Research Article

Sliding Mode Control for NSVs with Input Constraint Using Neural Network and Disturbance Observer

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The sliding mode control (SMC) scheme is proposed for near space vehicles (NSVs) with strong nonlinearity, high coupling, parameter uncertainty, and unknown time-varying disturbance based on radial basis function neural networks (RBFNNs) and the nonlinear disturbance observer (NDO). Considering saturation characteristic of rudders, RBFNNs are constructed as a compensator to overcome the saturation nonlinearity. The stability of the closed-loop system is proved, and the tracking error as well as the disturbance observer error can converge to the origin through the Lyapunov analysis. Simulation results are presented to demonstrate the effectiveness of the proposed flight control scheme.

1. Introduction

The NSV [1–3] is a kind of vehicles and flights above 20 km from the ground, which can be divided into low-speed NSV and high-speed NSV. Among them, the high-speed NSV has some significant advantages such as rapid response ability, difficulty of detecting, and strong penetration ability. Thus, it has drawn much attention around the world in recent years. Since the NSV has the large flight envelop as well as the changeable flight environment and it is susceptible to the severely external time-varying disturbance, it is necessary to develop the robust control scheme for the NSV. However, strong nonlinearity, high coupling, parameter uncertainty, and unknown time-varying disturbance will further increase the design difficulty of the robust flight control [4]. To efficiently handle the system uncertainty and the suffering unknown external disturbance, NDO can be employed to design the robust control scheme. According to the known information of the studied system, NDO can estimate the compounded disturbance on line and feedback the estimate value to the control system. As a result, the disturbance cancellation is guaranteed to improve the performance and robustness of the closed-loop system. Therefore, the technology of NDO has been extensively studied. In [4–7], NDO was used to estimate the system uncertainty and the unknown

external disturbance, and it was successfully applied to the robust control design of inverted pendulum and air vehicles, respectively.

Saturation as a common input nonlinearity exists in a wide range of practical systems, such as the NSV. The existence of input saturation will degrade the control system performance and even leads to the system instability if it is ignored in the control design. On the other hand, the control design considering input saturation is a challenging problem for uncertain nonlinear systems. Several control schemes for nonlinear systems with input saturation have been proposed in recent years. In [8–10], neural network was proposed to approximate the input-output difference of the actuator, and a compensator was designed to overcome the saturation nonlinearity. In [11, 12], considering the smoothness of a hyperbolic tangent function, the robust adaptive control was proposed based on the backstepping approach via using the special property of a Nussbaum function to handle input saturation. However, the rudder saturation of the NSV needs to be further studied in the robust attitude control design.

For the nonlinear systems, many robust control schemes were studied [13–18] in which SMC is an effective method. By applying a discontinuous control signal to change the system state, the system is forced to slide along a predesigned sliding mode. It has higher robustness because of the insensitivity to

the parameter uncertainty. However, the control input needs to be changed according to the system states, which causes the system trajectory passing across the sliding mode repeatedly. Thus, there exists chattering problem in the system [19]. By designing a new switching function which is related to the first-order or higher-order derivative of the control input, the continuous dynamic sliding mode control law can be obtained through integration. Therefore, the system chattering can be effectively reduced. In [20], a robust dynamic sliding mode control law was proposed to control a multi-degree-of-freedom humanoid robot arm, and satisfactory control results had been obtained through simulation. In [21], higher-order sliding mode control and sliding mode differentiator were studied. In [22], a second-order dynamic sliding control scheme was proposed to overcome chattering problem for the NSV. However, the sliding mode control considering input saturation should be further developed for the NSV based on NDO.

Motivated by above discussion and analysis, the robust attitude control is developed for the NSV in the presence of the system uncertainty, the unknown time-varying disturbance, and the input saturation. To handle the compounded disturbance, the NDO is proposed. Since the rudders of the NSV have the saturation characteristic, RBFNNs are constructed as a compensator to overcome the saturation nonlinearity. The stability of the closed-loop system is proved, and the tracking error as well as the disturbance observer error can converge to the origin through the Lyapunov analysis. The organization of the paper is as follows. In Section 2, the dynamic sliding mode control is reviewed. The robust attitude control is investigated for the NSV by considering the system uncertainty, the unknown external disturbance, and the input saturation in Section 3, and simulation results are given to illustrate the effectiveness of the proposed robust attitude control scheme. Section 4 concludes the remark.

For the convenience of description, the following notations are required.

Notations. Throughout this paper, $|\bullet|$ stands for absolute value of each element of the vector; $|\bullet|^c$ denotes power operation for the absolute value of each element of the vector, where c is a power exponent; $\text{sgn}(\bullet)$ stands for sign function operation of each element of the vector; $\|\bullet\|$ represents the Euclidean norm (or Frobenius norm for a matrix); $\int_0^t (\bullet) dt$ stands for integral operation of each element of the vector; $\text{diag}(\bullet)$ represents a diagonal matrix constructed by the elements of the vector; $\text{diag}\{\text{sgn}(\bullet)\}$ denotes constructing a diagonal matrix after sign function operation of the vector. Specifically, given a matrix W and a vector χ , then we have

$$\begin{aligned} |\chi| &= [|\chi_1|, |\chi_2|, \dots, |\chi_n|]^T, \\ |\chi|^c &= [|\chi_1|^c, |\chi_2|^c, \dots, |\chi_n|^c]^T, \\ \text{sgn}(\chi) &= [\text{sgn}(\chi_1), \text{sgn}(\chi_2), \dots, \text{sgn}(\chi_n)]^T, \\ \|\chi\| &= \sqrt{\chi_1^2 + \chi_2^2 + \dots + \chi_n^2}, \end{aligned}$$

$$\|W\| = \sqrt{\sum_{i=1}^n \sum_{j=1}^m w_{ij}^2}, W \in R^{n \times m},$$

$$\int_0^t \chi dt = \left[\int_0^t \chi_1 dt, \int_0^t \chi_2 dt, \dots, \int_0^t \chi_n dt \right]^T,$$

$$\text{diag}(\chi) = \begin{bmatrix} \chi_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \chi_n \end{bmatrix} \in R^{n \times n},$$

$$\text{diag}(\text{sgn}(\chi)) = \begin{bmatrix} \text{sgn}(\chi_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \text{sgn}(\chi_n) \end{bmatrix} \in R^{n \times n}.$$

(1)

2. Design of Dynamic Sliding Mode Control for MIMO System

In this paper, the dynamic sliding mode control scheme is used to the robust attitude control design of the NSV. For the convenience of control development, the dynamic sliding mode control scheme is described as follows according to [22, 23].

Consider that a class of multiinput and multioutput (MIMO) nonlinear affine systems is given by

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t), \\ y(t) &= h(x(t)), \end{aligned} \quad (2)$$

where $x \in R^n$ is the state vector of the nonlinear system, $y \in R^m$ is the output vector, $u \in R^m$ is the control input vector, and $f(x) \in R^n$, $g(x) \in R^{n \times m}$, and $h(x) \in R^m$ are continuous functions which are related to the state vector x . Suppose that the sliding mode surface for the MIMO nonlinear system (2) can be written as $\sigma = [\sigma_1(x), \sigma_2(x), \dots, \sigma_m(x)]^T \in R^m$.

