Research Article

Reliable Control for Time-Varying Delay Switched Fuzzy Systems with Faulty Actuators Based on Observers Switching Method

Le Zhang and Jinnan Wu

School of Information Engineering, Shenyang University, Shenyang 110044, China

Correspondence should be addressed to Le Zhang; snowise@126.com

Received 1 April 2013; Accepted 3 June 2013

Academic Editor: Rongni Yang

Copyright © 2013 L. Zhang and J. Wu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper deals with the reliable control problem of nonlinear systems represented by switched fuzzy systems (SFS) with time-varying delay, where each subsystem of switched system is a time-varying delay fuzzy system. A switched fuzzy system with a Takagi and Sugeno (T-S) fuzzy model, which differs from existing ones, is firstly employed to describe a nonlinear system. When the actuators are serious failure, the residual part of actuators cannot make original system stability, using switching technique depends on the states of observers, and the fuzzy reliable controller based on measured observers states instead of the original system states information is built. The stabilization criterion of the reliable control problem is given for the case that the state of original system is unmeasurable. The multi-Lyapunov functions method is utilized to the stability analysis and controller design for time-varying delay switched fuzzy systems with faulty actuators. Moreover, observers switching strategy achieving estimation errors decreasing uniformly asymptotically to zero of the switched fuzzy systems is considered. Finally, the stabilization criterion is transformed into the solvability of sufficient linear matrix inequality (LMI) conditions. To illustrate the effectiveness of the proposed stabilization criterion and controller design approaches, a designed numerical example is studied, and some simulations are provided.

1. Introduction

With the growing complexity of some control engineering problems, control techniques drawn from linear theory have shown their limits. Among nonlinear theory, new control approaches have appeared in the last decades such as fuzzy or hybrid techniques.

The fuzzy model-based control has been rapidly developed in recent years [1–7]. This approach is completely different from typical fuzzy controls [8–10]. By employing the Takagi-Sugeno (T-S) fuzzy model [11], which utilizes local linear system description for each rule, we can devise a control methodology to fully take advantages of linear control theory. A typical design procedure of fuzzy model-based control consists of two steps:

- (i) fuzzy model construction for a nonlinear system;
- (ii) fuzzy controller design.

Nevertheless, an inherent drawback remains since the number of fuzzy rules of a T-S model increase exponentially with

the number of nonlinearities constituting the matched nonlinear system. This makes controller design and implementation difficult as the complexity of the nonlinear system to be controlled increases [12], especially for reliable controller design. To outline the problem of rules explosion in T-S model, some authors have proposed to combine the merit of switched systems with T-S models to deal with nonlinear control problems.

A combination of hybrid systems and fuzzy multiple model systems for the continuous-time case was described, and an idea of the fuzzy switched hybrid control was put forward [13]. The new switching fuzzy systems model, which more approaches the complicated real systems such as multiple nonlinear systems, switched nonlinear hybrid systems, and second-order nonholonomic systems, was introduced based on the T-S fuzzy model [14–17]. This model is switching in local fuzzy rule level of the second level according to the premise variable in region rule level of the first level. So, as stated in [18], it is switching according to the same premise variable.

To obtain less conservative stability analysis results than the T-S fuzzy model-based control, in [19], we proposed a switched fuzzy (SF) model constructed by the idea of multidimensional sector nonlinearity [20] and a switching Lyapunov function [14]. A switched system is said to be a switched fuzzy system (SFS) if all of its subsystems are fuzzy systems. It may well be found that the results on switched fuzzy systems are very few. Differing from the conventional switching fuzzy systems which are all of two-level architecture, the new system consists of several subsystems, and they switch on/off each other. The resulting switched fuzzy system inherits some essential features of hybrid systems [21, 22] and maintains all the information and knowledge representation capacity of fuzzy systems. Continuous controllers for all switched subsystems and a switching law are designed to give robust asymptotic stability [19]. The method provides a kind of different premise variable switching directly. Yang et al. have compared the different features of switching fuzzy systems with switched fuzzy systems and introduced the sketch map to illustrate the construction of switched fuzzy system [18]. Among the earliest applications of switched fuzzy systems is the design problem of fuzzy switching control law for a helicopter, in the whole flight envelope [23]. Here H_∞ control approach is considered. In [24] the switching method for the fuzzy control of the overhead cranes is presented. One uses the information of the trolley position, load swing, and their differentiations to derive the proper control signal. The switching fuzzy algorithm to overcome the condition of dead-zone is especially investigated. Authors in [25] propose a switching control method based on fuzzy energy regions for this kind of systems, and the control method is applied to control a two degrees-of-freedom (dofs) under actuated manipulator with one active dof and one passive dof. From previously mentioned application areas of switched fuzzy systems, it can be concluded that these systems are ubiquitous and of significant practical application.

It should be noted that, since faults and even failures of control components often occur in real-world control systems, classical robust control methods may not provide satisfactory performance always and even drive closed-loop system unstable. A specific control design strategy named fault tolerant control (FTC) is the most popular method applied to systems in the case of unexpected faults, and passive and active approaches to FTC are available. The active FTC approach requires a fault detection and diagnosis (FDD) mechanism to detect and identify the faults in real time and then a mechanism to reconfigure the controllers according to the online fault information from the FDD. In particular, Wu and Ho propose a methodology for the design of both the fuzzy-rule independent and the fuzzy-rule dependent fault detection filters by using a general observer-based fault detection filter as a residual generator [26]. Compared to the passive approach, the active one needs more significantly computational power to implement. In [27], motivations to study controller failures are generalized. The first reason is that the signals are not transmitted perfectly or the controller itself is not available, for example, as in the case of packet dropout phenomenon in networked control systems (NCSs) [28]. The other is for a positive reason: for an economic or

system life consideration, where the controller is purposefully suspended from time to time.

Motivated by the aforementioned progress, reliable control has acquired wide attention, and considerable developments were achieved [29–31]. Different from traditional controller design, the objective of this study is to design an appropriate controller by using the information of residual actuators or sensors such that the closed-loop system can tolerate the abnormal operation of some specific control components and retain overall system stability with acceptable system performance. An abnormal operation may include degradation, amplification, and partial outage. In [32], the reliable controller design with actuator fault for the attitude control of an orbiting spacecraft equipped with six thrusters is considered. The application of the reliable control with actuator fault control schemes to the vertical takeoff and landing (VTOL) aircraft system is presented [33]. In the aforementioned manuscripts, the following two methods are widely used.

- (a) The fault model is depicted as a scaling factor. The outputs of controller without actuator failure are used as the inputs of controller in which actuator failures occurred. It is worth noting that these reliable control methods are all based on a basic assumption that the never failed actuators must stabilize the given system, which is obviously somehow impractical.
- (b) Decompose the matrix B (parameters of the control input) into two sets: the set of actuators that are susceptible to failures and the set of actuators that are robust to failure. In other words, actuators may suffer “serious faults” and even failures in some of them, yet the system can remain stable [34]. Motivated by this method, this paper studies the problem of reliable control where actuators suffer “serious faults” that tend towards failures and presents a more general reliable control method.

On the other hand, in almost all engineering control systems such as chemical processes and communication systems, delays are frequently encountered, which affect the performance of the system adversely, including causing instability. Hence, researchers have been paying remarkable attention to the problems of analysis and synthesis for time delay systems [4, 6, 35–42]. All the existing literature for stability analysis can be roughly divided into two types: time constant delay results [35] and time-varying delay ones [4, 6, 36–42]. It is obviously important to establish under what conditions the system is still stable subject to the “serious faults” and the time-varying delay. It is believed, in here, that a more general method that combines reliable control and switched fuzzy control of nonlinear systems, whose states are not available, in the sense of employing observers switching is addressed.

