Research Article

Cooperative Control for Uncertain Multiagent Systems via Distributed Output Regulation

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The distributed robust output regulation problem for multiagent systems is considered. For heterogeneous uncertain linear systems and a linear exosystem, the controlling aim is to stabilize the closed-loop system and meanwhile let the regulated outputs converge to the origin asymptotically, by the help of local interaction. The communication topology considered is directed acyclic graphs, which means directed graphs without loops. With distributed dynamic state feedback controller and output feedback controller, respectively, the solvability of the problem and the algorithm of controller design are both investigated. The solvability conditions are given in terms of linear matrix inequalities (LMIs). It is shown that, for polytopic uncertainties, the distributed controllers constructed by solving LMIs can satisfy the requirements of output regulation property.

1. Introduction

Recently, there are amounts of researches on cooperative control for multiagent systems (MASs) because of broad applications. An MAS is a practical model to describe dynamic agents which can exchange information by communication, such as unmanned air vehicles and sensor networks. According to different control objectives, problems of consensus, tracking, formation, flocking and the rest have been widely studied.

Among those cooperative control problems, the consensus problem and the tracking problem share some common characteristics. The consensus problem requires the MAS to reach an agreement by protocols based on local information. In [1, 2], the consensus problem is primarily studied, and the basic problem framework is formed. The consensus problem has been investigated for different kinds of agents, such as first-order integrators in [3], second-order integrators in [4, 5], linear systems in [6–8], and nonlinear systems in [9–12]. Recently, the output consensus problem for heterogeneous systems also attracted researchers. The dynamics and even dimensions of the agents are possibly different, so it is desirable to focus on the synchronization of outputs. In [13], the consensus of a class of second-order integrators

with unknown nonlinear dynamics is considered. As for high-order systems, the frequency domain approach is used to discuss the consensus of heterogeneous linear systems in [14, 15]. Uncertain minimum-phase linear MASs are studied in [16], by a low-gain approach. In [7], general uncertain linear MASs are considered, and a sufficient and necessary condition for the solvability of the output consensus problem is proposed. It is admirable that Wieland et al. introduce an important concept of internal model to cooperative control, which is also fundamental in output regulation theory.

For the leader-follower consensus problem, also called consensus tracking problem, it involves one or multiple leaders and several followers. A leader is usually the target to be tracked, or the agent that directly receives the information of the target. Distributed controllers are designed to help all the agents to track one or multiple leaders by cooperation. The tracking problem for MASs has been studied in a lot of papers, such as [17–22]. Note that for heterogeneous MASs, the outputs of all the agents are required to be synchronized in the issue of both consensus problem and tracking problem. To consider the two kinds of problems under a unified framework is one of the motivations to introduce distributed output regulation (DOR) problem.

According to [23], output regulation problem involves an exosystem and a regulated output defined by a combination of the measurement output and the output of the exosystem. Controllers are designed to stabilize the closed-loop system and modulate the regulated outputs to the origin. So, it has an attracting performance on solving a tracking problem in the presence of disturbances. The classic output regulation theory cannot directly be applied to MASs nevertheless. Actually the controllers obtained are probably not in a distributed form. So, a framework of DOR for MASs is introduced in [24–26]. What is mainly different from the classic theory is that the controllers have to be distributed, and only local information is available. In [24], homogenous linear MASs and a directed topology are considered, and dynamic state/output feedback controllers are designed. In [25], heterogenous MASs are challenged, and effective controllers are obtained under the directed acyclic topology. Different from the two works above, the limits on topology are dispelled in [26] by reconstructing the form of controllers. The communication of relative states of controllers replaces the communication of relative outputs.

This paper is basically motivated by [25]. In [25], the distributed controllers are robust to uncertain dynamics with sufficiently small uncertainties. However, more analyses based on information of the uncertainties are not involved in [25]. What we focus on in our paper is that how to design robust controllers if the uncertainty is structured. We suppose that the uncertainties are in a polytopic form. For the distributed robust output regulation problem, we give sufficient conditions of the solvability in terms of linear matrix inequalities (LMIs) and present an approach of controllers design.

An outline of this paper is as follows. In Section 2, some preliminaries and the problem statement are given. In Section 3, for distributed output regulation problem with polytopic uncertainties, the sufficient conditions of solvability and algorithms of both state and output feedback controllers design are proposed. In Section 4, a practical example is taken to show the control effect of our approach, which is compared with that of the algorithm given in [25]. In Section 5, a conclusion is given.

The following notations will be used throughout this paper. $\mathbb{R}$ is the set of real numbers. I_n is the n -dimensional identity matrix. $0_{m \times n}$ is the zero matrix with m rows and n columns, and 0 is the zero matrix with appropriate dimensions. For a symmetric matrix $M \in \mathbb{R}^{n \times n}$, $M > 0$ which means that M is positive definite. M^T is the transposition of the matrix M . $\text{diag block } (M_1, \dots, M_p)$ means a block diagonal matrix with $M_1, \dots, M_p$ as the diagonal entries. $\otimes$ denotes the Kronecker product.

2. Problem Statement

Let us begin with notations in graph theory [27]. A graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is the set of N nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. An edge from node v_j to node v_i is denoted by $(i, j) \in \mathcal{E}$. A subset of $\mathcal{E} \setminus \{(i_1, i_2), (i_2, i_3), \dots, (i_k, i_{k+1})\}$ is called a path from $v_{i_{k+1}}$ to v_{i_1} . If $i_1 = i_{k+1}$, the path is called a loop. N_i denotes the neighbor

set $\{j : (i, j) \in \mathcal{E}, 1 \leq j \leq N\}$, whose cardinality is $|N_i|$. A constant matrix $A_d = [a_{ij}] \in \mathbb{R}^{N \times N}$ is called the adjacency matrix of graph $\mathcal{G}$ if $a_{ij} = 1/|N_i|$ when $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ when $(i, j) \notin \mathcal{E}$. And a constant matrix $L = [l_{ij}] = I_N - A_d \in \mathbb{R}^{N \times N}$ is called the Laplacian matrix of graph $\mathcal{G}$. A graph is called an undirected graph if for all $1 \leq i, j \leq N$, $a_{ij} = a_{ji}$. Or else, it is called a directed graph. If a directed graph does not contain a loop, it is called a directed acyclic graph.

Consider an exosystem with $\omega \in \mathbb{R}^q$ as the state, whose dynamics can be described as follows:

$$\dot{\omega}(t) = S\omega(t), \quad (1)$$

where $S \in \mathbb{R}^{q \times q}$ is a constant matrix, satisfying the following assumption as that in [23, 25].

Assumption 1. S has no eigenvalues with negative real parts.

