Research Article

A New Constellation Configuration Scheme for Communicating Architecture in Cislunar Space

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A new constellation configuration is proposed to provide near-constant communications between any two points in the cislunar space. Four satellites were allocated on some natural orbits near LL_2 , LL_4 , LL_5 , and LL_3 points, the first three of which are expected to supply a global coverage on the Moon, and the last three afford a global coverage on the Earth. Considering the real dynamics and geometries, a full-ephemeris model is employed to refine the quasi-periodic trajectories near LL_2 , LL_3 , LL_4 , and LL_5 points by a multiple shooting corrector numerically. The revisiting intervals between the successive revisits on specified points by the constellation are demonstrated both on the geographical maps of the Earth and the Moon, with a consideration of equatorial inclinations with respect to the Earth-Moon orbital plane. Moreover, the coverage periodicity on the Moon is induced by Metonic cycle of 19 years. Cost analysis on stationkeeping and launching is implemented for this constellation in detail. Finally, some aspects about communications, such as satellite links, delays, ground stations, and communication devices, are surveyed preliminarily.

1. Introduction

With the increased interests in lunar exploration and development, autonomous and manned lunar surface missions will become more and more frequent in the future decades. In particular, with the construct of lunar laboratory and lunar base, a large number of data and images need to be transmitted from Moon to Earth (or reverse). Some of the missions, for example, the manned exploration on the lunar surface, require real-time communication. The communicating architecture in cislunar space, therefore, becomes a crucial problem.

Since 1960s many scholars have contributed a lot to the design of satellite configuration architecture. An important task of the constellation design is to make satellites own some desired coverage to the surface of the expected planet. Three satellites placed on the geostationary orbits equidistantly will almost cover the surfaces of the Earth [1]. To achieve the multiple coverage, Walker [2] developed a common methodology to design the magical Walker- δ constellation, which has

potential applications in the global position system (GPS). Ballard [3] designed rosette constellations with flowerlike orbital patterns to achieve invariant coverage on different deployment attitudes or orientations relative to Earth coordinates. Most of the significant works were employing Kepler orbit (or perturbed Kepler orbit) around the single planet to provide continuous communication between any points on the Earth. However, the constellation yielded in context of restricted two body problem is weak in building the link between the two separated planets, like Earth and Moon.

Because of the nature of the solutions, some useful architectures for lunar coverage may exist within the context of the restricted three-body problem (R3BP). Farquhar [4, 5] used a multibody model to design trajectories for communications between the far side of the Moon and the Earth. As one family of natural solution of R3BP, libration point orbits have potential uses in lunar constellation architecture to ensure adequate coverage. Chow et al. [6] considered that the most cost for the low Earth orbit constellation was the launch costs and reduced cost by using a single large launch vehicle

to carry an entire fleet of microsatellites to the LL_1 point. Grebow [7] created architectures by quasi-periodic orbits around LL_1 and LL_2 points for continuous coverage of the south pole of the Moon with only two spacecrafts. Bhasin and Hayden [8] adopted libration point orbits in deep space communicating architecture developed for NASA's future exploration and science missions. Kulkarni et al. [9] postulated a set of satellites placed in Halo orbits at Earth-Moon, Sun-Earth, and Sun-Mars libration points and a couple of relay satellites in a heliocentric earth orbit to support data and command communication for future space missions to Moon, Mars, and beyond. Lee et al. [10, 11] proposed a conceptual constellation to substitute geostationary orbit by three satellites placed on LL_3 , LL_4 , and LL_5 points; however, they did give the feasibility and coverage analysis in detail. Grebow et al. [12] investigated some natural orbits in full ephemeris model with periods ranging from 7 to 16 days and employed two spacecrafts in combination with Earth-Moon libration point orbits to achieve constant communications for lunar south pole exploration mission.

Quite different from the researches referred above, a new constellation configuration is proposed to provide near-constant communications between any two points in the cislunar space in this paper, with four satellites allocated on some natural orbits near LL_2 , LL_4 , LL_5 , and LL_3 points, respectively. The first three satellites in this communicating architecture are expected to supply a global coverage on Moon, while the last three will afford a global coverage on Earth. The constellation is beneficial for the communications between different points on the grounds of Earth or Moon, even for the case with no direct links as one is located at the far side of the Moon and another is placed on the Earth.