To analyze the nonlinear system (2), the following definitions and assumptions are required.

Definition 1 (see [24]). The gradient of the smooth scalar function $\sigma_i(x)$ with respect to the state vector x is defined as

$$\nabla \sigma_i(x) = \frac{\partial \sigma_i(x)}{\partial x} = \left[\frac{\partial \sigma_i(x)}{\partial x_1}, \frac{\partial \sigma_i(x)}{\partial x_2}, \dots, \frac{\partial \sigma_i(x)}{\partial x_n} \right]. \quad (3)$$

Definition 2 (see [24]). The Lie derivative of the smooth scalar function $\sigma_i(x)$ with respect to the vector field $f(x)$ is given by

$$L_f \sigma_i(x) = \nabla \sigma_i(x) f(x). \quad (4)$$

The multiple Lie derivative can be defined as the following recurrence relations:

$$\begin{aligned} L_f^0 \sigma_i(x) &= \sigma_i(x), \\ L_f^{k_i} \sigma_i(x) &= L_f L_f^{k_i-1} \sigma_i(x) = \nabla (L_f^{k_i-1} \sigma_i(x)) f(x), \\ k_i &= 1, 2, 3, \dots \end{aligned} \quad (5)$$

FIGURE 1: structure of the robust control scheme for the NSV.

Definition 3 (see [24]). If the Lie derivative of the smooth scalar function $\sigma_i(x)$ with respect to the vector field $f(x)$ satisfies the following two expressions:

$$\begin{aligned} L_f^{k_i} \sigma_i(x) &= 0, \quad k_i = 0, 1, \dots, r_i - 1, \\ L_f^{r_i} \sigma_i(x) &\neq 0, \end{aligned} \quad (6)$$

then the relative degree of $\sigma_i(x)$ with respect to $f(x)$ is r_i . Also, the relative degree of $\sigma = [\sigma_1(x), \sigma_2(x), \dots, \sigma_m(x)]^T$ with respect to $f(x)$ is r , where $r = \max\{r_1, r_2, \dots, r_m\}$.

Assumption 4 (see [22]). For the MIMO nonlinear system (2), the state x can converge to the origin when the system enters the sliding mode of $\sigma = 0$.

Assumption 5 (see [22]). For the MIMO nonlinear system (2), the relative degree vector of the sliding mode σ with respect to the input u is $[r_1, r_2, \dots, r_m]$. That is, the following equations are always held for all x in the domain:

$$\begin{aligned} \sum_{j=1}^m L_{g_j} L_f^{k_i} \sigma_i(x) &= 0, \quad k_i = 0, 1, \dots, r_i - 1, \\ \sum_{j=1}^m L_{g_j} L_f^{r_i-1} \sigma_i(x) &\neq 0, \end{aligned} \quad (7)$$

where g_j is the j th column vector of the matrix g , $i = 1, 2, \dots, m$.

Assumption 6 (see [22]). For the MIMO nonlinear system (2), the following matrix is invertible for all x in the domain:

$$\begin{bmatrix} L_{g_1} \left(L_f^{r_1-1} \sigma_1(x) \right) & L_{g_2} \left(L_f^{r_1-1} \sigma_1(x) \right) & \cdots & L_{g_m} \left(L_f^{r_1-1} \sigma_1(x) \right) \\ L_{g_1} \left(L_f^{r_2-1} \sigma_2(x) \right) & L_{g_2} \left(L_f^{r_2-1} \sigma_2(x) \right) & \cdots & L_{g_m} \left(L_f^{r_2-1} \sigma_2(x) \right) \\ \vdots & \vdots & \ddots & \vdots \\ L_{g_1} \left(L_f^{r_m-1} \sigma_m(x) \right) & L_{g_2} \left(L_f^{r_m-1} \sigma_m(x) \right) & \cdots & L_{g_m} \left(L_f^{r_m-1} \sigma_m(x) \right) \end{bmatrix} \in \mathbb{R}^{m \times m}. \quad (8)$$

The derivative of $\sigma_i(x)$ is given by

$$\sigma_i^{(k_i)} = L_f^{k_i} \sigma_i(x), \quad k_i = 0, 1, \dots, r_i - 1, \quad (9)$$

$$\sigma_i^{(r_i)} = L_f^{r_i} \sigma_i(x) + \sum_{j=1}^m L_{g_j} L_f^{r_i-1} \sigma_i(x) u_j, \quad (10)$$

$$\begin{aligned} \sigma_i^{(r_i+1)} &= L_f^{r_i+1} \sigma_i(x) + \sum_{j=1}^m L_{g_j} L_f^{r_i} \sigma_i(x) u_j \\ &+ \sum_{l=1}^m L_{g_l} \left(\sum_{j=1}^m L_{g_j} L_f^{r_i-1} \sigma_i(x) u_j \right) u_l \\ &+ \sum_{j=1}^m L_f L_{g_j} L_f^{r_i-1} \sigma_i(x) u_j \\ &+ \sum_{j=1}^m L_{g_j} L_f^{r_i-1} \sigma_i(x) \dot{u}_j, \end{aligned} \quad (11)$$

where $i = 1, 2, \dots, m$.

Design a new sliding mode as

$$\vartheta_i = \sigma_i^{(r_i)} + c_{i,1} \sigma_i^{(r_i-1)} + \cdots + c_{i,r_i-1} \dot{\sigma}_i + c_{i,r_i} \sigma_i + c_{i,r_i+1}, \quad (12)$$

where $c_{i,j}$ ($i = 1, 2, \dots, m$, $j = 1, 2, \dots, r_i + 1$) must make the polynomial (12) Hurwitz stable.

The derivative of $\dot{\vartheta}_i$ is given by

$$\begin{aligned} \dot{\vartheta}_i &= \sigma_i^{(r_i+1)} + c_{i,1} \sigma_i^{(r_i)} + \cdots + c_{i,r_i-1} \ddot{\sigma}_i + c_{i,r_i} \dot{\sigma}_i \\ &= \sigma_i^{(r_i+1)} + \sum_{j=1}^{r_i} c_{i,j} \sigma_i^{(r_i+1-j)}. \end{aligned} \quad (13)$$

