Research Article

Faired MISO B-Spline Fuzzy Systems and Its Applications

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We construct two classes of faired MISO B-spline fuzzy systems using the fairing method in computer-aided geometric design (CAGD) for reducing adverse effects of the inexact data. Towards this goal, we generalize the fairing method to high-dimension cases so that the fairing method only for SISO and DISO B-spline fuzzy systems is extended to fair the MISO ones. Then the problem to construct a faired MISO B-spline fuzzy systems is transformed into solving an optimization problem with a strictly convex quadratic objective function and the unique optimal solution vector is taken as linear combination coefficients of the basis functions for a certain B-spline fuzzy system to obtain a faired MISO B-spline fuzzy system. Furthermore, we design variable universe adaptive fuzzy controllers by B-spline fuzzy systems and faired B-spline fuzzy systems to stabilize the double inverted pendulum. The simulation results show that the controllers by faired B-spline fuzzy systems perform better than those by B-spline fuzzy systems, especially when the data for fuzzy systems are inexact.

1. Introduction

Since Zadeh introduced fuzzy theory in 1965, fuzzy systems have been utilized successfully in many areas, such as fuzzy control, classification, expert systems, and others. It is known that a fuzzy system is usually established by input-output data (I/O data) which can be obtained by experiments, expert knowledge, or observation records. However, the accuracy of these I/O data may be affected by hardware/software limitations, unavoidable round off, truncation error of a system, and some uncertainties [1]. This means that we cannot establish fuzzy systems on the exact I/O data. Therefore, it is important to construct appropriate fuzzy systems when the I/O data is inexact. Reference [2] gives the upper bounds of the output errors of two kinds of fuzzy systems affected by the perturbation of I/O data. However, the problem of performance improvement for fuzzy systems is seldom considered in the case of the inexact I/O data.

Fuzzy systems can be constructed by splines [3–6] because the design of a fuzzy system can be regarded as a function approximation problem [7–10] and spline functions have many nice structural properties and excellent approximation powers [11]. By investigating the relation

between fuzzy systems and splines, we proposed two classes of B-spline fuzzy systems (B-FSs) [12, 13], which are linear combination of B-spline basis functions and rational B-spline basis functions, respectively. Therefore, the single input single output (SISO) and double input single output (DISO) of these two classes of B-FSs can be regarded as curves and surfaces in CAGD. As curves and surfaces in CAGD, fairness is necessary. Though fairness is the property about geometric shapes, a fair curve can seek through the digitizing errors in its design process [14]. Hence, it is necessary to reduce adverse effects of the inexact I/O data on a fair fuzzy system. Since B-FSs can be regarded as curves and surfaces in CAGD, we can fair them and obtain good performance.

In this paper, we construct two classes of faired B-spline fuzzy systems (faired B-FSs) to reduce adverse effects of the inexact I/O data on fuzzy systems as well as improve their performance. For fairing these two classes of B-FSs, the energy extremum principle (energy method) based fairing method in CAGD is utilized for its overall modification nature. However, we note that the energy method in CAGD is only used to fair curves and surfaces, which means it can only fair the SISO and DISO B-FSs. So, we propose a regularization term taken as the energy function of the MISO B-FS. By using

this energy function, the energy method in CAGD, which can only be applied to fairing SISO and DISO B-FSs, is extended to fair the MISO ones. Therefore, based on the above preparations, the problem to construct a faired MISO B-FS is transformed into solving an optimization problem with a strictly convex quadratic objective function. In our proposed method, the faired MISO B-FS is available by taking the unique optimal solution vector as linear combination coefficients of the corresponding MISO B-FS.

As we all know, fuzzy controllers are a type of closed-loop fuzzy systems, while adaptive fuzzy controllers are closed-loop fuzzy systems with adaptive or training algorithms [15–19]. Especially, Professor Li advances the variable universe method [20–22], and this method succeeded in the experiment of controlling the simulation model and physical model of quadruple inverted pendulum with variable universe fuzzy controllers in 2001 [23] and 2002. In order to verify the ability of the faired B-FSs, we design variable universe adaptive fuzzy controllers by B-FSs and faired B-FSs to stabilize the double inverted pendulum. The simulation results show that the controllers by faired B-FSs perform better than those by B-FSs, especially when the I/O data for fuzzy systems are inexact.

The paper is organized as follows. Section 2 provides some preliminaries. The faired MISO B-FSs are constructed in Section 3. In Section 4, the variable universe adaptive fuzzy controllers by B-FSs and faired B-FSs are designed to demonstrate their ability. The final section contains some conclusions and prospects of our research.