Considering the real dynamics and geometries, a full ephemeris model is employed to refine the quasi-periodic trajectories near LL_2 , LL_3 , LL_4 , and LL_5 points by a multiple shooting corrector numerically. The revisiting intervals between the successive revisits on specified points by the constellation are demonstrated on the geographical maps of both the Earth and the Moon, with a consideration of equatorial inclinations with respect to the Earth-Moon orbital plane. Moreover, the coverage periodicity on the Moon is induced by Metonic cycle of 19 years.

Furthermore, cost analysis on stationkeeping and launching is implemented in detail in full ephemeris model. According to the stationkeeping and inserting strategies proposed in this paper, the total inserting cost of this constellation is 14.4731 km/s, and the total stationkeeping cost of this constellation is 261.8 m/s/year.

Compared with two geostationary orbit constellations that require six spacecrafts to cover both the Earth and Moon, this constellation architecture saves two spacecrafts to reduce the cost of the system a lot, which is considered as an alternative of limited geostationary orbit resources.

2. Design of Constellation Architecture

2.1. Constellation Configuration Scheme. To describe the global coverage constellation in cislunar space, four different

coordinates are referred to in this paper, as the syzygy frame in Earth-Moon system, the Earth-centered fixed frame, (i.e., WGS-84 frame), the Moon-centered fixed frame, and the Earth-centered inertial frame (i.e., J2000 frame), where the syzygy frame has the following features: the origin O is fixed at the barycenter of the Earth-Moon system; the z -axis is perpendicular to the lunar orbital plane; the x -axis points from Earth to Moon; and the y -axis is determined by the right-hand side rule.

Ignored other celestial bodies' gravitation and eccentricity of lunar orbit, the dynamics of system consisted of spacecraft, Earth, and Moon can be regarded as the circular restricted three-body problem (CR3BP). There are five libration points fixed in the syzygy frame, labeled as LL_i , $i = 1, \dots, 5$, where LL_2 and LL_3 locate at the far side of the Moon and Earth, respectively, and LL_4 and LL_5 form an equilateral triangle with Earth and Moon.

The combination of LL_2 , LL_4 , LL_5 , and LL_3 shapes a double-triangle configuration, where the first three points product one triangle focusing at the Moon, while the last three form the other triangle centering the Earth. By the inspiration of the global coverage constellation created by three satellites equispaced on the geostationary orbit, a new communicating architecture is proposed to provide some links (with time delay of 2-3 sec) between any two points in the cislunar space by LL_2 , LL_4 , LL_5 , and LL_3 points, shown in Figure 1. With the help of this constellation system, a direct or indirect link exists for any two points in cislunar space, even for the two points P_M and P_E located on the far side of Earth and Moon respectively (one of the feasible indirect links is as follows: $P_M \rightarrow LL_2 \rightarrow LL_4 \rightarrow LL_3 \rightarrow P_E$). Moreover, the quasi-halo orbit near LL_2 inheriting from halo family is employed to avoid some links from LL_2 point to Earth blocked by the Moon.

2.2. Numerical Refinements by Multiple Shooting Corrector. The eccentricity of the Moon's orbit and gravitational forces from other celestial bodies (like the Sun and the Jupiter) will make CR3BP different from the real dynamics, even the libration point cannot keep stationary due to this perturbations.

Therefore, a multiple shooting corrector is employed to create some natural orbits near the libration points in the full ephemeris model, which is useful for the computation of highly unstable periodic orbits with very long periods, as J_2 invariant relative orbits [13, 14] and solar sail's displaced orbits [15].

The general idea is to use a multiple shooting method similar to the one used for the numerical solution of boundary value problems. As in the iteration procedure, the total time span $[t_0, t_f]$ is split into a number of shorter subintervals by N equispaced points $t_0, t_1, t_2, \dots, t_{N-1}, t_f$. Denote the subinterval with equal length as $\Delta t = t_{i+1} - t_i$ ($i = 0, 1, 2, \dots, N-1$), and denote the point on the quasi-periodic trajectory as $\mathbf{Q}_i = [t_i, x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i]^T$ ($i = 0, 1, 2, \dots, N-1$). Let $\phi(\mathbf{Q}_i)$ be the image of the point $\mathbf{Q}_i$ under the flow associated to the full ephemeris dynamics after an amount of time Δt . To keep all the points $\mathbf{Q}_i$ staying on the same trajectory solved from

FIGURE 1: The communicating constellation architecture in cislunar space. LL_2 , LL_4 , and LL_5 form one triangle to supply a global coverage on Moon; LL_3 , LL_4 , and LL_5 form the other triangle points to supply a global coverage on Earth.

