Research Article

Fault Detection for Nonlinear Impulsive Switched Systems

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Received 3 April 2013; Revised 12 May 2013; Accepted 12 May 2013

Academic Editor: Xiaojie Su

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This paper is concerned with the fault detection problem for nonlinear impulsive switched systems. Fault detection filters are designed such that the augmented systems are stable, and the residual signal generated by the filters achieves the H_∞ -gain for disturbances and guarantees the H_- performance for faults. Sufficient conditions for the solvability of this problem are formulated in terms of linear matrix inequalities; furthermore, the filter gains are characterized by a convex optimization problem. A simulation on a continuous stirred tank reactor control system is given to demonstrate the effectiveness of the proposed methods.

1. Introduction

Switched systems are an important class of hybrid systems, which consist of a family of continuous-time or discrete-time subsystems and a switching law that specifies the switching between them. Study on this class of systems has attracted much attention in recent years for its theoretical significance [1–6] and engineering applications [7–9]. However, in a wide range of actual systems such as engineering, economics, and biology, there usually exist some impulses when the switched system is switching among subsystems. These special switched systems are called impulsive switched systems. As typical switched impulsive system, the circuit switching usually causes system state to change abruptly in some circuit systems [10]. The abrupt state at the time instant of switching could often lead to oscillations and instability and the lead poor performances. Recently, numbers of papers have focused on stability problem of switched impulsive systems [11–13].

On the other hand, fault detection (FD) is an important topic in system engineering from the viewpoint of the higher demands for safety and reliability of control systems [14–17]. Among these model-based approaches, the class of procedures are that observers or filters are firstly designed to construct a residual signal which is used to generate an alarm when the residual evaluation function has larger than the threshold. During the past decades, Many results,

using robust H_∞ technique [18–21] and H_∞/H_- technique, investigate this important issue [22, 23].

However, the problem of FD design in switched systems schemes is still in the early stage of development and a few results have been reported in the literatures [24–28]. It is worth noticing that all the aforementioned FD approaches for switched systems do not include impulsive switched systems. Moreover, since the FD approaches for switched systems without impulsive increments are not appropriate for switched systems with impulsive increments, the new FD technique need to solve the impulsive case. Because these impulsive switched systems exist widely, the FD problem for a general class of nonlinear impulsive switched systems is significant, both theoretically and practically.

In this paper, the problem of FD filters for a general class of nonlinear impulsive switched systems is presented. Different from [10, 29] which only involve linear impulsive switched systems with linear impulsive increments, nonlinear impulsive switched systems with nonlinear impulsive increments have been considered in this paper. The main contributions of this paper include (i) the H_- performance of nonlinear impulsive switched systems to directly reflect the effect on the residual signal from faults and obtain the better detection results and (ii) the smaller conservatism of sufficient conditions for the H_- performance and the H_∞ performance. Finally, the filters gains are characterized in terms of the solution of a convex optimization problem.

The paper is organized as follows. Section 2 introduces the problem under consideration and presents the design objectives. Section 3 illustrates the FD filter design approach in detail. The thresholds are given in Section 4. Two examples are given in Section 5 to demonstrate the proposed method. Conclusions of this paper are given in the last section.

Notation. For a matrix A , A^T denotes its transpose. For a symmetric matrix, $A > 0$ ($A \geq 0$) and $A < 0$ ($A \leq 0$) denote positive definiteness (positive semidefinite matrix) and negative definiteness (negative semidefinite matrix), respectively. The Hermitian part of a square matrix M is denoted by $\text{He}(M) := M + M^T$. The symbol $*$ within a matrix represents the symmetric entries.

2. Problem Formulation

2.1. System Model. Consider the following nonlinear impulsive switched systems:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)1}d(t) + B_{\sigma(t)2}f(t) \\ &\quad + Y_{\sigma(t)}(t, x(t)), \quad t \neq t_k, \\ \Delta x(t) &= H_{\sigma(t)}x(t) + \Omega_{\sigma(t)}x(t)(t, x(t)), \quad t = t_k, \quad (1) \\ y(t) &= C_{\sigma(t)}x(t) + D_{\sigma(t)1}d(t) + D_{\sigma(t)2}f(t), \\ x(t_0^+) &= x_0, \end{aligned}$$

where $x(t) \in R^n$ is the state, $y(t) \in R^m$ is the measured output, and $d(t) \in R^{n_d}$ and $f(t) \in R^{n_f}$ are the disturbance input and the fault input, respectively. It is assumed that both the disturbance input and the fault input are energy bounded; then it is demanded that they belong to $L_2[0, \infty)$. t_k , $k = 0, 1, 2, \dots$, are impulsive switching time points satisfying $0 = t_0 < t_1 < \dots < t_k < \dots < t_\infty = \infty$. The piecewise constant function $\sigma(t)$ is a switching rule which takes its values in the finite set $\mathcal{N} = \{1 \dots N\}$ ($N > 1$ is the number of subsystems). The index $\sigma(t)$ denotes the sequence of the active subsystem; that is, when $\sigma(t) = j$, the j th subsystems is activated at the time point $t = t_k$. $Y_{\sigma(t)}(t, x(t)) : [t_0, \infty) \times R^n \rightarrow R^n$, which is globally the Lipschitz continuous, and $Y_{\sigma(t)}(t, 0) \equiv 0$ for all $t \in [t_0, \infty)$. $\Delta x(t_k) = x(t_k^+) - x(t_k^-) = x(t_k^+) - x(t_k)$, with $x(t_k^+) = \lim_{t \rightarrow t_k^+} x(t)$ and $x(t_k^-) = \lim_{t \rightarrow t_k^-} x(t)$; that is, the solution $x(t)$ is left continuous. $\Omega_{\sigma(t)}(t, x(t)) : [t_0, \infty) \times R^n \rightarrow R^n$ is nonlinear function, and $\Omega_{\sigma(t)}(t, 0) \equiv 0$ for all $t \in [t_0, \infty)$. The j th subsystem is denoted by the matrices A_j , B_{j1} , B_{j2} , C_j , D_{j1} , and D_{j2} with appropriate dimensions.

We introduce the following assumptions for nonlinear impulsive switched systems (1).

Assumption 1. There exist nonnegative scalars $g_j > 0$, $j \in \mathcal{N}$, such that

$$Y_{\sigma(t)}^T(t, x(t)) Y_{\sigma(t)}(t, x(t)) \leq g_j x^T(t) x(t). \quad (2)$$

Assumption 2. Denote $\rho_{\sigma(t)}(\cdot)$ as the spectral radius for each subsystem. $\|\Omega_{\sigma(t)}x(t)(t, x(t))\| \leq \rho_{\sigma(t)}(H_{\sigma(t)} + 1)\|x(t)\|$ for $t \in [t_0, \infty)$.

For the purpose of the fault detection, the following FD filters are designed:

$$\begin{aligned} \dot{x}_f(t) &= A_{f\sigma(t)}x_f(t) + B_{f\sigma(t)}y(t), \\ r(t) &= C_{f\sigma(t)}x_f(t) + D_{f\sigma(t)}y(t), \end{aligned} \quad (3)$$

where $x_f(t)$ is the state of the filter and $r(t)$ is the residual signal. The matrices $A_{f\sigma(t)}$, $B_{f\sigma(t)}$, $C_{f\sigma(t)}$, and $D_{f\sigma(t)}$ with appropriate dimensions are to be determined.

