Research Article

Fault Detection for Non-Gaussian Stochastic Systems with Time-Varying Delay

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Fault detection (FD) for non-Gaussian stochastic systems with time-varying delay is studied. The available information for the addressed problem is the input and the measured output probability density functions (PDFs) of the system. In this framework, firstly, by constructing an augmented Lyapunov functional, which involves some slack variables and a tuning parameter, a delay-dependent condition for the existence of FD observer is derived in terms of linear matrix inequality (LMI) and the fault can be detected through a threshold. Secondly, in order to improve the detection sensitivity performance, the optimal algorithm is applied to minimize the threshold value. Finally, paper-making process example is given to demonstrate the applicability of the proposed approach.

1. Introduction

Fault detection for control systems has been of interest for many researchers during the past three decades (see [1–7] for surveys). Effective methodologies mainly include the filter- (or observer-) based approaches, the identification-based schemes, and the statistic approaches. However, most of the FD methodologies for stochastic systems only considered Gaussian systems. For example, the random process has been considered as Markov and Wiener processes in [8], respectively.

It has been shown that either the system variables are not Gaussian in [9, 10], existing methods may not be sufficient to characterize the non-Gaussian system behavior. Typical examples include fibre length distribution control in paper making, molecular weight distribution control, particle size distribution control in polymerization, and powder industries [8]. For such practical control problems, a new group of strategies that control the shape of PDFs for stochastic systems have been developed in the past few years (see [11, 12]), where the purpose is to design a controller so that the

PDF of the system output can track a prespecified desired PDF, as close as possible.

To simplify system modeling, B-spline neural networks had been initially used to approximate the output PDF [9, 13]. The motivation of FD via the output PDFs from the retention system in paper making was first studied in [10], where the weight dynamical system was supposed to be a precise linear model. However, linear mappings cannot change the shape of output PDFs, which implies that the fault cannot be detected through the shape change of the PDFs. To meet the requirement in complex processes, nonlinearity should be considered in the weighting dynamic behavior. Recently, a kind of observer-based FD algorithm has been established in [14], where the nonlinear weighting system was considered. However, only the uniform boundedness of the estimation error could be guaranteed in [14], which leads to some conservative criteria.

On the other hand, time delay exists commonly in dynamic systems and is frequently a source of instability and poor performance [15–17]; many works had been done about time-delay systems along the development of stochastic

theories [18]. Recently, FD problem has been studied for time-delay stochastic systems using PDF in [19–21]. But the criteria [19–21] are only available to systems with constant delay. Meanwhile, it is noted that in practice, time-varying delay is often encountered in dynamic systems. However, up to our knowledge, there have been few results in the literature of an investigation for the FD algorithm of dynamic systems with time-varying delay by using PDF.

In this paper, we provide a further contribution to FD for non-Gaussian stochastic systems with time-varying delay based on the method in [20, 21]. Firstly, by constructing an augmented Lyapunov functional, which involves some slack variables and a tuning parameter, a delay-dependent condition for the existence of FD observer is derived in terms of linear matrix inequality (LMI) and the fault can be detected through a threshold. Secondly, in order to improve the detection sensitivity performance, the optimal algorithm is applied to minimize the threshold value. Finally, paper-making process example is given to demonstrate the applicability of the proposed approach.

Notation. Throughout this paper, R^n denotes the n -dimensional Euclidean space. The superscripts “ τ ” and “ -1 ” stand for matrix transposition and matrix inverse, respectively; $P > (\geq 0)$ means that P is real symmetric and positive definite (semidefinite). In symmetric block matrices or complex matrix expressions, $\text{diag}\{\dots\}$ stands for a block-diagonal matrix, and $*$ represents a term that is induced by symmetry. For a vector $v(t)$, its norm is given by $\|v(t)\|_2^2 = \int_0^\infty v^\tau(t)v(t)dt$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for related algebraic operations.

2. Problem Formulation and Preliminaries

In this section, firstly, we briefly review square-root B-spline expansion technique presented in [19, 21], which is used to formulate the output PDFs with the dynamic weight and is essential in solving our FD problem.

For a dynamic stochastic system, its output PDFs is defined by $\gamma(z, u(t), F(t))$, where $u(t) \in R^m$ is control input and $F(t)$ is the fault vector to be detected. In [13, 19–21], the following square-root B-spline expansion model has been used to approximate $\gamma(z, u(t), F(t))$:

$$\sqrt{\gamma(z, u(t), F(t))} = \sum_{i=1}^n v_i(u, F) b_i(z), \quad (1)$$

where $b_i(z)$ ($i = 1, 2, \dots, n$) are prespecified basis functions defined on $[a, b]$ and $v_i(u(t), F(t))$ ($i = 1, 2, \dots, n$) are the corresponding weights of such an expansion. Denote $B_0(z) = [b_1(z) \ b_2(z) \ \dots \ b_{n-1}(z)]^\tau$ and $\bar{v}(t) := \bar{v}(u(t), F(t)) = [v_1 \ v_2 \ \dots \ v_{n-1}]^\tau$, and let $\Lambda_1 = \int_a^b B_0(z)B_0^\tau(z)dz$, $\Lambda_2 = \int_a^b B_0^\tau(z)b_n(z)dz$, $\Lambda_3 = \int_a^b b_n^2(z)dz \neq 0$, and $\Lambda_0 = \Lambda_1\Lambda_3 - \Lambda_2\Lambda_2^\tau$. Furthermore, it can be verified that (1) can be rewritten as (see [19] for details)