A MAS consists of N nonidentical dynamic agents which can exchange information among neighborhood. For $i = 1, \dots, N$, the i th agent can be expressed by

$$\begin{aligned} \dot{x}_i(t) &= \bar{A}_i x_i(t) + \bar{B}_i u_i(t) + \bar{E}_i \omega(t), \\ y_i(t) &= \bar{C}_i x_i(t), \end{aligned} \quad (2)$$

$$e_i(t) = \bar{C}_i x_i(t) + F\omega(t),$$

where $x_i \in \mathbb{R}^{p_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_i \in \mathbb{R}^n$, and $e_i \in \mathbb{R}^n$ are, respectively, the state, the control input, the measurement output, and the regulated output of the i th agent. $F \in \mathbb{R}^{n \times q}$ is a certain constant matrix, while those matrices $\bar{A}_i \in \mathbb{R}^{p_i \times p_i}$, $\bar{B}_i \in \mathbb{R}^{p_i \times m_i}$, and $\bar{C}_i \in \mathbb{R}^{n \times p_i}$ are uncertain matrices represented as

$$\begin{aligned} \bar{A}_i &= \sum_{k=1}^{g_i} \lambda_{ki} A_{ki}, \\ \bar{B}_i &= \sum_{k=1}^{g_i} \lambda_{ki} B_{ki}, \\ \bar{C}_i &= \sum_{k=1}^{g_i} \lambda_{ki} C_{ki}, \end{aligned} \quad (3)$$

where A_{ki} , B_{ki} , and C_{ki} are known constant matrices, and λ_{ki} are nonnegative constants satisfying that $\sum_{k=1}^{g_i} \lambda_{ki} = 1$, for $i = 1, \dots, N, k = 1, \dots, g_i$. $\bar{E}_i \in \mathbb{R}^{p_i \times q}$ is an arbitrary constant matrix.

The controlling aim is to stabilize the closed-loop system and also to regulate $e_1, \dots, e_N$ to the origin. For $i = 1, \dots, N$, if e_i is available to the i th agent, the output regulation of the i th agent is simple to be achieved by classic output regulation theory. However, only some of the agents can get information of their own regulated outputs, which are called leader nodes. The set of their serial numbers is denoted by $\mathcal{L}$, while other agents utilize the relative output among neighbors to accomplish the output regulation property. To ensure that all the agents can receive the information of the exosystem by local interaction, the communication graph of the MAS satisfies the following assumption.

Assumption 2. Graph $\mathcal{G}$ is a directed acyclic graph. And for each nonleader node v_i , there exists a leader node v_j such that a path from node v_j to node v_i exists.

In this note, two kinds of distributed dynamic feedback controllers are considered.

(I) Distributed dynamic state feedback controller:

$$\begin{aligned} \dot{z}_i(t) &= G_1 z_i(t) + G_2 e_{i\omega}(t), \\ u_i(t) &= K_{1i} x_i(t) + K_{2i} z_i(t), \\ z_i(t) &\in \mathbb{R}^c, \quad q \leq c, \quad i = 1, \dots, N, \end{aligned} \quad (4)$$

where

$$e_{i\omega}(t) = \begin{cases} e_i(t) & i \in \mathcal{L} \\ \frac{1}{|N_i|} \sum_{j \in N_i} (y_j(t) - y_i(t)) & i \notin \mathcal{L}. \end{cases} \quad (5)$$

K_{1i} , K_{2i} , G_1 , and G_2 are designed matrices with appropriate dimensions.

(II) Distributed dynamic output feedback controller:

$$\begin{aligned} \dot{z}_i(t) &= \bar{G}_{1i} z_i(t) + \bar{G}_{2i} e_{i\omega}(t), \\ u_i(t) &= K_i z_i(t), \\ z_i(t) &\in \mathbb{R}^{c+p_i}, \quad q \leq c, \quad i = 1, \dots, N. \end{aligned} \quad (6)$$

K_i , $\bar{G}_{1i}$, and $\bar{G}_{2i}$ are designed matrices with appropriate dimensions.

The variables $e_{i\omega}(t)$ in distributed feedback control laws (4) and (6) are measurable relative outputs. For $i \in \mathcal{L}$, $e_{i\omega}(t)$ is the regulated output $e_i(t)$. For $i \notin \mathcal{L}$, $e_{i\omega}(t)$ is the average of relative output errors between i th agent and its neighbors.

In the sequel, we rewrite the closed-loop system into a composite form. Let 1_N be a column vector with all the elements as 1. L_i and l_i , respectively, denote the i th row of L and $L 1_N$. For $i \in \mathcal{L}$, $l_i = 1$, while for $i \notin \mathcal{L}$, $l_i = 0$. Let $\chi = (x_1^T, z_1^T, \dots, x_N^T, z_N^T)^T$, $e = (e_1^T, \dots, e_N^T)^T$,

$$\begin{aligned} \mathcal{A}_1 &= \text{diag block}(\mathcal{A}_{11}, \dots, \mathcal{A}_{1N}), \\ \mathcal{A}_2 &= \text{diag block}(\mathcal{A}_{21}, \dots, \mathcal{A}_{2N}), \\ \mathcal{C}_1 &= \text{diag block}((\bar{C}_1 \ 0_{n \times c}), \dots, (\bar{C}_N \ 0_{n \times c})), \\ \mathcal{C}_2 &= \text{diag block}((\bar{C}_1 \ 0_{n \times (c+p_1)}), \dots, (\bar{C}_N \ 0_{n \times (c+p_N)})), \\ \mathcal{E}_1 &= \begin{pmatrix} L_1 \otimes G'_{21} \\ \vdots \\ L_N \otimes G'_{2N} \end{pmatrix}, \quad \mathcal{E}_2 = \begin{pmatrix} L_1 \otimes \bar{G}'_{21} \\ \vdots \\ L_N \otimes \bar{G}'_{2N} \end{pmatrix}, \\ \mathcal{E}_1 &= \begin{pmatrix} \bar{E}_1 \\ l_1 G_2 F \\ \vdots \\ \bar{E}_N \\ l_N G_2 F \end{pmatrix}, \quad \mathcal{E}_2 = \begin{pmatrix} \bar{E}_1 \\ l_1 \bar{G}_{21} F \\ \vdots \\ \bar{E}_N \\ l_N \bar{G}_{2N} F \end{pmatrix}, \end{aligned} \quad (7)$$

where $\mathcal{A}_{1i} = \begin{pmatrix} \bar{A}_i + \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ 0 & G_1 \end{pmatrix}$, $\mathcal{A}_{2i} = \begin{pmatrix} \bar{A}_i & \bar{B}_i K_i \\ 0 & \bar{G}_{1i} \end{pmatrix}$, $G'_{2i} = \begin{pmatrix} 0_{n \times p_i} \\ G_2 \end{pmatrix}$, and $\bar{G}'_{2i} = \begin{pmatrix} 0_{n \times p_i} \\ \bar{G}_{2i} \end{pmatrix}$, for $i = 1, \dots, N$. Then, the closed-loop system (1), (2) with controllers (4) or (6) can be rewritten as

$$\dot{\omega}(t) = S\omega(t),$$

$$\dot{\chi}(t) = A_c \chi(t) + B_c \omega(t), \quad (8)$$

$$e(t) = C_c \chi(t) + F_c \omega(t).$$

With the controller (4),

$$\begin{aligned} A_c &= \mathcal{A}_1 + \mathcal{E}_1 \mathcal{C}_1, & B_c &= \mathcal{E}_1, \\ C_c &= \mathcal{C}_1, & F_c &= 1_N \otimes F, \end{aligned} \quad (9)$$

while with the controller (6),

$$\begin{aligned} A_c &= \mathcal{A}_2 + \mathcal{E}_2 \mathcal{C}_2, & B_c &= \mathcal{E}_2, \\ C_c &= \mathcal{C}_2, & F_c &= 1_N \otimes F. \end{aligned} \quad (10)$$

As a result, the distributed output regulation problem studied in this paper is given as follows.

