarrow \Omega_z$.

The vector form of MEMS gyroscope dynamics model can be written as

$$\ddot{q} + D\dot{q} + Kq = u - 2\Omega\dot{q}, \quad (7)$$

where $q = \begin{bmatrix} x \\ y \end{bmatrix}$, $D = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}$, $K = \begin{bmatrix} \omega_x^2 & \omega_{xy} \\ \omega_{xy} & \omega_y^2 \end{bmatrix}$, $u = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$, $\Omega = \begin{bmatrix} 0 & -\Omega_z \\ \Omega_z & 0 \end{bmatrix}$.

Considering the system with parametric uncertainties and external disturbance, the dynamics of MEMS gyroscope (7) can be expressed as

$$\ddot{q} + (D + 2\Omega + \Delta D)\dot{q} + (K + \Delta K)q = u + d, \quad (8)$$

where ΔD is the unknown parameter uncertainties of the matrix $D + 2\Omega$, ΔK is the unknown parameter uncertainties of the matrix K , and d is an uncertain extraneous disturbance.

The control target for MEMS gyroscope is to find a control law so that the proof mass position $q(t)$ can track the desired trajectory $q_r(t)$. In addition, in the presence of model uncertainties and external disturbances, a robust control design is required to improve the robustness and tracking resolution of the control system. In this paper, adaptive global fast terminal sliding mode control is designed to make the system track the desired trajectory and improve the system robustness in the unknown model uncertainties and external disturbances.

3. Global Fast Terminal Sliding Mode Control

The traditional fast terminal sliding mode form can be defined as

$$s_a = \dot{x}_1 + \beta_0 x_1^{p_2/p_1} = 0, \quad (9)$$

where $x_1 \in R^1$ is state variable, $\beta_0 > 0$ is constant, and p_1 and p_2 are odd positive integers satisfying $p_1 > p_2$. Given an initial state $x_1(0) \neq 0$, the system will reach the equilibrium point $x_1 = 0$ along the sliding mode (9) in finite time t_{c1} . The time to reach $x_1 = 0$ from $x_1(0) \neq 0$ is determined by

$$t_{c1} = \frac{p_1}{\beta_0(p_1 - p_2)} |x_1(0)|^{(p_1 - p_2)/p_1}. \quad (10)$$

The global fast terminal sliding surface can be defined as

$$s_b = \dot{x}_1 + \alpha_0 x_1 + \beta_0 x_1^{p_2/p_1} = 0, \quad (11)$$

where $\alpha_0 > 0$ is constant. Given an initial state $x_1(0) \neq 0$, the system will reach the equilibrium point $x_1 = 0$ along the sliding mode (11) in finite time t_{c2} . The time to reach $x_1 = 0$ from $x_1(0) \neq 0$ is determined by

$$t_{c2} = \frac{p_1}{\alpha_0(p_1 - p_2)} \ln \frac{\alpha_0 x_1(0)^{(p_1 - p_2)/p_1} + \beta_0}{\beta_0}. \quad (12)$$

Referring to [17], we can demonstrate that $t_{c2} \leq t_{c1}$. Comparing (9) and (11), it can be seen that the convergence times of the two types of terminal sliding modes are all determined by the fast terminal attractor $\dot{x}_1 = -\beta_0 x_1^{p_2/p_1}$ when it is far from the equilibrium point. At this time, their convergence rates are basically the same. But when close to the equilibrium point, the convergence time of global fast terminal sliding mode is mainly determined by the linear sliding mode $\dot{x}_1 = -\alpha_0 x_1$ and the convergence rate accelerates exponentially. At this time, its convergence rate is much better than the traditional fast terminal sliding mode.

FIGURE 2: Block diagram of an adaptive global fast terminal sliding mode control for a MEMS gyroscope.

Remark 1. The global fast terminal sliding mode not only introduces the fast terminal attractor, making the system state converge in finite time, but also keeps the rapidity of the linear sliding mode when it is close to the equilibrium state, so as to realize the goal of that the system state will converge to the equilibrium state quickly and accurately.

4. Adaptive Global Fast Terminal Sliding Mode Control

The global fast terminal sliding mode control that is put forward has solved the optimal problem of convergence time. It integrates the advantages of traditional sliding mode control and terminal sliding mode control in the design of sliding mode and uses the concept of fast access to the equilibrium point at the arriving stage to ensure that the tracking error converges to zero in a shorter finite time. However, as a result of the design principle that is similar to the traditional sliding mode control, there still exists chattering in the global fast terminal sliding mode control signals. Also, the uncertainty boundary needs to be known in advance, and it is hard to accomplish that in actual MEMS gyroscopes. In allusion to the trajectory tracking control of MEMS gyroscopes with unknown model uncertainties and external disturbances, a new adaptive global fast terminal sliding mode control algorithm is synthesized in this section, which uses the respective advantages of adaptive and sliding mode control and the estimates of unknown uncertainty which can be obtained by adaptive control.