the dynamics, some corrections to $\mathbf{Q}_i$ are required to fulfill the matching conditions, as

$$\mathbf{F} \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \\ \vdots \\ \mathbf{Q}_N \end{pmatrix} = \begin{pmatrix} \phi(\mathbf{Q}_1) \\ \phi(\mathbf{Q}_2) \\ \vdots \\ \phi(\mathbf{Q}_{N-1}) \end{pmatrix} - \begin{pmatrix} \mathbf{Q}_2 \\ \mathbf{Q}_3 \\ \vdots \\ \mathbf{Q}_N \end{pmatrix} = 0. \quad (1)$$

In the j th iteration of this procedure, the matching equation can be written as

$$\left. \frac{\partial \mathbf{F}}{\partial \mathbf{Q}} \right|_{\mathbf{Q}^{(j)}} \cdot [\mathbf{Q}^{(j+1)} - \mathbf{Q}^{(j)}] = -\mathbf{F}|_{\mathbf{Q}^{(j)}}, \quad (2)$$

where $\mathbf{Q}^{(j)} = [\mathbf{Q}_1^{(j)} \mathbf{Q}_2^{(j)} \dots \mathbf{Q}_N^{(j)}]^T$. The differential function of $\mathbf{F}$ has the following expression:

$$\frac{\partial \mathbf{F}}{\partial \mathbf{Q}} = \begin{pmatrix} \left. \frac{\partial \phi}{\partial \mathbf{Q}} \right|_{\mathbf{Q}_1^{(j)}} & -\mathbf{I} & & & \\ & \left. \frac{\partial \phi}{\partial \mathbf{Q}} \right|_{\mathbf{Q}_2^{(j)}} & -\mathbf{I} & & \\ & & \ddots & \ddots & \\ & & & \left. \frac{\partial \phi}{\partial \mathbf{Q}} \right|_{\mathbf{Q}_{N-1}^{(j)}} & -\mathbf{I} \end{pmatrix}. \quad (3)$$

The iterations have been included in the software package OrbitalPredictor developed by Beihang University [16], which adapts JPL ephemeris model to generate some natural orbits near the libration points.

To accelerate the convergence of the algorithm, an initial iteration to the natural solution of the dynamics is required. For LL_3 , LL_4 , and LL_5 , the libration points themselves can work as the initial iterations to create bounded trajectories near these points; however, for a quasi-halo orbit near LL_2

point, an effective initial iteration can employ the following Richardson expansion developed from the Lindstedt-Poincaré method [17, 18]:

$$\begin{aligned} x &= a_{21}A_x^2 + a_{22}A_z^2 - A_x \cos \omega t + (a_{23}A_x^2 - a_{24}A_z^2) \cos 2\omega t \\ &\quad + (a_{31}A_z^3 - a_{32}A_xA_z^2) \cos 3\omega t, \\ y &= \kappa A_x \sin \omega t + (b_{21}A_x^2 - b_{22}A_z^2) \sin 2\omega t \\ &\quad + (b_{31}A_x^3 - b_{32}A_xA_z^2) \sin 3\omega t, \\ x &= \pm A_z \cos \omega t \pm d_{21}A_z [\cos 2\omega t - 3] \\ &\quad \pm (d_{32}A_zA_x^2 - d_{31}A_z^3) \cos 3\omega t, \end{aligned} \quad (4)$$

where the detailed discussion on the coefficients a , b , c , ω , and κ can be found in [17]. To keep the frequencies in z -axis equal to x - and y -axes, the amplitudes A_x and A_z are dependent on each other and also need to be no less than some critical values. Therefore, the amplitude A_x can be selected as the only parameter to identify different halo orbits, and then a halo orbit with the minimal amplitude $A_x = 11500$ km is created by the iteration algorithm in this paper.

Following the initial iterations proposed for LL_2 , LL_3 , LL_4 , and LL_5 referred to above, some bounded trajectories can be refined by the OrbitalPredictor full-ephemeris model shown in Figure 2 and then are targeted in Satellite Tool Kit (STK) using Astrogator tool [12, 19], which are shown in Figure 3 by STK.

3. Coverage Analysis of Constellation

The revisiting time is employed to parameterize the coverage of this constellation architecture, which is defined as the time intervals between the successive revisits on the specified ground point on the Earth or Moon. Therefore, this concept is measuring the interruption durations when communicating with any point.

