Research Article

Exact Solutions for Some Fractional Partial Differential Equations by the (G'/G) Method

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We apply the (G'/G) method to seek exact solutions for several fractional partial differential equations including the space-time fractional $(2+1)$ -dimensional dispersive long wave equations, the $(2+1)$ -dimensional space-time fractional Nizhnik-Novikov-Veselov system, and the time fractional fifth-order Sawada-Kotera equation. The fractional derivative is defined in the sense of modified Riemann-Liouville derivative. Based on a certain variable transformation, these fractional partial differential equations are transformed into ordinary differential equations of integer order. With the aid of mathematical software, a variety of exact solutions for them are obtained.

1. Introduction

In recent decades, fractional differential equations have gained much attention as they are widely used to describe various complex phenomena in many fields such as the fluid flow, signal processing, control theory, systems identification, biology, and other areas. Among the investigations for fractional differential equations, research for seeking exact solutions and approximate solutions of fractional differential equations is a hot topic. Many powerful and efficient methods have been proposed so far, for example, the fractional variational iteration method [1–5], the Adomian's decomposition method [6–8], the homotopy perturbation method [9], the finite difference method [10], the finite element method [11], the fractional subequation method [12–14], and so on. Using these methods, solutions with various forms for some given fractional differential equations have been established.

In this paper, we will apply the (G'/G) method [15–21] for solving some fractional partial differential equations in the sense of modified Riemann-Liouville derivative by Jumarie. The definition and some important properties for the Jumarie's modified Riemann-Liouville derivative of order α are listed as follows [22]:

$$D_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \times \int_0^t (t-\xi)^{-\alpha} \times (f(\xi) - f(0)) d\xi, & 0 < \alpha < 1, \\ (f^{(n)}(t))^{(\alpha-n)}, & n \leq \alpha < n+1, n \geq 1, \end{cases}$$

$$D_t^\alpha t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)} t^{r-\alpha}, \quad (1)$$

$$D_t^\alpha (f(t)g(t)) = g(t)D_t^\alpha f(t) + f(t)D_t^\alpha g(t), \quad (2)$$

$$D_t^\alpha f[g(t)] = f'_g[g(t)]D_t^\alpha g(t) = D_g^\alpha f[g(t)](g'(t))^\alpha. \quad (3)$$

The rest of this paper is organized as follows. In Section 2, we give the description of the (G'/G) method for solving fractional partial differential equations. Then in Section 3 we apply this method to establish exact solutions for the space-time fractional $(2+1)$ -dimensional dispersive long wave equations, the space-time fractional $(2+1)$ -dimensional

Nizhnik-Novikov-Veselov system, and the time fractional fifth-order Sawada-Kotera equation. Some conclusions are presented at the end of the paper.

2. Description of the (G'/G) Method for Fractional Partial Differential Equations

In this section we give the description of the (G'/G) method for solving fractional partial differential equations.

Suppose that a fractional partial differential equation, say in the independent variables $t, x_1, x_2, \dots, x_n$, is given by

$$P(u_1, \dots, u_k, D_t^\alpha u_1, \dots, D_t^\alpha u_k, D_{x_1}^\beta u_1, \dots, D_{x_1}^\beta u_k, \dots, D_{x_n}^\gamma u_1, \dots, D_{x_n}^\gamma u_k, \dots) = 0, \quad (4)$$

where $u_i = u_i(t, x_1, x_2, \dots, x_n)$, $i = 1, \dots, k$, are unknown functions and P is a polynomial in u_i and their various partial derivatives including fractional derivatives.

Step 1. Execute a certain variable transformation:

$$u_i(t, x_1, x_2, \dots, x_n) = U_i(\xi), \quad \xi = \xi(t, x_1, x_2, \dots, x_n), \quad (5)$$

such that (4) can be turned into the following ordinary differential equation of integer order with respect to the variable ξ :

$$\tilde{P}(U_1, \dots, U_k, U_1', \dots, U_k', U_1'', \dots, U_k'') = 0. \quad (6)$$

Step 2. Suppose that the solution of (6) can be expressed by a polynomial in (G'/G) as follows:

$$U_j(\xi) = \sum_{i=0}^{m_j} a_{j,i} \left(\frac{G'}{G} \right)^i, \quad j = 1, 2, \dots, k, \quad (7)$$

where $G = G(\xi)$ satisfies the second-order ODE in the form

$$G'' + \lambda G' + \mu G = 0, \quad (8)$$

and $\lambda, \mu, a_{j,i}$, $i = 0, 1, \dots, m_j$, $j = 1, 2, \dots, k$, are constants to be determined later, $a_{j,m} \neq 0$. The positive integer m_j can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (6).