The block diagram of an adaptive global fast terminal sliding mode control for a MEMS gyroscope is shown in Figure 2, the tracking error between expectation state and gyroscope state comes to the global fast terminal sliding mode controller. The adaptive global fast terminal sliding mode controller is proposed to control the MEMS gyroscope. The unknown upper bound of system uncertainties can be identified by adaptive estimator.

4.1. Design of Global Fast Terminal Sliding Mode Surface. The tracking control problem of a MEMS gyroscope is what this paper considers, so the control target is to design a suitable

control law to make the system output q track the state of the desired trajectory q_r completely in finite time.

For the trajectory tracking control of a MEMS gyroscope, two input and two output systems, the global fast terminal sliding surface is chosen as

$$s_c = [s_{c1}, s_{c2}]^T = \dot{e} + \alpha e + \beta e^{p_2/p_1}, \quad (13)$$

where $\alpha = \text{diag}(\alpha_1, \alpha_2)$ and $\beta = \text{diag}(\beta_1, \beta_2)$ are sliding mode surface constants, $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constants, $e = q - q_r = [q_1 - q_{r1}, q_2 - q_{r2}]^T$ is the system tracking error, and q_r is the desired trajectory. In the sliding mode $s_c = [s_{c1}, s_{c2}]^T$, the system reaches the equilibrium state $q = 0$ from any initial state $q(0) \neq 0$ in finite time $t_s = [t_{s1}, t_{s2}]^T$. The time t_s is determined by

$$t_s = \left[\frac{p_1}{\alpha_1(p_1 - p_2)} \ln \frac{\alpha_1 q_1(0)^{(p_1 - p_2)/p_1} + \beta_1}{\beta_1}, \frac{p_1}{\alpha_2(p_1 - p_2)} \ln \frac{\alpha_2 q_2(0)^{(p_1 - p_2)/p_1} + \beta_2}{\beta_2} \right]. \quad (14)$$

We can make the system reach equilibrium state in finite time t_s through setting the suitable parameters α, β, p_1 , and p_2 . After determining the sliding surface, the next step is to design a sliding mode controller to ensure the existence of the sliding mode stage.

4.2. Design of Global Fast Terminal Sliding Mode Controller. In the sliding mode control, the control input should be able to force all the system state trajectories to converge the sliding mode surface $s = 0$, so as to ensure the existence of the sliding mode stage. The behavior of the sliding mode is equivalent to that of the stability of the state trajectory in the sliding surface. In other words, the control law should be able to ensure that the tracking error converges to zero from any state.

Furthermore, rewriting (8) as

$$\ddot{q} + (D + 2\Omega)\dot{q} + Kq = u + f, \quad (15)$$

where f represents the matched lumped uncertainty and disturbance which is given by

$$f = d - \Delta D\dot{q} - \Delta Kq. \quad (16)$$

We make the following assumption prior to further discussion.

Assumption 2. The lumped uncertainty and disturbance f is input related and if the control input does not contain the acceleration signal, the system uncertainties will be bounded by a positive function of the position and velocity measurements in the following form:

$$\|f\| < b_0 + b_1 \|q\| + b_2 \|\dot{q}\|^2, \quad (17)$$

where b_0 , b_1 , and b_2 are unknown positive constants.

Remark 3. If lumped uncertainty and disturbance f is not related to with state position and velocity signals, the system uncertainty will be only bounded by a positive constant b_0 .

Using the sliding surface (13), the global fast sliding mode controller is proposed as follows:

$$u = u_0 + u_1 + u_2, \quad (18)$$

where u_0 is defined in (19), u_1 is defined in (20), and u_2 is defined in (21). Consider

$$u_0 = \ddot{q}_r + (D + 2\Omega) \dot{q} + Kq, \quad (19)$$

$$u_1 = -\alpha \dot{e} - \frac{p_2}{p_1} \beta \text{diag}(e_1^{p_2/p_1-1}, e_2^{p_2/p_1-1}) \dot{e}, \quad (20)$$

$$u_2 = -\rho \frac{s_c}{\|s_c\|}. \quad (21)$$

And ρ has the following expression:

$$\rho = \xi + b_0 + b_1 \|q\| + b_2 \|\dot{q}\|, \quad (22)$$

where ξ is a positive constant.