Combining (1) and (3), the nonlinear impulsive switched system can be written as

$$\begin{aligned} \dot{\tilde{x}}(t) &= \mathcal{A}_{\sigma(t)}\tilde{x}(t) + \mathcal{B}_{\sigma(t)1}d(t) + \mathcal{B}_{\sigma(t)2}f(t) \\ &\quad + \mathcal{Q}_{\sigma(t)}(t, \tilde{x}(t)), \quad t \neq t_k, \\ \Delta \tilde{x}(t) &= \mathcal{H}_{\sigma(t)}\tilde{x}(t) + \mathcal{O}_{\sigma(t)}(t, \tilde{x}(t)), \quad t = t_k, \\ r(t) &= \mathcal{C}_{\sigma}\tilde{x}(t) + \mathcal{D}_{\sigma 1}d(t) + \mathcal{D}_{\sigma 2}f(t), \end{aligned} \quad (4)$$

where

$$\begin{aligned} \tilde{x}(t) &= \begin{bmatrix} x^T(t) & x_f^T(t) \end{bmatrix}^T, \\ \Delta \tilde{x}(t_k) &= \tilde{x}(t_k^+) - \tilde{x}(t_k^-) = \tilde{x}(t_k^+) - \tilde{x}(t_k), \\ \mathcal{H}_{\sigma(t)} &= \begin{bmatrix} H_{\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{O}_j(t, \tilde{x}(t)) = \begin{bmatrix} \Omega_j(t, x(t)) \\ 0 \end{bmatrix} \end{aligned} \quad (5)$$

and when $\sigma(t) = j$,

$$\begin{aligned} &\left[\begin{array}{c|c|c|c} \mathcal{A}_j & \mathcal{B}_{j1} & \mathcal{B}_{j2} & \mathcal{Q}_j(t, \tilde{x}(t)) \\ \hline \mathcal{C}_j & \mathcal{D}_{j1} & \mathcal{D}_{j2} & 0 \end{array} \right] \\ &= \left[\begin{array}{c|c|c|c} A_j & 0 & B_{j1} & B_{j2} \\ B_{fj}C_j & A_{fj} & B_{fj}D_{j1} & B_{fj}D_{j2} \\ \hline D_{fj}C_j & C_{fj} & D_{fj}D_{j1} & D_{fj}D_{j2} \end{array} \middle| \begin{array}{c} Y_j(t, x(t)) \\ 0 \\ 0 \end{array} \right]. \end{aligned} \quad (6)$$

To present the purpose of this paper more precisely, the following definition is introduced.

Definition 3. Let Assumptions 1 and 2 be satisfied and $d(t) = 0$. Nonlinear impulsive switched system (4) under zero initial conditions is said to be stable with the H_- -gain β , if the condition holds that

$$\int_{t_0}^{\infty} \|r(s)\|^2 ds \geq \beta^2 \int_{t_0}^{\infty} \|f(s)\|^2 ds. \quad (7)$$

2.2. Problem Formulation. The design problem of the fault detection filters to be addressed in this paper can be expressed as follows.

The frameworks of FD filter design: given nonlinear impulsive switched system (1), the FD filters (3) are designed such that nonlinear impulsive switched system (4) is stable and the fault effects on the residual signal are maximized, while the disturbance effects on the residual signals are

minimized. Our design objective of the FD filters can now be formulated as the following performances:

$$\int_{t_0}^{\infty} \|r(s)\|^2 ds \geq \beta^2 \int_{t_0}^{\infty} \|f(s)\|^2 ds, \quad (8)$$

$$\int_{t_0}^{\infty} \|r(s)\|^2 ds \leq \gamma^2 \int_{t_0}^{\infty} \|d(s)\|^2 ds. \quad (9)$$

Remark 4. Condition (8) is expressed to maximize the effects of the fault $f(t)$ on the residual output $r(t)$ for impulsive switched system (4). That is, the residual output $r(t)$ is sensitive for the fault $f(t)$. Condition (9) is used for the disturbance attenuation performance, which minimizes the disturbance effects on the all residual outputs and ensures that the disturbances are not disastrous.

3. The Fault Detection Filter Design

Before beginning this section, the following lemmas are needed to present our main results.

Lemma 5 (see [30]). *Let $\epsilon > 0$ be a given scalar and $\Xi \in R^{p \times q}$ be a matrix such that $\Xi^T \Xi \leq I$, where I is an identity matrix with appropriate dimension. Then $2x^T \Xi y \leq \epsilon x^T x + \epsilon^{-1} y^T y$ for all $x \in R^p$ and $y \in R^q$.*

Lemma 6 (see [31]). *Let $P \in R^{n \times n}$ be a given symmetric positive definite matrix and let $Q \in R^{n \times n}$ be a given symmetric matrix. Then*

$$\begin{aligned} \lambda_{\min} \{P^{-1}Q\} x(t)^T P x(t) &\leq x(t)^T Q x(t) \\ &\leq \lambda_{\max} \{P^{-1}Q\} x(t)^T P x(t) \end{aligned} \quad (10)$$

for all $x(t) \in R^n$, while $\lambda_{\max}\{\cdot\}$ and $\lambda_{\min}\{\cdot\}$ denote, respectively, the largest and the smallest eigenvalues of the matrix inside the brackets.

In this section, sufficient conditions on the existence of the FD filters for nonlinear impulsive switched systems (4) would be given, and the desired filters can be obtained.

3.1. The Fault Sensitiveness Performance (8). Considering nonlinear impulsive switched system (4) with $d(t) = 0$, we have

$$\begin{aligned} \dot{\tilde{x}}(t) &= \mathcal{A}_{\sigma(t)} \tilde{x}(t) + \mathcal{B}_{\sigma(t)} f(t) \\ &\quad + \mathcal{Q}_{\sigma(t)}(t, \tilde{x}(t)), \quad t \neq t_k, \\ \Delta \tilde{x} &= \mathcal{H}_{\sigma(t)} \tilde{x}(t) + \mathcal{O}_{\sigma(t)}(t, \tilde{x}(t)), \quad t = t_k, \\ r(t) &= \mathcal{C}_{\sigma(t)} \tilde{x}(t) + \mathcal{D}_{\sigma(t)} f(t); \end{aligned} \quad (11)$$

H_- performance for nonlinear impulsive switched system (11) is given.

Lemma 7. *Let α_j, ϵ be constants satisfying $0 < \alpha_j < 1, \epsilon > 0$, $j \in \mathcal{N}$, and Assumptions 1 and 2 hold. Furthermore, suppose*

that nonlinear impulsive switched system (11) switches from i th subsystem to j th subsystem as switched time point t_k . If there exists the Lyapunov functions candidate $V_{\sigma(t)}(t) = V_j(t) = \tilde{x}(t)^T \mathcal{P}_j \tilde{x}(t)$, $j \in \mathcal{N}$, satisfying the following inequalities

$$\begin{bmatrix} \mathcal{A}_j^T \mathcal{P}_j + \mathcal{P}_j \mathcal{A}_j + \epsilon^{-1} \mathcal{P}_j^2 + \alpha_j \mathcal{P}_j + \epsilon g_j I - \mathcal{C}_j^T \mathcal{C}_j & \mathcal{P}_j \mathcal{B}_{j2} - \mathcal{C}_j^T \mathcal{D}_{j2} \\ \mathcal{B}_{j2}^T \mathcal{P}_j - \mathcal{D}_{j2}^T \mathcal{C}_j & \beta^2 I - \mathcal{D}_{j2}^T \mathcal{D}_{j2} \end{bmatrix} < 0, \quad j \in \mathcal{N}, \quad (12)$$

then nonlinear impulsive switched system (11) is stable with H_- gain β for any switching signal satisfying

$$\ln(\mu_{ij}|_k) - \alpha_i(t_k - t_{k-1}) < 0, \quad (13)$$

$$i \in \mathcal{N}, \quad j \in \mathcal{N}, \quad i \neq j, \quad k = 1, 2, \dots,$$

where $\mu_{ij}|_k = ((\delta + 1)\lambda_{\max}\{(\mathcal{H}_j + I)^T \mathcal{P}_j(\mathcal{H}_j + I)\} + (\delta^{-1} + 1)\rho_j^2 \lambda_{\max}\{\mathcal{P}_j\})/\lambda_{\min}\{\mathcal{P}_i\}$, $\delta > 0$.