By the generalized solutions of (8), we have

$$\left(\frac{G'}{G} \right) = \begin{cases} -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \\ \times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\ \left. \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right), & \lambda^2 - 4\mu > 0, \\ -\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \\ \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\ \left. \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \right), & \lambda^2 - 4\mu < 0, \\ -\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2 \xi}, & \lambda^2 - 4\mu = 0, \end{cases} \quad (9)$$

where C_1 and C_2 are arbitrary constants.

Step 3. Substituting (7) into (6) and using (8), collecting all terms with the same order of (G'/G) together, the left-hand side of (6) is converted into another polynomial in (G'/G) . Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $\lambda, \mu, a_{j,i}$, $i = 0, 1, \dots, m_j$, $j = 1, 2, \dots, k$.

Step 4. Solving the equations system in Step 3 and using (9), we can construct a variety of exact solutions for (4).

3. Applications of the Method

3.1. Space-Time Fractional (2 + 1)-Dimensional Dispersive Long Wave Equations. We consider the following space-time fractional (2 + 1)-dimensional dispersive long wave equations:

$$\begin{aligned} \frac{\partial^{2\alpha} u}{\partial y^\alpha \partial t^\alpha} + \frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} + \frac{\partial^\alpha (u(\partial^\alpha u / \partial x^\alpha))}{\partial y^\alpha} &= 0, \\ \frac{\partial^\alpha v}{\partial t^\alpha} + \frac{\partial^\alpha u}{\partial x^\alpha} + \frac{\partial^\alpha (uv)}{\partial x^\alpha} + \frac{\partial^{3\alpha} u}{\partial x^{2\alpha} \partial y^\alpha} &= 0, \end{aligned} \quad 0 < \alpha \leq 1, \quad (10)$$

which are a variation of the known (2 + 1)-dimensional dispersive long wave equations [23–36]:

$$\begin{aligned} u_{yt} + v_{xx} + (uu_x)_y &= 0, \\ v_t + u_x + (uv)_x + u_{xxy} &= 0. \end{aligned} \quad (11)$$

Some types of exact solutions for (11) have been obtained in [23–36] by the use of various methods including the Riccati subequation method [23, 24, 29], the nonlinear transformation method [25], Jacobi function method [27, 28, 36], (G'/G) -expansion method [26], modified CK's direct method [30], EXP-function method [31], Hopf-Cole transformation method [32], modified extended Fan's sub-equation method [33, 34], and generalized algebraic method [35]. But we notice so far that no research has been paid for (10). In the following we will apply the described method in Section 2 to (10).

To begin with, we suppose $u(x, y, t) = U(\xi)$, $v(x, y, t) = V(\xi)$, where $\xi = (c/\Gamma(1 + \alpha))t^\alpha + (k_1/\Gamma(1 + \alpha))x^\alpha + (k_2/\Gamma(1 + \alpha))y^\alpha + \xi_0$, k_1, k_2, c, ξ_0 are all constants with $k_1, k_2, c \neq 0$. Then by the use of the first equality in (3), we obtain $D_x^\alpha u = D_x^\alpha U(\xi) = U'(\xi)D_x^\alpha \xi = k_1 U'(\xi)$, $D_y^\alpha u = D_y^\alpha U(\xi) = U'(\xi)D_y^\alpha \xi = k_2 U'(\xi)$, $D_t^\alpha u = D_t^\alpha U(\xi) = U'(\xi)D_t^\alpha \xi = c U'(\xi)$, and then (10) can be turned into

$$\begin{aligned} ck_2 U'' + k_1^2 V'' + k_1 k_2 (UU'' + (U')^2) &= 0, \\ cV' + k_1 U' + k_1 (UV)' + k_1^2 k_2 U''' &= 0. \end{aligned} \quad (12)$$