Considering (11) and (13), we obtain

$$\begin{aligned} \dot{\vartheta} &= A(x, u) + B(x) \dot{u} + C(x, u), \\ A(x, u) &= A_1(x) + A_2(x) u + A_3(x) u + A_4(x) u, \end{aligned} \quad (14)$$

where $\dot{\vartheta} = [\dot{\vartheta}_1, \dot{\vartheta}_2, \dots, \dot{\vartheta}_m]^T$, and we have

$$\begin{aligned}
A_1(x) &= [L_f^{r_1+1}\sigma_1(x), L_f^{r_2+1}\sigma_2(x), \dots, L_f^{r_m+1}\sigma_m(x)]^T \in R^m, \\
A_2(x) &= \begin{bmatrix} L_{g_1}L_f^{r_1}\sigma_1(x) & \cdots & L_{g_m}L_f^{r_1}\sigma_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1}L_f^{r_m}\sigma_m(x) & \cdots & L_{g_m}L_f^{r_m}\sigma_m(x) \end{bmatrix} \in R^{m \times m}, \\
A_3(x) &= \begin{bmatrix} L_f L_{g_1} L_f^{r_1-1}\sigma_1(x) & \cdots & L_f L_{g_m} L_f^{r_1-1}\sigma_1(x) \\ \vdots & \ddots & \vdots \\ L_f L_{g_1} L_f^{r_m-1}\sigma_m(x) & \cdots & L_f L_{g_m} L_f^{r_m-1}\sigma_m(x) \end{bmatrix} \in R^{m \times m}, \\
A_4(x) &= \begin{bmatrix} L_{g_1} \left(\sum_{j=1}^m L_{g_j} L_f^{r_1-1}\sigma_1(x) u_j \right) & \cdots & L_{g_m} \left(\sum_{j=1}^m L_{g_j} L_f^{r_1-1}\sigma_1(x) u_j \right) \\ \vdots & \ddots & \vdots \\ L_{g_1} \left(\sum_{j=1}^m L_{g_j} L_f^{r_m-1}\sigma_m(x) u_j \right) & \cdots & L_{g_m} \left(\sum_{j=1}^m L_{g_j} L_f^{r_m-1}\sigma_m(x) u_j \right) \end{bmatrix} \in R^{m \times m}, \\
B(x) &= \begin{bmatrix} L_{g_1} L_f^{r_1-1}\sigma_1(x) & \cdots & L_{g_m} L_f^{r_1-1}\sigma_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m-1}\sigma_m(x) & \cdots & L_{g_m} L_f^{r_m-1}\sigma_m(x) \end{bmatrix} \in R^{m \times m}, \\
C(x, u) &= \left[\sum_{j=1}^{r_1} c_{1,j} \sigma_1^{(r_1+1-j)}, \sum_{j=1}^{r_2} c_{2,j} \sigma_2^{(r_2+1-j)}, \dots, \sum_{j=1}^{r_m} c_{m,j} \sigma_m^{(r_m+1-j)} \right]^T \in R^m.
\end{aligned} \tag{15}$$

To ensure the reaching condition of the sliding mode surface hold, the reaching law Φ_ϑ can take an exponential form which is described by

$$\Phi_\vartheta = -\bar{C}_1 \vartheta - \bar{C}_2 \operatorname{sgn}(\vartheta), \tag{16}$$

where $\bar{C}_i = \operatorname{diag}\{\bar{c}_{i,1}, \bar{c}_{i,2}, \dots, \bar{c}_{i,m}\} > 0, i = 1, 2$, are designed matrices.

Considering (14) and (16), we have

$$\dot{u} = B^{-1}(x) (\Phi_\vartheta - A(x, u) - C(x, u)). \tag{17}$$

Integration of (17) yields

$$u(t) = u(0) + \int_0^t (B^{-1}(x) (\Phi_\vartheta - A(x, u) - C(x, u))) dt. \tag{18}$$

Since the discontinuous term of the reaching law is taken into integration, we can obtain a continuous control input in

the whole time domain. Thus, the chattering of the system can be effectively reduced.

3. Design of Robust Attitude Control for the NSV

3.1. Attitude Dynamic Model for the NSV. Due to the fact that the change of attitude angular rate is faster than that of attitude angle, the attitude motion can be decomposed into two affine MIMO nonlinear systems [25] in accordance with singularly perturbed theory and time-scale separation principle:

$$\begin{aligned}
\dot{\Omega} &= f_s(\Omega) + g_s(\Omega) \omega + g_{s\delta}(\Omega) \delta(v(t)), \\
y_s &= \Omega, \\
\dot{\omega} &= f_f(\omega) + g_{f1} g_{f\delta}(\omega) \delta(v(t)), \\
y_f &= \omega,
\end{aligned} \tag{19}$$

where (19) are called slow-loop equation and fast-loop equation, respectively, and $\Omega = [\alpha, \beta, \mu]^T$ is the vector of attitude angles which are angle of attack, sideslip angle, and roll angle, $\omega = [p, q, r]^T$ is the vector of attitude angular rates which are roll angular rate, pitch angular rate, and yaw angular rate. In slow-loop equation, $f_s(\Omega) = [f_\alpha, f_\beta, f_\mu]^T$

is the state function vector, $g_s(\Omega)$ is the system matrix, and $g_{s\delta}(\Omega)$ is allocation matrix of rudders. In fast-loop equation, $f_f(\omega) = [f_p, f_q, f_r]^T$ is the state function vector, g_{f1} is the system matrix, $g_{f\delta}(\omega)$ is allocation matrix of rudders, $v(t) = [v_1, v_2, v_3, v_4, v_5]^T$ is the control input vector, and $\delta(v(t)) = [\delta_a, \delta_e, \delta_r, \delta_y, \delta_z]^T$ denotes the plant input vector subject to saturation nonlinearity which are ailerons, elevator, rudder, lateral deflection, and longitudinal deflection of the thrust vector control surface. The detailed expressions of corresponding terms in (19) can be found in [25].

The saturated function of $\delta_i(v_i)$ can be expressed as

$$\delta_i(v_i) = \text{sat}(v_i) = \begin{cases} \delta_{iM}, & v_i > \delta_{iM}, \\ v_i, & -\delta_{iM} \leq v_i \leq \delta_{iM}, \\ -\delta_{iM}, & v_i < -\delta_{iM}, \end{cases} \quad (20)$$

where δ_{iM} is the bound of $\delta_i(v_i)$, $i = 1, 2, 3, 4, 5$, $\delta(v) = [\delta_1(v_1), \delta_2(v_2), \delta_3(v_3), \delta_4(v_4), \delta_5(v_5)]^T$.

Considering that the control effect of Ω is mainly determined by $g_s\omega$ and the effect of $g_{s\delta}\delta$ on slow-loop system is very small, $g_{s\delta}\delta$ is taken as a part of the compounded disturbance D_s . On the other hand, there exists modeling error for the NSV, and the NSV is affected by the unknown time-varying external disturbance in flight. Thus, (19) can be rewritten as

$$\begin{aligned} \dot{\Omega} &= f_s + g_s\omega + D_s, \\ \gamma_s &= \Omega, \end{aligned} \quad (21)$$

$$\begin{aligned} \dot{\omega} &= f_f + g_f\delta(v) + D_f, \\ \gamma_f &= \omega, \end{aligned} \quad (22)$$

where $D_s = \Delta f_s + \Delta g_s\omega + g_{s\delta}\delta$ and $D_f = \Delta f_f + \Delta g_f\delta + d_f$ are the compounded disturbance of slowloop and fastloop, respectively; Δf_s , Δg_s , Δf_f , and Δg_f are modeling error of slowloop and fastloop, d_f is the time-varying external disturbance of fastloop, $g_f = g_{f1}g_{f\delta}$.