Proof. When $t \in (t_k, t_{k+1}]$ and j th subsystem is activated, evaluating the time derivative of $V_j(t)$ along the trajectory of nonlinear impulsive switched system (11) gives

$$\begin{aligned} \dot{V}_j(t) &= \tilde{x}^T(t) (\mathcal{A}_j^T \mathcal{P}_j + \mathcal{P}_j \mathcal{A}_j) \tilde{x}(t) + \tilde{x}^T(t) \mathcal{P}_j \mathcal{B}_{j2} f(t) \\ &\quad + f(t)^T \mathcal{B}_{j2}^T \mathcal{P}_j \tilde{x}(t) + \mathcal{Q}_j(t, \tilde{x}(t))^T \mathcal{P}_j \tilde{x}(t) \\ &\quad + \tilde{x}^T(t) \mathcal{P}_j \mathcal{Q}_j(t, \tilde{x}(t)). \end{aligned} \quad (14)$$

By Lemma 5, it is clear that

$$\begin{aligned} 2\mathcal{Q}_j(t, \tilde{x}(t))^T \mathcal{P}_j \tilde{x}(t) &\leq \epsilon \mathcal{Q}_j(t, \tilde{x}(t))^T \mathcal{Q}_j(t, \tilde{x}(t)) \\ &\quad + \epsilon^{-1} \tilde{x}^T(t) \mathcal{P}_j^2 \tilde{x}(t). \end{aligned} \quad (15)$$

Therefore, when assuming the zero input (i.e., $f(t) = 0$) and using Assumptions 1, we have the following condition from (14) and (15):

$$\dot{V}_j|_{f(t)=0}(t) \leq \tilde{x}^T(t) \mathcal{Q}_j \tilde{x}(t), \quad (16)$$

where $\mathcal{Q}_j = \mathcal{A}_j^T \mathcal{P}_j + \mathcal{P}_j \mathcal{A}_j + \epsilon g_j I + \epsilon^{-1} \mathcal{P}_j^2$. If (12) holds, then $\mathcal{Q}_j + \alpha_j \mathcal{P}_j \leq 0$, which implies that $\mathcal{Q}_j \leq -\alpha_j \mathcal{P}_j \leq 0$. Thus, from (16), we have $\dot{V}_j(t)|_{f(t)=0}(t) \leq -\alpha_j V_j(t) \leq 0$. Integrating this inequality gives

$$V_j(t) \leq e^{-\alpha_j(t-t_k)} V_j(t_k^+), \quad t \in (t_k, t_{k+1}]. \quad (17)$$

At the impulsive switching time point t_k , it has

$$\begin{aligned} V_j(t_k^+) &= (\Delta \tilde{x}(t_k) + \tilde{x}(t_k))^T \mathcal{P}_j (\Delta \tilde{x}(t_k) + \tilde{x}(t_k)) \\ &= [(\mathcal{H}_j + I) \tilde{x}(t_k)]^T \mathcal{P}_j [(\mathcal{H}_j + I) \tilde{x}(t_k)] \\ &\quad + 2\mathcal{O}_j^T(t, \tilde{x}(t)) \mathcal{P}_j [(\mathcal{H}_j + I) \tilde{x}(t_k)] \\ &\quad + \mathcal{O}_j^T(t, \tilde{x}(t)) \mathcal{O}_j(t, \tilde{x}(t)). \end{aligned} \quad (18)$$

By Lemma 6, it has

$$\begin{aligned}
 V_j(t_k^+) &\leq (\delta + 1) \left[(\mathcal{H}_j + I) \tilde{x}(t_k) \right]^T \mathcal{P}_j (\mathcal{H}_j + I) \tilde{x}(t_k) \\
 &\quad + (\delta^{-1} + 1) \mathcal{O}_j^T(t, \tilde{x}(t)) \mathcal{P}_j \mathcal{O}_j(t, \tilde{x}(t)) \\
 &\leq \left\{ (\delta + 1) \lambda_{\max} \left\{ (\mathcal{H}_j + I)^T \mathcal{P}_j (\mathcal{H}_j + I) \right\} \right. \\
 &\quad \left. + (\delta^{-1} + 1) \rho_j^2 \lambda_{\max} \left\{ \mathcal{P}_j \right\} \right\} \tilde{x}^T(t_k) \tilde{x}(t_k) \\
 &\leq \mu_{ij}|_k V_i(t_k),
 \end{aligned} \quad (19)$$

where $\mu_{ij}|_k = ((\delta + 1) \lambda_{\max} \{ (\mathcal{H}_j + I)^T \mathcal{P}_j (\mathcal{H}_j + I) \} + (\delta^{-1} + 1) \rho_j^2 \lambda_{\max} \{ \mathcal{P}_j \}) / \lambda_{\min} \{ \mathcal{P}_i \}$, $\delta > 0$. Therefore, from (17) and (19), we have

$$\begin{aligned}
 V_j(t) &\leq e^{-\alpha_j(t-t_k)} V_j(t_k^+) \\
 &\leq \mu_{ij}|_k e^{-\alpha_j(t-t_k)} V_i(t_k) \\
 &\leq \mu_{ij}|_k e^{-\alpha_j(t-t_k)} e^{-\alpha_i(t_k-t_{k-1})} V_i(t_{k-1}^+) \\
 &\leq \dots \leq \\
 &\leq \left(\prod_{\substack{l=1, q \neq p \\ q \in \mathcal{N}, p \in \mathcal{N}}}^k \mu_{qp}|_l e^{-\alpha_q(t_l-t_{l-1})} \right) e^{-\alpha_j(t-t_k)} \\
 &\quad \times V_{\sigma(t_0)}(t_0),
 \end{aligned} \quad (20)$$

where $\mu_{qp}|_l$ means the l th switching impulsive value from q th subsystem to p th subsystems, and α_q denotes the decay rate of the Lyapunov function for q th subsystem. Since (13) holds; that is, there exists $\mu_{qp}|_l > 0$ such that $\ln(\mu_{qp}|_l) - \alpha_q(t_l - t_{l-1}) < 0$, then

$$\begin{aligned}
 g(t - t_0) &= -\alpha_j(t - t_k) + \sum_{\substack{l=1, q \neq p \\ q \in \mathcal{N}, p \in \mathcal{N}}}^k \ln(\mu_{qp}|_l) \\
 &\quad - \alpha_q(t_l - t_{l-1}) < 0.
 \end{aligned} \quad (21)$$

It follows that

$$V_{\sigma(t)}(t) \leq e^{g(t-t_0)} V_{\sigma(t_0)}(t_0). \quad (22)$$

Therefore, we conclude that $V_{\sigma(t)}(t)$ converges to zero as $t \rightarrow \infty$ then nonlinear impulsive switched system (II) with $f(t) = 0$ is stable.