Suppose that the solutions of (12), can be expressed by

$$\begin{aligned} U(\xi) &= \sum_{i=0}^{m_1} a_i \left(\frac{G'}{G} \right)^i, \\ V(\xi) &= \sum_{i=0}^{m_2} b_i \left(\frac{G'}{G} \right)^i. \end{aligned} \quad (13)$$

Balancing the order of V'' and UU'' , $(UV)'$ and U''' in (12) we can obtain $m_1 = 1, m_2 = 2$. So we have

$$\begin{aligned} U(\xi) &= a_0 + a_1 \left(\frac{G'}{G} \right), \\ V(\xi) &= b_0 + b_1 \left(\frac{G'}{G} \right) + b_2 \left(\frac{G'}{G} \right)^2. \end{aligned} \quad (14)$$

Substituting (14) into (12), using (8), collecting all the terms with the same power of (G'/G) together, and equating each coefficient to zero, yields a set of algebraic equations. Solving these equations yields

$$\begin{aligned} a_0 &= \frac{\pm k_1^2 \lambda - c}{k_1}, & a_1 &= \pm 2k_1, \\ b_0 &= -2k_1 k_2 \mu - 1, & b_1 &= -2k_1 k_2 \lambda, \\ b_2 &= -2k_1 k_2. \end{aligned} \quad (15)$$

Substituting the result above into (14) and combining it with (9) we can obtain the following exact solutions to (10).

When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned} u_1(x, y, t) &= \frac{\pm k_1^2 \lambda - c}{k_1} \\ &\quad \pm 2k_1 \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \right. \\ &\quad \times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\ &\quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\ &\quad \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\ &\quad \left. \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right) \left. \right], \\ &\quad - \frac{3}{2} k_1^2 \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \right. \\ &\quad \times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\ &\quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \end{aligned}$$

$$\begin{aligned}
& \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\
& \quad \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \Bigg]^2, \\
v_1(x, y, t) &= -2k_1 k_2 \mu - 1 - 2k_1 k_2 \lambda \\
& \times \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \right. \\
& \times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\
& \quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\
& \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\
& \quad \left. \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right) \Bigg] \\
& - 2k_1 k_2 \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \right. \\
& \times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\
& \quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\
& \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\
& \quad \left. \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right) \Bigg]^2, \\
\end{aligned} \tag{16}$$

where $\xi = (c/\Gamma(1+\alpha))t^\alpha + (k_1/\Gamma(1+\alpha))x^\alpha + (k_2/\Gamma(1+\alpha))y^\alpha + \xi_0$.

In particular, if we take $C_2 = 0$, then we obtain the following solitary wave solutions, which are shown in Figures 1 and 2.

When $\lambda^2 - 4\mu < 0$,

$$\begin{aligned}
u_2(x, y, t) &= \frac{\pm k_1^2 \lambda - c}{k_1} \\
& \pm 2k_1 \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \right. \\
& \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
& \quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
& \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
& \quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \right) \Bigg] \\
& - \frac{3}{2} k_1^2 \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \right. \\
& \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
& \quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
& \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
& \quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \right) \Bigg]^2, \\
v_2(x, y, t) &= -2k_1 k_2 \mu - 1
\end{aligned}$$

$$\begin{aligned}
& -2k_1k_2\lambda \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \right. \\
& \quad \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
& \quad \quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
& \quad \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
& \quad \quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right)^{-1} \left. \right] \\
& -2k_1k_2 \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \right. \\
& \quad \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
& \quad \quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
& \quad \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
& \quad \quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right)^2 \left. \right], \\
\end{aligned} \tag{17}$$

where $\xi = (c/\Gamma(1+\alpha))t^\alpha + (k_1/\Gamma(1+\alpha))x^\alpha + (k_2/\Gamma(1+\alpha))y^\alpha + \xi_0$.

In Figures 3 and 4, the periodic function solutions (17) are demonstrated with some certain parameters.