In this paper, the control objective is that the robust attitude control is designed to render γ_s to follow the given desired trajectory Ω_c in the presence of the system uncertainty, unknown external disturbance, and input saturation. The external disturbance only acts on fast-loop system in the form of torque. On the other hand, the slow-loop system is only affected by the system uncertainty. The block diagram of the developed robust control scheme is shown in Figure 1.

To proceed the design of flight control scheme for the attitude motion (21) and (22) of the NSV, the following assumptions are required.

Assumption 7. For the attitude motion (21) and (22) of the NSV, the compounded disturbance of slow-loop system and its first-order derivative are bounded, that is, $|D_{s,i}| \leq \beta_{s,i}$, $\beta_{s,i} > 0$, $|\dot{D}_{s,i}| \leq \beta_{ds,i}$, $\beta_{ds,i} > 0$, $D_s = [D_{s,1}, D_{s,2}, D_{s,3}]^T$, $\dot{D}_s = [\dot{D}_{s,1}, \dot{D}_{s,2}, \dot{D}_{s,3}]^T$, $\beta_s = [\beta_{s,1}, \beta_{s,2}, \beta_{s,3}]^T$, $\beta_{ds} = [\beta_{ds,1}, \beta_{ds,2}, \beta_{ds,3}]^T$. At the same time, the compounded disturbance of fastloop is bounded, that is, $|D_{f,i}| \leq \beta_{f,i}$, $\beta_{f,i} > 0$, $D_f = [D_{f,1}, D_{f,2}, D_{f,3}]^T$.

Assumption 8 (see [22]). For the attitude motion (21) and (22) of the NSV, the desired attitude angle Ω_c and its second-order derivative are known, bounded, and continuous.

Assumption 9 (see [26]). For the attitude motion (21) and (22) of the NSV, generalized inverse matrices of the control gain matrices g_s and g_f exist.

Remark 10. For a practical system, the external disturbance is bounded since it actually exists. In addition, the parameter uncertainty $\Delta(\bullet)$ is the function of system state Ω or ω . Thus, we assume that the boundaries of D_s , $\dot{D}_s$, and D_f satisfy $|D_{s,i}| \leq \beta_{s,i}$, $|\dot{D}_{s,i}| \leq \beta_{ds,i}$, and $|D_{f,i}| \leq \beta_{f,i}$, respectively. Accordingly, the Assumption 7 is reasonable.

3.2. Design of Slow-Loop System for the NSV. In slow-loop system, only the system uncertainty is considered. The adaptive control approach is used to estimate the upper bound of the system uncertainty. The robust control scheme of slow-loop system is proposed based on the dynamic sliding mode control. To obtain the corresponding function's derivative, the higher-order sliding mode differentiator (HOSMD) [27–29] is employed because of the ability of its arbitrary approximation.

Theorem 11. Consider slow-loop system (21) of the NSV satisfied Assumptions 7–9. Under sliding modes (23) and (24), exponential reaching law (25), parameter updated law (26), and the dynamic sliding mode control law (27), the tracking error of slow-loop system asymptotically converges to the origin:

$$\sigma_{s1} = e_s + \int_0^t A_{s1} e_s dt, \quad (23)$$

$$\sigma_{s2} = \dot{\sigma}_{s1} + A_{s2} \sigma_{s1}, \quad (24)$$

$$\Phi_{\sigma_{s2}} = -B_{s1} \sigma_{s2} - B_{s2} \text{sgn}(\sigma_{s2}), \quad (25)$$

$$\dot{\hat{\beta}}_{ds} = B_{s3} |\sigma_{s2}|, \quad (26)$$

$$\omega_c = g_s^{-1} \int_0^t (\psi_1 + \psi_2) dt, \quad (27)$$

$$\psi_1 = -\dot{f}_s + \ddot{\Omega}_c - (A_{s1} + A_{s2}) \dot{e}_s - A_{s2} A_{s1} e_s,$$

$$\psi_2 = -B_{s1} \sigma_{s2} - B_{s2} \text{sgn}(\sigma_{s2}) - \text{diag}\{\text{sgn}(\sigma_{s2})\} \hat{\beta}_{ds},$$

where $e_s = \Omega - \Omega_c$ is the tracking error of slowloop, $A_{s1} = \text{diag}\{a_{s1,1}, a_{s1,2}, a_{s1,3}\} > 0$, $A_{s2} = \text{diag}\{a_{s2,1}, a_{s2,2}, a_{s2,3}\} > 0$, $B_{s1} = \text{diag}\{b_{s1,1}, b_{s1,2}, b_{s1,3}\} > 0$, $B_{s2} = \text{diag}\{b_{s2,1}, b_{s2,2}, b_{s2,3}\} > 0$, and $B_{s3} = \text{diag}\{b_{s3,1}, b_{s3,2}, b_{s3,3}\} > 0$ are designed matrices, and $\hat{\beta}_{ds}$ is the estimate value of β_{ds} .

Proof. Consider the Lyapunov function as

$$V_s = \frac{1}{2} \sigma_{s2}^T \sigma_{s2} + \frac{1}{2} \tilde{\beta}_{ds}^T B_{s3}^{-1} \tilde{\beta}_{ds}, \quad (28)$$

where $\tilde{\beta}_{ds} = \beta_{ds} - \hat{\beta}_{ds}$ and $\dot{\tilde{\beta}}_{ds} = \dot{\beta}_{ds} - \dot{\hat{\beta}}_{ds} = -\dot{\hat{\beta}}_{ds}$.

Differentiating (24) and considering (21), (23), and (27), we have

$$\begin{aligned}\dot{\sigma}_{s2} &= \dot{f}_s + \psi_1 + \psi_2 + \dot{D}_s - \ddot{\Omega}_c + A_{s1}\dot{e}_s + A_{s2}\dot{e}_s + A_{s2}A_{s1}e_s \\ &= \dot{D}_s - B_{s1}\sigma_{s2} - B_{s2} \operatorname{sgn}(\sigma_{s2}) - \operatorname{diag}\{\operatorname{sgn}(\sigma_{s2})\} \hat{\beta}_{ds}.\end{aligned}\quad (29)$$

Invoking (26) and (29), the time derivative of V_s is given by

$$\begin{aligned}\dot{V}_s &= \sigma_{s2}^T \dot{\sigma}_{s2} - \tilde{\beta}_{ds}^T B_{s3}^{-1} \dot{\beta}_{ds} \\ &= \sigma_{s2}^T (\dot{D}_s - B_{s1}\sigma_{s2} - B_{s2} \operatorname{sgn}(\sigma_{s2}) \\ &\quad - \operatorname{diag}\{\operatorname{sgn}(\sigma_{s2})\} \hat{\beta}_{ds}) - \tilde{\beta}_{ds}^T |\sigma_{s2}| \\ &\leq -\sigma_{s2}^T B_{s1}\sigma_{s2} - \sigma_{s2}^T B_{s2} \operatorname{sgn}(\sigma_{s2}) + |\sigma_{s2}|^T \tilde{\beta}_{ds} \\ &\quad - |\sigma_{s2}|^T \hat{\beta}_{ds} - \tilde{\beta}_{ds}^T |\sigma_{s2}| \\ &= -\sum_{i=1}^3 b_{s1,i} \sigma_{s2,i}^2 - \sum_{i=1}^3 b_{s2,i} |\sigma_{s2,i}|,\end{aligned}\quad (30)$$

where $\sigma_{s2} = [\sigma_{s2,1}, \sigma_{s2,2}, \sigma_{s2,3}]$.