FIGURE 2: The bounded trajectories near LL_2 , LL_3 , LL_4 , and LL_5 created by OrbitalPredictor with the epoch time of 12:00, July 1, 2015. The quasi-periodic trajectories near (a) LL_2 point, (b) LL_3 point, (c) LL_4 point, and (d) LL_5 point.

3.1. Coverage to Moon by the Constellation. As the preliminary investigation on the coverage to Moon, some hypotheses can be adopted to plot the covering region, including the following: the inclination of the equator plane with respect to the lunar orbital plane is neglected as 0° , instead of its true value of $6^\circ 41'$; the eccentricity of lunar orbit and the gravitation from the Sun are ignored to place the spacecrafts on LL_3 , LL_4 , and LL_5 points and the halo orbit near LL_2 without any stationkeeping maneuver; that is, the libration point orbits are yielded from CR3BP. Due to the osculating position on halo orbit, the maximal and common regions are achieved in Figure 4, where the former is defined as the region covered by the constellation at any time, while the latter is defined as the region covered all the time. Therefore, the constellation can cover the lunar surface by 99.45% (maximal region) and 98.95% (common region) except the south and north poles.

In a full investigation on the coverage to Moon, no hypothesis is employed, including the real geometries and the full dynamics. All the bounded trajectories in this constellation are refined by the OrbitalPredictor full-ephemeris model, and the revisiting intervals are counted statistically in real geometrical scenario by STK.

The Moon is in synchronous rotation: it rotates about its axis in about the same time it takes to orbit the Earth; however, due to the inclination between lunar orbital and equator planes and some others, there are several librations to change the angle from which the Moon is seen, allowing about 59% of its surface to be seen from the Earth.

Moreover, the 19-year Metonic cycle of the equatorial inclination, with respect to lunar orbital plane, makes the Moon go overhead at latitudes over the equator from 18.5° to 28.5° , so it is necessary to give results in term of time in a whole cycle. Figure 5 depicts the revisiting times on the lunar

FIGURE 3: The quasi-periodic trajectories near LL_2 , LL_3 , LL_4 , and LL_5 created by STK with the epoch time of 12:00, July 1, 2015. The bounded trajectories are shown in (a) the syzygy frame in Earth-Moon system and (b) the Earth-centered inertial frame. The ground tracks of the trajectories are shown in (c) the Earth-centered fixed frames and (d) the Moon-centered fixed frames.

FIGURE 4: The preliminary analysis on covering region by the constellation on the lunar surface. (a) The maximal region covered by the constellation at any time; (b) the common region covered by the constellation all the time.

surface by the constellations created from different epoch times in a Metonic cycle. The maximal coverage for full geometries is investigated by the maximal revisiting times during this cycle on the specified point; however, the common coverage is yielded by the minimal revisiting times during this cycle, and then both the maximal and minimal revisiting times are revealed in Figure 6 on the global surface.

Different from the preliminary results, some conclusions can be induced from Figures 5 and 6. Firstly, all the points on the lunar surface can be covered by this constellation, where most of them are accessed without any time interval (i.e., the blank regions in the top of Figure 6), and few of them are

revisited after some intervals less than 5 days (i.e., the colored regions in the top of Figure 6).

Secondly, due to the lunar synchronous rotation, this constellation is weak in covering some longitude lines located near 150°W and 150°E shown in Figure 5. The discontinuous communications are interrupted by some intervals less than 2 hours.

Thirdly, only some sites keep discontinuous communications during a whole Metonic cycle, which are marked in the bottom of Figure 6 from $20^\circ\text{N}140^\circ\text{W}$ to $70^\circ\text{N}140^\circ\text{W}$, from $30^\circ\text{S}150^\circ\text{W}$ to $2^\circ\text{N}150^\circ\text{W}$, from $85^\circ\text{S}55^\circ\text{E}$ to $85^\circ\text{S}100^\circ\text{E}$, and from $80^\circ\text{S}170^\circ\text{E}$ to $80^\circ\text{S}130^\circ\text{W}$.

FIGURE 5: The full analysis on revisiting time on the lunar surface during a whole Metonic cycle from 2015 to 2034. The coverage analyses are implemented by the epoch times of (a) 12:00, July 1, 2015, (b) 12:00, July 1, 2018, (c) 12:00, July 1, 2021, (d) 12:00, July 1, 2024, (e) 12:00, July 1, 2027, (f) 12:00, July 1, 2030, and (g) 12:00, July 1, 2033.