In this paper, the reliable control problem of switched fuzzy systems is considered for the case that the state of system is unmeasurable. The problem begins with the representation models for time-varying delay switched fuzzy system. The derivative of time-varying delay is allowed to be bounded with an unknown constant. Both fuzzy reliable controllers for subsystems and observers switching laws

which make switched fuzzy system asymptotic stability are designed. Sufficient conditions are derived via multiple Lyapunov function method. The remainder of this paper is organized as follows. Section 2 presents an outline for switched fuzzy systems model and preliminaries. In Section 3, stabilization and the reliable controller are derived. An illustrative example is given in Section 4. Finally, some concluding remarks are included in Section 5.

2. Model Description and Preliminaries

In this section, we present a time-varying delay switched fuzzy model with faulty actuators. The main novelty is the combination of reliable control and switched fuzzy control. Consider the continuous time-varying delay switched fuzzy model in which the delays are changed depending on time, namely, every subsystem of switched systems is time-varying delay fuzzy system:

$$R_{\sigma(t)}^l : \text{if } \xi_1 \text{ is } M_{\sigma(t)1}^l \cdots \text{ and } \xi_p \text{ is } M_{\sigma(t)p}^l, \text{ then} \quad (1)$$

$$\dot{x}(t) = A_{\sigma(t)l}x(t) + A_{h\sigma(t)l}x(t - d_{\sigma(t)}(t)) + B_{\sigma(t)l}u_{\sigma(t)}(t), \quad (2a)$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0], \quad (2b)$$

$$y(t) = C_{\sigma(t)l}x(t), \quad l = 1, 2, \dots, N_{\sigma(t)}. \quad (2c)$$

In here the symbols represent the following: $\sigma : \overline{M} = \{1, 2, \dots, m\}$ is the switching signal to be designed. $\xi_1, \xi_2, \dots, \xi_p$ are the premise variables. $M_{\sigma(t)1}^l, \dots, M_{\sigma(t)p}^l$ denote fuzzy sets in the σ th switched subsystem. $R_{\sigma(t)}^l$ denote the l th fuzzy inference rule in the σ th switched subsystem. $N_{\sigma(t)}$ is the number of inference rules in the σ th switched subsystem, and fuzzy rules are selected in every switched subsystem. $u_{\sigma(t)}(t)$ is the input variable of the σ th switched subsystem. $x(t) = [x_1(t) \ x_2(t) \ \cdots \ x_n(t)]^T \in \mathbb{R}^n$ is the state variable vector. $A_{\sigma(t)l}, A_{h\sigma(t)l}, B_{\sigma(t)l}$, and $C_{\sigma(t)l}$ are known constant matrices of appropriate dimensions of the σ th switched subsystem. $\varphi(t)$ is a continuously differentiable vector-valued initial function, and $\tau > 0$. $d_{\sigma(t)}(t)$ denote the time-varying delay of the σ th switched subsystem satisfying Assumption 1.

Assumption 1. $d_{\sigma(t)}(t)$ is a differentiable function, satisfying $0 \leq d_{\sigma(t)}(t) \leq \tau$ and $\dot{d}_{\sigma(t)}(t) \leq d < 1$ for some known constant d .

The i th switched subsystem is

$$R_i^l : \text{if } \xi_1 \text{ is } M_{i1}^l \cdots \text{ and } \xi_p \text{ is } M_{ip}^l, \text{ then} \quad (3)$$

$$\dot{x}(t) = A_{il}x(t) + A_{hil}x(t - d_i(t)) + B_{il}u_i(t), \quad (4a)$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0], \quad (4b)$$

$$y(t) = C_{il}x(t), \quad i = 1, \dots, m, \quad l = 1, 2, \dots, N_i. \quad (4c)$$

Therefore, the global model of the i th switched subsystem is described by

$$\dot{x}(t) = \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) [A_{il}x(t) + A_{hil}x(t - d_i(t)) + B_{il}u_i(t)], \quad (5a)$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0], \quad (5b)$$

$$y(t) = \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) C_{il}x(t), \quad i = 1, \dots, m, \quad (5c)$$

along with

$$0 \leq \eta_{il}(\xi(t)) \leq 1, \quad \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) = 1, \quad (6)$$

in which

$$\omega_{il}(\xi(t)) = \prod_{\rho=1}^p M_{i\rho}^l(\xi_{\rho}(t)), \quad \eta_{il}(\xi(t)) = \frac{\omega_{il}(\xi(t))}{\sum_{l=1}^{N_i} \omega_{il}(\xi(t))}, \quad (7)$$

where $M_{i\rho}^l(\xi_{\rho}(t))$ denotes the membership function and $\xi_{\rho}(t)$ belongs to the fuzzy set $M_{i\rho}^l$.

Since the system states often are not directly measurable, we consider the switching signal $\sigma = \sigma(\hat{x}(t))$, which depends on observer states. $\hat{x}(t)$ is the observers signal of $x(t)$. Suppose $\overline{\Omega}_1, \overline{\Omega}_2, \dots, \overline{\Omega}_m$ is a partition of $\mathbb{R}^n$, that is, $\bigcup_{i=1}^m \overline{\Omega}_i = \mathbb{R}^n \setminus \{0\}$ and $\overline{\Omega}_i \cap \overline{\Omega}_j = \Phi$, $i \neq j$. The design of switching signal is

$$\sigma = \sigma(\hat{x}(t)) = i : [0, +\infty) \longrightarrow \overline{M} = \{1, 2, \dots, m\} \quad (8)$$

which depends on $\overline{\Omega}_1, \overline{\Omega}_2, \dots, \overline{\Omega}_m$. When $\hat{x}(t) \in \overline{\Omega}_i$, the switching signal subjects to

$$v_i(\hat{x}(t)) = \begin{cases} 1 & \hat{x}(t) \in \overline{\Omega}_i, \\ 0 & \hat{x}(t) \notin \overline{\Omega}_i, \end{cases} \quad i \in \overline{M}. \quad (9)$$

That is, if and only if $\sigma = \sigma(\hat{x}(t)) = i$, then $v_i(\hat{x}(t)) = 1$.

From $v_i(\hat{x}(t))$, the system (2a), (2b), and (2c) is represented by

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^m \sum_{l=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \\ &\quad \times [A_{il}x(t) + A_{hil}x(t - d_i(t)) + B_{il}u_i(t)], \end{aligned} \quad (10a)$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0], \quad (10b)$$

$$y(t) = \sum_{i=1}^m \sum_{l=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) C_{il}x(t). \quad (10c)$$

In order to investigate the reliable control problem, we classify actuators of i th switched subsystem (5a), (5b), and

(5c) into two groups [34]. One is a set of actuators susceptible to failures, denoted by $\Theta_i = \{1, 2, \dots, k_i\}$, ($i = 1, 2, \dots, m$). The other is a set of actuators robust to failures, denoted by $\bar{\Theta}_i = \{1, 2, \dots, k_i\} - \Theta_i$, ($i = 1, 2, \dots, m$). Therefore, introduce the decomposition of B_{il} as

$$B_{il} = B_{\Theta_{il}} + B_{\bar{\Theta}_{il}}, \quad i = 1, 2, \dots, m, \quad l = 1, 2, \dots, N_i, \quad (11)$$

where $B_{\Theta_{il}}$ and $B_{\bar{\Theta}_{il}}$ are gained by making the column elements of B_{il} which correspond to Θ_{il} and $\bar{\Theta}_{il}$, respectively, zero. Chose ω_i as set of actuators susceptible to failures in practice, $\omega_i \subseteq \Theta_i$, then

$$B_i = B_{\bar{\omega}_i} + B_{\omega_i}, \quad i = 1, 2, \dots, m, \quad (12)$$

where $B_{\bar{\omega}_i}$ and B_{ω_i} are gained by making the column elements of B_i which correspond to ω_i and $\bar{\omega}_i$, respectively, zero; the following is obvious

$$\begin{aligned} B_{\bar{\Theta}_i} B_{\bar{\Theta}_i}^T &\leq B_{\bar{\omega}_i} B_{\bar{\omega}_i}^T, \quad B_{\omega_i} B_{\omega_i}^T \leq B_{\Theta_i} B_{\Theta_i}^T \\ B_{il} B_{il}^T &= B_{\Theta_{il}} B_{\Theta_{il}}^T + B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{il}}^T, \quad i = 1, 2, \dots, m. \end{aligned} \quad (13)$$

Remark 2. In the existing standard reliable control problem, the condition $(A_{il}, B_{\bar{\Theta}_{il}})$ must be a stable pair requisite. This strong condition is no longer needed here for switched fuzzy systems because we can design the switching law to make the origin system stabilizable. In fact, if $(A_{jl}, B_{\bar{\Theta}_{jl}})$ is a stable pair for any $j \in \bar{M}$, then we can design state feedback controller for the j th subsystem that makes the system (10a), (10b), and (10c) stabilizable.