It is obvious that if $\sigma_{s2} \neq 0$, then $\dot{V}_s < 0$. Thus, the sliding mode σ_{s2} satisfies the reaching condition, and σ_{s2} can asymptotically converge to the origin. From (23) and (24), we can know that the sliding mode σ_{s1} and the tracking error e_s converge to the origin when σ_{s2} converges to the origin. This concludes the proof. $\square$

Remark 12. ω_c in (27) is the controller of slow-loop system as well as the desired input of fast-loop system.

Remark 13. In the control law of the slow-loop system, the differential terms, such as $\dot{f}_s$, $\dot{e}_s$, and $\dot{\sigma}_{s1}$, are difficult to obtain through derivative method. Thus, HOSMD is employed to get the corresponding derivatives. The form of n th-order HOSMD is given by [27–29]

$$\begin{aligned}\dot{z}_0 &= c_0 = -\lambda_0 |z_0 - f(t)|^{n/(n+1)} \operatorname{sgn}(z_0 - f(t)) + z_1 \\ &\vdots \\ \dot{z}_i &= c_i = -\lambda_i |z_i - c_{i-1}|^{(n-i)/(n-i+1)} \operatorname{sgn}(z_i - c_{i-1}) + z_{i+1} \\ &\vdots \\ \dot{z}_{n-1} &= c_{n-1} = -\lambda_{n-1} |z_{n-1} - c_{n-2}|^{(1/2)} \operatorname{sgn}(z_{n-1} - c_{n-2}) + z_n \\ \dot{z}_n &= -\lambda_n \operatorname{sgn}(z_n - c_{n-1}),\end{aligned}\quad (31)$$

where z_i and c_j are states of the system (31), $\lambda_0, \lambda_1, \dots, \lambda_n$ are designed parameters, and $f(t)$ is the known function. The aim of HOSMD is to make c_j approximate the differential term $f(t)^{(j+1)}$ to arbitrary accuracy, $i = 0, 1, \dots, n$, $j = 0, 1, \dots, n-1$.

3.3. Design of Fast-Loop System for the NSV. In fast-loop system, the compounded disturbance is subjected to the system uncertainty as well as the unknown time-varying external disturbance. In general, the order of magnitude of the external disturbance is much larger than that of the system uncertainty. If we still use the adaptive scheme to handle the compounded disturbance of fast-loop system, the system trajectory will pass through the equilibrium point repeatedly. Thus, the designed disturbance cancellation can not accurately compensate the suffering disturbance and leads to the performance of the control system degeneracy. Here, the nonlinear disturbance observer is proposed to estimate the compounded disturbance. However, the robust control design of fast-loop system is different from that of slow-loop system based on DSMC and NDO. The reason is that the output derivative of NDO will be contained in the control law and it is more complex and difficult to prove the closed-loop system stability as well as the boundedness of disturbance observer error. Therefore, the traditional sliding mode control is used in fast-loop system, and the double power reaching law is designed to reduce the chattering of the system.

Considering that rudders are subject to saturation nonlinearity in fast-loop system and RBFNNs can smoothly approximate any continuous function over the compact set to arbitrary accuracy [30, 31]. Here, RBFNNs are constructed as a compensator to overcome the saturation nonlinearity. Specifically, we employ RBFNNs to estimate the limited part exceeding the bound of saturation and use it to design the robust control law. As a result, the actuator can drop out the saturation nonlinearity.

Considering (20), we obtain

$$\delta(v) = \operatorname{sat}(v) = v + \xi(v), \quad (32)$$

where $\xi(v)$ is the limited part exceeding the actuator.

Using RBFNNs to approximate $\xi(v)$, we have

$$\xi(v) = W_\xi^{*T} s_\xi(z) + \varepsilon_\xi, \quad (33)$$

where $W_\xi^* \in R^{l \times 5}$ are the optimal weights and satisfy $\|W_\xi^*\| \leq W_M$, l is the number of nodes; $z = [\omega_c, e_f]^T$ is the input vector of RBFNNs; $\varepsilon_\xi = [\varepsilon_{\xi,1}, \varepsilon_{\xi,2}, \varepsilon_{\xi,3}, \varepsilon_{\xi,4}, \varepsilon_{\xi,5}]^T$ is the smallest approximated error, and ε_ξ can be arbitrarily small through turning weights and nodes; suppose $|\varepsilon_{\xi,i}| \leq \varepsilon_{M\xi,i}$ and $\varepsilon_{M\xi,i} > 0$ in this paper; $s_\xi(z) = [s_{\xi,1}, s_{\xi,2}, \dots, s_{\xi,l}]^T$ is the radial basis function vector, and $s_{\xi k}$ has the form as follows:

$$s_{\xi k} = \exp\left(-\frac{\|z - c_k\|^2}{2b_k^2}\right), \quad (34)$$

where c_k and b_k are the center and width of the neural cell of the k th hidden layer, $k = 1, 2, \dots, l$.

Design the control law as

$$v = v_0 - v_\xi + v_r, \quad (35)$$

where v_0 is the controller by neglecting saturation nonlinearity, $v_\xi = \widehat{W}_\xi^T s_\xi(z)$ is the output of RBFNNs, and v_r is the robust term.

Substituting (33) and (35) into (32), we obtain

$$\begin{aligned}\delta(v) &= v_0 - v_\xi + v_r + W_\xi^{*T} s_\xi(z) + \varepsilon_\xi \\ &= v_0 + \widetilde{W}_\xi^T s_\xi(z) + v_r + \varepsilon_\xi,\end{aligned}\quad (36)$$

where $\widetilde{W}_\xi = W_\xi^* - \widehat{W}_\xi$.

Considering the unknown compounded disturbance D_f of the fast-loop system (22), it cannot be directly used in the design of fast-loop control scheme. To estimate the compounded disturbance, the nonlinear disturbance observer is presented below.