According to [18], it is known that the control system is stable in this control law and reaches the stable point in finite time. But the control law requires that ρ should be known. We usually choose the larger ρ conservatively to compensate the system model uncertainties and the upper bound of the disturbances fully, but this often enhances the chattering.

In order to overcome this problem, we introduce a proper adaptation law combining with global fast terminal sliding mode control to realize the adaptive global fast terminal sliding mode control in the next part. This method achieves the goal of tracking system model errors and uncertain disturbances automatically, weakening the chattering and ensuring the stability of the control system.

4.3. Design of Adaptive Global Fast Terminal Sliding Mode Controller. Supposing that $\hat{b}_0$, $\hat{b}_1$, and $\hat{b}_2$ are the estimates of b_0 , b_1 , and b_2 , respectively, the following simple adaptive control laws can realize the estimation of the unknown parameters b_0 , b_1 , and b_2 . Consider

$$\begin{aligned} \dot{\hat{b}}_0 &= m_0 \|s_c\|, \\ \dot{\hat{b}}_1 &= m_1 \|s_c\| \|q\|, \quad \dot{\hat{b}}_2 = m_2 \|s_c\| \|\dot{q}\|^2, \end{aligned} \quad (23)$$

where m_0 , m_1 , and m_2 are arbitrary positive constants representing the adaptive gains.

Based on the given adaptation law, for the MEMS gyroscope system with the dynamics (15), the adaptive global fast terminal sliding mode controller is designed synthetically as follows, which uses the sliding surface (13):

$$u = u_0 + u_1 + u_2, \quad (24)$$

where u_0 is defined in (25), u_1 is defined in (26), and u_2 is defined in (27).

$$u_0 = a + (D + 2\Omega) v + Kq, \quad (25)$$

$$u_1 = -W \frac{s_c}{\|s_c\|}, \quad (26)$$

$$u_2 = -\frac{s_c}{\|s_c\|} (\hat{b}_0 + \hat{b}_1 \|q\| + \hat{b}_2 \|\dot{q}\|^2), \quad (27)$$

where $W = \text{diag}(w_1, w_2)$, $w_i > 0$, $i = 1, 2$, is a designed parameter matrix, and

$$v = \dot{q}_r - \alpha \dot{e} - \beta e^{p_2/p_1}, \quad (28)$$

$$a = \dot{v} = \ddot{q}_r - \alpha \ddot{e} - \frac{p_2}{p_1} \beta \text{diag}(e_1^{p_2/p_1-1}, e_2^{p_2/p_1-1}) \ddot{e}.$$

Theorem 4. If the control law (24), with the sliding surface (13) and the adaptation law (23), is applied to the nonlinear uncertain system defined by (15), then the system's tracking error can converge to zero in finite time.

Proof. Let us consider the following positive definite function as a Lyapunov function candidate:

$$V = \frac{1}{2} s_c^T s_c + \frac{1}{2} \sum_{i=0}^2 m_i^{-1} \tilde{b}_i^2, \quad (29)$$

where $\tilde{b}_i = b_i - \hat{b}_i$ and $i = 0, 1, 2$ are the estimation errors between actual values and their estimated values, respectively.

Since b_0 , b_1 , and b_2 are constants, we have $\dot{\tilde{b}}_i = -\dot{\hat{b}}_i$.

Substituting the control u in (24) into the dynamic of MEMS gyroscope (15) yields

$$\begin{aligned} \ddot{q} + (D + 2\Omega + \Delta D) \dot{q} + (K + \Delta K) q \\ = a + (D + 2\Omega) v + Kq + u_1 + u_2 + d. \end{aligned} \quad (30)$$

Rewriting (30) yields

$$\begin{aligned} (\ddot{q} - a) + (D + 2\Omega) (\dot{q} - v) + \Delta D \dot{q} + \Delta K q \\ = u_1 + u_2 + d. \end{aligned} \quad (31)$$

Substituting (28) into (31) generates

$$\dot{s}_c + (D + 2\Omega) s_c + \Delta D \dot{q} + \Delta K q = u_1 + u_2 + d. \quad (32)$$