3. Main New Results

In particular, the main attention of this paper is concentrated on actuators of switched fuzzy system suffering serious failures, which has great significance in theoretical study and engineering applications. Moreover, according to PDC (Parallel Distributed Compensation), the observers and fuzzy output feedback controllers are designed.

$$\begin{aligned} \hat{\dot{x}}(t) &= \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ig}(\xi(t)) \\ &\quad \times \left[A_{il} \hat{x}(t) + A_{hil} \hat{x}(t - d_i(t)) + B_{il} u_i(t) \right. \\ &\quad \left. + L_{il} [y(t) - C_{ig} \hat{x}(t)] \right], \end{aligned} \quad (14a)$$

$$u_i(t) = - \sum_{l=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) K_{il} \hat{x}(t). \quad (14b)$$

It ensures that the system (2a), (2b), and (2c) is global asymptotic stability, where $\hat{x}(t) \in \mathbb{R}^n$ is state vectors of fuzzy observer, and L_{il} represents observer gain matrix for

the l th fuzzy rule of the i th switching subsystem. L_{il} can be determined by solving conditions (18). K_{il} denote the controllers feedback gain matrix, and K_{il} can be determined by solving controller design conditions (19).

In this section, we derive the sufficient conditions for global asymptotic stability of the time-varying delay switched fuzzy system (2a), (2b), and (2c). Choosing observer error $e(t) = x(t) - \hat{x}(t)$, we can obtain

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \\ &\quad \times \left[\left[A_{il} - B_{il} K_{ij} \right] x(t) + A_{hil} x(t - d_i(t)) \right. \\ &\quad \left. + B_{il} K_{ij} e(t) \right], \end{aligned} \quad (15a)$$

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\ &\quad \times \left[\left(A_{il} - B_{il} K_{ij} \right) \hat{x}(t) + A_{hil} \hat{x}(t - d_i(t)) \right. \\ &\quad \left. + L_{il} C_{ig} e(t) \right], \end{aligned} \quad (15b)$$

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ig}(\xi(t)) \\ &\quad \times \left[\left(A_{il} - L_{il} C_{ig} \right) e(t) + A_{hil} e(t - d_i(t)) \right]. \end{aligned} \quad (15c)$$

Theorem 3. Suppose that the switched fuzzy system (10a), (10b), and (10c) satisfies the Assumption 1. If there exist constants β_{ir} ($i, r \in \bar{M}$) (either all nonnegative or all nonpositive), positive definite matrixes P_i , R_i , Q_i , P_e , R_e , Q_e , and a group of positive scalars constants α_{ij} , ξ_{il} , ε_1 , ε_2 , ε_3 , satisfying the inequalities

$$\begin{aligned} &\left[\begin{array}{cccc} \pi_{11} + \sum_{r=1}^m \beta_{ir} (P_i - P_r) & \pi_{12} & \pi_{13} & 0 \\ * & \pi_{22} & 0 & 0 \\ * & * & \pi_{33} & \pi_{34} \\ * & * & * & \pi_{44} \end{array} \right] < 0, \\ &i = 1, \dots, m, \quad l, j, g = 1, 2, \dots, N_i, \end{aligned} \quad (16)$$

where $*$ denotes the symmetric term, where

$$\begin{aligned} \pi_{11} &= A_{il}^T P_i + P_i A_{il} - \alpha_{il} P_i B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{il}}^T P_i - \alpha_{ij} P_i B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \\ &\quad + \tau \left(A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \right)^T R_i \left(A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \right) \end{aligned}$$

$$\begin{aligned}
& + \tau (\varepsilon_1 + \varepsilon_2) \left(A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \right)^T \\
& \times \left(A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \right) + Q_i, \\
\pi_{12} & = P_i A_{hil}, \\
\pi_{13} & = P_i L_{il} C_{ig}, \\
\pi_{22} & = \tau \left[A_{hil}^T R_i A_{hil} + \varepsilon_1^{-1} A_{hil}^T R_i R_i A_{hil} + \varepsilon_3 A_{hil}^T A_{hil} \right] \\
& - (1 - d_i) Q_i, \\
\pi_{33} & = \tau \left[C_{ig}^T L_{il}^T R_i L_{il} C_{ig} + (\varepsilon_2^{-1} + \varepsilon_3^{-1}) C_{ig}^T L_{il}^T R_i R_i L_{il} C_{ig} \right] \\
& + (A_{il} - L_{il} C_{ig})^T P_e + P_e (A_{il} - L_{il} C_{ig}) \\
& + \tau (A_{il} - L_{il} C_{ig})^T R_e (A_{il} - L_{il} C_{ig}) + Q_e, \\
\pi_{34} & = P_e A_{hil} + \tau (A_{il} - L_{il} C_{ig})^T R_e A_{hil}, \\
\pi_{44} & = \tau A_{hil}^T R_e A_{hil} - (1 - d_i) Q_e,
\end{aligned} \tag{17}$$

then the closed-loop system (10a), (10b), and (10c) is asymptotically stable in zero by the reliable controllers (14b). The parameters of formula (14a), (14b) satisfy

$$L_{il} = \xi_{il} P_e^{-1} C_{il}^T, \quad i = 1, \dots, m, \quad l = 1, 2, \dots, N_i, \tag{18}$$

$$K_{ij} = \alpha_{ij} B_{\bar{\Theta}_{ij}}^T P_i, \quad i = 1, \dots, m, \quad j = 1, 2, \dots, N_i. \tag{19}$$

Remark 4. The controllers are reliable controller because of the controllers feedback gain matrix K_{ij} that are the functions of the actuators failures matrix $B_{\bar{\Theta}_{ij}}$. Moreover, differing from traditional switching system, the switching signal (8) in this paper is dependency on the state of observer.

Proof. Without loss of generality, suppose $\beta_{ir} \geq 0$. For any $i \in \bar{M}$, if

$$\hat{x}^T(t) (P_i - P_r) \hat{x}(t) \geq 0, \quad \forall r \in \bar{M}. \tag{20}$$

From (16), we can gain

$$\begin{aligned}
& \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
& \times \Xi^T(t) \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & 0 \\ * & \pi_{22} & 0 & 0 \\ * & * & \pi_{33} & \pi_{34} \\ * & * & * & \pi_{44} \end{bmatrix} \Xi(t) < 0,
\end{aligned} \tag{21}$$

where

$$\Xi(t) = \begin{bmatrix} \hat{x}(t) \\ \hat{x}(t - d_i(t)) \\ e(t) \\ e(t - d_i(t)) \end{bmatrix}. \tag{22}$$

Obviously, for $\forall \hat{x}(t) \in \mathbb{R}^n \setminus \{0\}$, there certainly is an $i \in \bar{M}$ such that

$$x^T(t) (P_i - P_r) x(t) \geq 0, \quad \forall r \in \bar{M}. \tag{23}$$

For arbitrary $i \in \bar{M}$, let

$$\Omega_i = \{ \hat{x}(t) \in \mathbb{R}^n \setminus \{0\} \mid \hat{x}^T(t) (P_i - P_r) \hat{x}(t) \geq 0, \forall r \in \bar{M} \}, \tag{24}$$

then $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n \setminus \{0\}$. Further, let construct the sets $\bar{\Omega}_1 = \Omega_1, \dots, \bar{\Omega}_i = \Omega_i - \bigcup_{j=1}^{i-1} \bar{\Omega}_j, \dots, \bar{\Omega}_m = \Omega_m - \bigcup_{j=1}^{m-1} \bar{\Omega}_j$. Obviously, we have $\bigcup_{i=1}^m \bar{\Omega}_i = \mathbb{R}^n \setminus \{0\}$ and $\bar{\Omega}_i \cap \bar{\Omega}_j = \Phi, i \neq j$.