Fourthly, the worst intervals occur at the high-latitude areas instead of the poles because the maximal revisiting time is 0.96 day at the north pole while 1.21 days at the south pole. Thus this constellation has a good coverage performance at the poles.

3.2. Coverage to Earth by the Constellation. As the preliminary investigation on the coverage, the irrotational Earth is assumed in combination with no consideration of the eccentricity of lunar orbit and the gravitation from the Sun. Therefore, the constellation has the fixed configuration in the syzygy frame, which results in the maximal coverage region

equal to the common coverage region shown in Figure 7. From the hypothesis referred above, the constellation can cover the Earth's surface by 99.1% except the south and north poles.

In a full investigation on the coverage to Earth, the hypothesis is unloaded with some considerations of the real geometries and full dynamics. All the quasi-periodic trajectories near LL_3 , LL_4 , and LL_5 are refined by the OrbitalPredictor full-ephemeris model, and the revisiting intervals are counted statistically in real geometrical scenario by STK.

The 19-year Metonic cycle of the equatorial inclination, with respect to lunar orbital plane, has also an influence

FIGURE 6: The full analysis on the maximal and common coverages during a whole Metonic cycle from 2015 to 2034. The maximal revisiting times on the global surface during a whole Metonic cycle are implemented in (a), and the minimal revisiting times in (b).

FIGURE 7: The preliminary analysis on covering region by the constellation on the Earth's surface.

on the coverage by the constellation. However, because of rotating more quickly around its own axis, the Earth owns the constant revisiting time on the same latitude lines located near 70°S and 70°N shown in Figure 8, where the discontinuous communications are interrupted by some intervals less than 2.5 hours. It is quite different from the Moon where the discontinuous communications are just locating on the longitude lines near 150°W and 150°E . Moreover, the regions on other latitudes are accessed without any time interval (i.e., the blank regions in the top of Figure 8).

In addition, the conclusion that the revisiting time keeps invariant on the same latitude line will be true for a whole Metonic cycle. Therefore, the relationship between revisiting time and the latitudes will be investigated in the following section just on the longitude line of 0°E , shown in Figures 9 and 10.

Different from the preliminary results, some conclusions can be induced from Figures 9 and 10. Firstly, all the points on the Earth's surface can be covered by this constellation, and the discontinuous communications keep the invariant revisiting time on the same latitudes during a whole Metonic cycle.

Secondly, due to the rotation of the Earth, few of the regions located at the latitudes from 80°S to 52°S and from 55°N to 79°N are revisited after some intervals less than 2.5 hours, while the other regions are accessed without any time interval.

Thirdly, the worst intervals occur at the regions located from 71°S to 65°S and from 62°N to 70°N , which keep discontinuous communications during a whole Metonic cycle. Moreover, this constellation has a good coverage performance at the poles with no time interval.

TABLE 1: Stationkeeping cost of the constellation from 12:00, July 1, 2015, to 12:00, July 1, 2016.

Libration point	Number of maneuvers	Position error (km)	Average ΔV (m/s)	Total ΔV (m/s)
LL_2	52	954.2	1.96	102.3
LL_3	52	1102.4	1.7	88.5
LL_4	52	1464.6	0.6	31.2
LL_5	52	1398.7	0.77	39.8

FIGURE 8: The full analysis on revisiting time on the Earth's surface with the epoch time of 12:00, July 1, 2015. The conclusion that the revisiting time keeps invariant on the same latitude line will be true for a whole Metonic cycle.

3.3. Multiple Links to Earth and Moon by LL_4 and LL_5 . In this constellation, the LL_4 point takes simultaneously charge in the communications with Earth and Moon, as well as LL_5 . Therefore, two communication devices need to be equipped on the spacecrafts, with their antennas pointing to the Earth and Moon, respectively. However, due to the Metonic cycle of the equatorial inclination with respect to lunar orbital plane, the boundaries of the angle between the two antennas are illustrated points in Figure 11, where the angle between the two antennas near LL_4 point varies from 47° to 81° , and from 34° to 85° near LL_5 point.