Problem 3. Distributed output regulation problem: design controllers in the form of (4) or (6) such that the closed-loop system (8) has the following properties.

- (i) It is exponentially stable at the origin with $\omega = 0$.
- (ii) For all initial values $\chi(0)$ and $\omega(0)$, $\lim_{t \rightarrow \infty} e(t) = 0$.

3. Main Results

In this section, we give two theorems about the solvability of Problem 3 and the approach of controller design. First of all, we introduce the concept of quadratic stability and related lemmas, which will be used later.

Definition 4 (Amato [28]). Consider a parametric uncertain linear system given by

$$\dot{x}(t) = A(p_0) x(t), \quad t \in [0, +\infty), \quad (11)$$

where $x(t) \in \mathbb{R}^{n_0}$, $p_0 \in H \subset \mathbb{R}^{m_0}$ is the vector of uncertain parameters, where H is a hyperbox, and $A(\cdot)$ is continuous. This system is said to be quadratically stable (QS) in H if and only if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n_0 \times n_0}$ such that for all $p_0 \in H$, $A^T(p_0)P + PA(p_0) < 0$.

Lemma 5 (Amato [28]). Assume that the system (11) is QS. Then, for any function $p_0(\cdot)$ that is piecewise continuous on $\mathbb{R}^+$ and valued on W , the linear time-varying system $\dot{x}(t) = A(p_0(t))x(t)$ is exponentially stable.

Lemma 6 (Amato [28]). Assume that the uncertain system is in a polytopic form; that is,

$$\dot{x}(t) = A(t) x(t), \quad A(t) \in \text{conv}\{A_1, A_2, \dots, A_g\}, \quad (12)$$

where $A_1, \dots, A_g \in \mathbb{R}^{n_0 \times n_0}$ and $\text{conv}\{\cdot\}$ represents the convex hull of the following matrices. It is QS if and only if there exists a positive definite symmetric matrix P such that for $k = 1, \dots, g$, $A_k^T P + P A_k < 0$.

Second, we need to recall the concept of internal model and its property.

Definition 7 (Huang [23]). Given any square matrix S , a pair of matrices $(\mathcal{G}_1, \mathcal{G}_2)$ is said to incorporate a p -copy internal model of the matrix S if the pair satisfies that

$$\mathcal{G}_1 = T \begin{pmatrix} S_1 & S_2 \\ 0 & G_1 \end{pmatrix} T^{-1}, \quad \mathcal{G}_2 = T \begin{pmatrix} S_3 \\ G_2 \end{pmatrix}, \quad (13)$$

where S_1, S_2 , and S_3 are arbitrary constant matrices of appropriate dimensions, T is any nonsingular matrix with the same dimension as $\mathcal{G}_1$, and G_1, G_2 are described as follows:

$$\begin{aligned} G_1 &= \text{block diag}(\beta_1, \dots, \beta_p), \\ G_2 &= \text{block diag}(\sigma_1, \dots, \sigma_p), \end{aligned} \quad (14)$$

where for $j = 1, \dots, p$, β_j is a constant square matrix of dimension d_j for some integer d_j and σ_j is a constant column vector of dimension d_j such that

- (i) β_i and σ_i are controllable,
- (ii) the minimal polynomial of S divides the characteristic polynomial of β_i .

Lemma 8 (Huang [23]). Under Assumption 1, assume that $(\mathcal{G}_1, \mathcal{G}_2)$ incorporates a p -copy internal model of S . Let

$$\begin{bmatrix} \widehat{A} & \widehat{B} \\ \mathcal{G}_2 \widehat{C} & \mathcal{G}_1 \end{bmatrix} \quad (15)$$

be exponentially stable, where $\widehat{A}$, $\widehat{B}$, and $\widehat{C}$ are any matrices with appropriate dimensions. Then, for any matrices $\widehat{E}$ and $\widehat{F}$ of appropriate dimensions, the following matrix equations

$$\begin{aligned} XS &= \widehat{A}X + \widehat{B}Z + \widehat{E}, \\ ZS &= \mathcal{G}_1 Z + \mathcal{G}_2 (\widehat{C}X + \widehat{F}) \end{aligned} \quad (16)$$

have a unique solution X and Z . Moreover, X satisfies $\widehat{C}X + \widehat{F} = 0$.

Based on these preparations, theorems on solvability of Problem 3 with state/output feedback controllers are given as follows.

Theorem 9. Suppose that Assumptions 1 and 2 hold and the pair (G_1, G_2) is a p -copy internal model of the exosystem (1). For $i = 1, \dots, N$, if there exist a matrix Y_i and a positive definite matrix $\tilde{P}_i$ satisfying that

$$\widetilde{A}_{ki} \tilde{P}_i + \tilde{P}_i \widetilde{A}_{ki}^T + \widetilde{B}_{ki} Y_i + Y_i^T \widetilde{B}_{ki}^T < 0, \quad k = 1, \dots, g_i, \quad (17)$$

where

$$\widetilde{A}_{ki} = \begin{pmatrix} A_{ki} & 0 \\ G_2 C_{ki} & G_1 \end{pmatrix}, \quad \widetilde{B}_{ki} = \begin{pmatrix} B_{ki} \\ 0 \end{pmatrix}, \quad (18)$$

Then, Problem 3 is solvable by a dynamic state feedback controller (4), where $(K_{1i}, K_{2i}) = K_i = Y_i \tilde{P}_i^{-1}$.

Proof. Let $\gamma_i(t) = (x_i^T, z_i^T)^T$. Then, the subsystems of (8) with controller (4) can be written as follows: for $i \in \mathcal{L}$,

$$\dot{\gamma}_i(t) = \begin{pmatrix} \bar{A}_i + \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ G_2 \bar{C}_i & G_1 \end{pmatrix} \gamma_i(t) + \begin{pmatrix} \bar{E}_i \\ G_2 F \end{pmatrix} w(t), \quad (19)$$

and for $i \notin \mathcal{L}$,

$$\begin{aligned} \dot{\gamma}_i(t) &= \begin{pmatrix} \bar{A}_i + \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ G_2 \bar{C}_i & G_1 \end{pmatrix} \gamma_i(t) + \begin{pmatrix} \bar{E}_i \\ 0 \end{pmatrix} w(t) \\ &\quad + \frac{1}{|N_i|} \sum_{j \in N_i} \begin{pmatrix} 0 & 0 \\ -G_2 \bar{C}_j & 0 \end{pmatrix} \gamma_j(t). \end{aligned} \quad (20)$$