To proceed further, choose the Lyapunov functional candidate of the following form:

$$\begin{aligned}
V_i(\hat{x}(t), e(t)) & = V_1(t) + V_2(t) + V_3(t) + V_4(t) \\
& + V_5(t) + V_6(t),
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
V_1(\hat{x}(t)) & = \hat{x}^T(t) P_i \hat{x}(t), \\
V_2(\hat{x}(t)) & = \int_{-\tau}^0 \int_{t+\theta}^t \hat{x}^T(s) R_i \dot{\hat{x}}(s) ds d\theta, \\
V_3(\hat{x}(t)) & = \int_{t-d_i(t)}^t \hat{x}^T(s) Q_i \hat{x}(s) ds, \\
V_4(e(t)) & = e^T(t) P_e e(t), \\
V_5(e(t)) & = \int_{-\tau}^0 \int_{t+\theta}^t e^T(s) R_e \dot{e}(s) ds d\theta, \\
V_6(e(t)) & = \int_{t-d_i(t)}^t e^T(s) Q_e e(s) ds,
\end{aligned} \tag{26}$$

where P_i, R_i, Q_i, P_e, R_e , and Q_e are positive definite matrixes satisfying inequalities (16).

Thereafter, we will show that under the switching law (8), along any nonzero solutions of system (2a), (2b), and (2c), $\dot{V}_i(\hat{x}(t), e(t)) < 0$. When $\hat{x}(t) \in \bar{\Omega}_i$, from switching law (8), it is known that the i th subsystem of system (10a), (10b), and (10c) is active. Thus, we have

$$\begin{aligned}
& \dot{V}_1(\hat{x}(t)) \\
& = \dot{\hat{x}}^T(t) P_i \hat{x}(t) + \hat{x}^T(t) P_i \dot{\hat{x}}(t) \\
& = \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
& \times \left\{ \hat{x}^T(t) \left[(A_{il} - B_{il} K_{ij})^T P_i + P_i (A_{il} - B_{il} K_{ij}) \right] \hat{x}(t) \right. \\
& + e^T(t) (L_{il} C_{ig})^T P_i \hat{x}(t) + \hat{x}^T(t) P_i L_{il} C_{ig} e(t) \\
& + \hat{x}^T(t - d_i(t)) A_{hil}^T P_i \hat{x}(t) + \hat{x}^T(t) P_i A_{hil} \hat{x}(t - d_i(t)) \left. \right\} \\
& \leq \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t))
\end{aligned}$$

$$\begin{aligned}
& \times \left\{ \hat{x}^T(t) \left[\left(A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \right)^T P_i + P_i \right. \right. \\
& \quad \times \left. \left(A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \right) \right] \hat{x}(t) + e^T(t) (L_{il} C_{ig})^T \\
& \quad \times P_i \hat{x}(t) + \hat{x}^T(t) P_i L_{il} C_{ig} e(t) + \hat{x}^T(t - d_i(t)) A_{hil}^T \\
& \quad \times P_i \hat{x}(t) + \hat{x}^T(t) P_i A_{hil} \hat{x}(t - d_i(t)) \left. \right\}. \tag{27}
\end{aligned}$$

Thus, it follows further that

$$\begin{aligned}
& \dot{V}_2(\hat{x}(t)) \\
& = \tau \dot{\hat{x}}^T(t) R_i \dot{\hat{x}}(t) - \int_{t-\tau}^t \dot{\hat{x}}^T(s) R_i \dot{\hat{x}}(s) ds \\
& \leq \tau \dot{\hat{x}}^T(t) R_i \dot{\hat{x}}(t) - \int_{t-d_i(t)}^t \dot{\hat{x}}^T(s) R_i \dot{\hat{x}}(s) ds \\
& \leq \tau \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
& \quad \times \left\{ \hat{x}^T(t) (A_{il} - B_{il} K_{ij})^T R_i (A_{il} - B_{il} K_{ij}) \hat{x}(t) \right. \\
& \quad + \hat{x}^T(t) (A_{il} - B_{il} K_{ij})^T R_i A_{hil} \hat{x}(t - d_i(t)) \\
& \quad + \hat{x}^T(t - d_i(t)) A_{hil}^T R_i (A_{il} - B_{il} K_{ij}) \hat{x}(t) \\
& \quad + \hat{x}^T(t) (A_{il} - B_{il} K_{ij})^T R_i L_{il} C_{ig} e(t) \\
& \quad + \hat{x}^T(t - d_i(t)) A_{hil}^T R_i A_{hil} \hat{x}(t - d_i(t)) \\
& \quad + \hat{x}^T(t - d_i(t)) A_{hil}^T R_i L_{il} C_{ig} e(t) \\
& \quad + e^T(t) C_{ig}^T L_{il}^T R_i (A_{il} - B_{il} K_{ij}) \hat{x}(t) \\
& \quad + e^T(t) C_{ig}^T L_{il}^T R_i A_{hil} \hat{x}(t - d_i(t)) \\
& \quad \left. + e^T(t) C_{ig}^T L_{il}^T R_i L_{il} C_{ig} e(t) \right\} - \int_{t-d_i(t)}^t \dot{\hat{x}}^T(s) R_i \dot{\hat{x}}(s) ds \\
& \leq \tau \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
& \quad \times \left\{ \hat{x}^T(t) \left[\left(A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \right)^T R_i (A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i) \right. \right. \\
& \quad \left. \left. + (\varepsilon_1 + \varepsilon_2) (A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i)^T \right. \right. \\
& \quad \left. \left. \times A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \right] \hat{x}(t) + \hat{x}^T(t - d_i(t)) \right. \\
& \quad \left. \times [A_{hil}^T R_i A_{hil} + \varepsilon_1^{-1} A_{hil}^T R_i R_i A_{hil} + \varepsilon_3 A_{hil}^T A_{hil}] \right. \\
& \quad \left. \times \hat{x}(t - d_i(t)) + e^T(t) \right\}
\end{aligned}$$

$$\begin{aligned}
& \times [C_{ig}^T L_{il}^T R_i L_{il} C_{ig} + (\varepsilon_2^{-1} + \varepsilon_3^{-1}) C_{ig}^T L_{il}^T R_i R_i L_{il} C_{ig}] \\
& \times e(t) \left. \right\} - \int_{t-d_i(t)}^t \dot{\hat{x}}^T(s) R_i \dot{\hat{x}}(s) ds. \tag{28}
\end{aligned}$$

Notice that from Assumption 1, the following can be obtained:

$$\begin{aligned}
& \dot{V}_3(\hat{x}(t)) \\
& = \hat{x}^T(t) Q_i \hat{x}(t) - (1 - \dot{d}_i(t)) \hat{x}^T(t - d_i(t)) Q_i \hat{x}(t - d_i(t)) \\
& \leq \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
& \quad \times \left\{ \hat{x}^T(t) Q_i \hat{x}(t) - (1 - \dot{d}_i(t)) \hat{x}^T(t - d_i(t)) Q_i \hat{x}(t - d_i(t)) \right\}. \tag{29}
\end{aligned}$$

It holds that

$$\begin{aligned}
& \dot{V}_4(e(t)) \\
& = \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
& \quad \times \left\{ e^T(t - d_i(t)) A_{hil}^T P_e e(t) + e^T(t) P_e A_{hil} e(t - d_i(t)) \right. \\
& \quad \left. + e^T(t) \left[(A_{il} - L_{il} C_{ig})^T P_e + P_e (A_{il} - L_{il} C_{ig}) \right] e(t) \right\}. \tag{30}
\end{aligned}$$