When $\lambda^2 - 4\mu = 0$,

$$\begin{aligned}
u_3(x, y, t) = & \frac{\pm k_1^2 \lambda - c}{k_1} \pm 2k_1 \left[-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2 \xi} \right] \\
& - \frac{3}{2} k_1^2 \left[-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2 \xi} \right]^2,
\end{aligned}$$

FIGURE 1: The solution u_1 with the positive symbol selected in $\pm$ and $\lambda = 4$, $\mu = 3$, $c = k_1 = k_2 = 1$, $C_2 = \xi_0 = 0$, $\alpha = 1/2$, and $t = 0.1$.

FIGURE 2: The solution v_1 with $\lambda = 4$, $\mu = 3$, $c = k_1 = k_2 = 1$, $C_2 = \xi_0 = 0$, $\alpha = 1/2$, and $t = 0.1$.

$$v_3(x, y, t) = -2k_1k_2\mu - 1$$

$$\begin{aligned}
& -2k_1k_2\lambda \left[-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2 \xi} \right] \\
& -2k_1k_2 \left[-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2 \xi} \right]^2,
\end{aligned}$$

(18)

where $\xi = (c/\Gamma(1+\alpha))t^\alpha + (k_1/\Gamma(1+\alpha))x^\alpha + (k_2/\Gamma(1+\alpha))y^\alpha + \xi_0$.

Remark 1. The established solutions in (16)–(18) are new exact solutions for the space-time fractional $(2 + 1)$ -dimensional dispersive long wave equations.

FIGURE 3: The solution u_2 with the positive symbol selected in $\pm$ and $\lambda = 2, \mu = 2, c = k_1 = k_2 = 1, C_2 = \xi_0 = 0, \alpha = 1/2$, and $t = 0.1$.

FIGURE 4: The solution v_2 with $\lambda = 2, \mu = 2, c = k_1 = k_2 = 1, C_2 = \xi_0 = 0, \alpha = 1/2$, and $t = 0.1$.

3.2. Space-Time Fractional (2 + 1)-Dimensional Nizhnik-Novikov-Veselov System. We will consider the space-time fractional (2 + 1)-dimensional Nizhnik-Novikov-Veselov (NNV) system:

$$\begin{aligned}
 D_t^\alpha u + aD_x^{3\beta} u + bD_y^{3\gamma} u + cD_x^\beta u + dD_y^\gamma u \\
 = 3aD_x^\beta (uv) + 3bD_y^\gamma (uw), \\
 D_x^\beta u = D_y^\gamma v, \\
 D_y^\gamma u = D_x^\beta w, \\
 0 < \alpha, \beta, \gamma \leq 1,
 \end{aligned} \tag{19}$$

which is a variation of the following (2 + 1)-dimensional Nizhnik-Novikov-Veselov (NNV) system [21, 37, 38]:

$$\begin{aligned}
 u_t + au_{xxx} + bu_{yyy} + cu_x + du_y \\
 = 3a(uv)_x + 3b(uw)_y, \\
 u_x = v_y, \\
 u_y = w_x.
 \end{aligned} \tag{20}$$

Suppose $u(x, y, t) = U(\xi)$, $v(x, y, t) = V(\xi)$, $w(x, y, t) = W(\xi)$, where $\xi = (c/\Gamma(1 + \alpha))t^\alpha + (k/\Gamma(1 + \beta))x^\beta + (l/\Gamma(1 + \gamma))y^\gamma + \xi_0$, c, k, l, ξ_0 are all constants with $k_1, k_2, c \neq 0$. Then by the use of the first equality in (3), (19) can be turned into the following forms:

$$\begin{aligned}
 cu' + ak^3 u''' + bl^3 u''' + cku' + dlu' \\
 = 3ak(uv)' + 3bl(uw)', \\
 ku' = lv', \\
 lu' = kw'.
 \end{aligned} \tag{21}$$

Integrating (21) once, we have

$$\begin{aligned}
 cu + ak^3 u'' + bl^3 u'' + cku + dlu \\
 = 3akuv + 3bluw + g_1, \\
 ku = lv + g_2, \\
 lu = kw + g_3,
 \end{aligned} \tag{22}$$

where g_1, g_2 , and g_3 are the integration constants.