2. Preliminaries

In this section, we will introduce the definition of B-spline basis functions, the Frobenius norm, and briefly review the two classes of MISO B-FSs in [12, 13].

Definition 1 (B-spline basis functions [24]). Let $U = \{u_0, \dots, u_m\}$ be a nondecreasing sequence of real numbers, that is, $u_i \leq u_{i+1}$, $i = 0, \dots, m-1$. The u_i is called knots, and U is the knot vector. The i th B-spline basis function of p -degree (order $p+1$), denoted by $N_{i,p}(u)$, is defined as

$$N_{i,0}(u) = \begin{cases} 1, & \text{if } u_i \leq u < u_{i+1}, \\ 0, & \text{otherwise,} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u). \quad (1)$$

Definition 2 (Frobenius norm [25]). The Frobenius norm on $\mathbb{R}^{n \times n}$ is defined as

$$\|A\|_F = \left(\sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2 \right)^{1/2}, \quad A = (a_{ij}) \in \mathbb{R}^{n \times n}. \quad (2)$$

Obviously,

$$\|A\|_F = \left(\text{Tr}(A^T A) \right)^{1/2}. \quad (3)$$

In fuzzy inference, to acquire a group of inference rules and to acquire a group of I/O data are the same thing [10]. In this paper, we always denote the I/O data for MISO fuzzy systems as

$$\{(u_{1,j_1}, u_{2,j_2}, \dots, u_{n,j_n}, v_{j_1 j_2 \dots j_n}) \mid j_i = 0, 1, \dots, p_i, i = 1, \dots, n\}, \quad (4)$$

where $u_{i,0} = a_i$, $u_{i,p_i} = b_i$, and $u_{i,0} < u_{i,1} < \dots < u_{i,p_i}$, $i = 1, \dots, n$.

Let $A_{i,j_i}(x) = N_{j_i-2,3}(x)$, where $N_{j_i-2,3}(x)$ is the cubic B-spline basis function with knots u_{i,j_i-2} , u_{i,j_i-1} , u_{i,j_i} , u_{i,j_i+1} , and u_{i,j_i+2} . The two classes of MISO B-FSs we proposed in [12, 13] are listed below.

- (1) The MISO first class of B-spline fuzzy system (1-B-FS) is

$$F_1(x_1, x_2, \dots, x_n) = \sum_{j_1=-1}^{p_1+1} \sum_{j_2=-1}^{p_2+1} \dots \sum_{j_n=-1}^{p_n+1} A_{1,j_1}(x_1) A_{2,j_2}(x_2) \dots A_{n,j_n}(x_n) \omega_{j_1 j_2 \dots j_n}, \quad (5)$$

where $\omega_{j_1 j_2 \dots j_n}$, $j_i = -1, 0, \dots, p_i + 1$, $i = 1, \dots, n$ are obtained by solving equations

$$F_1(u_{1,j_1}, u_{2,j_2}, \dots, u_{n,j_n}) = v_{j_1 j_2 \dots j_n}, \quad (6)$$

$$j_i = -1, 0, \dots, p_i + 1, i = 1, \dots, n,$$

after extrapolating some points.

- (2) The MISO second class of B-spline fuzzy system (2-B-FS) is

$$F_2(x_1, x_2, \dots, x_n) = \sum_{j_1=0}^{p_1} \sum_{j_2=0}^{p_2} \dots \sum_{j_n=0}^{p_n} \frac{A_{1,j_1}(x_1) A_{2,j_2}(x_2) \dots A_{n,j_n}(x_n)}{\sum_{l_1=0}^{p_1} \sum_{l_2=0}^{p_2} \dots \sum_{l_n=0}^{p_n} A_{1,l_1}(x_1) A_{2,l_2}(x_2) \dots A_{n,l_n}(x_n)} \times v_{j_1 j_2 \dots j_n}. \quad (7)$$

3. The Faired MISO B-FSs

In this section, two classes of the faired MISO B-FSs are constructed. In order to fair them together, we write them in the unification SISO form. By analyzing the energy

functions of SISO and DISO B-FS, the energy function of MISO B-FS is proposed, which is a regularization term essentially. Consequently, the energy method is suitable for fairing the MISO B-FSs. Then, we transform the problem to construct a faired MISO B-FS into solving an unconstrained optimization problem. Based on the unification SISO form, the objective function of the unconstrained optimization problem can be reduced to a quadratic function which is turned to a strictly convex quadratic function via using a proper weight. Therefore, the unique optimal solution is available through solving linear equations of the first-order optimality condition. Consequently, the faired MISO B-FS is obtained by taking the unique optimal solution vector as linear combination coefficients of the corresponding B-FS. In the following, we will describe the above procedure in details.