From $0 \leq d_\sigma(t) \leq \tau$, it is clear that

$$\begin{aligned}
& \dot{V}_5(e(t)) \\
& = \tau \dot{e}^T(t) R_e \dot{e}(t) - \int_{t-\tau}^t \dot{e}^T(s) R_e \dot{e}(s) ds \\
& \leq \tau \dot{e}^T(t) R_e \dot{e}(t) - \int_{t-d_i(t)}^t \dot{e}^T(s) R_e \dot{e}(s) ds \\
& \leq \tau \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} \sum_{g=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
& \quad \times \left\{ e^T(t) (A_{il} - L_{il} C_{ig})^T R_e (A_{il} - L_{il} C_{ig}) e(t) \right. \\
& \quad + e^T(t) (A_{il} - L_{il} C_{ig})^T R_e A_{hil} e(t - d_i(t)) \\
& \quad + e^T(t - d_i(t)) A_{hil}^T R_e (A_{il} - L_{il} C_{ig}) e(t) \\
& \quad + e^T(t - d_i(t)) A_{hil}^T R_e A_{hil} e(t - d_i(t)) \left. \right\} \\
& \quad - \int_{t-d_i(t)}^t \dot{e}^T(s) R_e \dot{e}(s) ds. \tag{31}
\end{aligned}$$

It is easy to derive the following:

$$\begin{aligned}
 \dot{V}_6(e(t)) &= e^T(t) Q_e e(t) - (1 - d_i(t)) e^T(t - d_i(t)) Q_e e(t - d_i(t)) \\
 &\leq \sum_{i=1}^m \sum_{l=1}^{N_i} \sum_{j=1}^{N_i} v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
 &\quad \times \{e^T(t) Q_e e(t) - (1 - d_i(t)) e^T(t - d_i(t)) Q_e e(t - d_i(t))\}. \quad (32)
 \end{aligned}$$

At this point of the argument, we should note that the output of the failure actuators is zero for $w_i \in \Theta_i$. Namely, only the residual normal actuators contribute to the time-delay switched fuzzy system. It holds true that

$$B_{il} K_{ij} = B_{\bar{w}_{il}} K_{ij}, \quad \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \leq B_{\bar{w}_{il}} K_{ij}. \quad (33)$$

Thus, when $\hat{x}(t) \in \bar{\Omega}_i$, we have

$$\begin{aligned}
 \dot{V}_i(\hat{x}(t), e(t)) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) \\
 &\quad + \dot{V}_5(t) + \dot{V}_6(t). \quad (34)
 \end{aligned}$$

Substituting (27), (28), (29), (30), (31), and (32) into (34) yields

$$\begin{aligned}
 \dot{V}_i(\hat{x}(t), e(t)) &\leq \Psi_i - \int_{t-d_i(t)}^t \hat{\hat{x}}^T(s) R_i \hat{\hat{x}}(s) ds \\
 &\quad - \int_{t-d_i(t)}^t \dot{e}^T(s) R_e \dot{e}(s) ds, \quad (35)
 \end{aligned}$$

where

$$\begin{aligned}
 \Psi_i &= \Xi^T(t) \Pi_i \Xi(t), \\
 \Pi_i &= \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & 0 \\ * & \pi_{22} & 0 & 0 \\ * & * & \pi_{33} & \pi_{34} \\ * & * & * & \pi_{44} \end{bmatrix}, \quad (36) \\
 \pi_{11} &= A_{il}^T P_i + P_i A_{il} - \alpha_{il} P_i B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i - \alpha_{ij} P_i B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i \\
 &\quad + \tau (A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i)^T R_i (A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i) \\
 &\quad + \tau (\varepsilon_1 + \varepsilon_2) (A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i)^T \\
 &\quad \times (A_{il} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T P_i) + Q_i, \\
 \pi_{12} &= P_i A_{hil},
 \end{aligned}$$

$$\pi_{13} = P_i L_{il} C_{ig},$$

$$\begin{aligned}
 \pi_{22} &= \tau [A_{hil}^T R_i A_{hil} + \varepsilon_1^{-1} A_{hil}^T R_i R_i A_{hil} + \varepsilon_3 A_{hil}^T A_{hil}] \\
 &\quad - (1 - d_i) Q_i,
 \end{aligned}$$

$$\begin{aligned}
 \pi_{33} &= \tau [C_{ig}^T L_{il}^T R_i L_{il} C_{ig} + (\varepsilon_2^{-1} + \varepsilon_3^{-1}) C_{ig}^T L_{il}^T R_i R_i L_{il} C_{ig}] \\
 &\quad + (A_{il} - L_{il} C_{ig})^T P_e + P_e (A_{il} - L_{il} C_{ig}) \\
 &\quad + \tau (A_{il} - L_{il} C_{ig})^T R_e (A_{il} - L_{il} C_{ig}) + Q_e,
 \end{aligned}$$

$$\pi_{34} = P_e A_{hil} + \tau (A_{il} - L_{il} C_{ig})^T R_e A_{hil},$$

$$\pi_{44} = \tau A_{hil}^T R_e A_{hil} - (1 - d_i) Q_e.$$

(37)

From (16) and (35), we know that $\dot{V}_i(\hat{x}(t), e(t)) < 0$ are well defined under the switching law (8) for arbitrary $\hat{x}(t) \neq 0$ and $e(t) \neq 0$. According to (35), we can conclude that the closed-loop system of (10a), (10b), and (10c) is asymptotically stable via the state-feedback controller (14b) under the switching law (8) when the actuators are serious failure, and the observer error $e(t)$ asymptotically converges to zero. This ends the proof. $\square$

Moreover, we reformulate (36) into LMI problem. Considering (36) and multiplying both sides of (36) by the matrix $\text{diag}\{P_i^{-1}, I, I, I\}$, (36) can be reformulated as follows:

$$\Pi_i = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & 0 \\ * & \gamma_{22} & 0 & 0 \\ * & * & \gamma_{33} & \gamma_{34} \\ * & * & * & \gamma_{44} \end{bmatrix}, \quad (38)$$

$$\gamma_{11} = P_i^{-1} A_{il}^T + A_{il} P_i^{-1} - \alpha_{il} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T$$

$$+ \tau (A_{il} P_i^{-1} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T)^T R_i (A_{il} P_i^{-1} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T)$$

$$+ \tau (\varepsilon_1 + \varepsilon_2) (A_{il} P_i^{-1} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T)^T$$

$$\times (A_{il} P_i^{-1} - \alpha_{ij} B_{\bar{\Theta}_{il}} B_{\bar{\Theta}_{ij}}^T) + P_i^{-1} Q_i P_i^{-1},$$

$$\gamma_{12} = A_{hil},$$

$$\gamma_{13} = L_{il} C_{ig},$$

$$\gamma_{22} = \tau [A_{hil}^T R_i A_{hil} + \varepsilon_1^{-1} A_{hil}^T R_i R_i A_{hil} + \varepsilon_3 A_{hil}^T A_{hil}]$$

$$- (1 - d_i) Q_i,$$

$$\begin{aligned}
\gamma_{33} &= \tau \left[C_{ig}^T L_{il}^T R_i L_{il} C_{ig} + (\varepsilon_2^{-1} + \varepsilon_3^{-1}) C_{ig}^T L_{il}^T R_i L_{il} C_{ig} \right] \\
&+ (A_{il} - L_{il} C_{ig})^T P_e + P_e (A_{il} - L_{il} C_{ig}) \\
&+ \tau (A_{il} - L_{il} C_{ig})^T R_e (A_{il} - L_{il} C_{ig}) + Q_e, \\
\gamma_{34} &= P_e A_{hil} + \tau (A_{il} - L_{il} C_{ig})^T R_e A_{hil}, \\
\gamma_{44} &= \tau A_{hil}^T R_e A_{hil} - (1 - d_i) Q_e.
\end{aligned} \tag{39}$$

We restate as follows.