4. Cost Analysis of Constellation

4.1. Stationkeeping Strategies and Costs. Stationkeeping correction is necessary because the true trajectory will diverge from the nominal one under the effects of the navigation error E_1 , the initial error from the launch rocket E_2 , and the error from inserting the nominal trajectory on velocity E_3 [20, 21]. However, the presence of the trajectory correction will cause velocity errors E_4 . These error resources are assumed to be Gauss distributed and their average values are zero. E_1 and E_2 have the same position error variances of 10 km and velocity error variances of 0.01 m/s (1σ); E_3 and E_4 have the velocity error variances of 3% and their velocity errors in the direction of radius are $2.06^\circ(3\sigma)$ [22, 23]. The velocity error in the normal direction is uniformly distributed.

Assume that there are k corrections during the flight on nominal trajectory, which is divided into $k + 1$ sections according to the amount of corrections, as it is shown in Figure 12.

The state errors between the actual and nominal trajectories at time $\bar{t}_m$ are $\delta\mathbf{R}_m$ and $\delta\mathbf{V}_m$ from the errors E_1 , E_2 , E_3 ,

and E_4 , respectively, and the Taylor expansion of the position vector at time $\bar{t}_m$ is obtained

$$\begin{aligned} & \mathbf{R}(\bar{t}_{m+1}, \bar{\mathbf{R}}_m + \delta\mathbf{R}_m, \bar{\mathbf{V}}_m + \delta\mathbf{V}_m + \Delta\mathbf{V}_m, \bar{t}_m) \\ &= \mathbf{R}(\bar{t}_{m+1}, \bar{\mathbf{R}}_m, \bar{\mathbf{V}}_m, \bar{t}_m) + \frac{\partial \bar{\mathbf{R}}_{m+1}}{\partial \bar{\mathbf{R}}_m} \cdot \delta\mathbf{R}_m + \frac{\partial \bar{\mathbf{R}}_{m+1}}{\partial \bar{\mathbf{V}}_m} \cdot (\delta\mathbf{V}_m + \Delta\mathbf{V}_m), \end{aligned} \quad (5)$$

where the correction velocity $\Delta\mathbf{V}_m$ is used to correct the trajectory aiming at achieving the wanted position $\mathbf{R}(\bar{t}_{m+1}, \bar{\mathbf{R}}_m, \bar{\mathbf{V}}_m, \bar{t}_m)$ at time $\bar{t}_m$. According to (5), $\Delta\mathbf{V}_m$ can be obtained

$$\Delta\mathbf{V}_m = -\left(\frac{\partial \bar{\mathbf{R}}_{m+1}}{\partial \bar{\mathbf{V}}_m}\right)^{-1} \cdot \frac{\partial \bar{\mathbf{R}}_{m+1}}{\partial \bar{\mathbf{R}}_m} \cdot \delta\mathbf{R}_m - \delta\mathbf{V}_m. \quad (6)$$

The correction velocity $\Delta\mathbf{V}_m$ is a linear comparison of $\delta\mathbf{R}_m$ and $\delta\mathbf{V}_m$, and this characteristic is definitely suitable for trajectory correction during the flight.

A correction schedule with an equi-interval correction time of 7 days is adopted in this paper to investigate the stationkeeping costs for this constellation in the full ephemeris model. The maneuver results are obtained in Table 1 for one complete year. Therefore, the total stationkeeping cost of this constellation is 261.8 m/s/year, which is accepted for the aerospace engineering.

Furthermore, some optimization technique to decrease the stationkeeping cost can be implemented in the future, including some selections for the correction schedule, and the application of the stochastic optimal control into the correction strategy.

FIGURE 9: The full analysis on revisiting time on the Earth's surface during a whole Metonic cycle from 2015 to 2034. The relationship between the latitudes and the revisiting times is implemented by the epoch times of (a) 12:00, July 1, 2015, (b) 12:00, July 1, 2018, (c) 12:00, July 1, 2021, (d) 12:00, July 1, 2024, (e) 12:00, July 1, 2027, (f) 12:00, July 1, 2030, and (g) 12:00, July 1, 2033.

4.2. Inserting Strategies and Costs into Nominal Trajectories. After being launching into the parking orbit around the Earth, some maneuvers are required to guidance the spacecrafts to nominal trajectories located near LL_2 , LL_3 , LL_4 , and LL_5 , respectively, which are refined in Section 2.2. Some inserting strategies are included in this paper to analyze the inserting costs for this constellation.