Secondly, establish the H_- performance defined in (7) for nonlinear impulsive switched system (II). Consider the following performance index:

$$J = \int_{t_0}^{\infty} (\beta^2 \|f(s)\|^2 - \|r(s)\|^2) ds. \quad (23)$$

For any nonzero $f(t) \in L_2[t_0, \infty)$ and zero initial condition $\tilde{x}(t_0)$, let $\Gamma_1(t) = \beta^2 f^T(t) f(t) - r^T(t) r(t)$ and

consider the Lyapunov functions as $V_j(t) = \tilde{x}(t)^T \mathcal{P}_j \tilde{x}(t)$. For j th subsystem, it has

$$V_j(\tilde{x}(t)) + \alpha_j V_j(\tilde{x}(t)) + \Gamma_1(t) \leq \mathcal{X}_1^T \mathcal{R}_{j1} \mathcal{X}_1, \quad j \in \mathcal{N}, \quad (24)$$

where

$$\mathcal{X}_1 = [\tilde{x}(t)^T \quad f(t)^T]^T,$$

$\mathcal{R}_{j1}$

$$= \begin{bmatrix} \mathcal{A}_j^T \mathcal{P}_j + \mathcal{P}_j \mathcal{A}_j + \varepsilon^{-1} \mathcal{P}_j^2 + \alpha_j \mathcal{P}_j + \varepsilon g_j I - \mathcal{C}_j^T \mathcal{C}_j & \mathcal{P}_j \mathcal{B}_{j2} - \mathcal{C}_j^T \mathcal{D}_{j2} \\ \mathcal{B}_{j2}^T \mathcal{P}_j - \mathcal{D}_{j2}^T \mathcal{C}_j & \beta^2 I - \mathcal{D}_{j2}^T \mathcal{D}_{j2} \end{bmatrix}. \quad (25)$$

Equation (12) is equivalent to $\mathcal{R}_{j1} < 0$. Then, it has $V_j(\tilde{x}(t)) + \alpha_j V_j(\tilde{x}(t)) + \Gamma_1(t) \leq 0$. By iteration operation on the above inequality for $t \in (t_k, t_{k+1}]$, we have

$$V_j(t) \leq e^{-\alpha_j(t-t_k)} V_j(t_k^+) - \int_{t_k}^t \Gamma_1(s) ds. \quad (26)$$

Along the same lines as the impulsive switching time point t_k , it has from (19) and (26) that

$$\begin{aligned}
 V_j(t) &\leq e^{-\alpha_j(t-t_k)} V_j(t_k^+) - \int_{t_k}^t \Gamma_1(s) ds \\
 &\leq \mu_{ij}|_k e^{-\alpha_j(t-t_k)} V_i(t_k) - \int_{t_k}^t \Gamma_1(s) ds \\
 &\leq \mu_{ij}|_k e^{-\alpha_j(t-t_{k-1})} V_i(t_{k-1}^+) - \mu_{ij}|_k e^{-\alpha_j(t-t_k)} \\
 &\quad \times \int_{t_{k-1}}^{t_k} \Gamma_1(s) ds - \int_{t_k}^t \Gamma_1(s) ds \\
 &\leq \dots \leq \\
 &\leq e^{g(t-t_0)} V_{\sigma(t_0)}(t_0) - \int_{t_0}^t e^{g(t-s)} \Gamma_1(s) ds,
 \end{aligned} \quad (27)$$

where $g(t - t_0)$ and $g(t - s)$ are defined as (21).

Under the zero initial condition, (27) implies $\int_{t_0}^t e^{g(t-s)} \Gamma_1(s) ds < 0$. Because $0 < g(t - s) < 1$, $s \in [t_0, t]$, it has $\int_{t_0}^t \Gamma_1(s) ds < \int_{t_0}^t e^{g(t-s)} \Gamma_1(s) ds < 0$; that is,

$$\beta^2 \int_{t_0}^t f^T(s) f(s) ds \leq \int_{t_0}^t r^T(s) r(s) ds. \quad (28)$$

When $t \rightarrow \infty$, nonlinear impulsive switched system (II) has H_- performance, which completes the proof. $\square$

Remark 8. For switched systems, the Lyapunov function is discontinuous at the time instant of switching; that is to say, this Lyapunov function does not have its derivative in the whole time domain. Then the piecewise Lyapunov function was utilized in Lemma 7. Each subsystem has one corresponding Lyapunov function, and each Lyapunov function has its derivative in the corresponding subsystem time domain.

Based on Lemma 7, the following theorem is given to obtain the fault sensitiveness performance conditions in terms of linear matrix inequalities.

Theorem 9. Let α_j , β , and ε be constants satisfying $0 < \alpha_j < 1$, $\beta > 0$, $\varepsilon > 0$ and Assumptions 1 and 2 hold. If there exist matrix

variables Y_{aj} , X_{aj} , M_{aj} , S_{aj} , Z_{aj} , N_j , R_j , $\widehat{A}_{fj}$, $\widehat{B}_{fj}$, $\widehat{C}_{fj}$, $\widehat{D}_{fj}$, and symmetric positive-definite matrices

$$\mathcal{P}_{aj} = \begin{bmatrix} P_{aj1} & P_{aj2} \\ * & P_{aj3} \end{bmatrix} > 0, \quad j \in \mathcal{N}, \quad (29)$$

satisfying the following inequalities

$$\begin{bmatrix} \Theta_{a1} & \Theta_{a2} & \Theta_{a3} & 0 & 0 \\ * & \Theta_{a4} & \Theta_{a5} & \mathcal{P}_{aj} & 0 \\ * & * & \Theta_{a6} & 0 & R_j^T \\ * & * & * & -\varepsilon^{-1}I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad j \in \mathcal{N}, \quad (30)$$

where

$$\begin{aligned} \Theta_{a1} &= \begin{bmatrix} -He(X_{aj}) & -Y_{aj} - a_1 N_j^T \\ * & -He(a_2 N_j) \end{bmatrix}, \\ \Theta_{a3} &= \begin{bmatrix} -Z_{aj} + X_{aj}^T B_{j2} + a_1 \widehat{B}_{fj} D_{j2} \\ -a_5 N_j v_j + Y_{aj}^T B_{j2} + a_2 \widehat{B}_{fj} D_{j2} \end{bmatrix}, \\ \Theta_{a2} &= \begin{bmatrix} P_{aj1} - M_{aj} + X_{aj}^T A_j + a_1 \widehat{B}_{fj} C_j & P_{aj2} - S_{aj} + a_1 \widehat{A}_{fj} \\ P_{aj2}^T - a_3 N_j + Y_{aj}^T A_j + a_2 \widehat{B}_{fj} C_j & P_{aj3} - a_4 N_j + a_2 \widehat{A}_{fj} \end{bmatrix}, \\ \Theta_{a4} &= \begin{bmatrix} \alpha_j P_{aj1} + \varepsilon g_j I + He(M_{aj}^T A_j + a_3 \widehat{B}_{fj} C_j) & \alpha_j P_{aj2} + a_4 \widehat{A}_{fj} + A_j^T S_{aj} + a_4 C_j^T \widehat{B}_{fj}^T \\ * & \alpha_j P_{aj3} + \varepsilon g_j I + He(a_4 \widehat{A}_{fj}) \end{bmatrix}, \\ \Theta_{a5} &= \begin{bmatrix} M_{aj}^T B_{j2} + a_3 \widehat{B}_{fj} D_{j2} + A_j^T Z_{aj} + a_5 C_j^T \widehat{B}_{fj}^T v_j - C_j^T \widehat{D}_{fj}^T \\ S_{aj}^T B_{j2} + a_4 \widehat{B}_{fj} D_{j2} + a_5 \widehat{A}_{fj}^T v_j - \widehat{C}_{fj}^T \end{bmatrix}, \\ \Theta_{a6} &= \beta^2 I + He(B_{j2}^T Z_{aj} + a_5 D_{j2}^T \widehat{B}_{fj}^T v_j - \widehat{D}_{fj} D_{j2}), \end{aligned} \quad (31)$$

then, for any switching signal satisfying (13), nonlinear impulsive switched system (11) is stable and guarantees the H_- performance (8). Moreover, if (30) is feasible, then the FD filter gains in form of (3) can be given by

$$\begin{bmatrix} A_{fj} & B_{fj} \\ C_{fj} & D_{fj} \end{bmatrix} = \begin{bmatrix} N_j & 0 \\ 0 & R_j \end{bmatrix}^{-T} \begin{bmatrix} \widehat{A}_{fj} & \widehat{B}_{fj} \\ \widehat{C}_{fj} & \widehat{D}_{fj} \end{bmatrix}. \quad (32)$$

Proof. Now, to establish the convex condition, (12) can be rewritten as

$$\mathcal{U}^T \mathcal{M}_{aj} \mathcal{U} < 0, \quad (33)$$

where

$$\mathcal{U} = \begin{bmatrix} \mathcal{A}_j & \mathcal{B}_{j2} \\ I & 0 \\ 0 & I \end{bmatrix},$$