Theorem 5. Suppose that the switched fuzzy system (10a), (10b), and (10c) satisfies Assumption 1. If there exist constants β_{ir} ($i, r \in \bar{M}$) (either all nonnegative or all non-positive),

positive definite matrixes P_i , R_i , Q_i , P_e , R_e , Q_e , and a group of positive scalars constants α_{ij} , ξ_{il} , ε_1 , ε_2 , ε_3 , satisfying the inequalities

$$\begin{bmatrix} \gamma_{11} + \sum_{r=1}^m \beta_{ir} (P_i - P_r) & \gamma_{12} & \gamma_{13} & 0 \\ * & \gamma_{22} & 0 & 0 \\ * & * & \gamma_{33} & \gamma_{34} \\ * & * & * & \gamma_{44} \end{bmatrix} < 0, \tag{40}$$

$$i = 1, \dots, m, \quad l, j, g = 1, 2, \dots, N_i,$$

where $*$ denotes the symmetric term, then the closed-loop system (10a), (10b), and (10c) is asymptotically stable in zero by the reliable controllers (14b). The parameters of formulae (14a) and (14b) satisfy (18) and (19).

It should be noted that the stability conditions (40) can be transformed into the linear matrix inequalities by using Schur's complement. Consider the following:

$$\begin{bmatrix} \sigma_{11} + \sum_{r=1}^m \beta_{ir} (P_i - P_r) & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} & 0 & \sigma_{17} & 0 & 0 & 0 & 0 \\ * & \sigma_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \sigma_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \sigma_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \sigma_{55} & \sigma_{56} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \sigma_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \sigma_{77} & \sigma_{78} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \sigma_{88} & \sigma_{89} & 0 & 0 \\ * & * & * & * & * & * & * & * & \sigma_{99} & \sigma_{910} & 0 \\ * & * & * & * & * & * & * & * & * & \sigma_{1010} & \sigma_{1011} \\ * & * & * & * & * & * & * & * & * & * & \sigma_{1111} \end{bmatrix} < 0, \tag{41}$$

$$i = 1, \dots, m, \quad l, j, g = 1, 2, \dots, N_i,$$

where $*$ denotes the symmetric term, where

$$\begin{aligned}
\sigma_{11} &= P_i^{-1} A_{il}^T + A_{il} P_i^{-1} - \alpha_{il} B_{\Theta_{il}}^T B_{\Theta_{il}}^T - \alpha_{ij} B_{\Theta_{il}}^T B_{\Theta_{ij}}^T, \\
\sigma_{12} &= P_i^{-1}, \\
\sigma_{13} &= (A_{il} P_i^{-1} - \alpha_{ij} B_{\Theta_{il}}^T B_{\Theta_{ij}}^T)^T, \\
\sigma_{14} &= (A_{il} P_i^{-1} - \alpha_{ij} B_{\Theta_{il}}^T B_{\Theta_{ij}}^T)^T, \\
\sigma_{15} &= A_{hil}, \\
\sigma_{56} &= A_{hil}^T R_i, \\
\sigma_{17} &= L_{il} C_{ig}, \\
\sigma_{78} &= C_{ig}^T L_{il}^T, \\
\sigma_{89} &= C_{ig}^T L_{il}^T R_i, \\
\sigma_{910} &= (A_{il} - L_{il} C_{ig})^T,
\end{aligned}$$

$$\sigma_{22} = -Q_i^{-1},$$

$$\sigma_{33} = -\tau^{-1} R_i^{-1},$$

$$\sigma_{44} = -\frac{I}{\tau(\varepsilon_1 + \varepsilon_2)},$$

$$\sigma_{55} = \tau [A_{hil}^T R_i A_{hil} + \varepsilon_3 A_{hil}^T A_{hil}] - (1 - d_i) Q_i,$$

$$\sigma_{66} = -\tau^{-1} \varepsilon_1 I,$$

$$\sigma_{77} = (A_{il} - L_{il} C_{ig})^T P_e + P_e (A_{il} - L_{il} C_{ig}) + Q_e,$$

$$\sigma_{88} = -\tau^{-1} R_i^{-1},$$

$$\sigma_{99} = -\tau^{-1} (\varepsilon_2^{-1} + \varepsilon_3^{-1})^{-1} I,$$

$$\sigma_{1010} = -\tau^{-1} R_e^{-1},$$

$$\sigma_{1011} = P_e A_{hil} + \tau (A_{il} - L_{il} C_{ig})^T R_e A_{hil},$$

$$\sigma_{1111} = \tau A_{hil}^T R_e A_{hil} - (1 - d_i) Q_e.$$

(42)

Remark 6. The Lyapunov function (34) is multi-Lyapunov containing positive definite matrixes P_i, R_i, Q_i, P_e, R_e , and Q_e . There will be some difficulty to seek positive definite matrixes, which induce some possible conservativeness different from the partition definite matrix method in [38]. However, we adopt the following methods to overcome the main sources of conservativeness.

- (1) Notice that the stable conditions in Theorem 5 are all of LMI form due to (41), we can solve the problem of stabilization converting to the LMIs' problem, and the method of augmenting matrix norm is avoided.
- (2) The maximum delay bound is obtained by solving LMIs containing variables P_i, R_i, Q_i, P_e, R_e , and Q_e instead of preselecting P_i, R_i, Q_i, P_e, R_e , and Q_e . Thus, optimizing positive definite matrices in order to obtain a less conservative delay bound can be easily realized.

4. Illustrative Numerical Example

To illustrate the effectiveness of proposed approach, the following switched fuzzy system with time-varying delay is considered:

$$\begin{aligned}
 &R_1^1: \text{ if } z \text{ is } M_{11}^1, \text{ then} \\
 &\dot{x}(t) = A_{11}x(t) + A_{h11}x(t - d_1(t)) + B_{11}u_1(t), \\
 &y(t) = C_{11}x(t), \\
 &R_1^2: \text{ if } z \text{ is } M_{11}^2, \text{ then} \\
 &\dot{x}(t) = A_{12}x(t) + A_{h12}x(t - d_1(t)) + B_{12}u_1(t), \\
 &y(t) = C_{12}x(t), \\
 &R_2^1: \text{ if } \delta \text{ is } M_{21}^1, \text{ then} \\
 &\dot{x}(t) = A_{21}x(t) + A_{h21}x(t - d_2(t)) + B_{21}u_2(t), \\
 &y(t) = C_{21}x(t), \\
 &R_2^2: \text{ if } \delta \text{ is } M_{21}^2, \text{ then} \\
 &\dot{x}(t) = A_{22}x(t) + A_{h22}x(t - d_2(t)) + B_{22}u_2(t), \\
 &y(t) = C_{22}x(t)
 \end{aligned} \tag{43}$$

along with system matrices

$$\begin{aligned}
 A_{11} &= \begin{bmatrix} -4.5 & 3 \\ 0 & 2 \end{bmatrix}, & A_{12} &= \begin{bmatrix} -18 & 2.5 \\ 0 & -11 \end{bmatrix}, \\
 A_{21} &= \begin{bmatrix} 1.5 & 0 \\ 1 & -4.5 \end{bmatrix}, & A_{22} &= \begin{bmatrix} -12 & 0 \\ -4 & -5 \end{bmatrix}, \\
 A_{h11} &= A_{h12} = \begin{bmatrix} 0.6 & 0.5 \\ 0 & 0.2 \end{bmatrix}, & A_{h21} &= A_{h22} = \begin{bmatrix} 0.1 & 0.6 \\ 0.4 & 0.3 \end{bmatrix},
 \end{aligned}$$

$$\begin{aligned}
 B_{11} &= B_{12} = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, & B_{21} &= B_{22} = \begin{bmatrix} 2 & 0 \\ 2 & 5 \end{bmatrix}, \\
 B_{\bar{\Theta}_{11}} &= B_{\bar{\Theta}_{12}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, & B_{\bar{\Theta}_{21}} &= B_{\bar{\Theta}_{22}} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}, \\
 C_{11} &= C_{12} = C_{21} = C_{22} = \begin{bmatrix} 2 & 0 \end{bmatrix}.
 \end{aligned} \tag{44}$$

It should be noted that $(A_{il}, B_{\bar{\Theta}_{il}})$ is not a stable pair system matrices. In turn, a subsystem is unstable when failures of actuators occur. Nonetheless, the reliable control problem is solvable via switching between unstable subsystems as proved in the previous section (see Theorem 3). If the time-varying delays are chosen as

$$d_1(t) = 0.003 - 0.003 \sin(t), \tag{45a}$$

$$d_2(t) = 0.01 - 0.01 \sin(t), \tag{45b}$$

then it is clear that $d_1 \leq 0.003$, $d_2 \leq 0.01$, and $\tau \leq 0.02$.