3.1. The Unification SISO Form of Two Classes of B-FSs. The uniform form of (5) and (7) is,

$$F(\mathbf{x}) = \sum_{k=1}^N C_k(\mathbf{x}) \alpha_k = \alpha^T \mathbf{C}(\mathbf{x}), \quad (8)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_n)$, $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_N)^T$, and $\mathbf{C}(\mathbf{x}) = (C_1(\mathbf{x}), C_2(\mathbf{x}), \dots, C_N(\mathbf{x}))^T$.

If we have

$$\begin{aligned} C_k(\mathbf{x}) &= A_{1,j_1}(x_1) A_{2,j_2}(x_2) \cdots A_{n,j_n}(x_n), \\ \alpha_k &= \omega_{j_1 j_2 \cdots j_n}, \\ k &= (j_1 + 2) + (j_2 + 1)(p_1 + 3) + \cdots \\ &\quad + (j_n + 1)(p_1 + 3) \cdots (p_{n-1} + 3), \\ j_i &= -1, 0, \dots, p_i + 1, \quad i = 1, 2, \dots, n, \\ N &= (p_1 + 3)(p_2 + 3) \cdots (p_n + 3), \end{aligned} \quad (9)$$

then (8) will turn to the 1-B-FS (5).

While, if we have

$$\begin{aligned} C_k(\mathbf{x}) &= \frac{A_{1,j_1}(x_1) A_{2,j_2}(x_2) \cdots A_{n,j_n}(x_n)}{\sum_{l_1=0}^{p_1} \sum_{l_2=0}^{p_2} \cdots \sum_{l_n=0}^{p_n} A_{1,l_1}(x_1) A_{2,l_2}(x_2) \cdots A_{n,l_n}(x_n)}, \\ \alpha_k &= v_{j_1 j_2 \cdots j_n}, \\ k &= (j_1 + 1) + j_2(p_1 + 1) + \cdots \\ &\quad + j_n(p_1 + 1) \cdots (p_{n-1} + 1), \\ j_i &= 0, 1, \dots, p_i, \quad i = 1, 2, \dots, n, \\ N &= (p_1 + 1)(p_2 + 1) \cdots (p_n + 1), \end{aligned} \quad (10)$$

then (8) is exactly the 2-B-FS (7). Consequently, (8) is named as the unification SISO form of the two classes of B-FSs.

3.2. Generalizing the Energy Method to High-Dimension Cases.

The objects studied in CAGD are curves and surfaces which are parametric equations with single parameter and double parameters, respectively. The problem of fairing a curve (surface) by the energy method can be transformed into the following optimal one [26]:

$$\min_V E(V) + wD(V), \quad (11)$$

where the variable V is the vector of control points, $E(V)$ is the energy function, $D(V)$ is the difference between the faired data points and the original ones, and w is the weight which is assigned in advance. In CAGD, an approximated or simplified strain energy is used as energy function $E(V)$ [26]. For SISO and DISO B-FS, which can be regarded as curve and surface in CAGD, the energy functions can be written as

$$E_1 = \int_a^b (F''(x))^2 dx, \quad (12)$$

$$E_2 = \iint_D \left(\left(\frac{\partial^2 F}{\partial x_1^2} \right)^2 + 2 \left(\frac{\partial^2 F}{\partial x_1 \partial x_2} \right)^2 + \left(\frac{\partial^2 F}{\partial x_2^2} \right)^2 \right) dx_1 dx_2. \quad (13)$$

By (2) and (3), the integrand of E in (12) or (13) is the square of the Frobenius norm of $\nabla^2 F(\mathbf{x})$ (the Hessian matrix of $F(\mathbf{x})$). That is,

$$\begin{aligned} E &= \int_D \|\nabla^2 F(\mathbf{x})\|_F^2 d\mathbf{x} \\ &= \int_D \text{Tr} \left[(\nabla^2 F(\mathbf{x}))^T \nabla^2 F(\mathbf{x}) \right] d\mathbf{x}. \end{aligned} \quad (14)$$

When the number of input variable is more than 2, it is difficult to get an energy function with specific geometric meaning or physical meaning. In order to fair the B-FSs with more than 2 input variables, we have to generalize the energy method to the high-dimension cases. In fact, the energy method is a kind of regularization method with the energy function as its regularization term. Especially, when the observation data is inexact, the regularization method can identify a meaningful and stable solution [27]. This coincides with our motive to fair. It is noted that (14) can be viewed as a regularization term. Consequently, we call the regularization method with (14) as its regularization term the generalized energy method. It is known that (14) is the energy function for MIMO B-FS. Therefore, we generalize the energy method to the high-dimension cases.