The invariant manifold theory developed in CR3BP has potential applications in creating the inserting trajectories into a halo orbit near L_1 point [24, 25]. However, According to the technique developed by Koon et al. [26], the insertion into the quasi-periodic orbit near LL_2 point needs some

combinations between unstable manifolds stemming from EL_1 (or EL_2) point and stable manifolds stemming from LL_2 point. In fact, the similar results can be achieved by the weak stability boundary theory (WSB) proposed by Belbruno and Miller [27] to create some low-energy cislunar trajectory with the help of the Sun's gravitation. Therefore, following on the initial guess obtained in CR3BP, the iteration procedure defined in Section 2.2 can be used to product the real inserting trajectory to the quasi-periodic orbit near LL_2 point in the full ephemeris model, shown in Figure 13. Only one maneuver is required on the parking orbit with its altitude of 200 km to insert the nominal orbit, without other

FIGURE 10: The relationship between the latitudes and the maximal and common revisiting times during a whole Metonic cycle from 2015 to 2034. The maximal revisiting times on the latitudes during a whole Metonic cycle are implemented in (a), and the minimal revisiting times in (b).

FIGURE 11: The angle between the two antennas equipped on the LL_4 and LL_5 spacecrafts. The maximal angle during a Metonic cycle is indicated by the upper boundary, while the minimal angle during a Metonic cycle is indicated by the lower boundary.

FIGURE 12: Stationkeeping corrections on the nominal trajectory.

maneuver at the end of the transfer trajectory. Thus, during this transfer, the total transfer duration is 91.3 days, and the total cost of maneuvers is 3.2018 km/s.

For the bounded trajectories near LL_3 , LL_4 , and LL_5 , all of the stable manifolds stemming from these libration points are quite far from the Earth, which is different from LL_2 point. The Hohmann transfer is employed in this paper to guide the spacecrafts from the parking orbits to their stable manifolds, and then to insert the nominal trajectory following its stable manifolds. Based on the initial guess obtained in CR3BP, the iteration procedure defined in Section 2.2 can be used to product the real inserting trajectory to the bounded orbit near LL_3 , LL_4 , and LL_5 points in the full ephemeris model,

shown in Figures 14, 15, and 16, respectively. Therefore, the whole transfer trajectory includes the Hohmann segment and coasting segment along the stable manifold, and two maneuvers are required on the parking orbit with its altitude of 200 km to insert the nominal orbit; the first one is imposed on the parking orbit, and the last one is imposed at the end of the Hohmann segment.

For LL_3 point, the total transfer duration is 45.1 days, and the total cost of maneuvers is 3.7518 km/s. For LL_4 point, the total transfer duration is 47.57 days, and the total cost of maneuvers is 3.7585 km/s. For LL_5 point, the total transfer duration is 63.4 days, and the total cost of maneuvers is 3.761 km/s.

FIGURE 13: The inserting trajectory into the nominal orbit near LL_2 point from the parking orbit with its altitude of 200 km in the syzygy frame. The quasi-periodic halo orbit near LL_2 point created in full ephemeris model with the epoch time of 12:00, July 1, 2015. Only one maneuver $\Delta V = 3.2018$ km/s on the parking orbit is required to insert the nominal orbit without other maneuvers at the end of the transfer trajectory. The transfer duration is 91.3 days.

FIGURE 14: The inserting trajectory into the nominal orbit near LL_3 point from the parking orbit with its altitude of 200 km in the syzygy frame. The bounded orbit near LL_2 point created in full ephemeris model with the epoch time of 12:00, July 1, 2015. Two maneuvers of 3.7518 km/s are required to insert the nominal orbit on the parking orbit and at the end of the Hohmann segment. The transfer duration is 45.1 days.

Therefore, the total inserting cost of this constellation is 14.4731 km/s, which is accepted for the aerospace engineering. Furthermore, the inserting costs by Hohmann segments in the cases of LL_3 , LL_4 , and LL_5 points can be optimized by some techniques developed in space manifold dynamics in the future.

5. Communications

There are several communicating links inside the constellation, which is shown in Table 2. The links make the

constellation more functional than the single sum of the four satellites. Furthermore, the links make the constellation more robust, so the constellation will still work depending on the links at the event that one or two satellites happen to be broken.

The distance from Earth to Moon brings in a long time delay of 1.28 s. Due to this, direct voice communications may be impossible. Instead, data communications are preferred for this constellation architecture, including some links between ground sites on the Earth and the far side on the Moon or some data exchanges between the points on the Earth, and so on.

TABLE 2: Links inside the constellation ($\checkmark$: link, $\times$: no link).