Equation (32) can be expressed as

$$\dot{s}_c = -(D + 2\Omega) s_c - \Delta D \dot{q} - \Delta K q + u_1 + u_2 + d, \quad (33)$$

from the definition of f in (16), we have

$$\dot{s}_c = -(D + 2\Omega)s_c + u_1 + u_2 + f. \quad (34)$$

Differentiating V with respect to time yields, we have

$$\begin{aligned} \dot{V} &= s_c^T \dot{s}_c - \sum_{i=0}^2 m_i^{-1} \tilde{b}_i \dot{\hat{b}}_i \\ &= s_c^T [-(D + 2\Omega)s_c + u_1 + u_2 + f] - m_0^{-1} \tilde{b}_0 \dot{\hat{b}}_0 \\ &\quad - m_1^{-1} \tilde{b}_1 \dot{\hat{b}}_1 - m_2^{-1} \tilde{b}_2 \dot{\hat{b}}_2 \\ &= s_c^T (u_1 + u_2 + f) + s_c^T [-(D + 2\Omega)s_c \\ &\quad - m_0^{-1} \tilde{b}_0 \dot{\hat{b}}_0 - m_1^{-1} \tilde{b}_1 \dot{\hat{b}}_1 - m_2^{-1} \tilde{b}_2 \dot{\hat{b}}_2] \\ &\leq s_c^T (u_1 + u_2 + f) - m_0^{-1} \tilde{b}_0 \dot{\hat{b}}_0 - m_1^{-1} \tilde{b}_1 \dot{\hat{b}}_1 - m_2^{-1} \tilde{b}_2 \dot{\hat{b}}_2 \\ &\leq -\lambda_{\min}(W) \|s_c\| + s_c^T (u_2 + f) - m_0^{-1} \tilde{b}_0 \dot{\hat{b}}_0 \\ &\quad - m_1^{-1} \tilde{b}_1 \dot{\hat{b}}_1 - m_2^{-1} \tilde{b}_2 \dot{\hat{b}}_2 = -\lambda_{\min}(W) \|s_c\| + s_c^T f \\ &\quad - \|s_c\| (\hat{b}_0 + \hat{b}_1 \|q\| + \hat{b}_2 \|\dot{q}\|^2) \\ &\quad - \tilde{b}_0 \|s_c\| - \tilde{b}_1 \|s_c\| \|q\| - \tilde{b}_2 \|s_c\| \|\dot{q}\|^2 \\ &= -\lambda_{\min}(W) \|s_c\| + s_c^T f - \|s_c\| (\hat{b}_0 + \hat{b}_1 \|q\| + \hat{b}_2 \|\dot{q}\|^2) \\ &\leq -\lambda_{\min}(W) \|s_c\| + \|s_c\| f \\ &\quad - \|s_c\| (\hat{b}_0 + \hat{b}_1 \|q\| + \hat{b}_2 \|\dot{q}\|^2) \leq -\eta \|s_c\| < 0, \end{aligned} \quad (35)$$

where $\eta = \lambda_{\min}(W) - \|f\| + (\hat{b}_0 + \hat{b}_1 \|q\| + \hat{b}_2 \|\dot{q}\|^2) > 0$ and $\|s_c\| \neq 0$ [15].

Therefore, according to Lyapunov's second method, the designed controller can guarantee the globally asymptotical stability of the system and make the output tracking error converge to zero in finite time. On the other hand, in the global fast terminal sliding mode $s_c = \dot{e} + \alpha e + \beta e^{p_2/p_1} = 0$, the output tracking error $e = q - q_r$ converges to zero in finite time. $\square$

Remark 5. In the proof of Theorem 4, we used the Rayleigh principle, namely,

$$\lambda_{\min}(W) \|s\|^2 \leq s^T W s \leq \lambda_{\max}(W) \|s\|^2, \quad (36)$$

where $\lambda_{\min}(W)$ and $\lambda_{\max}(W)$ are the minimum and maximum eigenvalues of W , respectively. Since W is a diagnose matrix, that is, $W = \text{diag}\{w_1, \dots, w_n\}$, $\lambda_{\min}(W)$ and $\lambda_{\max}(W)$ have the simple types described as follows:

$$\begin{aligned} \lambda_{\min}(W) &= \min_{i=1, \dots, n} \{w_i\}, \\ \lambda_{\max}(W) &= \max_{i=1, \dots, n} \{w_i\}. \end{aligned} \quad (37)$$