3.3. The Faired MISO B-FSs. In general, the curves (surfaces) in CAGD are referred to the parametric B-spline curves (surfaces), while the B-FSs are the B-spline functions. When the parametric B-spline curves (surfaces) degenerate into B-spline functions, the control points will be the coefficients of B-spline basis functions. Write the original B-FS and the faired B-FS as

$$F_0(\mathbf{x}) = \alpha_0^T \mathbf{C}(\mathbf{x}), \quad (15)$$

$$F(\mathbf{x}) = \alpha^T \mathbf{C}(\mathbf{x}), \quad (16)$$

respectively. Then, by (11), we can transform the problem to construct a faired MISO B-FS into solving the following unconstrained optimization problem as

$$\min_{\alpha \in \mathbb{R}^N} H(\alpha) \triangleq E + w(\alpha^T - \alpha_0^T)(\alpha - \alpha_0), \quad (17)$$

where w , the positive weight, is assigned in advance, and E is defined as in (14).

From the following two extreme cases, we can recognize the concrete significance of w .

(1) When $w \rightarrow \infty$, since $H = \min$, we have

$$\alpha = \alpha_0. \quad (18)$$

Thus the faired B-FS $F(\mathbf{x})$ turns to the original B-FS $F_0(\mathbf{x})$, which implies that the B-FS $F_0(\mathbf{x})$ is not modified after the fairing process.

(2) When $w = 0$, we immediately get that $E = 0$ from $H = \min$. Therefore, the B-FS $F(\mathbf{x})$ turns to be the fairest one in this case.

For the cases in between, when w is set to be a small number, E becomes small and the fuzzy system (16) is fair at the cost of much difference between $F(\mathbf{x})$ and $F_0(\mathbf{x})$ as well as much difference between $F(\mathbf{u}_s)$ and v_s . On the other hand, the larger w is, the less the difference is between $F(\mathbf{x})$ and $F_0(\mathbf{x})$. Thus there is less difference between $F(\mathbf{u}_s)$ and v_s . And the fairness of the fuzzy system (16) may be poor. So when the I/O data is inexact, a smaller w is needed to make a larger difference between $F(\mathbf{x})$ and $F_0(\mathbf{x})$, and then the original B-FS $F_0(\mathbf{x})$ may be improved. On the contrary, a larger w might be appropriate.

Let $D = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$, and substitute (8) into (14), we have,

$$\begin{aligned} E &= \int_D \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial^2 F}{\partial x_i \partial x_j} \right)^2 dx \\ &= \alpha^T \left(\int_D \sum_{i=1}^n \sum_{j=1}^n \frac{\partial C}{\partial x_i \partial x_j} \left(\frac{\partial C}{\partial x_i \partial x_j} \right)^T dx \right) \alpha. \end{aligned} \quad (19)$$

Then, the objective function of the optimal problem (17) is

$$H(\alpha) = \alpha^T (A + wI) \alpha - 2w\alpha^T \alpha_0 + w\alpha_0^T \alpha_0, \quad (20)$$

where

$$A = \int_D \sum_{i=1}^n \sum_{j=1}^n \frac{\partial C}{\partial x_i \partial x_j} \left(\frac{\partial C}{\partial x_i \partial x_j} \right)^T dx. \quad (21)$$

From (20), we know that $H(\alpha)$ is a quadratic function. Moreover, by (21), $A + wI$ is a symmetric matrix. Obviously, there always exists weight w to make $A + wI$ a symmetric positive definite matrix. Thus, $H(\alpha)$ becomes a strictly convex quadratic function. Consequently, through solving the linear

FIGURE 1: A simplified model of the double inverted pendulum.

equations of the first-order optimality condition as shown in the following:

$$(A + wI) \alpha^* = w\alpha_0, \quad (22)$$

we can get the unique optimal solution α^* of (17). Let α^* serve as the linear combination coefficients of (16), the faired MISO B-FS is available.