According to [29], by relabeling the nodes, a directed acyclic graph could be put into an ordered form. That is to say, for all edges $(i, j) \in \mathcal{E}$, $i > j$ holds. Notice that matrix A_c is consequently a block lower triangular matrix and system (8) with $w = 0$ is asymptotically stable if and only if all the subsystems below are asymptotically stable

$$\begin{aligned} \dot{\gamma}_i(t) &= \begin{pmatrix} \bar{A}_i + \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ G_2 \bar{C}_i & G_1 \end{pmatrix} \gamma_i(t) \\ &= \sum_{k=1}^{g_i} \lambda_{ki} \begin{pmatrix} A_{ki} + B_{ki} K_{1i} & B_{ki} K_{2i} \\ G_2 C_{ki} & G_1 \end{pmatrix} \gamma_i(t) \\ &= \sum_{k=1}^{g_i} \lambda_{ki} (\widetilde{A}_{ki} + \widetilde{B}_{ki} K_i) \gamma_i(t). \end{aligned} \quad (21)$$

Consider LMI (17). When it holds for a symmetric positive definite matrix $\tilde{P}_i$ and a matrix Y_i , let $P_i = \tilde{P}_i^{-1}$, and let $K_i = Y_i P_i$. Pre- and postmultiplied by P_i , (17) is equivalent to the following inequality:

$$(\widetilde{A}_{ki} + \widetilde{B}_{ki} K_i)^T P_i + P_i (\widetilde{A}_{ki} + \widetilde{B}_{ki} K_i) < 0. \quad (22)$$

According to Lemma 6, when it holds, the subsystems (21) are all QS. And according to Lemma 5, for any polytopic uncertainties, the subsystems are exponentially stable. The system (8) with $w = 0$ is consequently exponentially stable. The condition (i) of Problem 3 has been satisfied. In the following, the error $e(t)$ is proved to converge to zero.

Since the matrix $\begin{pmatrix} \bar{A}_i + \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ G_2 \bar{C}_i & G_1 \end{pmatrix}$ is Hurwitz, according to Lemma 8, for any $\bar{E}_i$ and F_i , the following matrix equations

$$\begin{aligned} X_i S &= (\bar{A}_i + \bar{B}_i K_{1i}) X_i + \bar{B}_i K_{2i} Z_i + \bar{E}_i, \\ Z_i S &= G_1 Z_i + G_2 (\bar{C}_i X_i + F_i) \end{aligned} \quad (23)$$

have a unique solution X_i and Z_i , and at the same time, $\bar{C}_i X_i + F_i = 0$. For $i \in \mathcal{L}$, $F_i = F$, otherwise, $F_i = (-1/|N_i|) \sum_{j \in N_i} \bar{C}_j X_j$. Therefore, the N coupled matrix equations have a unique solution $(X_1, Z_1, \dots, X_N, Z_N)$.

The proof will be given by induction. As mentioned earlier, each agent can only receive information from the agents with smaller labels, after appropriately relabeling the

directed acyclic graph. That is to say, for $i = 1, 2, \dots, N$, $F_i = F$ or $F_i = (-1/|N_i|) \sum_{j=1, j \in N_i}^{i-1} \bar{C}_j X_j$. For $i = 1$, the first agent does not communicate with any other agents but the exosystem, so $F_1 = F$. And we can obtain a unique pair of (X_1, Z_1) that satisfies (23) and $\bar{C}_1 X_1 + F_1 = 0$. For $i = 2$, if it is a leader, then $F_2 = F$; or else, it can only communicate with the first agent, which means that $F_2 = -\bar{C}_1 X_1$. In both cases F_2 is a certain and known matrix, so there exists a unique solution (X_2, Z_2) of (23) and $\bar{C}_2 X_2 + F_2 = 0$. Suppose that for $i = 1, 2, \dots, m$, the solution (X_i, Z_i) has already been obtained. And then for $i = m + 1$, either $F_i = F$ or $F_i = (-1/|N_i|) \sum_{j=1, j \in N_i}^{i-1} \bar{C}_j X_j$ is a certain and known matrix. So, the unique solution (X_{m+1}, Z_{m+1}) also exists. By induction, we can obtain the unique solution $(X_1, Z_1, \dots, X_N, Z_N)$, and at the same time, $\bar{C}_1 X_1 + F_1 = \dots = \bar{C}_N X_N + F_N = 0$.

Take notations $\Gamma_i = \begin{pmatrix} X_i \\ Z_i \end{pmatrix}$ and $\tilde{\gamma}_i = \gamma_i - \Gamma_i \omega$, and substitute (23) into (19) and (20). It is obtained that

$$\begin{aligned} \dot{\tilde{\gamma}}_i(t) &= \dot{\gamma}_i(t) - \Gamma_i S \omega(t) \\ &= \begin{pmatrix} \bar{A}_i + \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ G_2 \bar{C}_i & G_1 \end{pmatrix} \tilde{\gamma}_i(t), \quad i \in \mathcal{L}, \\ \dot{\tilde{\gamma}}_i(t) &= \dot{\gamma}_i(t) - \Gamma_i S \omega(t) \\ &= \begin{pmatrix} \bar{A}_i + \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ G_2 \bar{C}_i & G_1 \end{pmatrix} \tilde{\gamma}_i(t) \\ &\quad - \frac{1}{|N_i|} \sum_{j \in N_i} \begin{pmatrix} 0 & 0 \\ G_2 \bar{C}_j & 0 \end{pmatrix} \tilde{\gamma}_j(t), \quad i \notin \mathcal{L}. \end{aligned} \quad (24)$$

By the first part of the proof, the system above is exponentially stable; that is,

$$\lim_{t \rightarrow \infty} \tilde{\gamma}_i(t) = 0, \quad i = 1, \dots, N. \quad (25)$$

Since

$$\begin{aligned} e_{i\omega}(t) &= \bar{C}_i x_i(t) + F \omega(t) \\ &= (\bar{C}_i \ 0) \tilde{\gamma}_i(t) + (\bar{C}_i X_i + F) \omega(t), \quad i \in \mathcal{L}, \\ e_{i\omega}(t) &= \bar{C}_i x_i(t) - \frac{1}{|N_i|} \sum_{j \in N_i} \bar{C}_j x_j(t) \\ &= (\bar{C}_i \ 0) \tilde{\gamma}_i(t) - \frac{1}{|N_i|} \sum_{j \in N_i} (\bar{C}_j \ 0) \tilde{\gamma}_j(t) \\ &\quad + (\bar{C}_i X_i + F_i) \omega(t), \quad i \notin \mathcal{L}, \end{aligned} \quad (26)$$

it is obvious that

$$\lim_{t \rightarrow \infty} e_{i\omega}(t) = 0, \quad i = 1, \dots, N. \quad (27)$$

Under Assumption 2, the statement (27) is equivalent to $\lim_{t \rightarrow \infty} e(t) = 0$. This completes the proof. $\square$