An auxiliary variable is given by

$$\sigma = \eta - \omega. \quad (37)$$

And the derivative of η is

$$\dot{\eta} = -K\sigma - \text{diag}\{\text{sgn}(\sigma)\} \widehat{\beta}_f + f_f + g_f \delta(v) - K\sigma_f, \quad (38)$$

where $K = \text{diag}\{k_1, k_2, k_3\} > 0$ is a designed matrix, $\widehat{\beta}_f = [\widehat{\beta}_{f,1}, \widehat{\beta}_{f,2}, \widehat{\beta}_{f,3}]^T$ is the estimate value of $\beta_f = [\beta_{f,1}, \beta_{f,2}, \beta_{f,3}]^T$, $\sigma = [\sigma_1, \sigma_2, \sigma_3]^T$ is the auxiliary variable, and $\sigma_f = [\sigma_{f,1}, \sigma_{f,2}, \sigma_{f,3}]^T$ is the sliding mode surface of fast-loop system, which will be introduced later.

The estimate value of D_f can be written as

$$\begin{aligned}\widehat{D}_f &= -K\sigma - \text{diag}\{\text{sgn}(\sigma)\} \widehat{\beta}_f + \text{diag}\{\text{sgn}(\sigma_f)\} |\widehat{\beta}_f| \\ &\quad + \text{diag}\{\text{sgn}(\sigma_f)\} \widehat{\beta}_f.\end{aligned}\quad (39)$$

Differentiating (37) and considering (22) and (38), we have

$$\dot{\sigma} = \dot{\eta} - \dot{\omega} = -K\sigma - \text{diag}\{\text{sgn}(\sigma)\} \widehat{\beta}_f - K\sigma_f - D_f. \quad (40)$$

The disturbance approximation error is given by

$$\widetilde{D}_f = D_f - \widehat{D}_f. \quad (41)$$

Substituting (39) and (40) into (41), we obtain

$$\begin{aligned}\widetilde{D}_f &= K\sigma + \text{diag}\{\text{sgn}(\sigma)\} \widehat{\beta}_f \\ &\quad + D_f - \text{diag}\{\text{sgn}(\sigma_f)\} |\widehat{\beta}_f| \\ &\quad - \text{diag}\{\text{sgn}(\sigma_f)\} \widehat{\beta}_f \\ &= -\dot{\sigma} - K\sigma_f - \text{diag}\{\text{sgn}(\sigma_f)\} |\widehat{\beta}_f| \\ &\quad - \text{diag}\{\text{sgn}(\sigma_f)\} \widehat{\beta}_f.\end{aligned}\quad (42)$$

Theorem 14. Consider that fast-loop system (22) of the NSV satisfied Assumptions 7–9. The sliding mode is designed as (43), and the reaching law is chosen as (44). The nonlinear disturbance observer is proposed as (39), and the parameter adaptation law is designed as (45). RBFNNs are constructed as a compensator to overcome the saturation nonlinearity, and the updated law of the neural network weight is chosen as

(46). Under the control law (47), the tracking error of fast-loop system asymptotically converges to the origin, and the error of the disturbance observer also converges to the origin:

$$\sigma_f = e_f + \int_0^t A_f e_f dt, \quad (43)$$

$$\begin{aligned}\Phi_{\sigma_f} &= -B_{f1} \text{diag}\{\text{sgn}(\sigma_f)\} |\sigma_f|^{c_1} \\ &\quad - B_{f2} \text{diag}\{\text{sgn}(\sigma_f)\} |\sigma_f|^{c_2},\end{aligned}\quad (44)$$

$$\dot{\beta}_f = \Gamma_\beta (|\sigma| + |\sigma_f|), \quad (45)$$

$$\dot{\widehat{W}}_\xi = \Gamma_\xi s_\xi(z) \sigma_f^T g_f - \gamma \|\sigma_f^T g_f\| \Gamma_\xi \widehat{W}_\xi, \quad (46)$$

$$v = v_0 - v_\xi + v_r,$$

$$\begin{aligned}v_0 &= g_f^T (g_f g_f^T)^{-1} (-f_f + \dot{\omega}_c - A_f e_f - \widehat{D}_f \\ &\quad - B_{f1} \text{diag}\{\text{sgn}(\sigma_f)\} |\sigma_f|^{c_1} \\ &\quad - B_{f2} \text{diag}\{\text{sgn}(\sigma_f)\} |\sigma_f|^{c_2}), \\ v_\xi &= \widehat{W}_\xi^T s_\xi(z), \\ v_r &= -K_r \text{sgn}(g_f^T \sigma_f),\end{aligned}\quad (47)$$

where $e_f = \omega - \omega_c$ is the tracking error of fastloop; $A_f = \text{diag}\{a_{f,1}, a_{f,2}, a_{f,3}\} > 0$, $B_{f1} = \text{diag}\{b_{f,1,1}, b_{f,1,2}, b_{f,1,3}\} > 0$, $B_{f2} = \text{diag}\{b_{f,2,1}, b_{f,2,2}, b_{f,2,3}\} > 0$, $\Gamma_\beta = \Gamma_\beta^T > 0$, $\Gamma_\xi = \Gamma_\xi^T > 0$, and $K_r = \text{diag}\{k_{r,1}, k_{r,2}, k_{r,3}, k_{r,4}, k_{r,5}\}$ are designed matrices, and $k_{r,i} - \varepsilon_{M\xi,i} - W_M^2 \gamma / 4 > 0$; $\gamma > 0$, $c_1 > 1$ and $0 < c_2 < 1$ are designed parameters.

Proof. Consider the Lyapunov function as

$$V_f = \frac{1}{2} \sigma_f^T \sigma_f + \frac{1}{2} \sigma^T \sigma + \frac{1}{2} \widetilde{\beta}_f^T \Gamma_\beta^{-1} \widetilde{\beta}_f + \frac{1}{2} \text{tr}(\widetilde{W}_\xi^T \Gamma_\xi^{-1} \widetilde{W}_\xi), \quad (48)$$

where $\widetilde{\beta}_f = \beta_f - \widehat{\beta}_f$, $\widetilde{W}_\xi = W_\xi^* - \widehat{W}_\xi$, and $\dot{\widetilde{\beta}}_f = \dot{\beta}_f - \dot{\widehat{\beta}}_f = -\dot{\widehat{\beta}}_f$ and $\dot{\widetilde{W}}_\xi = \dot{W}_\xi^* - \dot{\widehat{W}}_\xi = -\dot{\widehat{W}}_\xi$.