The membership functions, respectively, are chosen as,

$$\begin{aligned}
 \mu_{M_{11}^1}(z(t)) &= \frac{1}{1 + e^{-2z(t)}}, & \mu_{M_{11}^2}(z(t)) &= 1 - \mu_{M_{11}^1}, \\
 \mu_{M_{21}^1}(\delta(t)) &= \frac{1}{1 + e^{(-2(\delta(t)-0.3))}}, & \mu_{M_{21}^2}(\delta(t)) &= 1 - \mu_{M_{21}^1}.
 \end{aligned} \tag{46}$$

Then the parameters of controllers are defined:

$$\xi_{11} = \xi_{12} = 1, \quad \xi_{21} = \xi_{22} = 0.8, \quad \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.01. \tag{47}$$

Define the state observers as

$$\begin{aligned}
 \hat{\dot{x}}(t) &= \sum_{i=1}^2 \sum_{l=1}^2 \sum_{j=1}^2 \sum_{g=1}^2 v_i(\hat{x}(t)) \eta_{il}(\xi(t)) \eta_{ij}(\xi(t)) \eta_{ig}(\xi(t)) \\
 &\quad \times \left[(A_{il} - B_{il}K_{ij}) \hat{x}(t) + A_{hil} \hat{x}(t - d_i(t)) \right. \\
 &\quad \left. + L_{il}C_{ig}e(t) \right],
 \end{aligned} \tag{48}$$

upon choosing

$$\begin{aligned}
 \alpha_{11} &= \alpha_{12} = 3, & \alpha_{21} &= \alpha_{22} = 4, \\
 \beta_{ir} &= 1 \quad (i, r = 1, 2).
 \end{aligned} \tag{49}$$

Solving inequality (16), one can find that

$$\begin{aligned}
 P_1 &= \begin{bmatrix} 0.2034 & -0.1141 \\ -0.1141 & 0.9462 \end{bmatrix}, & P_2 &= \begin{bmatrix} 0.5998 & -0.0412 \\ -0.0412 & 1.5313 \end{bmatrix}, \\
 R_1 &= \begin{bmatrix} 0.3148 & -0.1214 \\ -0.1214 & 0.9789 \end{bmatrix}, & R_2 &= \begin{bmatrix} 0.6112 & -0.0291 \\ -0.0291 & 0.9891 \end{bmatrix},
 \end{aligned}$$

FIGURE 1: (a) The observer error trajectories of SFS with time-varying delay and (b) control signal of the reliable controller.

$$\begin{aligned}
 Q_1 &= \begin{bmatrix} 0.6471 & -0.2438 \\ -0.2438 & 1.1231 \end{bmatrix}, & Q_2 &= \begin{bmatrix} 0.9876 & -0.1128 \\ -0.1128 & 0.8532 \end{bmatrix}, \\
 P_e &= \begin{bmatrix} 0.4924 & -0.2089 \\ -0.2089 & 0.9479 \end{bmatrix}, & R_e &= \begin{bmatrix} 0.5133 & -0.2087 \\ -0.2087 & 0.8921 \end{bmatrix}, \\
 Q_e &= \begin{bmatrix} 0.6127 & -0.0921 \\ -0.0921 & 0.9432 \end{bmatrix}.
 \end{aligned} \tag{50}$$

From (18), the observer gain matrices are derived as follows:

$$L_{11} = L_{12} = \begin{bmatrix} 4.4807 \\ 0.9875 \end{bmatrix}, \quad L_{21} = L_{22} = \begin{bmatrix} 3.5845 \\ 0.7900 \end{bmatrix}. \tag{51}$$

Next, define

$$\begin{aligned}
 \Omega_1 &= \{ \hat{x}(t) \in \mathbb{R}^2 \mid \hat{x}(t)^T (P_1 - P_2) \hat{x}(t) \geq 0, \hat{x}(t) \neq 0 \}, \\
 \Omega_2 &= \{ \hat{x}(t) \in \mathbb{R}^2 \mid \hat{x}(t)^T (P_2 - P_1) \hat{x}(t) \geq 0, \hat{x}(t) \neq 0 \}.
 \end{aligned} \tag{52}$$

Then, $\Omega_1 \cup \Omega_2 = \mathbb{R}^2 \setminus \{0\}$. Thus the closed-loop system is asymptotic stability under the following switching law:

$$\sigma(t) = \begin{cases} 1 & \hat{x}(t) \in \Omega_1, \\ 2 & \hat{x}(t) \in \Omega_2 \setminus \Omega_1. \end{cases} \tag{53}$$

From (19), we obtain output feedback controller

$$\begin{aligned}
 K_{11} &= K_{12} = \begin{bmatrix} 0.6102 & -0.3423 \\ 0 & 0 \end{bmatrix}, \\
 K_{21} &= K_{22} = \begin{bmatrix} 0 & 0 \\ -0.8240 & 30.6260 \end{bmatrix}.
 \end{aligned} \tag{54}$$

A simulation has been performed with the initial states $x(0) = [-15, 3]^T$ and the time-varying delay chosen as (45a), (45b). The simulation results for the observer error and control signal are depicted in Figure 1.

5. Conclusion

In this paper, the problem of reliable control for nonlinear systems represented by time-varying delay switched fuzzy systems has been investigated. Firstly, a model of switched fuzzy systems which combines reliable control and switched fuzzy control was presented, and all subsystems of the switching system are time-varying delay fuzzy systems. In particular, attention was focused on actuators that may suffer “serious faults”.

Reliable controllers and switching law are designed such that they make the switched fuzzy systems asymptotic stability, based on the switching strategy, that is, switching between faulty actuators. Moreover, the switching strategy is dependent on the state of observer error. Finally, a numerical example has been provided to illustrate the efficiency of the proposed reliable controller design and observer switching law methodology.

Acknowledgments

This work is supported by National Nature Science Foundation under Grant 61004039, by the Science Foundation of Educational Department of Liaoning Province under Grant L2010374, and also by the key laboratory for manufacturing industrial integrated automation of Liaoning Province. The authors would like to thank Professor Ximing Sun for his valuable comments.

References

- [1] H.-N. Wu, J.-W. Wang, and H.-X. Li, “Design of distributed H_∞ fuzzy controllers with constraint for nonlinear hyperbolic PDE systems,” *Automatica*, vol. 48, no. 10, pp. 2535–2543, 2012.