Remark 3. Since it is convenient to deal with a uniform cubic B-spline, when the knots of the B-spline basis functions of a B-FS are arbitrary (non-uniform), we can approximate to this B-FS by one fuzzy system with uniform cubic B-spline basis functions to fair it.

Algorithm. Constructing a Faired MISO B-FS.

Step 1. Extract I/O data (4) from the fuzzy inference rules;

Step 2. Given the initial weight w ;

Step 3. Calculate A by (21);

Step 4. If $A + wI$ is positive definite, the optimal solution is available by solving linear equations (22). Otherwise, let $w = 2w$, go to Step 4;

Step 5. Evaluate the obtained faired B-FS, $F(\mathbf{x})$, if it works, end, if not, tuning the weight w , go to Step 4.

4. Simulation Results

In this section, we design variable universe adaptive fuzzy controllers by B-FSs (hereinafter abbreviated as B-FCs) and faired B-FSs (hereinafter abbreviated as faired B-FCs) to stabilize the double inverted pendulum. Moreover, the control effect between them is compared. As the analysis

TABLE 1: Control rules for double inverted pendulum.

ec	e			e			
	A_1	A_2	A_3	A_4	A_5	A_6	A_7
B_1	-0.8333	-0.8333	-0.6333	-0.5	-0.3333	-0.1667	0
B_2	-0.8333	-0.6333	-0.5	-0.3333	-0.1667	0	0.1667
B_3	-0.6333	-0.5	-0.3333	-0.1667	0	0.1667	0.3333
B_4	-0.5	-0.3333	-0.1667	0	0.1667	0.3333	0.5
B_5	-0.3333	-0.1667	0	0.1667	0.3333	0.5	0.6333
B_6	-0.1667	0	0.1667	0.3333	0.5	0.6333	0.8333
B_7	0	0.1667	0.3333	0.5	0.6333	0.8333	0.8333

FIGURE 2: Response of controllers by 1-B-FS, 2-B-FS, faired 1-B-FS, and faired 2-B-FS with I/O data DISODS.

in Section 3.3, the weight w can affect the fairness and difference of the faired B-FSs. Further, it may affect the control effect of the faired B-FCs, too. Therefore, the control effect incited by different weights is demonstrated in the following simulations.

The double inverted pendulum is mainly made up of a cart, two rods which are freely linked together. The case where they are put in a coordinate system is shown in Figure 1.

Let the clockwise angle and moment in Figure 1 be in positive direction. And we assume that u is the outer force of system, x the displacement of the cart, θ_i the angle between rod i and vertical direction, O_i and G_i the linking point and centroid of rod i , m_0 the mass of the cart, m_i the mass of rod i , J_i the moment of inertia of rod i around O_i , l_i the distance from O_i to G_i , L_i the length of rod i , f_0 the fricative coefficient between the cart and its orbit, and f_i the fricative coefficient of rod i around O_i ($i = 1, 2$). Then the differential equations

to describe the locomotion of the double inverted pendulum are

$$H_1(z)\dot{z} = H_2(z)z + H_3(z) + H_0u, \quad (23)$$

where $z = (x, \theta_1, \theta_2, \dot{x}, \dot{\theta}_1, \dot{\theta}_2)^T$, $a_1 = m_1l_1 + m_2L_1$, $a_2 = m_2l_2$, $b_1 = J_1 + m_2L_1^2$, $b_2 = J_2$, and

$$H_1(z) =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & a_1 \cos \theta_1 & b_1 & a_2 L_1 \cos(\theta_2 - \theta_1) \\ 0 & 0 & 0 & a_2 \cos \theta_2 & a_2 L_1 \cos(\theta_2 - \theta_1) & b_2 \end{pmatrix},$$

$$H_2(z) =$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -f_1 - f_2 & a_2 L_1 \dot{\theta}_2 \sin(\theta_2 - \theta_1) + f_2 \\ 0 & 0 & 0 & 0 & -a_2 L_1 \dot{\theta}_1 \sin(\theta_2 - \theta_1) + f_2 & -f_2 \end{pmatrix}, \quad (24)$$

$H_3(z) = (0, 0, 0, 0, a_1 g \sin \theta_1, a_2 g \sin \theta_2)^T$, and $H_0 = (0, 0, 0, 1, 0, 0)^T$.