Differentiating (43) and considering (22), we obtain

$$\dot{\sigma}_f = f_f + g_f \delta(v) + D_f - \dot{\omega}_c + A_f e_f. \quad (49)$$

Substituting (36), (41), and (47) into (49), we have

$$\begin{aligned}\dot{\sigma}_f &= f_f + D_f - \dot{\omega}_c + A_f e_f \\ &\quad + g_f (v_0 + \widetilde{W}_\xi^T s_\xi(z) + v_r + \varepsilon_\xi) \\ &= \widetilde{D}_f - B_{f1} \text{diag}\{\text{sgn}(\sigma_f)\} |\sigma_f|^{c_1} \\ &\quad - B_{f2} \text{diag}\{\text{sgn}(\sigma_f)\} |\sigma_f|^{c_2} \\ &\quad + g_f \widetilde{W}_\xi^T s_\xi(z) - g_f K_r \text{sgn}(g_f^T \sigma_f) + g_f \varepsilon_\xi.\end{aligned}\quad (50)$$

Invoking (40) and (50), the time derivative of V_f is given by

$$\begin{aligned}\dot{V}_f &= \sigma_f^T \dot{\sigma}_f + \sigma_f^T \dot{\sigma} - \tilde{\beta}_f^T \Gamma_\beta^{-1} \dot{\tilde{\beta}}_f - \text{tr} \left(\tilde{W}_\xi^T \Gamma_\xi^{-1} \dot{\tilde{W}}_\xi \right) \\ &= \sigma_f^T \tilde{D}_f - \sum_{i=1}^3 b_{f1,i} |\sigma_{f,i}|^{1+c_1} - \sum_{i=1}^3 b_{f2,i} |\sigma_{f,i}|^{1+c_2} \\ &\quad + \sigma_f^T g_f \tilde{W}_\xi^T s_\xi(z) - \sigma_f^T g_f K_r \text{sgn}(\sigma_f) \\ &\quad + \sigma_f^T g_f \varepsilon_\xi - \sum_{i=1}^3 k_i \sigma_i^2 - |\sigma|^T \hat{\beta}_f - \sigma^T K \sigma_f \\ &\quad - \sigma^T D_f - \tilde{\beta}_f^T \Gamma_\beta^{-1} \dot{\tilde{\beta}}_f - \text{tr} \left(\tilde{W}_\xi^T \Gamma_\xi^{-1} \dot{\tilde{W}}_\xi \right).\end{aligned}\quad (51)$$

Considering (42), we obtain

$$\begin{aligned}\sigma_f^T \tilde{D}_f &= \sigma_f^T K \sigma + \sigma_f^T \text{diag} \{ \text{sgn}(\sigma) \} \hat{\beta}_f \\ &\quad + \sigma_f^T D_f - |\sigma_f|^T |\hat{\beta}_f| - |\sigma_f|^T \hat{\beta}_f \\ &\leq \sigma_f^T K \sigma + |\sigma_f|^T |\hat{\beta}_f| + |\sigma_f|^T \hat{\beta}_f \\ &\quad - |\sigma_f|^T |\hat{\beta}_f| - |\sigma_f|^T \hat{\beta}_f \\ &= \sigma_f^T K \sigma + |\sigma_f|^T \tilde{\beta}_f.\end{aligned}\quad (52)$$

Substituting (52) into (51) yields

$$\begin{aligned}\dot{V}_f &\leq -\sum_{i=1}^3 b_{f1,i} |\sigma_{f,i}|^{1+c_1} - \sum_{i=1}^3 b_{f2,i} |\sigma_{f,i}|^{1+c_2} \\ &\quad - \sum_{i=1}^3 k_i \sigma_i^2 - \sigma_f^T g_f K_r \text{sgn}(\sigma_f) + \sigma_f^T g_f \varepsilon_\xi \\ &\quad + |\sigma_f|^T \tilde{\beta}_f - |\sigma|^T \hat{\beta}_f + |\sigma|^T \hat{\beta}_f + \sigma_f^T g_f \tilde{W}_\xi^T s_\xi(z) \\ &\quad - \tilde{\beta}_f^T \Gamma_\beta^{-1} \dot{\tilde{\beta}}_f - \text{tr} \left(\tilde{W}_\xi^T \Gamma_\xi^{-1} \dot{\tilde{W}}_\xi \right) \\ &\leq -\sum_{i=1}^3 b_{f1,i} |\sigma_{f,i}|^{1+c_1} - \sum_{i=1}^3 b_{f2,i} |\sigma_{f,i}|^{1+c_2} \\ &\quad - \sum_{i=1}^3 k_i \sigma_i^2 - \sum_{i=1}^3 (k_{r,i} - \varepsilon_{M\xi,i}) |\sigma_f^T g_{f,i}| + |\sigma_f|^T \tilde{\beta}_f \\ &\quad + |\sigma|^T \tilde{\beta}_f - \tilde{\beta}_f^T \Gamma_\beta^{-1} \dot{\tilde{\beta}}_f \\ &\quad - \text{tr} \left(\tilde{W}_\xi^T \Gamma_\xi^{-1} \dot{\tilde{W}}_\xi - \tilde{W}_\xi^T s_\xi(z) \sigma_f^T g_f \right),\end{aligned}\quad (53)$$

where $g_{f,i}$ is the i th column vector of the matrix g_f . $\square$

Considering parameter updated laws (45) and (46), we have

$$\begin{aligned}\dot{V}_f &\leq -\sum_{i=1}^3 b_{f1,i} |\sigma_{f,i}|^{1+c_1} - \sum_{i=1}^3 b_{f2,i} |\sigma_{f,i}|^{1+c_2} - \sum_{i=1}^3 k_i \sigma_i^2 \\ &\quad - \sum_{i=1}^3 (k_{r,i} - \varepsilon_{M\xi,i}) |\sigma_f^T g_{f,i}| + \text{tr} \left(\gamma \|\sigma_f^T g_f\| \tilde{W}_\xi^T \tilde{W}_\xi \right).\end{aligned}\quad (54)$$

Considering the following fact that

$$\text{tr} \left(\tilde{W}_\xi^T (W_\xi^* - \tilde{W}_\xi) \right) \leq \|\tilde{W}_\xi\| \|W_\xi^*\| - \|\tilde{W}_\xi\|^2, \quad (55)$$

we have

$$\begin{aligned}\text{tr} \left(\gamma \|\sigma_f^T g_f\| \tilde{W}_\xi^T \tilde{W}_\xi \right) &= \gamma \|\sigma_f^T g_f\| \text{tr} \left(\tilde{W}_\xi^T (W_\xi^* - \tilde{W}_\xi) \right) \\ &\leq \gamma \|\sigma_f^T g_f\| \left(\|\tilde{W}_\xi\| W_M - \|\tilde{W}_\xi\|^2 \right) \\ &= \gamma \|\sigma_f^T g_f\| \left(- \left(\|\tilde{W}_\xi\| - \frac{1}{2} W_M \right)^2 + \frac{1}{4} W_M^2 \right).\end{aligned}\quad (56)$$