$\mathcal{M}_{aj}$

$$= \begin{bmatrix} 0 & \mathcal{P}_{aj} & 0 \\ \mathcal{P}_{aj} & \alpha_j \mathcal{P}_{aj} + \varepsilon^{-1} \mathcal{P}_{aj}^2 + \varepsilon g_j I - \mathcal{C}_j^T \mathcal{C}_j & -\mathcal{C}_j^T \mathcal{D}_{j2} \\ 0 & -\mathcal{D}_{j2}^T \mathcal{C}_j & \beta^2 I - \mathcal{D}_{j2}^T \mathcal{D}_{j2} \end{bmatrix}. \quad (34)$$

On the other hand, we also can have

$$[-I \quad \mathcal{A}_j \quad \mathcal{B}_{j2}] \mathcal{U} = 0. \quad (35)$$

Based on Finsler's Lemma, it has

$$\mathcal{M}_{aj} + He \left(\begin{bmatrix} -I \\ \widehat{A}_j^T \\ \widehat{B}_{j2}^T \end{bmatrix} \mathcal{W}_{aj} \right) < 0, \quad (36)$$

where $\mathcal{W}_{aj}$ introduced by Finsler's Lemma is the matrix variable of appropriate dimensions. Partition $\mathcal{W}_{aj}$ as $\mathcal{W}_{aj} = [\mathcal{W}_{aj1} \ \mathcal{W}_{aj2} \ \mathcal{W}_{aj3}]$, and

$$\mathcal{W}_{aj1} = \begin{bmatrix} X_{aj} & Y_{aj} \\ a_1 N_j & a_2 N_j \end{bmatrix}, \quad \mathcal{W}_{aj2} = \begin{bmatrix} M_{aj} & S_{aj} \\ a_3 N_j & a_4 N_j \end{bmatrix}, \quad (37)$$

$$\mathcal{W}_{aj3} = \begin{bmatrix} Z_{aj} \\ a_5 N_j v_j \end{bmatrix}.$$

Due to $-\mathcal{C}_j^T \mathcal{C}_j$, $-\mathcal{C}_j^T \mathcal{D}_{j2}$ and $-\mathcal{D}_{j2}^T \mathcal{D}_{j2}$ in $\mathcal{M}_{aj}$, then the condition (36) is non-convex. To establish the convex condition, (36) can be rewritten as

$$\overline{\mathcal{U}}^T \overline{\mathcal{M}}_{aj} \overline{\mathcal{U}} < 0, \quad (38)$$

where

$$\overline{\mathcal{U}} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ 0 & \mathcal{C}_j & \mathcal{D}_{j2} \end{bmatrix}, \quad (39)$$

$$\overline{\mathcal{M}}_{aj} = \begin{bmatrix} -\text{He}(\mathcal{W}_{aj1}) & \mathcal{P}_{aj} - \mathcal{W}_{aj2} + \mathcal{W}_{aj1}^T \mathcal{A}_j & -\mathcal{W}_{aj3} + \mathcal{W}_{aj1}^T \mathcal{B}_{j2} & 0 \\ * & \alpha_j \mathcal{P}_{aj} + \varepsilon g_j I + \varepsilon^{-1} \mathcal{P}_{aj}^2 + \text{He}(\mathcal{A}_j^T \mathcal{W}_{aj2}) & \mathcal{A}_j^T \mathcal{W}_{aj3} + \mathcal{W}_{aj2}^T \mathcal{B}_{j2} & 0 \\ * & * & \beta^2 I + \text{He}(\mathcal{B}_{j2}^T \mathcal{W}_{aj3}) & 0 \\ * & * & * & 0 \\ * & * & * & -I \end{bmatrix}.$$

Using Finsler's Lemma again, (38) holds if

$$\overline{\mathcal{M}}_{aj} + \text{He}(\mathcal{R}\mathcal{L}) < 0 \quad (40)$$

holds for any $\mathcal{R}$ with appropriate dimension and $\mathcal{L} = [0, -\mathcal{C}_j, -\mathcal{D}_{j2}, I]$. Let $\mathcal{R} = [0, 0, R_j, 0]^T$ and define $\widehat{A}_{fj} = N_j^T A_{fj}$, $\widehat{B}_{fj} = N_j^T B_{fj}$, $\widehat{C}_{fj} = R_j^T C_{fj}$, $\widehat{D}_{fj} = R_j^T D_{fj}$; then (30) is reached by the Schur complement on (40). Hence if the conditions (30) holds, nonlinear impulsive switched system (11) is stable and guarantees the H_- performance (8), which completes the proof. $\square$

Remark 10. Scalars $a_1 \sim a_5$ are used to reduce conservatism for the same variable N_j in matrix variables $\mathcal{W}_{aj1} \sim \mathcal{W}_{aj3}$. On the other hand, the coefficient matrices v_j are given to adjust the variable dimension. When we set $a_1 \sim a_5$, v_j as the fixed parameters, the conditions in Theorem 9 become convex.

3.2. The Disturbance Attenuation Performance (8). Consider nonlinear impulsive switched system (4) with $f(t) = 0$, we have

$$\dot{\tilde{x}}(t) = \mathcal{A}_{\sigma(t)} \tilde{x}(t) + \mathcal{B}_{\sigma(t)} d(t) + \mathcal{Q}_{\sigma(t)}(t, \tilde{x}(t)), \quad t \neq t_k,$$

$$\Delta \tilde{x} = \mathcal{H}_{\sigma(t)} \tilde{x}(t) + \mathcal{O}_{\sigma(t)}(t, \tilde{x}(t)), \quad t = t_k, \quad (41)$$

$$r(t) = \mathcal{C}_{\sigma(t)} \tilde{x}(t) + \mathcal{D}_{\sigma(t)} d(t);$$

H_∞ performance for nonlinear impulsive switched system (41) is given.

Lemma 11. Let α_j, ε be constants satisfying $0 < \alpha_j < 1$, $\varepsilon > 0$, $j \in \mathcal{N}$, and Assumptions 1 and 2 hold. Furthermore, suppose that nonlinear impulsive switched system (41) switches from i th subsystem to j th subsystem as switched time point t_k . If there exists the Lyapunov functions candidate $V_{\sigma(t)}(t) = V_j(t) = \tilde{x}(t)^T \mathcal{P}_j \tilde{x}(t)$, $j \in \mathcal{N}$, satisfying

$$\begin{bmatrix} \mathcal{A}_j^T \mathcal{P}_j + \mathcal{P}_j \mathcal{A}_j + \varepsilon^{-1} \mathcal{P}_j^2 + \alpha_j \mathcal{P}_j + \varepsilon g_j I + \mathcal{C}_j^T \mathcal{C}_j & \mathcal{P}_j \mathcal{B}_{j1} + \mathcal{C}_j^T \mathcal{D}_{j1} \\ \mathcal{B}_{j1}^T \mathcal{P}_j + \mathcal{D}_{j1}^T \mathcal{C}_j & -\gamma^2 I + \mathcal{D}_{j1}^T \mathcal{D}_{j1} \end{bmatrix} < 0, \quad j \in \mathcal{N}, \quad (42)$$

then nonlinear impulsive switched system (41) is stable with H_∞ -gain γ for any switching signal satisfying (13).