- [2] L. G. Wu, X. J. Su, P. Shi, and J. Qiu, "Model approximation for discrete-time state-delay systems in the TS fuzzy framework," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 2, pp. 366–378, 2011.
- [3] X. J. Su, L. G. Wu, and P. Shi, "Sensor networks with random link failures: distributed filtering for T-S fuzzy systems," *IEEE Transactions on Industrial Informatics*, no. 99, 2012.
- [4] L. G. Wu, X. J. Su, P. Shi, and J. Qiu, "A new approach to stability analysis and stabilization of discrete-time T-S fuzzy time-varying delay systems," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 41, no. 1, pp. 273–286, 2011.
- [5] X. Liu and Q. Zhang, "New approaches to H_∞ controller designs based on fuzzy observers for T-S fuzzy systems via LMI," *Automatica*, vol. 39, no. 9, pp. 1571–1582, 2003.
- [6] X. J. Su, P. Shi, L. G. Wu, and Y. D. Song, "A novel control design on discrete-time Takagi-Sugeno fuzzy systems with time-varying delays," *IEEE Transactions on Fuzzy Systems*, no. 99, 2012.
- [7] K. Tanaka, T. Hori, and H. O. Wang, "A multiple Lyapunov function approach to stabilization of fuzzy control systems," *IEEE Transactions on Fuzzy Systems*, vol. 11, no. 4, pp. 582–589, 2003.
- [8] G. Chen and D. Zhang, "Back-driving a truck with suboptimal distance trajectories: a fuzzy logic control approach," *IEEE Transactions on Fuzzy Systems*, vol. 5, no. 3, pp. 369–380, 1997.
- [9] D. Filev and P. Angelov, "Fuzzy optimal control," *Fuzzy Sets and Systems*, vol. 47, no. 2, pp. 151–156, 1992.
- [10] M. Sugeno, "On stability of fuzzy systems expressed by fuzzy rules with singleton consequents," *IEEE Transactions on Fuzzy Systems*, vol. 7, no. 2, pp. 201–224, 1999.
- [11] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.
- [12] F. Delmotte, T. M. Guerra, and M. Ksantini, "Continuous Takagi-Sugeno's models: reduction of the number of LMI conditions in various fuzzy control design technics," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 3, pp. 426–438, 2007.
- [13] R. Palm and D. Driankov, "Fuzzy switched hybrid systems - modeling and identification," in *Proceedings of the IEEE International Symposium on Intelligent Control (ISIC '98)*, pp. 130–135, September 1998.
- [14] K. Tanaka, M. Iwasaki, and H. O. Wang, "Switching control of an R/C hovercraft: stabilization and smooth switching," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 31, no. 6, pp. 853–863, 2001.
- [15] K. Tanaka, M. Iwasaki, and H. O. Wang, "Stability and smoothness conditions for switching fuzzy systems," in *Proceedings of the American Control Conference*, pp. 2474–2478, June 2000.
- [16] K. Tanaka, M. Iwasaki, and H. O. Wang, "Stable switching fuzzy control and its application to a hovercraft type vehicle," in *Proceedings of the 9th IEEE International Conference on Fuzzy Systems (FUZZ-IEEE '00)*, pp. 804–809, May 2000.
- [17] W. J. Wang, Y. J. Chen, and C. H. Sun, "Relaxed stabilization criteria for discrete-time T-S fuzzy control systems based on a switching fuzzy model and piecewise Lyapunov function," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 37, no. 3, pp. 551–559, 2007.
- [18] H. Yang, L. Zhang, and X. Liu, "Robust tracking control for switched fuzzy systems with fast switching controller," *Mathematical Problems in Engineering*, vol. 2012, Article ID 872826, 21 pages, 2012.
- [19] L. Zhang and Y. W. Jing, "Robust adaptive reliable tracking control of uncertain switched fuzzy systems using T-S models," in *Proceedings of the International Conference on Impulsive and Hybrid Dynamical Systems*, pp. 910–913.
- [20] H. Ohtake, K. Tanaka, and H. O. Wang, "Fuzzy modeling via sector nonlinearity concept," *Integrated Computer-Aided Engineering*, vol. 10, no. 4, pp. 333–341, 2003.
- [21] G. Zhang, C. Han, Y. Guan, and L. Wu, "Exponential stability analysis and stabilization of discrete-time nonlinear switched systems with time delays," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 3A, pp. 1973–1986, 2012.
- [22] S. B. Attia, S. Salhi, and M. Ksouri, "Static switched output feedback stabilization for linear discrete-time switched systems," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 5, pp. 3203–3213, 2012.
- [23] J. Dai, J. Mao, W. Wan, Y. Li, and M. J. Er, "Fuzzy switching control law design for helicopters," in *Proceedings of the IEEE Region 10 Conference on Computers, Communications, Control and Power Engineering*, pp. 1634–1637, October 2002.
- [24] C. Y. Chang, S. W. Hsu, and K. H. Chiang, "The switching fuzzy controller of the overhead crane system," in *Proceedings of the IEEE International Conference on Control Applications*, pp. 753–758, Taipei, Taiwan, September 2004.
- [25] K. Izumi, K. Watanabe, K. Ichida, and Y. Tachibana, "The design of fuzzy energy regions optimized by GA for a switching control of multi-link underactuated manipulators," in *Proceedings of the International Conference on Control, Automation and Systems (ICCAS '07)*, pp. 24–29, October 2007.
- [26] L. Wu and D. W. C. Ho, "Fuzzy filter design for Itô stochastic systems with application to sensor fault detection," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 1, pp. 233–242, 2009.
- [27] G. S. Zhai and H. Lin, "Controller failure time analysis for symmetric H_∞ control systems," *International Journal of Control*, vol. 77, no. 6, pp. 598–605, 2004.
- [28] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–172, 2007.
- [29] B. Jiang, M. Staroswiecki, and V. Cocquempot, "Fault diagnosis based on adaptive observer for a class of non-linear systems with unknown parameters," *International Journal of Control*, vol. 77, no. 4, pp. 415–426, 2004.
- [30] Z. Sun, "A robust stabilizing law for switched linear systems," *International Journal of Control*, vol. 77, no. 4, pp. 389–398, 2004.
- [31] H. Wu, "Reliable LQ fuzzy control for continuous-time nonlinear systems with actuator faults," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 34, no. 4, pp. 1743–1752, 2004.
- [32] Q. Hu, B. Xiao, and M. I. Friswell, "Robust fault-tolerant control for spacecraft attitude stabilisation subject to input saturation," *IET Control Theory & Applications*, vol. 5, no. 2, pp. 271–282, 2011.
- [33] E. Tian, D. Yue, and C. Peng, "Reliable control for networked control systems with probabilistic sensors and actuators faults," *IET Control Theory and Applications*, vol. 4, no. 8, pp. 1478–1488, 2010.
- [34] R. Wang, J. Zhao, and G. M. Dimirovski, "Reliable H_∞ control for a class of switched nonlinear systems," in *Proceedings of the 16th IFAC World Congress*, pp. 1–6, IEEE, Prague, Czech Republic, 2005.
- [35] L. Zhang, X. Liu, and H. Yang, "Robust reliable control for a class of uncertain switched fuzzy systems based on observers

- switching,” in *Proceedings of the International Conference on Intelligent Control and Information Processing (ICICIP '10)*, pp. 207–212, August 2010.
- [36] L. Wu and W. X. Zheng, “Weighted H_∞ model reduction for linear switched systems with time-varying delay,” *Automatica*, vol. 45, no. 1, pp. 186–193, 2009.
 - [37] L. Wu, Z. Feng, and W. X. Zheng, “Exponential stability analysis for delayed neural networks with switching parameters: average dwell time approach,” *IEEE Transactions on Neural Networks*, vol. 21, no. 9, pp. 1396–1407, 2010.
 - [38] L. Wu and W. X. Zheng, “ L_2 - L_∞ control of nonlinear fuzzy itô stochastic delay systems via dynamic output feedback,” *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 39, no. 5, pp. 1308–1315, 2009.
 - [39] R. Yang, P. Shi, G.-P. Liu, and H. Gao, “Network-based feedback control for systems with mixed delays based on quantization and dropout compensation,” *Automatica*, vol. 47, no. 12, pp. 2805–2809, 2011.
 - [40] X.-M. Sun, G.-P. Liu, D. Rees, and W. Wang, “Stability of systems with controller failure and time-varying delay,” *Institute of Electrical and Electronics Engineers*, vol. 53, no. 10, pp. 2391–2396, 2008.
 - [41] X. Sun, W. Wang, G. Liu, and J. Zhao, “Stability analysis for linear switched systems with time-varying delay,” *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 38, no. 2, pp. 528–533, 2008.
 - [42] X.-M. Sun, G.-P. Liu, W. Wang, and D. Rees, “Stability analysis for networked control systems based on average dwell time method,” *International Journal of Robust and Nonlinear Control*, vol. 20, no. 15, pp. 1774–1784, 2010.