Substituting (56) into (54) yields

$$\begin{aligned}\dot{V}_f &\leq -\sum_{i=1}^3 b_{f1,i} |\sigma_{f,i}|^{1+c_1} - \sum_{i=1}^3 b_{f2,i} |\sigma_{f,i}|^{1+c_2} \\ &\quad - \sum_{i=1}^3 k_i \sigma_i^2 - \sum_{i=1}^3 (k_{r,i} - \varepsilon_{M\xi,i}) |\sigma_f^T g_{f,i}| \\ &\quad + \frac{1}{4} W_M^2 \gamma \|\sigma_f^T g_f\| - \gamma \|\sigma_f^T g_f\| \left(\|\tilde{W}_\xi\| - \frac{1}{2} W_M \right)^2 \\ &\leq -\sum_{i=1}^3 b_{f1,i} |\sigma_{f,i}|^{1+c_1} - \sum_{i=1}^3 b_{f2,i} |\sigma_{f,i}|^{1+c_2} \\ &\quad - \sum_{i=1}^3 k_i \sigma_i^2 - \sum_{i=1}^3 \left(k_{r,i} - \varepsilon_{M\xi,i} - \frac{1}{4} W_M^2 \gamma \right) |\sigma_f^T g_{f,i}| \\ &\quad - \gamma \|\sigma_f^T g_f\| \left(\|\tilde{W}_\xi\| - \frac{1}{2} W_M \right)^2.\end{aligned}\quad (57)$$

If $k_{r,i} - \varepsilon_{M\xi,i} - W_M^2 \gamma / 4 > 0$, then $\dot{V}_f < 0$. Thus, σ_f and σ converge to the origin. According to (43), we obtain that the tracking error e_f converges to the origin. When σ converges to the origin, we have $\tilde{\sigma} = 0$. Thus, we can know that the approximation error $\tilde{D}_f$ converges to the origin from (42). This includes the proof.

Remark 15. For the double power reaching law [32–34], the first term of (44) plays a key role when the system state is far away from the sliding mode ($|\sigma_{f,i}| > 1$), and the speed of the

FIGURE 2: Angle of attack.

FIGURE 4: Roll angle.

FIGURE 3: Sideslip angle.

FIGURE 5: Roll angular rate.

double power reaching law is faster than that of the standard reaching law. On the other hand, when the system state is close to the sliding mode ($|\sigma_{f,i}| < 1$), the second term of (44) plays a key role, and the speed of the double power reaching law is lower than that of the standard reaching law. Therefore, the chattering phenomenon can be attenuated through the double power reaching law.

3.4. Simulation Study for the NSV. In this section, the simulation results are given to illustrate the effectiveness of the proposed attitude control schemes for the NSV. The initial values are chosen as $\alpha_0 = 2^\circ$, $\beta_0 = 1^\circ$, $\mu_0 = 0^\circ$, $p_0 = q_0 = r_0 = 0$ rad/s, $H_0 = 22000$ m, $V_0 = 600$ m/s, $\delta_{aM} = \delta_{eM} = \delta_{rM} = 30^\circ$, and $\delta_{yM} = \delta_{zM} = 15^\circ$. The desired flight attitudes are

chosen as

$$\begin{aligned} \alpha_c &= \begin{cases} 2^\circ, & 3k \leq t < 3(k+1), \\ 5^\circ, & 3(k+1) \leq t < 3(k+2), \end{cases} \\ \beta_c &= 2^\circ, \\ \mu_c &= \begin{cases} 0^\circ, & 3k \leq t < 3(k+1), \\ 3^\circ, & 3(k+1) \leq t < 3(k+2), \end{cases} \\ k &= 0, 1, 2, \dots \end{aligned} \quad (58)$$

Suppose that there are +20% and −20% uncertainties on aerodynamic coefficients and aerodynamic moment coefficients, respectively. On the other hand, the unknown external

FIGURE 6: Pitch angular rate.

FIGURE 8: Ailerons.

FIGURE 7: Yaw angular rate.

FIGURE 9: Elevator.

disturbance moments in fast-loop system of the NSV are given by

$$\begin{aligned} d_{Mf,1}(t) &= 4 \times 10^5 (\sin(5t) + 0.2) Nm, \\ d_{Mf,2}(t) &= 4 \times 10^6 (\cos(5t) + 0.1) Nm, \\ d_{Mf,3}(t) &= 4 \times 10^6 \sin(5t) Nm. \end{aligned} \quad (59)$$

In order to obtain continuous and bounded derivatives of the desired flight attitudes, two-order reference model $G(s) = \omega_s^2 / (s^2 + 2\xi\omega_s s + \omega_s^2)$ is employed, where ω_s and ξ are designed parameters. All parameters are given as $A_{s1} = \text{diag}\{1.5, 1, 1\}$, $A_{s2} = \text{diag}\{1, 1, 1\}$, $B_{s1} = \text{diag}\{0.2, 0.2, 0.2\}$,

$B_{s2} = \text{diag}\{0.2, 0.2, 0.2\}$, $B_{s3} = \text{diag}\{0.1, 0.1, 0.1\}$, $A_f = \text{diag}\{0.5, 0.5, 0.5\}$, $B_{f1} = \text{diag}\{0.18, 0.18, 0.18\}$, $B_{f2} = \text{diag}\{0.18, 0.18, 0.18\}$, $K = \text{diag}\{6, 6, 6\}$, $\Gamma_\beta = \text{diag}\{4, 4, 4\}$, $\Gamma_\xi = \text{diag}\{50\}_{15 \times 15}$, $K_r = \text{diag}\{20, 20, 20, 20, 20\}$, $\gamma = 0.001$, $c_1 = 2.5$, $c_2 = 0.5$, $\omega_s = 4$, and $\xi = 0.8$.

The simulation results are shown in Figures 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12, where dash dot lines (with subscript “c”) represent the desired attitudes and dot lines (with subscript “1”) stand for the responses without NDO and saturation compensation, while solid lines (with subscript “2”) represent the responses with NDO and saturation compensation.

From Figures 2 to 4, we can observe that the small tracking errors of attitude angles are obtained and the attitude angular rates tend to be stable from Figures 5 to 7 based on the proposed sliding mode control scheme while the oscillation phenomena emerge without the nonlinear

FIGURE 10: Rudder.

FIGURE 12: Longitudinal deflection of the thrust vectoring control surface.

FIGURE 11: Lateral deflection of the thrust vectoring control surface.

disturbance observer and RBFNNs saturation compensator. On the other hand, the control inputs are presented from Figures 8 to 12. Under the proposed robust control scheme, it is clear that all deflections of rudders are in the saturation range. The deflections of ailerons and elevator reach the saturation limits without the nonlinear disturbance observer and RBFNNs saturation compensator. That is, the developed robust control scheme for the NSV can prevent the control signals from reaching saturation limits. From the previously stated analysis, the satisfactory attitude tracking control performance is obtained under the proposed robust control scheme for the NSV in the presence of the unknown external disturbance and the input saturation. Thus, the proposed robust control scheme is valid for the NSV.

4. Conclusion

An effective control scheme is proposed for the NSV with strong nonlinearity, high coupling, parameter uncertainty, and unknown time-varying disturbance based on SMC and NDO. Firstly, the nonlinear disturbance observer is designed to handle the compounded disturbance. Secondly, considering saturation characteristic of rudders, RBFNNs are constructed as a compensator to overcome the saturation nonlinearity. The stability of the closed-loop system is proved, and the tracking error as well as the disturbance observer error can converge to the origin through the Lyapunov analysis. Finally, Simulation results demonstrate the effectiveness of the proposed flight control scheme.

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