Proof. Following the same lines as those for Lemma 7, nonlinear impulsive switched system (41) is stable, for any nonzero $d(t) \in l_2[0, \infty)$ and zero initial condition $\tilde{x}(t_0)$. Let $\Gamma_2(t) = r^T(t)r(t) - \gamma^2 d^T(t)d(t)$ and consider the Lyapunov function as Lemma 7; we can have

$$V_j(\tilde{x}(t)) + \alpha_j V_j(\tilde{x}(t)) + \Gamma_2(t) \leq \mathcal{X}_2^T \mathcal{R}_{j2} \mathcal{X}_2, \quad j \in \mathcal{N}, \quad (43)$$

where

$$\mathcal{X}_2 = [\tilde{x}(t)^T \ d(t)^T]^T,$$

$$\mathcal{R}_{j2} = \begin{bmatrix} \mathcal{A}_j^T \mathcal{P}_j + \mathcal{P}_j \mathcal{A}_j + \varepsilon^{-1} \mathcal{P}_j^2 + \alpha_j \mathcal{P}_j + \varepsilon g_j I + \mathcal{C}_j^T \mathcal{C}_j & \mathcal{P}_j \mathcal{B}_{j1} + \mathcal{C}_j^T \mathcal{D}_{j1} \\ \mathcal{B}_{j1}^T \mathcal{P}_j + \mathcal{D}_{j1}^T \mathcal{C}_j & -\gamma^2 I + \mathcal{D}_{j1}^T \mathcal{D}_{j1} \end{bmatrix}. \quad (44)$$

Equation (42) is equivalent to $\mathcal{R}_{j2} < 0$. Thus, it has

$$V_j(\tilde{x}(t)) + \alpha_j V_j(\tilde{x}(t)) + \Gamma_2(t) \leq 0. \quad (45)$$

By iteration operation on the above inequality for $t \in (t_k, t_{k+1}]$, we have $V_j(t) \leq e^{-\alpha_j(t-t_k)} V_j(t_k^+) - \int_{t_k}^t \Gamma_2(s) ds$. As the same lines as those for Lemma 7, it has

$$\begin{aligned} V_j(t) &\leq e^{-\alpha_j(t-t_k)} V_j(t_k^+) - \int_{t_k}^t \Gamma_2(s) ds \\ &\leq \dots \leq \\ &\leq e^{g(t-t_0)} V_{\sigma(t_0)}(t_0) - \int_{t_0}^t e^{g(t-s)} \Gamma_2(s) ds \\ &\leq e^{g(t-t_0)} V_{\sigma(t_0)}(t_0) - \int_{t_0}^t \Gamma_1(s) ds. \end{aligned} \quad (46)$$

Under the zero initial condition, (46) implies $\int_{t_0}^t e^{g(t-s)} \Gamma_2(s) ds < 0$. Because $0 < g(t-s) < 1$, $s \in [t_0, t)$, it can further have

$$\int_{t_0}^t r^T(s) r(s) ds \leq \gamma^2 \int_{t_0}^t d^T(s) d(s) ds. \quad (47)$$

When $t \rightarrow \infty$, the nonlinear impulsive switched system (41) has H_∞ performance, which completes the proof. $\square$

Based on Lemma 11, the following theorem is given to obtain sufficient conditions by linear matrix inequalities.

Theorem 12. Let α_j , γ , and ε be constants satisfying $0 < \alpha_j < 1$, $\gamma > 0$, and $\varepsilon > 0$ and Assumptions 1 and 2 hold. If there exist matrix variables Y_{bj} , X_{bj} , M_{bj} , S_{bj} , N_j , R_j , $\hat{A}_{fj}$, $\hat{B}_{fj}$, $\hat{C}_{fj}$, and $\hat{D}_{fj}$ and symmetric positive-definite matrices

$$\mathcal{P}_{bj} = \begin{bmatrix} P_{bj1} & P_{bj2} \\ * & P_{bj3} \end{bmatrix} > 0, \quad j \in \mathcal{N}, \quad (48)$$

satisfying the following inequalities

$$\begin{bmatrix} \Theta_{b1} & \Theta_{b2} & \Theta_{b3} & 0 & 0 \\ * & \Theta_{b4} & \Theta_{b5} & \mathcal{P}_{bj} & \Theta_{b6} \\ * & * & -\gamma^2 I & 0 & D_{j1}^T \hat{D}_{fj}^T \\ * & * & * & -\varepsilon^{-1} I & 0 \\ * & * & * & * & I - \text{He}(R_j) \end{bmatrix} < 0, \quad j \in \mathcal{N}, \quad (49)$$

where

$$\begin{aligned} \Theta_{b1} &= \begin{bmatrix} -\text{He}(X_{bj}) & -Y_{bj} - b_1 N_j^T \\ * & -\text{He}(b_2 N_j) \end{bmatrix}, \quad \Theta_{b3} = \begin{bmatrix} X_{bj}^T B_{j1} + b_1 \hat{B}_{fj} D_{j1} \\ Y_{bj}^T B_{j1} + b_2 \hat{B}_{fj} D_{j1} \end{bmatrix}, \\ \Theta_{b2} &= \begin{bmatrix} P_{bj1} - M_{bj} + X_{bj}^T A_j + b_1 \hat{B}_{fj} C_j & P_{bj2} - S_{bj} + b_1 \hat{A}_{fj} \\ P_{bj2}^T - b_3 N_j + Y_{bj}^T A_j + b_2 \hat{B}_{fj} C_j & P_{bj3} - b_4 N_j + b_2 \hat{A}_{fj} \end{bmatrix}, \\ \Theta_{b4} &= \begin{bmatrix} \alpha_j P_{bj1} + \text{He}(M_{bj}^T A_j + b_3 \hat{B}_{fj} C_j) + \varepsilon g_j I & \alpha_j P_{bj2} + b_3 \hat{A}_{fj} + A_j^T S_{bj} + b_4 C_j^T \hat{B}_{fj}^T \\ * & \alpha_j P_{bj3} + \varepsilon g_j I + \text{He}(b_4 \hat{A}_{fj}) \end{bmatrix}, \\ \Theta_{b5} &= \begin{bmatrix} M_{bj}^T B_{j1} + b_3 \hat{B}_{fj} D_{j1} \\ S_{bj}^T B_{j1} + b_4 \hat{B}_{fj} D_{j1} \end{bmatrix}, \quad \Theta_{b6} = \begin{bmatrix} C_j^T \hat{D}_{fj}^T \\ \hat{C}_{fj}^T \end{bmatrix}, \end{aligned} \quad (50)$$

then, for any switching signal satisfying (13), nonlinear impulsive switched system (41) is stable and guarantees the H_∞ performance (9). Moreover, if (49) is feasible, then the FD filter gains in form of (3) can be given by

$$\begin{bmatrix} A_{fj} & B_{fj} \\ C_{fj} & D_{fj} \end{bmatrix} = \begin{bmatrix} N_j & 0 \\ 0 & R_j \end{bmatrix}^{-T} \begin{bmatrix} \hat{A}_{fj} & \hat{B}_{fj} \\ \hat{C}_{fj} & \hat{D}_{fj} \end{bmatrix}. \quad (51)$$

Proof. To establish the convex condition, (42) can be rewritten as follows:

$$\begin{aligned} &\begin{bmatrix} \mathcal{A}_j & \mathcal{B}_{j1} \\ I & 0 \end{bmatrix}^T \begin{bmatrix} 0 & \mathcal{P}_{bj} \\ \mathcal{P}_{bj} & \alpha \mathcal{P}_{bj} \end{bmatrix} \begin{bmatrix} \mathcal{A}_j & \mathcal{B}_{j1} \\ I & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \varepsilon^{-1} \mathcal{P}_{bj}^2 + \varepsilon g_j I + \mathcal{C}_j^T \mathcal{C}_j & \mathcal{C}_j^T \mathcal{D}_{j1} \\ \mathcal{D}_{j1}^T \mathcal{C}_j & -\gamma^2 I + \mathcal{D}_{j1}^T \mathcal{D}_{j1} \end{bmatrix} < 0. \end{aligned} \quad (52)$$