Suppose that the solutions of (22) can be expressed by a polynomial in (G'/G) as follows:

$$\begin{aligned}
 U(\xi) &= \sum_{i=0}^{m_1} a_i \left(\frac{G'}{G} \right)^i, \\
 V(\xi) &= \sum_{i=0}^{m_2} b_i \left(\frac{G'}{G} \right)^i, \\
 W(\xi) &= \sum_{i=0}^{m_3} c_i \left(\frac{G'}{G} \right)^i,
 \end{aligned} \tag{23}$$

where a_i, b_i , and c_i are constants. Balancing the order of u'' and uv , and the order of u and v , the order of u and w in (22), we can obtain $m_1 = m_2 = m_3 = 2$. So we have

$$\begin{aligned}
 U(\xi) &= a_0 + a_1 \left(\frac{G'}{G} \right)^1 + a_2 \left(\frac{G'}{G} \right)^2, \\
 V(\xi) &= b_0 + b_1 \left(\frac{G'}{G} \right)^1 + b_2 \left(\frac{G'}{G} \right)^2, \\
 W(\xi) &= c_0 + c_1 \left(\frac{G'}{G} \right)^1 + c_2 \left(\frac{G'}{G} \right)^2.
 \end{aligned} \tag{24}$$

Substituting (24) into (22), using (8), collecting all the terms with the same power of (G'/G) together, and equating each coefficient to zero, yields a set of algebraic equations. Solving these equations yields.

Case I. Consider

$$\begin{aligned} a_2 &= 2kl, & a_1 &= 2kl\lambda, & a_0 &= 2kl\mu, \\ b_2 &= 2k^2, & b_1 &= 2k^2\lambda, & b_0 &= 2k^2\mu, \\ c_2 &= 2l^2, & c_1 &= 2l^2\lambda, & c_0 &= 2l^2\mu, \\ c &= 4ak^3\mu - ak^3\lambda^2 - bl^3\lambda^2 - dl + 4bl^3\mu - ck, \\ g_1 &= g_2 = g_3 = 0, \end{aligned} \quad (25)$$

where k and l are arbitrary constants.

Substituting the result above into (24) and combining it with (9) we can obtain the corresponding exact solutions for (19).

When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned} u_1(x, y, t) &= 2kl\mu - \frac{kl\lambda^2}{2} + \frac{kl(\lambda^2 - 4\mu)}{2} \\ &\times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\ &\quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right)^2 \\ &\times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\ &\quad \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \Big)^2, \\ v_1(x, y, t) &= 2k^2\mu - \frac{k^2\lambda^2}{2} + \frac{k^2(\lambda^2 - 4\mu)}{2} \\ &\times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\ &\quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right)^2 \end{aligned}$$

$$\begin{aligned} &\times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\ &\quad \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \Big)^2, \\ w_1(x, y, t) &= 2l^2\mu - \frac{l^2\lambda^2}{2} + \frac{l^2(\lambda^2 - 4\mu)}{2} \\ &\times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\ &\quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right)^2 \\ &\times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\ &\quad \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \Big)^2, \end{aligned} \quad (26)$$

where $\xi = ((4ak^3\mu - ak^3\lambda^2 - bl^3\lambda^2 - dl + 4bl^3\mu - ck)/\Gamma(1 + \alpha))t^\alpha + (k/\Gamma(1 + \beta))x^\beta + (l/\Gamma(1 + \gamma))y^\gamma + \xi_0$.

When $\lambda^2 - 4\mu < 0$,

$$\begin{aligned} u_2(x, y, t) &= 2kl\mu - \frac{kl\lambda^2}{2} + \frac{kl(4\mu - \lambda^2)}{2} \\ &\times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\ &\quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \right)^2 \\ &\times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\ &\quad \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \Big)^2, \end{aligned}$$

$$\begin{aligned}
v_2(x, y, t) &= 2k^2\mu - \frac{k^2\lambda^2}{2} + \frac{k^2}{2}(4\mu - \lambda^2) \\
&\times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
&\quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
&\quad \left. \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
&\quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \right)^2,
\end{aligned}$$

$$\begin{aligned}
w_2(x, y, t) &= 2l^2\mu - \frac{l^2\lambda^2}{2} + \frac{l^2}{2}(4\mu - \lambda^2) \\
&\times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
&\quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
&\quad \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
&\quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \right)^2,
\end{aligned} \tag{27}$$

where $\xi = ((4ak^3\mu - ak^3\lambda^2 - bl^3\lambda^2 - dl + 4bl^3\mu - ck)/\Gamma(1 + \alpha))t^\alpha + (k/\Gamma(1 + \beta))x^\beta + (l/\Gamma(1 + \gamma))y^\gamma + \xi_0$.