	LL_2	LL_3	LL_4	LL_5	Earth	Moon
LL_2	$\times$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
LL_3	$\times$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$
LL_4	$\checkmark$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\checkmark$
LL_5	$\checkmark$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\checkmark$
Earth	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$
Moon	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$

FIGURE 15: The inserting trajectory into the nominal orbit near LL_3 point from the parking orbit with its altitude of 200 km in the syzygy frame. The bounded orbit near LL_2 point created in full ephemeris model with the epoch time of 12:00, July 1, 2015. Two maneuvers of 3.7585 km/s are required to insert the nominal orbit on the parking orbit and at the end of the Hohmann segment. The transfer duration is 47.57 days.

FIGURE 16: The inserting trajectory into the nominal orbit near LL_3 point from the parking orbit with its altitude of 200 km in the syzygy frame. The bounded orbit near LL_2 point created in full ephemeris model with the epoch time of 12:00, July 1, 2015. Two maneuvers of 3.761 km/s are required to insert the nominal orbit on the parking orbit and at the end of the Hohmann segment. The transfer duration is 63.4 days.

Furthermore, some ground stations are required to track the four spacecrafts in the constellation, and also to deal with orbital determination. Due to the data rates and Bands, the diameter of antenna is not less than 50 m. According to the coverage region defined in Figure 8, the ground stations should be located at the latitudes from 80°S to 52°S and from 55°N to 79°N to track and manage this constellation.

6. Conclusion

To provide near-constant communications between any two points in the cislunar space, a new constellation configuration is proposed in this paper. Four satellites were allocated on some natural orbits near LL_2 , LL_4 , LL_5 , and LL_3 points, the first three of which are expected to supply a global coverage on the Moon, while the last three afford a global coverage on the Earth. Considering the real dynamics and geometries, a full ephemeris model is employed to refine the quasi-periodic trajectories near LL_2 , LL_3 , LL_4 , and LL_5 points by a multiple shooting corrector numerically.

The revisiting intervals between the successive revisits are demonstrated on the geographical maps of the Earth and the Moon, with a consideration of the Metonic cycle of the equatorial inclination with respect to lunar orbital plane. All the points on the lunar surface can be covered by this constellation, where most of them are accessed without any time interval, and few of them are revisited after some intervals less than 5 days. This constellation is weak in covering some longitude lines located near 150°W and 150°E . The discontinuous communications are interrupted by some intervals less than 2 hours. Only some sites keep discontinuous communications during a whole Metonic cycle, which are located from $20^{\circ}\text{N}140^{\circ}\text{W}$ to $70^{\circ}\text{N}140^{\circ}\text{W}$, from $30^{\circ}\text{S}150^{\circ}\text{W}$ to $2^{\circ}\text{N}150^{\circ}\text{W}$, from $85^{\circ}\text{S}55^{\circ}\text{E}$ to $85^{\circ}\text{S}100^{\circ}\text{E}$, and from $80^{\circ}\text{S}170^{\circ}\text{E}$ to $80^{\circ}\text{S}130^{\circ}\text{W}$. The worst intervals occur at the high-latitude areas instead of the poles because the maximal revisiting time is 0.96 day at the north pole while 1.21 days at the south pole. This constellation has a good coverage performance at the lunar poles.

All the points on the Earth's surface can be covered by this constellation, and the discontinuous communications keep the invariant revisiting time on the same latitudes during a whole Metonic cycle. Few of regions located at the latitudes from 80°S to 52°S and from 55°N to 79°N are revisited after some intervals less than 2.5 hours, while the other regions are accessed without any time interval. The worst intervals occur at the regions located from 71°S to 65°S and from 62°N to 70°N , which keep discontinuous communications during a whole Metonic cycle. This constellation has a good coverage performance at the Earth's poles with no time interval.

Moreover, cost analysis on stationkeeping and launching is implemented in detail in full ephemeris model. According to the stationkeeping and inserting strategies proposed in this paper, the total inserting cost of this constellation is 14.4731 km/s, and the total stationkeeping cost of this constellation is 261.8 m/s/year.

Furthermore, some optimizations on this constellation can be implemented in the future, including optimizing the orbital elements in this constellation to promote the coverage

on the Earth and Moon, selecting the correction schedule from viewpoint of the stochastic optimal control to decrease the stationkeeping cost, and optimizing the Hohmann segments for LL_3 , LL_4 , and LL_5 points to decrease the launching cost based on space manifold dynamics.

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