Denote

$$\mathcal{M}_{bj} = \begin{bmatrix} 0 & \mathcal{P}_{bj} & 0 \\ \mathcal{P}_{bj} & \alpha \mathcal{P}_{bj} + \varepsilon^{-1} \mathcal{P}_{bj}^2 + \varepsilon g_j I + \mathcal{E}_j^T \mathcal{E}_j & \mathcal{E}_j^T \mathcal{D}_{j1} \\ 0 & \mathcal{D}_{j1}^T \mathcal{E}_j & -\gamma^2 I + \mathcal{D}_{j1}^T \mathcal{D}_{j1} \end{bmatrix}. \quad (53)$$

From (52), we have

$$\begin{bmatrix} \mathcal{A}_j & \mathcal{B}_{j1} \\ I & 0 \\ 0 & I \end{bmatrix}^T \mathcal{M}_{bj} \begin{bmatrix} \mathcal{A}_j & \mathcal{B}_{j1} \\ I & 0 \\ 0 & I \end{bmatrix} < 0. \quad (54)$$

On the other hand,

$$\begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}^T \mathcal{M}_{bj} \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} = -\gamma^2 I + \mathcal{D}_{j1}^T \mathcal{D}_{j1} < 0. \quad (55)$$

Based on Projection Lemma, it follows from (54) and (55) that

$$\mathcal{M}_{bj} + \text{He} \left(\begin{bmatrix} -I \\ \widetilde{A}_j^T \\ \widetilde{B}_{j1}^T \end{bmatrix} \mathcal{W}_{bj} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \right) < 0, \quad (56)$$

where $\mathcal{W}_{bj}$ introduced by Projection Lemma is the matrix variable of appropriate dimensions. Partition $\mathcal{W}_{bj}$ as $\mathcal{W}_{bj} = [\mathcal{W}_{bj1} \ \mathcal{W}_{bj2}]$. By the Schur complement, (56) is equivalent to

$$\begin{bmatrix} -\text{He}(\mathcal{W}_{bj1}) & \mathcal{P}_{bj} - \mathcal{W}_{bj2} + \mathcal{W}_{bj1}^T \mathcal{A}_j & \mathcal{W}_{bj1}^T \mathcal{B}_{j1} & 0 & 0 \\ * & \alpha_j \mathcal{P}_{bj} + \varepsilon g_j I + \text{He}(\mathcal{A}_j^T \mathcal{W}_{bj2}) & \mathcal{W}_{bj2}^T \mathcal{B}_{j1} & \mathcal{P}_{bj} & \mathcal{E}_j^T \\ * & * & -\gamma^2 I & 0 & \mathcal{D}_{j1}^T \\ * & * & * & -\varepsilon^{-1} I & 0 \\ * & * & * & * & -1 \end{bmatrix} < 0. \quad (57)$$

Then pre- and postmultiply $\text{diag}\{I, I, I, I, R_j^T\}$ to (57); one obtains

$$\begin{bmatrix} -\text{He}(\mathcal{W}_{bj1}) & \mathcal{P}_{bj} - \mathcal{W}_{bj2} + \mathcal{W}_{bj1}^T \mathcal{A}_j & \mathcal{W}_{bj1}^T \mathcal{B}_{j1} & 0 & 0 \\ * & \alpha_j \mathcal{P}_{bj} + \varepsilon g_j I + \text{He}(\mathcal{A}_j^T \mathcal{W}_{bj2}) & \mathcal{W}_{bj2}^T \mathcal{B}_{j1} & \mathcal{P}_{bj} & \mathcal{E}_j^T R_j \\ * & * & -\gamma^2 I & 0 & \mathcal{D}_{j1}^T R_j \\ * & * & * & -\varepsilon^{-1} I & 0 \\ * & * & * & * & -R_j^T R_j \end{bmatrix} < 0. \quad (58)$$

we partition $\mathcal{W}_{bj1}$, $\mathcal{W}_{bj2}$, respectively, as

$$\mathcal{W}_{bj1} = \begin{bmatrix} X_{bj} & Y_{bj} \\ a_1 N_j & a_2 N_j \end{bmatrix}, \quad \mathcal{W}_{bj2} = \begin{bmatrix} M_{bj} & S_{bj} \\ a_3 N_j & a_4 N_j \end{bmatrix} \quad (59)$$

and $b_1 \sim b_4$ are set to be fixed parameters. Define $\widehat{A}_{fj} = N_j^T A_{fj}$, $\widehat{B}_{fj} = N_j^T B_{fj}$, $\widehat{C}_{fj} = R_j^T C_{fj}$, and $\widehat{D}_{fj} = R_j^T D_{fj}$; $-R_j^T R_j \leq I - \text{He}(R_j)$ then (58) becomes (49). Hence if the condition (30) holds, nonlinear impulsive switched system (41) is stable and guarantees the H_∞ performance (8), which completes the proof. $\square$

Remark 13. Scalars $b_1 \sim b_4$ are used to reduce conservatism for the same variable N_j in matrix variables $\mathcal{W}_{bj1}$ and $\mathcal{W}_{bj2}$. When we set $b_1 \sim b_4$ as the fixed parameters, the conditions in Theorem 12 become convex.

3.3. Algorithm. In the previous sections, Theorems 9 and 12 have formulated the inequality conditions for the performances (8)-(9), respectively. Summarily, we have the following algorithm.

It is noted that conditions (30) and (49) are all convex. Hence, the problem of FD filter design can directly translate into the following optimization problem:

$$\begin{aligned} \max \quad & \beta \\ \text{s.t.} \quad & (30) \text{ and } (49), \quad j \in \mathcal{N}. \end{aligned} \quad (60)$$

Remark 14. The H_- performance and H_∞ performance are considered to describe the fault sensitiveness performance and the disturbance attenuation performance, respectively. Due to the multiobjective optimization problem, each performance has some LMI conditions. However, by analyzing the theorems and their proofs, it can be discovered that $\mathcal{W}_{aj1}$, $\mathcal{W}_{aj2}$ in Theorem 9 and $\mathcal{W}_{bj1}$, $\mathcal{W}_{bj2}$ in Theorem 12 can be

defined as the same variable to decrease the computational complexity.

Finally, the gain matrices A_{fj} , B_{fj} , C_{fj} , and D_{fj} of the filters can be derived as

$$\begin{aligned} A_{fj} &= N_j^{-T} \hat{A}_{fj}, & B_{fj} &= N_j^{-T} \hat{B}_{fj}, \\ C_{fj} &= R_j^{-T} \hat{C}_{fj}, & D_{fj} &= R_j^{-T} \hat{D}_{fj}. \end{aligned} \quad (61)$$

4. Thresholds Computation

After the gain matrices of the filters A_{fj} , B_{fj} , C_{fj} , and D_{fj} are designed, similarly to that proposed in [32], the residual evaluation function $J_r(\tau)$ can be chosen as

$$J_r(\tau) = \sqrt{\frac{1}{\tau} \int_{s_l}^{s_k} r^T(s) r(s) ds}, \quad (62)$$

where $J_r(\tau)$ is square value which means the average energy of residual signal over a time interval (s_l, s_k) , s_l denotes the initial evaluation time instant, and s_k stands for the evaluation time.

We propose to use the following threshold:

$$J_{th} = \sup_{\substack{d(t) \in L_2[0, \infty) \\ f(t)=0, j \in \mathcal{N}}} J_r. \quad (63)$$

Consequently, the occurrence of faults can be detected by the following logic rule:

$$\begin{aligned} \|J_r(\tau)\| \leq J_{th} &\implies \text{no alarm} \\ \|J_r(\tau)\| > J_{th} &\implies \text{alarm.} \end{aligned} \quad (64)$$

5. Examples

In this section, we present examples derived from a liquid level control system to illustrate the effectiveness of FD design approach.

Consider a continuous stirred tank reactor (CSTR) control system [33] shown in Figure 1. It consists of a constant-volume CSTR fed by a single inlet stream through a selector valve which is connected to two different source streams.