When $\lambda^2 - 4\mu = 0$,

$$\begin{aligned}
u_3(x, y, t) &= -\frac{kl\lambda^2}{2} + \frac{2klC_2^2}{(C_1 + C_2\xi)^2} + 2kl\mu, \\
v_3(x, y, t) &= -\frac{k^2\lambda^2}{2} + \frac{2k^2C_2^2}{(C_1 + C_2\xi)^2} + 2k^2\mu, \\
w_3(x, y, t) &= -\frac{l^2\lambda^2}{2} + \frac{2l^2C_2^2}{(C_1 + C_2\xi)^2} + 2l^2\mu,
\end{aligned} \tag{28}$$

where $\xi = ((4ak^3\mu - ak^3\lambda^2 - bl^3\lambda^2 - dl + 4bl^3\mu - ck)/\Gamma(1 + \alpha))t^\alpha + (k/\Gamma(1 + \beta))x^\beta + (l/\Gamma(1 + \gamma))y^\gamma + \xi_0$.

Case 2. Consider

$$\begin{aligned}
a_2 &= 2kl, & a_1 &= 2kl\lambda, & a_0 &= \frac{1}{3}kl(\lambda^2 + 2\mu), \\
b_2 &= 2k^2, & b_1 &= 2k^2\lambda, & b_0 &= \frac{1}{3}k^2(\lambda^2 + 2\mu), \\
c_2 &= 2l^2, & c_1 &= 2l^2\lambda, & c_0 &= \frac{1}{3}l^2(\lambda^2 + 2\mu), \\
c &= -4ak^3\mu + ak^3\lambda^2 + bl^3\lambda^2 - dl - 4bl^3\mu - ck, \\
g_1 &= g_2 = g_3 = 0.
\end{aligned} \tag{29}$$

Substituting the result above into (24) and combining it with (9) we can obtain the corresponding exact solutions for (19).

When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned}
u_4(x, y, t) &= \frac{1}{3}kl(\lambda^2 + 2\mu) - \frac{kl\lambda^2}{2} + \frac{kl(\lambda^2 - 4\mu)}{2} \\
&\times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\
&\quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\
&\quad \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\
&\quad \left. \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right)^2,
\end{aligned}$$

$$\begin{aligned}
v_4(x, y, t) &= \frac{1}{3}k^2(\lambda^2 + 2\mu) - \frac{k^2\lambda^2}{2} + \frac{k^2}{2}(\lambda^2 - 4\mu) \\
&\times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\
&\quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\
& \quad \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \Bigg)^2, \\
w_4(x, y, txi) \\
& = \frac{1}{3} l^2 (\lambda^2 + 2\mu) - \frac{l^2 \lambda^2}{2} + \frac{l^2}{2} (\lambda^2 - 4\mu) \\
& \quad \times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\
& \quad \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\
& \quad \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\
& \quad \left. \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \Bigg)^2, \tag{30}
\end{aligned}$$

where $\xi = ((-4ak^3\mu + ak^3\lambda^2 + bl^3\lambda^2 - dl - 4bl^3\mu - ck)/\Gamma(1 + \alpha))t^\alpha + (k/\Gamma(1 + \beta))x^\beta + (l/\Gamma(1 + \gamma))y^\gamma + \xi_0$.