Remark 10. When there is no uncertainty in the system (2), $\bar{A}_i = A_{1i}$, $\bar{B}_i = B_{1i}$, and $\bar{C}_i = C_{1i}$, $i = 1, \dots, N$. Then,

the conclusion of Theorem 9 still holds if the solvability of LMI (17) is replaced by the statement that the pair (A_{1i}, B_{1i}) is stabilizable. In fact, according to [23], from the two statements,

- (i) the pair (A_{1i}, B_{1i}) is stabilizable,
- (ii) Assumption 1 holds, and the pair (G_1, G_2) is an internal model of S ; it is followed that the pair

$$\left(\begin{pmatrix} A_{1i} & 0 \\ G_2 C_{1i} & G_1 \end{pmatrix}, \begin{pmatrix} B_{1i} \\ 0 \end{pmatrix} \right) \quad (28)$$

is stabilizable. It is equivalent to LMI (17) that holds with $g_i = 1$, $i = 1, \dots, N$. Specially, if for $i = 1, \dots, N$, $A_{1i} = A$, $B_{2i} = B$, and $C_{1i} = C$, $g_i = 1$, then in Theorem 9, the solvability of LMI (17) can be replaced by the statement that the pair (A, B) is stabilizable.

Theorem 11. Suppose that Assumptions 1 and 2 hold and the pair (G_1, G_2) is an internal model of the exosystem (1) if the following conditions are satisfied:

- (i) For $i = 1, \dots, N$, there exist a matrix Y_i and a symmetric positive definite matrix $\tilde{P}_i$ satisfying LMI (17),
- (ii) For $i = 1, \dots, N$, and the matrices $Y_i, \tilde{P}_i$ obtained in (i), let $P_i = \tilde{P}_i^{-1}$, and let $K_i = (K_{1i}, K_{2i}) = Y_i P_i$, and there exist matrices M_{1i}, M_{2i}, M_{3i} and a symmetric positive definite matrix Q_i satisfying the following LMI:

$$\begin{pmatrix} \bar{\Pi}_{11,ki} & \bar{\Pi}_{12,ki} \\ \bar{\Pi}_{12,ki}^T & \bar{\Pi}_{22,ki} \end{pmatrix} < 0, \quad (29)$$

where

$$\begin{aligned} \bar{\Pi}_{11,ki} &= P_i \bar{A}_{ki} + \bar{A}_{ki}^T P_i + \bar{B}_{ki} K_i P_i + P_i K_i^T \bar{B}_{ki}^T, \\ \bar{\Pi}_{12,ki} &= P_i \begin{pmatrix} B_{ki} K_{1i} \\ 0 \end{pmatrix} \\ &\quad + \begin{pmatrix} -A_{ki}^T Q_i - K_{1i}^T B_{ki} Q_i + M_{1i}^T + C_{ki}^T M_{3i}^T \\ -K_{2i}^T B_{ki} Q_i + M_{2i}^T \end{pmatrix}, \\ \bar{\Pi}_{22,ki} &= -Q_i B_{ki} K_{1i} - K_{1i}^T B_{ki}^T Q_i + M_{1i} + M_{1i}^T, \\ &\quad k = 1, \dots, g_i. \end{aligned} \quad (30)$$

Then, Problem 3 is solvable by a dynamic output feedback controller (6) with

$$\bar{G}_{1i} = \begin{pmatrix} T_{1i} & T_{2i} \\ 0 & G_1 \end{pmatrix}, \quad \bar{G}_{2i} = \begin{pmatrix} T_{3i} \\ G_2 \end{pmatrix}, \quad (31)$$

where $T_{1i} = Q_i^{-1} M_{1i}$, $T_{2i} = Q_i^{-1} M_{2i}$, and $T_{3i} = Q_i^{-1} M_{3i}$.

Proof. For $i = 1, \dots, N$, the subsystems of (8) with (6) and (31) can be written in such a form: for $i \in \mathcal{L}$,

$$\dot{\gamma}_i(t) = \begin{pmatrix} \bar{A}_i & \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ T_{3i} \bar{C}_i & T_{1i} & T_{2i} \\ G_2 \bar{C}_i & 0 & G_1 \end{pmatrix} \gamma_i(t) + \begin{pmatrix} \bar{E}_i \\ T_{3i} F \\ G_2 F \end{pmatrix} \omega(t), \quad (32)$$

and for $i \notin \mathcal{L}$,

$$\begin{aligned} \dot{\gamma}_i(t) = & \begin{pmatrix} \bar{A}_i & \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ T_{3i} \bar{C}_i & T_{1i} & T_{2i} \\ G_2 \bar{C}_i & 0 & G_1 \end{pmatrix} \gamma_i(t) + \begin{pmatrix} \bar{E}_i \\ 0 \\ 0 \end{pmatrix} \omega(t) \\ & - \frac{1}{|N_i|} \sum_{j \in N_i} \begin{pmatrix} 0 & 0 & 0 \\ T_{3i} \bar{C}_j & 0 & 0 \\ G_2 \bar{C}_j & 0 & 0 \end{pmatrix} \gamma_j(t). \end{aligned} \quad (33)$$

Similar to the proof of the previous theorem, the stability of the system (8) with $\omega = 0$ is dependent on the following subsystems:

$$\begin{aligned} \dot{\gamma}_i(t) = & \begin{pmatrix} \bar{A}_i & \bar{B}_i K_{1i} & \bar{B}_i K_{2i} \\ T_{3i} \bar{C}_i & T_{1i} & T_{2i} \\ G_2 \bar{C}_i & 0 & G_1 \end{pmatrix} \gamma_i(t) \\ = & \sum_{k=1}^{g_i} \lambda_{ki} \begin{pmatrix} A_{ki} & B_{ki} K_{1i} & B_{ki} K_{2i} \\ T_{3i} C_{ki} & T_{1i} & T_{2i} \\ G_2 C_{ki} & 0 & G_1 \end{pmatrix} \gamma_i(t). \end{aligned} \quad (34)$$

Let

$$\xi_i(t) = \begin{pmatrix} I_{p_i} & 0 & 0 \\ 0 & 0 & I_c \\ -I_{p_i} & I_{p_i} & 0 \end{pmatrix} \gamma_i(t). \quad (35)$$

Then, the subsystem (34) is similar to the following system:

$$\dot{\xi}_i(t) = \sum_{k=1}^{g_i} \lambda_{ki} \begin{pmatrix} A_{ki} + B_{ki} K_{1i} & B_{ki} K_{2i} & B_{ki} K_{1i} \\ G_2 C_{ki} & G_1 & 0 \\ U_{ki} & V_{ki} & W_{ki} \end{pmatrix} \xi_i(t), \quad (36)$$

where

$$\begin{aligned} U_{ki} &= T_{1i} + T_{3i} C_{ki} - A_{ki} - B_{ki} K_{1i}, \\ V_{ki} &= T_{2i} - B_{ki} K_{2i}, \quad W_{ki} = T_{1i} - B_{ki} K_{1i}. \end{aligned} \quad (37)$$