Assuming constant liquid volume, negligible heat losses, perfectly mixing, and a first-order reaction in reactant A, the dynamical equations of the CSTR at each operating mode are described by

$$\begin{aligned} \dot{C}_A &= \frac{q_i}{V} (C_{Afi} - C_A) - a_0 \exp\left(-\frac{E}{RT}\right) C_A, \\ \dot{T} &= \frac{q_i}{V} (T_{fi} - T) - a_1 \exp\left(-\frac{E}{RT}\right) C_A + a_2 (T_a - T), \end{aligned} \quad (65)$$

FIGURE 1: Schematic diagram of the process.

where C_A is the reactant A concentration (mol/L), q_i is the i th mode's feed flow rate (L/min), C_{Afi} is the i th mode's concentration of the feed stream (mol/L), V is the volume of the reactor (L), E is the activation energy, R is the gas constant (J/(mol·K)), T is the reactor temperature (K), T_{fi} is the i th mode's reactor temperature of the feed stream (K), and T_a is the coolant temperature (K). These nominal values of the parameters are described in [34], and we choose the nominal operating conditions corresponding to an unstable equilibrium point as $T_A^* = 300$ K, $C_A^* = 0.5$ mol/L, and $T^* = 350$ K for both modes.

Since the objective in the example is to testify the FD filter design techniques for the switched system and sustain the main theoretical results, we make design as [33] and the closed-loop system can be obtained with matrices

$$A_{a1} = \begin{bmatrix} 0.5 & 1 \\ -3 & -3 \end{bmatrix}, \quad A_{a2} = \begin{bmatrix} 2 & 1 \\ -8 & -3 \end{bmatrix}. \quad (66)$$

Suppose other system matrices in (1) as

$$\begin{aligned} B_{11} &= \begin{bmatrix} 0.24 \\ -0.10 \end{bmatrix}, & B_{12} &= \begin{bmatrix} 0.95 \\ -1.29 \end{bmatrix}, \\ B_{21} &= \begin{bmatrix} 0.1 \\ -0.31 \end{bmatrix}, & B_{22} &= \begin{bmatrix} -0.52 \\ 1.34 \end{bmatrix}, \\ \Upsilon_1(t, x(t)) &= \begin{bmatrix} \sin(x_1(t)) e^{-0.05t} \\ 0 \end{bmatrix}, \\ \Upsilon_2(t, x(t)) &= \begin{bmatrix} \sin(x_1(t)) e^{-0.05t} \\ \sin(x_2(t)) e^{-0.05t} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
H_1 &= H_2 = - \begin{bmatrix} 0.58 & 0 \\ 0 & 0.58 \end{bmatrix}, \\
\Omega_1(t, x(t)) &= \Omega_2(t, x(t)) = \begin{bmatrix} 0.3 \sin(x_1(t_i)) \\ 0.3 \sin(x_2(t_i)) \end{bmatrix}, \\
C_1 &= C_2 = [1, 1], \quad D_{11} = D_{21} = 0.1, \\
D_{12} &= D_{22} = 0.94.
\end{aligned} \tag{67}$$

Let $q_1 = q_3 = -1$, $q_2 = q_4 = 1$, $q \in \{a, b\}$, $v_j = [1, 1]^T$, $j \in \mathcal{N}$. By giving $\gamma = 1$, $\alpha_j = 0.5$, and $\varepsilon = 0.02$ and obtaining $g_j = 1$, $\rho_j = 0.32$, we solve the convex optimization problem (60) and get the optimal sensitivity performance gain $\beta = 1.0884$. The gain matrices of the fault detection filters and the matrix in the Lyapunov function are

$$\begin{aligned}
\begin{bmatrix} A_{f1} & B_{f1} \\ C_{f1} & D_{f1} \end{bmatrix} &= \begin{bmatrix} -0.9011 & -0.2686 & 1.2677 \\ -0.3316 & -0.7128 & -1.5227 \\ 0.2561 & -0.4982 & 3.2893 \end{bmatrix}, \\
\begin{bmatrix} A_{f2} & B_{f2} \\ C_{f2} & D_{f2} \end{bmatrix} &= \begin{bmatrix} 2.9038 & 1.6812 & -0.6576 \\ -10.1086 & -4.6603 & 1.6406 \\ -1.9562 & -1.3863 & 1.4977 \end{bmatrix}, \\
\mathcal{P}_1 &= \begin{bmatrix} 35.2676 & 24.1501 & -6.6667 & -1.7260 \\ 24.1501 & 22.2633 & -3.1844 & -4.7123 \\ -6.6667 & -3.1844 & 19.7912 & 11.8843 \\ -1.7260 & -4.7123 & 11.8843 & 12.3490 \end{bmatrix}, \\
\mathcal{P}_2 &= \begin{bmatrix} 28.4151 & 10.2415 & -22.8368 & -8.1969 \\ 10.2415 & 4.8161 & -9.0122 & -4.0940 \\ -22.8368 & -9.0122 & 24.4790 & 9.1073 \\ -8.1969 & -4.0940 & 9.1073 & 4.1803 \end{bmatrix}.
\end{aligned} \tag{68}$$

Then by (13), the dwell time for each subsystem is obtained with $\delta = 1$:

$$\begin{aligned}
\text{dwell time for subsystem 1} &\Rightarrow \tau_1 > \frac{\ln(\mu_{12}|_{k_1})}{\alpha_1} = 9.6813, \\
\text{dwell time for subsystem 2} &\Rightarrow \tau_2 > \frac{\ln(\mu_{21}|_{k_2})}{\alpha_2} = 13.8568 \\
&\quad k_1, k_2 \in \{1, 2, \dots\}, \quad k_1 \neq k_2.
\end{aligned} \tag{69}$$

To illustrate the simulation results of the FD objective, the two cases which include the fault for subsystem 1 and

FIGURE 2: Switch signal.

subsystem 2, respectively, are considered. The disturbance is assumed to be $d(t) = 0.8 \cos(0.53t) \exp(-0.05t)$. The switching signal in this paper is shown in Figure 2.

Case 1. The fault for subsystem 1 with the unit amplitude occurs from 100 sec to 200 sec. The generated residual $r(t)$ and the evolution of the residual evaluation function $J_{r(t)}$ are shown in Figure 3. The simulation results show that when the fault for subsystem 1 occurs, the residual signal varies sharply, and $J_{r(t)} > J_{th}$ at 109 sec, which means that the fault for subsystem 1 can be detected 9 sec after the fault of subsystem 1 occurs. Hence, the fault for subsystem 1 can be detected.

Case 2. The fault for subsystem 2 with the unit amplitude also occurs from 200 sec to 300 sec. The generated residual $r(t)$ and the evolution of the residual evaluation function $J_{r(t)}$ are shown in Figure 4. It can be seen that when subsystem 2 are activated at 210 sec, the residual signal is changed sharply and $J_{r(t)} > J_{th}$ at 234 sec. Thus, the fault for subsystem 2 can be detected.

From Cases 1 and 2, we see that both the faults for subsystem 1 and subsystem 2, respectively, can be detected, and they can demonstrate the effectiveness of the proposed design method.

6. Conclusion

In this paper, the problem of FD filter design for nonlinear impulsive switched systems has been investigated. Firstly, the H_- performance and the H_∞ performance are presented for nonlinear impulsive switched systems, and sufficient conditions to characterize given performances have been obtained. Subsequently, the design of FD filters is formulated as a multiobjective optimization problem, and the filter gains are characterized in term of the solution of LMI conditions. Finally, an example has been given to illustrate the effectiveness of the proposed method.

Future works on fault detection for nonlinear impulsive switched systems are to focus on how to cope with stochastic

FIGURE 3: (a) The residual signal. (b) The residual evaluation function and threshold when the fault for subsystem 1 occurs from 100 sec to 200 sec.

FIGURE 4: (a) The residual signal. (b) The residual evaluation function and threshold when the fault for subsystem 2 occurs from 200 sec to 300 sec.

switched systems as in [35, 36] and switched systems with time delay as in [37–39].

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