When $\lambda^2 - 4\mu < 0$,

$$\begin{aligned}
u_5(x, y, t) \\
& = \frac{1}{3} kl (\lambda^2 + 2\mu) - \frac{kl\lambda^2}{2} + \frac{kl(4\mu - \lambda^2)}{2} \\
& \quad \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
& \quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
& \quad \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
& \quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \Bigg)^2,
\end{aligned}$$

$$\begin{aligned}
v_5(x, y, t) \\
& = \frac{1}{3} k^2 (\lambda^2 + 2\mu) - \frac{k^2 \lambda^2}{2} + \frac{k^2}{2} (4\mu - \lambda^2) \\
& \quad \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
& \quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
& \quad \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
& \quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \Bigg)^2,
\end{aligned}$$

$$\begin{aligned}
w_5(x, y, t) \\
& = \frac{1}{3} l^2 (\lambda^2 + 2\mu) - \frac{l^2 \lambda^2}{2} + \frac{l^2}{2} (4\mu - \lambda^2) \\
& \quad \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
& \quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
& \quad \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
& \quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \Bigg)^2, \tag{31}
\end{aligned}$$

where $\xi = ((-4ak^3\mu + ak^3\lambda^2 + bl^3\lambda^2 - dl - 4bl^3\mu - ck)/\Gamma(1 + \alpha))t^\alpha + (k/\Gamma(1 + \beta))x^\beta + (l/\Gamma(1 + \gamma))y^\gamma + \xi_0$.

When $\lambda^2 - 4\mu = 0$,

$$\begin{aligned}
u_6(x, y, t) & = -\frac{kl\lambda^2}{2} + \frac{2klC_2^2}{(C_1 + C_2\xi)^2} + \frac{1}{3} kl (\lambda^2 + 2\mu), \\
v_6(x, y, t) & = -\frac{k^2\lambda^2}{2} + \frac{2k^2C_2^2}{(C_1 + C_2\xi)^2} + \frac{1}{3} k^2 (\lambda^2 + 2\mu), \tag{32} \\
w_6(x, y, t) & = -\frac{l^2\lambda^2}{2} + \frac{2l^2C_2^2}{(C_1 + C_2\xi)^2} + \frac{1}{3} l^2 (\lambda^2 + 2\mu),
\end{aligned}$$

where $\xi = ((-4ak^3\mu + ak^3\lambda^2 + bl^3\lambda^2 - dl - 4bl^3\mu - ck)/\Gamma(1 + \alpha))t^\alpha + (k/\Gamma(1 + \beta))x^\beta + (l/\Gamma(1 + \gamma))y^\gamma + \xi_0$.

Remark 2. The established solutions in (26)–(32) for the space-time fractional (2 + 1)-dimensional Nizhnik-Novikov-Veselov system have not been reported by other authors so far.

Remark 3. In [21], Zayed solved the (2 + 1)-dimensional Nizhnik-Novikov-Veselov (NNV) system (20) by the (G'/G) -expansion method. We note that if we put $\alpha = 1$, $k = l = 1$ in (26)–(32), then our results reduce to the results in [21] despite the slight difference of the symbols. So the solutions presented here are of more general forms.

3.3. Time Fractional Fifth-Order Sawada-Kotera Equation. We consider the time fractional fifth-order Sawada-Kotera equation:

$$D_t^\alpha u + u_{xxxxx} + 45u_x u^2 + 15(u_x u_{xx} + uu_{xxx}) = 0, \quad 0 < \alpha \leq 1, \quad (33)$$

which is the variation of the fifth-order Sawada-Kotera equation [39]:

$$u_t + u_{xxxxx} + 45u_x u^2 + 15(u_x u_{xx} + uu_{xxx}) = 0. \quad (34)$$

Now we apply the proposed method to (33). To begin with, we suppose $u(x, t) = U(\xi)$, where $\xi = kx + (c/\Gamma(1 + \alpha))t^\alpha + \xi_0$, k, c, ξ_0 are all constants with $k, c \neq 0$. Then by the use of the first equality in (3), (33) can be turned into

$$cU' + k^5 U^{(5)} + 45kUU' + 15k^3 (U'U'' + UU''') = 0. \quad (35)$$

Suppose that the solution of (35) can be expressed by

$$U(\xi) = \sum_{i=0}^m a_i \left(\frac{G'}{G} \right)^i. \quad (36)$$

Balancing the order of $U^{(5)}$ and UU''' in (36) we have $m = 2$. So

$$U(\xi) = a_0 + a_1 \left(\frac{G'}{G} \right) + a_2 \left(\frac{G'}{G} \right)^2. \quad (37)$$