For the control system of the double inverted pendulum, our control aim is to make the angles θ_1 , θ_2 , respectively, converge to 0 and, at the same time, drive the cart to the point x_d which is pointed out by us in advance. In this simulation experiment, the parameters in the double inverted pendulum are taken as $m_1 = 0.373$, $m_2 = 0.088$ (unit: kg), $L_1 = 0.397$, $l_1 = 0.31815$, $L_2 = 0.345$, $l_2 = 0.15205$ (unit: m), $J_1 = 0.044048$, $J_2 = 0.00297947$ (unit: $\text{kg} \cdot \text{m}^2$), $f_1 = 0$, $f_2 = 0$ (unit: $\text{N} \cdot \text{s} \cdot \text{m}$), and $g = 9.81$ (unit: m/s^2).

The variable universe adaptive fuzzy controller for the double inverted pendulum is designed by the method in [23]. Firstly, we linearize (23) at the equilibrium point $z = (0, 0, 0, 0, 0, 0)$. Then, let $Q = \text{diag}([1, 1, 1, 1, 1, 1])$, $R = 0.1$, and solve LQR by Matlab. And then we can get a state feedback matrix $k = (3.1623, -154.66, 192.58, 6.3854, -6.7368, 26.806)$ and

TABLE 2: Performance index of control systems by 1-B-FS, faired 1-B-FSs with I/O data DISODS.

State	Index	1-B-FS		Faired 1-B-FS	
	w		0.0002	0.3	2
	Diff	0	4.65%	1.19%	0.953%
x	Steady-state error (10^{-9})	65.6	5.10	1.93	23.8
θ_1		150	16.4	0.461	52.7
θ_2		33.0	9.24	3.65	13.5
x	Maximum overshoot	0.231	0.230	0.226	0.227
θ_1		0.118	0.111	0.113	0.115
θ_2		0.0570	0.0561	0.0554	0.0560
x	Settling	3.36	3.44	3.33	3.34
θ_1		0.822	0.826	0.820	0.839
θ_2		0.713	0.319	0.313	0.315
	$\int u^2 dt$	6.27	4.94	5.27	5.61

TABLE 3: Performance index of control systems by 2-B-FS, faired 2-B-FSs with I/O data DISODS.

State	Index	2-B-FS		Faired 2-B-FS	
	w		3	5	10
	Diff	0.332%	1.20%	0.853%	0.610%
x	Steady-state error (10^{-9})	13.4	2.45	3.66	0.587
θ_1		14.4	2.45	6.18	0.973
θ_2		17.5	3.31	3.34	0.549
x	Maximum overshoot	0.229	0.228	0.227	0.227
θ_1		0.117	0.116	0.115	0.114
θ_2		0.0565	0.0563	0.0559	0.0557
x	Settling time	3.35	3.35	3.34	3.35
θ_1		0.823	0.825	0.802	0.812
θ_2		0.31	0.308	0.315	0.314
	$\int u^2 dt$	5.90	5.80	5.55	5.44

$\|k\|_2 = 248.64$. Let k be the coefficient vector to reduce the dimensions and

$$e \triangleq \frac{k(1)z(1) + k(2)z(2) + k(3)z(3)}{\|k\|_2}, \quad (25)$$

$$ec \triangleq \frac{k(4)z(4) + k(5)z(5) + k(6)z(6)}{\|k\|_2}$$

be a kind of integrated error and integrated error change rate. The variable universe adaptive fuzzy controller is

$$u = \|k\|_2 \left(\int_0^t 5(e + ec) F\left(\frac{e}{\alpha(e)}, \frac{ec}{\alpha(ec)}\right) \|k\|_2 dt + 1 \right) \times F\left(\frac{e}{\alpha(e)}, \frac{ec}{\alpha(ec)}\right), \quad (26)$$

where $F(\cdot)$ is a fuzzy system of two input variables.

The universe of $e(t)$ and $ec(t)$ are both taken as $[-1, 1]$. The fuzzy partition of $e(t)$ is $A_1 = \text{NB}$, $A_2 = \text{NM}$, $A_3 = \text{NS}$, $A_4 = \text{ZO}$, $A_5 = \text{PS}$, $A_6 = \text{PM}$, $A_7 = \text{PB}$, and the fuzzy partition of $ec(t)$ is $B_1 = \text{NB}$, $B_2 = \text{NM}$, $B_3 = \text{NS}$, $B_4 = \text{ZO}$, $B_5 = \text{PS}$, $B_6 = \text{PM}$, $B_7 = \text{PB}$. The control rules are given in Table 1 [23].