Remark 6. Since there are two switching functions sgn in adaptive terminal sliding mode control (26) and (27), there

exists chattering in the global fast terminal sliding mode control signals. In order to reduce the chattering, we can use adaptive global fast terminal quasi-sliding mode control instead of adaptive global fast terminal sliding mode control. The so-called adaptive global fast terminal quasi-sliding mode control is a control method that the system trajectory is confined to a neighborhood δ within the ideal sliding mode. The quasi-sliding mode control adopts normal sliding mode control in the boundary layer outside and continuous state feedback control in the boundary layer, so as to avoid or weaken the chattering. Therefore, the controller (24) is improved as follows:

$$\begin{aligned} u_1 &= \begin{cases} -W \frac{s_c}{\|s_c\|}, & \text{if } \|s_c\| > \delta, \\ -W \frac{s_c}{\delta}, & \text{if } \|s_c\| \leq \delta, \end{cases} \\ u_2 &= \begin{cases} -\frac{s_c}{\|s_c\|} (\hat{b}_0 + \hat{b}_1 \|q\| + \hat{b}_2 \|\dot{q}\|^2), & \text{if } \|s_c\| > \delta, \\ -\frac{s_c}{\delta} (\hat{b}_0 + \hat{b}_1 \|q\| + \hat{b}_2 \|\dot{q}\|^2), & \text{if } \|s_c\| \leq \delta, \end{cases} \end{aligned} \quad (38)$$

where δ is a designed positive parameter; then, the chattering will be eliminated.

5. Simulation Example

In this section, we will evaluate the proposed adaptive global fast terminal sliding mode control scheme on the lumped MEMS gyroscope sensor model by using Matlab/Simulink. The control objective is to design an adaptive global fast terminal sliding mode controller so that the trajectory $q(t)$ can track the state of the desired model $q_r(t)$ completely in finite time. In addition, the unpredictably upper bound of system uncertainties can be estimated by adaptive estimator. The parameters of the MEMS gyroscope are as follows:

$$\begin{aligned} m &= 1.8 \times 10^{-7} \text{ kg}, & k_{xx} &= 63.955 \text{ N/m}, \\ k_{yy} &= 95.92 \text{ N/m}, & k_{xy} &= 12.779 \text{ N/m}, \\ d_{xx} &= 1.8 \times 10^{-6} \text{ Ns/m}, & d_{yy} &= 1.8 \times 10^{-6} \text{ Ns/m}, \\ d_{xy} &= 3.6 \times 10^{-7} \text{ Ns/m}. \end{aligned} \quad (39)$$

Since the general displacement range of the MEMS gyroscope in each axis is sub-micrometer level, it is reasonable to choose $1 \mu\text{m}$ as the reference length q_0 . Given that the usual natural frequency of each axis of a MEMS gyroscope is in the KHz range, so, choose the ω_0 as 1 KHz. The unknown angular velocity is assumed $\Omega_z = 100 \text{ rad/s}$. Then, the nondimensional values of the MEMS gyroscope parameters are listed as follows:

$$\begin{aligned} \omega_x^2 &= 355.3, & \omega_y^2 &= 532.9, \\ \omega_{xy} &= 70.99, & d_{xx} &= 0.01, \\ d_{yy} &= 0.01, & d_{xy} &= 0.002, & \Omega_z &= 0.1. \end{aligned} \quad (40)$$

FIGURE 3: Position tracking of x-axis.

FIGURE 4: Position tracking of y-axis.

In this simulation example, the initial values of the system are selected as $q_1(0) = 1.0$, $\dot{q}_1(0) = 0$, $q_2(0) = 0.5$, and $\dot{q}_2(0) = 0$. The desired motion trajectories are described by $q_{r1} = \sin(\pi t)$ and $q_{r2} = \cos(\pi t)$. The lumped parametric uncertainties and external disturbances given by $f = d - \Delta D\dot{q} - \Delta Kq$ are composed of model uncertainties and external disturbances. As for model uncertainties, there are $\pm 10\%$ parameter variations for the spring and damping coefficients with respect to their nominal values and $\pm 10\%$ magnitude changes in the coupling terms, that is, d_{xy} and ω_{xy} with respect to their nominal values. Random signal $d = [0.5 * \text{randn}(1, 1); 0.5 * \text{randn}(1, 1)]$ is considered as external disturbances. The chosen sliding surface parameters are $p_1 = 5$, $p_2 = 3$, $\alpha_1 = \alpha_2 = 0.25$, and $\beta_1 = \beta_2 = 0.5$ and the initial conditions of the upper bound of the uncertainty are $b_{00} = 0.1$, $b_{10} = 0.2$, and $b_{20} = 0.3$. The adaptive gains are chosen as $m_0 = 1.0$, $m_1 = 1.6$, and $m_2 = 1.6$. The sliding mode controller parameter of (26) is $W = \text{diag}(2, 4)$ ($w_1 = 2$, $w_2 = 4$). In order to have a small boundary layer around the sliding surface and reduce the chattering, we have chosen the designed positive parameter δ equal to 0.06 and used the adaptive global fast terminal quasi-sliding mode control (36). The simulation results are shown in Figures 3–9.