Substituting (37) into (35), using (8), collecting all the terms with the same power of (G'/G) together, and equating each coefficient to zero, yields a set of algebraic equations. Solving these equations yields

$$\begin{aligned} a_0 = & - \left(\left(40k^3\mu + 5k^3\lambda^2 \right. \right. \\ & \left. \left. \pm \sqrt{80k^6\mu^2 - 40k^6\mu\lambda^2 + 5k^6\lambda^4 - 20kc} \right) \right. \\ & \left. \times (30k)^{-1} \right), \\ a_1 = & -2k^2\lambda, \quad a_2 = -2k^2. \end{aligned} \quad (38)$$

Substituting the result above into (37) and combining it with (9) we can obtain the corresponding exact solutions to (33).

When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned} u_1(x, t) = & - \left(\left(40k^3\mu + 5k^3\lambda^2 \right. \right. \\ & \left. \left. \pm \sqrt{80k^6\mu^2 - 40k^6\mu\lambda^2 + 5k^6\lambda^4 - 20kc} \right) \right. \\ & \left. \times (30k)^{-1} \right) \\ & - 2k^2\lambda \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \right. \\ & \times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\ & \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\ & \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\ & \left. \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right)^{-1} \Big] \\ & - 2k^2 \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \right. \\ & \times \left(\left(C_1 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \right. \\ & \left. \left. + C_2 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\ & \times \left(C_1 \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right. \\ & \left. \left. + C_2 \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right)^{-1} \right)^{-1} \Big]^2, \end{aligned} \quad (39)$$

where $\xi = kx + (c/\Gamma(1 + \alpha))t^\alpha + \xi_0$.

When $\lambda^2 - 4\mu < 0$,

$$\begin{aligned}
 u_2(x, t) &= - \left(\left(40k^3\mu + 5k^3\lambda^2 \right. \right. \\
 &\quad \left. \left. \pm \sqrt{80k^6\mu^2 - 40k^6\mu\lambda^2 + 5k^6\lambda^4 - 20kc} \right) \right. \\
 &\quad \left. \times (30k) \right)^{-1} \\
 &\quad - 2k^2\lambda \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \right. \\
 &\quad \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
 &\quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
 &\quad \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
 &\quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \right) \right] \\
 &\quad - 2k^2 \left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \right. \\
 &\quad \times \left(\left(-C_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \right. \\
 &\quad \left. \left. + C_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right. \\
 &\quad \times \left(C_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right. \\
 &\quad \left. \left. + C_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right)^{-1} \right) \right]^2,
 \end{aligned} \tag{40}$$

where $\xi = kx + (c/\Gamma(1 + \alpha))t^\alpha + \xi_0$.

When $\lambda^2 - 4\mu = 0$,

$$\begin{aligned}
 u_3(x, t) &= - \left(\left(40k^3\mu + 5k^3\lambda^2 \right. \right. \\
 &\quad \left. \left. \pm \sqrt{80k^6\mu^2 - 40k^6\mu\lambda^2 + 5k^6\lambda^4 - 20kc} \right) \right. \\
 &\quad \left. \times (30k)^{-1} \right) \\
 &\quad - 2k^2\lambda \left[-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi} \right] \\
 &\quad - 2k^2 \left[-\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi} \right]^2,
 \end{aligned} \tag{41}$$

where $\xi = kx + (c/\Gamma(1 + \alpha))t^\alpha + \xi_0$.

Remark 4. The established solutions in (39)–(41) are new exact solutions for the time fractional fifth-order Sawada-Kotera equation.

4. Conclusions

We have solved the space-time fractional $(2 + 1)$ -dimensional dispersive long wave equations, the space-time fractional $(2 + 1)$ -dimensional Nizhnik-Novikov-Veselov system, and the time fractional fifth-order Sawada-Kotera equation by the known (G'/G) method successfully. In the process, the transformation for ξ is very important, which ensures that certain fractional partial differential equations can be turned into ordinary differential equations of integer order. With the aid of mathematical software, a variety of exact solutions for these equations are obtained, which may help to understand better physical phenomenon related to these equations. Finally, we note that this approach can also be applied to solve other fractional partial differential equations.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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