So the relatively exact I/O data for $F(\cdot)$ is that

$$\text{DISODS} \triangleq \{(x_i, y_j, z_{ij}) \mid i = 1, 2, \dots, 7, j = 1, 2, \dots, 7\}, \quad (27)$$

where $(x_1, x_2, \dots, x_7) = (-1, -2/3, -1/3, 0, 1/3, 2/3, 1)$, $(y_1, y_2, \dots, y_7) = (-1, -2/3, -1/3, 0, 1/3, 2/3, 1)$, and matrix $(z_{ij})_{7 \times 7}$ is formed by the data in Table 1.

When the I/O data is inexact, we consider the data with noise only which is obtained by adding the Gaussian white noise. In this simulation, the inexact I/O data is obtained by adding the Gaussian white noise with mean 0 and variance $(0.001)^2$ to I/O data DISODS, and the obtained inexact I/O data is denoted as DISODS¹.

Let $\alpha(e) = 1$, $\alpha(ec) = 1$, $z_0 = (0, 0.03, -0.03, 0, 0, 0)^T$, and $x_d = 0.1$. Figures 2 and 3 show the control effect of B-FCs and faired B-FCs respectively, where Figure 3 is the average result of 100 independent runs. Obviously,

- (1) when the I/O data is DISODS, the control effect of the B-FCs is nearly as good as that of the faired ones (Figure 2);
- (2) when the I/O data is DISODS¹, the faired B-FCs outperform that of the B-FCs (Figure 3).

TABLE 4: Performance index of control systems by 1-B-FS, faired 1-B-FSs with I/O data DISODS¹.

State	Index	1-B-FS	Faired 1-B-FS		
	w		0.001	0.5	4
	Diff	0	1.24%	1.21%	0.689%
x	Steady-state error	1.77	0.0843	0.0899	0.277
θ_1		58.8×10^{-9}	34.6×10^{-9}	37.2×10^{-9}	56.0×10^{-9}
θ_2		26.2×10^{-9}	50.0×10^{-9}	16.9×10^{-9}	25.5×10^{-9}
x	Maximum overshoot	2.01	0.228	0.225	0.495
θ_1		0.219	0.115	0.113	0.118
θ_2		0.149	0.0566	0.0560	0.0644
x	Settling time	4.04	3.25	3.24	3.59
θ_1		1.68	0.825	0.818	0.846
θ_2		1.68	0.431	0.395	0.796
	$\int u^2 dt$	15.2	5.52	5.23	6.84

TABLE 5: Performance index of control systems by 2-B-FS, faired 2-B-FSs with I/O data DISODS¹.

State	Index	2-B-FS	Faired 2-B-FS		
	w		0.4	1.5	11
	Diff	$0.213 \times 10^{-3}\%$	8.63%	1.35%	0.146%
x	Steady-state error	1.71	0.0967	0.110	0.447
θ_1		62.8×10^{-9}	50.7×10^{-9}	44.0×10^{-9}	53.4×10^{-9}
θ_2		30.1×10^{-9}	19.4×10^{-9}	17.1×10^{-9}	24.6×10^{-9}
x	Maximum overshoot	1.94	0.268	0.322	0.664
θ_1		0.206	0.157	0.1166	0.1173
θ_2		0.143	0.0735	0.0570	0.0695
x	Settling time	4.00	3.30	3.45	3.66
θ_1		1.62	1.46	0.827	0.881
θ_2		1.60	0.806	0.682	0.866
	$\int u^2 dt$	13.9	13.2	6.24	6.64

The control performance of control systems in terms of different weights is shown in Tables 2, 3, 4, and 5, where the control performance includes dynamical performance such as maximum overshoot and settling time, and steady performance such as steady-state error; $T = 100$ s; diff = $(\sum(F(\mathbf{x}_s) - v_s)^2 / \sum(v_s)^2) \times 100\%$ is defined as the relatively adjustment between fuzzy system and its corresponding I/O data; the setting time is defined as the time required for the system to settle within 5% of the steady value, and $\int u^2 dt$ shows the consumption of energy. In particular, the results shown in Tables 4 and 5 are the average result of 100 independent runs. The following results can be seen from the above tables.

- (1) The smaller (larger) the weight is, the larger (smaller) the relatively adjustment between the fuzzy system and its corresponding I/O data is (seen from Tables 2, 3, 4 and 5). This agrees with our analysis in Section 3.3.
- (2) For I/O data DISODS, only small adjustment can make the faired B-FCs have good control effect (seen from Tables 2 and 3). Since the variance of Gaussian

white noise of DISODS¹ is a small value, $(0.001)^2$, the good control effect is also available by small adjustment (seen from Tables 4 and 5).