According to Lemma 6, the system (36) is QS if there exists a symmetric positive definite matrix $R_i > 0$ such that

$$\begin{aligned} & \begin{pmatrix} A_{ki} + B_{ki} K_{1i} & B_{ki} K_{2i} & B_{ki} K_{1i} \\ G_2 C_{ki} & G_1 & 0 \\ U_{ki} & V_{ki} & W_{ki} \end{pmatrix}^T R_i \\ & + R_i \begin{pmatrix} A_{ki} + B_{ki} K_{1i} & B_{ki} K_{2i} & B_{ki} K_{1i} \\ G_2 C_{ki} & G_1 & 0 \\ U_{ki} & V_{ki} & W_{ki} \end{pmatrix} < 0. \end{aligned} \quad (38)$$

Suppose that R_i is a block diagonal matrix, which means that $R_i = \begin{pmatrix} P_i & 0 \\ 0 & Q_i \end{pmatrix}$, $P_i \in \mathbb{R}^{(p_i+c) \times (p_i+c)}$, and $Q_i \in \mathbb{R}^{p_i \times p_i}$. Then, the inequality above can be rewritten as

$$\begin{pmatrix} \Pi_{11,ki} & \Pi_{12,ki} \\ \Pi_{12,ki}^T & \Pi_{22,ki} \end{pmatrix} < 0, \quad (39)$$

where

$$\begin{aligned} \Pi_{11,ki} &= (\bar{A}_{ki} + \bar{B}_{ki} K_i)^T P_i + P_i (\bar{A}_{ki} + \bar{B}_{ki} K_i), \\ \Pi_{12,ki} &= P_i \begin{pmatrix} B_{ki} K_{1i} \\ 0 \end{pmatrix} + \begin{pmatrix} U_{ki}^T \\ V_{ki}^T \end{pmatrix} Q_i, \end{aligned} \quad (40)$$

$$\Pi_{22,ki} = W_{ki}^T Q_i + Q_i W_{ki}.$$

Notice that $\Pi_{11,ki} < 0$ is just the inequality (22), which is equivalent to LMI (17). When $\Pi_{11,ki} < 0$ holds, the matrices $\bar{P}_i$, Y_i , $P_i = \bar{P}_i^{-1}$, $K_i = (K_{1i}, K_{2i}) = Y_i \bar{P}_i$ have already been obtained. Let $Q_i T_{1i} = M_{1i}$, $Q_i T_{2i} = M_{2i}$, and $Q_i T_{3i} = M_{3i}$. We turn the inequality (39) into LMI (29). This implies that the system (8) with $\omega = 0$ is asymptotically stable.

The rest part of the proof is similar to that in Theorem 9. Recalling that $\begin{pmatrix} \bar{A}_i & \bar{B}_i K_i \\ \bar{G}_2 \bar{C}_i & \bar{G}_{1i} \end{pmatrix}$ is Hurwitz and $(\bar{G}_{1i}, \bar{G}_{2i})$ incorporates a p -copy internal model of S , we can obtain that for any matrices $\bar{E}_i$ and F_i , the following matrix equations

$$\begin{aligned} \bar{X}_i S &= \bar{A}_i \bar{X}_i + \bar{B}_i K_i \bar{Z}_i + \bar{E}_i, \\ \bar{Z}_i S &= \bar{G}_{1i} \bar{Z}_i + \bar{G}_{2i} (\bar{C}_i \bar{X}_i + F_i) \end{aligned} \quad (41)$$

have a unique solution $\bar{X}_i$ and $\bar{Z}_i$. And meanwhile, $\bar{C}_i \bar{X}_i + F_i = 0$. For $i \in \mathcal{L}$, $F_i = F$, otherwise $F_i = (-1/|N_i|) \sum_{j \in N_i} \bar{C}_j \bar{X}_j$.

Take notations $\bar{\Gamma}_i = \begin{pmatrix} \bar{X}_i \\ \bar{Z}_i \end{pmatrix}$ and $\bar{\gamma}_i(t) = \gamma_i(t) - \bar{\Gamma}_i \omega(t)$. By calculation, it can be verified that $\lim_{t \rightarrow \infty} \bar{\gamma}_i(t) = 0$. Consequently, $\lim_{t \rightarrow \infty} e_{i\omega}(t) = 0$. When Assumption 2 holds, the above statement is equivalent to $\lim_{t \rightarrow \infty} e(t) = 0$. This completes the proof. $\square$

Remark 12. Consider the robust tracking problem for heterogeneous MAS in the presence of disturbances. Suppose that an active leader is described as

$$\dot{x}_k(t) = A_k x_k(t), \quad y_k(t) = C_k x_k(t), \quad (42)$$

and there is an environmental disturbance $d(t)$ satisfying

$$\dot{d}(t) = A_d d(t). \quad (43)$$

The model of the MAS can be given in

$$\begin{aligned} \dot{x}_i(t) &= \bar{A}_i x_i(t) + \bar{B}_i u_i(t) + E_{di} d(t), \\ y_i(t) &= \bar{C}_i x_i(t), \\ i &= 1, \dots, N, \end{aligned} \quad (44)$$

where $\bar{A}_i$, $\bar{B}_i$, $\bar{C}_i$, and E_{di} are uncertain matrices with polytopic uncertainties. The control target is to design distributed controllers such that the outputs of followers track the output of the leader. That is to say, for $i = 1, \dots, N$, $e_i(t) = y_i(t) - y_k(t)$ converges to zero. This problem is just Problem 3 with $\omega = (x_k^T, d^T)^T$, $S = \text{diag block } (A_k, A_d)$, $\bar{E} = (0 \ E_{di})$, and $F = (-C_k \ 0)$. The robust tracking problem for the systems (44), therefore, can be studied under the framework of DOR problem. It can be solved by designing the controller (4) or (6) if the conditions in Theorems 9 or 11 are satisfied.

Remark 13. By Theorems 9 and 11, the sufficient condition of the solvability for Problem 3 depends on the dynamics of each agent. For a heterogenous MAS, N LMIs have to be solved to obtain appropriate distributed controllers, which limits scalability. When the agents share some common characteristics, such as nominal parts, the complexity of LMIs will decrease. Specially, if agents are identical in the sense of common nominal parts and common bounds of the uncertainties, the solution to one LMI will construct the required control law.

4. A Numerical Example

In this section, a numerical example of a heterogenous MAS consisting seven agents is given to show the control effect of controllers obtained by Theorems 9 and 11. A comparison is also given between our controllers and that of [25]. In the example, suppose that there is a constant environmental noise and the reference signal (the output of the tracking target) is a sinusoidal signal. Let $\omega = (\omega_1, \omega_2, \omega_3)^T$, where ω_1, ω_2 serve as the reference signals and ω_3 as the exogenous disturbance. Obviously, the differential equations about them are $\dot{\omega}_1(t) = -\omega_1(t)$, $\dot{\omega}_2(t) = -\omega_2(t)$, and $\dot{\omega}_3(t) = 0$. Therefore, the exosystem can be described by (1) with

$$S = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (45)$$

According to [23], we can immediately obtain a 1-copy internal model as follows:

$$G_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad G_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \quad (46)$$

In the example, three kinds of linear agents with 2, 3, and 4 orders are considered. 2 or 3 basement matrices are randomly valued for each kind. We present the detailed description of basement matrices in the appendix.