FIGURE 5: Position tracking error of x-axis comparison between AGFTSMC and SMC.

FIGURE 6: Position tracking error of y-axis comparison between AGFTSMC and SMC.

Figures 3 and 4 depict the MEMS gyroscope along x-axis and y-axis tracking trajectories using the adaptive global fast terminal sliding mode control law, respectively. From Figures 3 and 4, it can be observed intuitively that after about 3 seconds, the actual motion trajectory of the MEMS gyroscope is consistent with the desired reference trajectory, making the system state track the state of the desired trajectory completely in a shorter finite time.

Subsequently, in order to demonstrate the advantages of the paper, a comparable investigation on MEMS gyroscope model (15) is accomplished between the proposed adaptive global fast terminal sliding mode control and conventional sliding mode control, and the results are shown in Figures 5 and 6, where the tracking errors of x-axis and y-axis corresponding to adaptive global fast terminal sliding mode control more severely decrease in contrast with the conventional sliding mode control due to the proposed finite time convergence algorithm. Besides, from the Figures 5 and 6, it can be also seen that the tracking errors of adaptive global

FIGURE 7: Convergence of the terminal sliding surface $s(t)$. FIGURE 8: Control input signal of x -axis.

fast terminal sliding mode control converge to zero in about 3 seconds, which improves the dynamic behavior of the MEMS gyroscope and verify that the adaptive global fast terminal sliding mode control can ensure the global stability of the control system.

Moreover, we can set the appropriate sliding mode surface parameters to adjust the convergence rate, so as to improve the work efficiency. Figure 7 describes the convergence of the global fast terminal sliding mode surface $s(t)$. It is shown that the sliding mode surface converges to zero in a very short time. The global fast terminal sliding mode control has solved the optimal problem of convergence time compared with the traditional terminal sliding mode so that it can make the system state converge to the equilibrium state quickly and accurately. It should be noted that we can also choose appropriate reaching law to improve the convergence rate further, such as ideal reaching law and adaptive reaching law, which have been proposed in recent years. Sliding mode control system utilized these two types of reaching laws that not only can reach the region of the switching surface in finite time but also enhance the dynamic performance of the control system.

Figures 8 and 9 depict the smooth sliding mode control forces of the MEMS gyroscope along x -axis and y -axis with the adaptive global fast terminal sliding mode controller, respectively. It is clearly observed from Figures 8 and 9 that there is no chattering using the proposed controller. Figure 10

FIGURE 9: Control input signal of y -axis. FIGURE 10: Adaptively estimated parameters $\hat{b}_0$, $\hat{b}_1$, and $\hat{b}_2$.

presents the adaptively estimated parameters $\hat{b}_0$, $\hat{b}_1$, and $\hat{b}_2$. These results show a stable convergence of the upper bound parameters.

In the presence of large uncertainties, the global fast terminal sliding mode control can give large tracking error which is not the case in adaptive global fast terminal sliding mode control. Therefore, the introduction of adaptive control technique can adapt to the changes of external environment and control structure parameters, which maintains the optimal performance of the control system. In addition, adaptive technology can effectively solve the robustness problem in the presence of unknown model parameters and external disturbances, so as to improve the MEMS gyroscope's reliability and disturbance rejection ability.

6. Conclusion

In this paper, we proposed the design of a new adaptive global fast terminal sliding mode controller for trajectory tracking control of MEMS vibratory gyroscopes, which enables the robustness in the presence of parameter uncertainties and the external disturbances. The main feature of this design is that it combines the global fast terminal sliding mode control with a boundary layer and the adaptive approach.

Based on the Lyapunov's stability theory, the system state with the designed global fast terminal sliding mode controller is able to converge to equilibrium point in a shorter finite time. In order to reduce the chattering, we present a softening control approach. Considering the unknown upper bound of system uncertainties, an adaptive global fast terminal sliding mode controller with unpredictably upper bound of parameter estimation is derived. The adaptive algorithm is used to estimate the bounds of uncertainties and external disturbances. Simulation results are provided to demonstrate the validity and reliability of the proposed control scheme.

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