- (3) For the relatively exact I/O data DISODS, we can see from Tables 2 and 3 that, almost all the performance index of control systems by faired B-FCs are slightly better than those by B-FCs. Moreover, for faired 1-B-FCs (faired 2-B-FCs), we should note that the energy consumption gets higher (lower) as the weight increasing.
- (4) For I/O data DISODS¹, the control performance of control systems with faired B-FCs is much better than that of control systems with B-FCs (seen from Tables 4 and 5), only except the steady-state error of θ_1 of $w = 0.001$ in Table 4. Especially, we point out that the energy consumption of faired B-FCs is less than that of the B-FCs.
- (5) From Tables 4 and 5, we also found that, for I/O data DISODS¹, neither faired B-FCs with larger weights

FIGURE 3: Response of controllers by 1-B-FS, 2-B-FS, faired 1-B-FS, and faired 2-B-FS with I/O data DISODS¹.

nor those with smaller ones can stabilize the double inverted pendulum (figures not shown). In Table 4, when the weight is 0.5, we can obtain almost the best control performance, especially the energy consumption is the least, while the weight is 1.5 in Table 5 for the same goal. Therefore, one can conclude that the larger (smaller) weights lead to smaller (larger) adjustment to the inexact I/O data DISODS¹, and both cases are not suitable for the faired B-FSs which are used to construct controllers.

In summary, when the I/O data for fuzzy system is relatively exact, the control effect of the faired B-FCs is slightly better than that of the B-FCs, which means the faired B-FSs for the faired B-FCs improve the B-FSs for the B-FCs slightly. While the I/O data for fuzzy system is inexact, the control effect of the faired B-FCs outperforms that of the B-FCs, in this case, the corresponding faired B-FSs reduce adverse effects of the inexact I/O data on the corresponding B-FSs as well as improve them significantly.

Remark 4. (1) To compare the control effect of faired B-FCs and B-FCs with inexact I/O data, the variance of the Gaussian white noise added to DISODS should be as small as $(0.001)^2$. Otherwise, the corresponding B-FCs cannot stabilize the double inverted pendulum. Actually, when the variance of Gaussian white noise is as large as $(0.1)^2$, the corresponding faired B-FCs can also stabilize the double inverted pendulum.

FIGURE 4: Response of controllers by faired 1-B-FS and faired 2-B-FS with I/O data DISODS².

(2) In order to investigate the control capability of the faired B-FCs, we choose

$$(z_{ij}^2)_{7 \times 7} = \begin{pmatrix} -0.8333 & -0.8333 & -0.6333 & -0.5 & -0.3333 & -0.1667 & 0 \\ -0.8333 & 0 & 0 & 0 & 0 & 0 & 0.1667 \\ -0.6333 & 0 & 0 & 0 & 0 & 0 & 0.3333 \\ -0.5 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ -0.3333 & 0 & 0 & 0 & 0 & 0 & 0.6333 \\ -0.1667 & 0 & 0 & 0 & 0 & 0 & 0.8333 \\ 0 & 0.1667 & 0.3333 & 0.5 & 0.6333 & 0.8333 & 0.8333 \end{pmatrix}, \quad (28)$$

and use $\text{DISODS}^2 \triangleq \{(x_i, y_j, z_{ij}^2) \mid i = 1, 2, \dots, 7, j = 1, 2, \dots, 7\}$ denote the I/O data. In this case, the faired B-FCs can stabilize the double pendulum as well as locate the cart (Figure 4). However, the B-FCs cannot do these.

5. Conclusion

In this paper, the energy method in CAGD was utilized to design the faired MISO B-FSs. Based on our generalized approach, the construction of a faired MISO B-FS is equivalent to solve an optimization problem with a strictly convex quadratic objective function. By taking the unique optimal solution vector as the linear combination coefficients of a certain B-FS, a faired MISO B-FS is obtained. For the faired MISO B-FSs, the fairness and difference can be adjusted by modifying the weights in the objective function. This gives us the opportunity to improve the performance of fuzzy systems and fuzzy controllers. Moreover, we use the obtained faired MISO B-FSs to stabilize the double inverted pendulum by modifying the weights. It is concluded that the faired B-FCs

outperform the B-FCs in the case of exact and inexact I/O data. In fact, there are many fairing methods in CAGD. We only choose the energy approach. Moreover, the faired MISO B-FSs are fuzzy systems with robustness. In the future, we will try to fair the B-FSs by other fairing methods and investigate their robustness.

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