And as for $\bar{E}_1, \dots, \bar{E}_7$, their first two columns are zeros, noticing that only ω_3 refers to the disturbance, while the other elements are valued randomly. Finally, the regulated output, regarded as the tracking error between $y_i(t)$ and the sine reference signal $\omega_1(t)$, is defined as $e_i(t) = y_i(t) - \omega_1(t) = \bar{C}_i x_i(t) + F\omega(t)$, so, value $F = (-1 \ 0 \ 0)$.

The communication network is described by the directed acyclic graph shown in Figure 1, which satisfies Assumption 2. According to Theorem 11, by solving three kinds of LMIs as (17), we get control gains $K_1 = K_2 = (-6.3230, 28.5507, 2.6636, 0.9019, 6.8977)$, $K_3 = K_4 = K_5 = (0.9071, -9.9908, 10.4217, -1.2503, 0.9513, -3.1324)$, $K_6 = K_7 = (226.6539, 117.7418, -20.0749, -91.1218, 4.6064, -2.6648, 11.6774)$. If the states of each agent are measurable, then a candidate distributed controller can be chosen as (4) with the coefficients $K_1, \dots, K_7$. Or else, one more group of LMIs (29) need to be solved to construct the output feedback controllers. In this situation, we obtain T_{1i}, T_{2i} and T_{3i} for $i = 1, \dots, 7$, which are also listed in the appendix due to

FIGURE 1: Information flow among exosystem and agents.

space limitation. And then the distributed dynamic output feedback controllers are given as (6). For simplicity, we denote this control law as controller C_a .

On the other hand, we translate the polytopic uncertainty into an equivalent form that consists of a nominal part and an uncertain part. In this case, the nominal part $A_1, \dots, A_7, B_1, \dots, B_7$, and $C_1, \dots, C_7$ can be valued as the average of the basement matrices. Then, by the algorithm in [25], for $i = 1, \dots, 7$, the candidate control law can be chosen as (6) with

$$\bar{G}_{1i} = \begin{pmatrix} A_i + B_i K_{1i} + L_i C_i & B_i K_{2i} \\ 0 & G_i \end{pmatrix}, \quad (47)$$

$$\bar{G}_{2i} = \begin{pmatrix} -L_i \\ G_i \end{pmatrix}, \quad J_i = Q_i^{-1} Z_i,$$

where $K_1 = K_2 = (-10, 0, 2, 1, -1)$, $L_1 = L_2 = (9.5, 10)^T$, $K_3 = K_4 = K_5 = (-9, -2, -4, 3, 2, 4)$, $L_3 = L_4 = L_5 = (30, -70, 28)^T$, $K_6 = K_7 = (10, 10, -2, 5, 1, 2, -2)^T$, $L_6 = L_7 = (-69, -41, -10, -10)^T$. For simplicity, this control law obtained by [25] is denoted by controller C_b .

Note that both controllers C_a and C_b are robust controllers against the model uncertainty. To compare the effects, two cases of uncertainties are considered in the example. For one case, the model uncertainties denoted by Δ_1 are rather close to the “nominal part” set earlier. For $i = 1, \dots, 7$, $k = 1, \dots, g_i$, the values of λ_{ki} are randomly chosen around $1/g_i$, whose values are shown in the appendix. For another case, the model uncertainties denoted by Δ_2 are valued freely with reasonable coefficients, which are relatively farther from the “nominal part.” With the same initial values, the numerical results of the regulated output for each agent are demonstrated in Figure 2 and Figure 3. Our controller C_a is effective in the presence of either Δ_1 or Δ_2 . However, as for Controller C_b , when the uncertainty is relatively small, it can help to achieve the output regulation property. While in the case of Δ_2 , the regulated errors diverge rapidly as shown in Figure 3(b). As a consequence of the above, for Problem 3 with polytopic uncertainties, Theorems 9 and 11 help construct distributed dynamic state/output feedback controllers of stronger robustness, at the price of increasing the calculation complexity.

5. Conclusions

In this paper, we consider the distributed robust output regulation problem for MASs under the topology of directed

FIGURE 2: Regulated output of the MAS with the controller C_a . FIGURE 3: Regulated output of the MAS with the controller C_b .

acyclic graphs. As an extension of the existing results, we focus on a special class of parametric uncertainty. Assume that the dynamics of heterogeneous agents are in a polytopic form, and we study the solvability of the problem and the design of distributed controllers. Both dynamic state feedback controller and dynamic output feedback controller are under consideration. At last, a practical example is presented to validate our results. By local interaction, a team of heterogeneous agents is required to achieve a common output, which is generated by an exosystem. The controller generated by our theorem can realize the property of output regulation and the performance of tracking the reference signal and rejecting the disturbances as a special case.

Appendix

Here are some detailed descriptions of the example in Section 4. Agents v_1 and v_2 are systems with 2 orders. And there are three basement matrices for an uncertain system, which means $g_1 = g_2 = 3$

$$A_{11} = A_{12} = \begin{pmatrix} -4.8512 & -8.9578 \\ -8.7108 & 7.4003 \end{pmatrix},$$

$$A_{21} = A_{22} = \begin{pmatrix} -4.5841 & -7.5263 \\ -9.3135 & 7.0596 \end{pmatrix},$$

$$A_{31} = A_{32} = \begin{pmatrix} -5.7140 & -8.2750 \\ -9.1175 & 7.0544 \end{pmatrix},$$

$$\begin{aligned}
B_{11} &= B_{12} = \begin{pmatrix} 1.6684 \\ -0.0861 \end{pmatrix}, \\
B_{21} &= B_{22} = \begin{pmatrix} 1.9897 \\ -0.7854 \end{pmatrix}, \quad B_{31} = B_{32} = \begin{pmatrix} 1.8509 \\ -0.2403 \end{pmatrix}, \\
C_{11} &= C_{12} = (1.0614 \quad 0.8390), \\
C_{21} &= C_{22} = (1.0102 \quad 0.9982), \\
C_{31} &= C_{32} = (0.9283 \quad 0.7936).
\end{aligned} \tag{A.1}$$

Agents v_3 , v_4 , and v_5 are systems with 3 orders. And $g_3 = g_4 = g_5 = 2$

$$\begin{aligned}
A_{13} &= A_{14} = A_{15} = \begin{pmatrix} 2.5419 & -1.2319 & 4.7038 \\ 1.9598 & -1.9843 & -4.6719 \\ -2.6654 & -2.4159 & -4.6598 \end{pmatrix}, \\
A_{23} &= A_{24} = A_{25} = \begin{pmatrix} 1.1025 & -2.1701 & 3.6488 \\ 0.0927 & -0.1010 & -2.6299 \\ -3.4064 & -3.1707 & -3.2973 \end{pmatrix},
\end{aligned} \tag{A.2}$$

$$B_{13} = B_{14} = B_{15} = \begin{pmatrix} 3.2738 \\ 2.3681 \\ -0.0088 \end{pmatrix},$$

$$B_{23} = B_{24} = B_{25} = \begin{pmatrix} 4.3570 \\ 1.9363 \\ -0.0489 \end{pmatrix},$$

$$C_{13} = C_{14} = C_{15} = (-2.9090 \quad -1.6423 \quad -0.6208), \tag{A.3}$$

$$C_{23} = C_{24} = C_{25} = (-3.5357 \quad -1.0246 \quad -2.3913). \tag{A.4}$$

Agents v_6 and v_7 are systems with 4 orders. And $g_6 = g_7 = 3$

$$A_{16} = A_{17} = \begin{pmatrix} 7.3858 & 7.0606 & -1.9638 & -6.3218 \\ 1.5941 & 2.4411 & -8.4807 & -5.2009 \\ 0.9972 & -2.9810 & -5.2017 & -1.6547 \\ -7.1009 & 0.2650 & -7.5336 & -9.0069 \end{pmatrix},$$

$$A_{26} = A_{27} = \begin{pmatrix} 5.7750 & 7.7097 & -3.0848 & -4.7077 \\ -0.1851 & 0.8409 & -8.0396 & -3.7288 \\ 1.0337 & -2.4579 & -4.1684 & -3.4229 \\ -7.0579 & 1.8202 & -7.1493 & -10.8314 \end{pmatrix},$$

$$A_{36} = A_{37} = \begin{pmatrix} 7.6867 & 8.3454 & -1.3674 & -6.1338 \\ -0.1668 & 0.5027 & -7.5538 & -6.0157 \\ -0.0637 & -4.8089 & -4.6107 & -0.6759 \\ -7.6883 & -1.0590 & -7.7299 & -10.2511 \end{pmatrix},$$

$$B_{16} = B_{17} = \begin{pmatrix} 0.0681 \\ 0.0862 \\ 2.5410 \\ 2.3587 \end{pmatrix},$$

$$B_{26} = B_{27} = \begin{pmatrix} -0.2206 \\ 0.3290 \\ 1.9179 \\ 2.2930 \end{pmatrix},$$

$$B_{36} = B_{37} = \begin{pmatrix} -0.2305 \\ 0.9642 \\ 3.2929 \\ 2.4590 \end{pmatrix},$$

$$C_{16} = C_{17} = (1.4942 \quad -2.5319 \quad -1.0521 \quad 1.0049),$$

$$C_{26} = C_{27} = (0.9337 \quad -1.6942 \quad -1.9109 \quad 0.4535),$$

$$C_{36} = C_{37} = (1.4678 \quad -2.6602 \quad -1.1586 \quad 0.6176). \tag{A.5}$$

Next, these are the matrices obtained according to the solution of LMIs (29)

$$T_{11} = T_{12} = \begin{pmatrix} 37.5608 & 109.7053 \\ -118.3196 & -149.5498 \end{pmatrix},$$

$$T_{21} = T_{22} = \begin{pmatrix} 4.8984 & 1.6586 & 12.6849 \\ -1.7007 & -0.5759 & -4.4042 \end{pmatrix},$$

$$T_{31} = T_{32} = \begin{pmatrix} -43.5409 & -74.3826 \\ 91.9349 & 157.0558 \end{pmatrix},$$

$$T_{13} = T_{14} = T_{15} = \begin{pmatrix} -6.8197 & -42.1931 & 33.9132 \\ 9.9034 & -14.7875 & 19.6377 \\ -2.6741 & -1.6089 & -3.5333 \end{pmatrix},$$

$$T_{23} = T_{24} = T_{25} = \begin{pmatrix} -4.2895 & 3.2637 & -10.7466 \\ -2.3038 & 1.7529 & -5.7718 \\ 0.1139 & -0.0867 & 0.2854 \end{pmatrix},$$

$$T_{33} = T_{34} = T_{35} = \begin{pmatrix} 11.1566 & 5.9291 & 5.2847 \\ -7.3616 & -3.9123 & -3.4871 \\ -0.3312 & -0.1760 & -0.1569 \end{pmatrix},$$

$$T_{16} = T_{17}$$

$$= \begin{pmatrix} -167.8742 & 206.0977 & 254.4031 & -44.6630 \\ 15.0769 & 147.1908 & 106.7331 & -62.5135 \\ 462.5231 & 174.7488 & -97.4980 & -165.1735 \\ 532.2860 & 236.1899 & -93.3484 & -210.2411 \end{pmatrix},$$

$$T_{26} = T_{27} = \begin{pmatrix} -1.0162 & 0.5879 & -2.5760 \\ 1.5155 & -0.8767 & 3.8419 \\ 8.8346 & -5.1108 & 22.3961 \\ 10.5625 & -6.1104 & 26.7763 \end{pmatrix},$$

$$T_{36} = T_{37}$$

$$= \begin{pmatrix} 123.6493 & -224.3619 & -253.0593 & 60.0567 \\ 59.3071 & -107.6129 & -121.3773 & 28.8056 \\ -26.7899 & 48.6103 & 54.8279 & -13.0119 \\ -19.6265 & 35.6123 & 40.1673 & -9.5326 \end{pmatrix}. \tag{A.6}$$

The coefficients in case of the uncertainty Δ_1 are as follows:

$$\begin{aligned} \lambda_{11} &= 0.33, & \lambda_{21} &= 0.33, & \lambda_{31} &= 0.34, \\ \lambda_{12} &= 0.4, & \lambda_{22} &= 0.3, & \lambda_{32} &= 0.3, \\ \lambda_{13} &= 0.6, & \lambda_{23} &= 0.4, \\ \lambda_{14} &= 0.45, & \lambda_{24} &= 0.55, & (A.7) \\ \lambda_{15} &= 0.33, & \lambda_{25} &= 0.37, & \lambda_{35} &= 0.4, \\ \lambda_{16} &= 0.35, & \lambda_{26} &= 0.36, & \lambda_{36} &= 0.39, \\ \lambda_{17} &= 0.41, & \lambda_{27} &= 0.3, & \lambda_{37} &= 0.39. \end{aligned}$$

The coefficients in case of the uncertainty Δ_2 are as follows:

$$\begin{aligned} \lambda_{11} &= 0.1, & \lambda_{21} &= 0.2, & \lambda_{31} &= 0.7, \\ \lambda_{12} &= 0.5, & \lambda_{22} &= 0, & \lambda_{32} &= 0.5, \\ \lambda_{13} &= 0, & \lambda_{23} &= 1, \\ \lambda_{14} &= 0.81, & \lambda_{24} &= 0.19, & (A.8) \\ \lambda_{15} &= 0.3, & \lambda_{25} &= 0.5, & \lambda_{35} &= 0.2, \\ \lambda_{16} &= 0.02, & \lambda_{26} &= 0.13, & \lambda_{36} &= 0.85, \\ \lambda_{17} &= 0.3, & \lambda_{27} &= 0.1, & \lambda_{37} &= 0.6. \end{aligned}$$

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