$$\sqrt{\gamma(z, u(t), F)} = B^\tau(z) \bar{v}(t) + h(\bar{v}(t)) b_n(z), \quad (2)$$

where

$$\begin{aligned} B^\tau(z) &= B_0^\tau(z) - \frac{\Lambda_2}{\Lambda_3} b_n(z), \\ h(\bar{v}(t)) &= \frac{\sqrt{\Lambda_3 - \bar{v}^\tau(t) \Lambda_0 \bar{v}^\tau(t)}}{\Lambda_3}. \end{aligned} \quad (3)$$

The relationship between the input and the weights related to the PDFs can be described by the following model:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d(t)) + Gg(x(t)) \\ &\quad + Hu(t) + J_f(t), \\ \bar{v}(t) &= Ex(t), \end{aligned} \quad (4)$$

where $x(t) \in R^m$ is the unmeasured state and $F(t) \in R^m$ is the fault to be detected. $A \in R^{m \times m}$, $A_d \in R^{m \times m}$, $G \in R^{m \times m}$, $H \in R^{m \times 1}$, $J_f \in R^{m \times m}$, and $E \in R^{m \times m}$ represent the known parametric matrices of the dynamic part of the weight system. In fact, these matrices can be estimated by using the scaling parameter identification algorithms in [22]. Similar model can be found in [21]. However, only constant delay is considered in [21]. In this paper, the time delay $d(t)$ is a differential function, satisfying $0 \leq d(t) \leq h$, $\dot{d}(t) \leq \mu$, and the initial condition is defined by $x(t) = \phi(t)$ ($-h \leq t \leq 0$). In addition, similar to [19, 21], the following assumptions are needed.

Assumption 1. For any $x_1(t)$ and $x_2(t)$, $g(x(t))$ satisfies $g(0) = 0$ and

$$\|g(x_1(t)) - g(x_2(t))\| \leq \|U_2(x_1(t) - x_2(t))\|, \quad (5)$$

where U_2 is a known matrix.

Assumption 2. There is a known matrix U_1 ; for any $\bar{v}_1(t)$ and $\bar{v}_2(t)$, $h(\bar{v}(t))$ denoted by (3) satisfies the following condition:

$$\|h(\bar{v}_1(t)) - h(\bar{v}_2(t))\| \leq \|U_1(\bar{v}_1(t) - \bar{v}_2(t))\|, \quad (6)$$

where $\|\cdot\|$ is denoted as the Euclidean norm.

Inequalities (5) and (6) are typically required in the literature on FD for nonlinear systems, for example, [23, 24], which will help to simplify the design of algorithms later on.

Generally speaking a fault detection system consists of a residual generator, and a residual evaluator including an evaluation function and a threshold. We will consider two parts of fault detection systems by using the information of PDF in the following section.

2.1. Residual Generator. For the purpose of residual generation, we construct the following nonlinear observer:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + A_d\hat{x}(t-d(t)) + Gg(\hat{x}(t)) \\ &\quad + Hu(t) + L\hat{\xi}(t), \end{aligned} \quad (7)$$

FIGURE 1: The classical residual generator.

where $\hat{x}(t)$ is the estimated state and $L \in R^{m \times p}$ is the gain to be determined. $\xi(t)$ is the residual signal which is defined in terms of output PDFs as

$$\xi(t) = \int_a^b \sigma(z) \left(\sqrt{\gamma(z, u(t), F)} - \sqrt{\hat{\gamma}(z, u(t))} \right) dz, \quad (8)$$

$$\sqrt{\hat{\gamma}(z, u(t))} = B^T(z) E \hat{x}(t) + h(E \hat{x}(t)) b_n(z).$$

Remark 3. The classical residual generator design methods (such as [25–27]) are formulated in Figure 1. Different from the classical residual generator, residual $\xi(t)$ in (7) is formulated as an integral with respect to the difference of the measured PDFs and the estimated PDFs.

In order to describe the dynamics of (7), we first consider $e(t) = x(t) - \hat{x}(t)$ and $\xi(t)$; it can be shown that

$$\begin{aligned} \dot{e}(t) &= (A - L\Gamma_1) e(t) + A_d e(t - d(t)) \\ &\quad + G[g(x(t)) - g(\hat{x}(t))] \\ &\quad - L\Gamma_2 [h(Ex(t)) - h(E\hat{x}(t))] + J_f(t), \end{aligned} \quad (9)$$

$$\xi(t) = \Gamma_1 e(t) + \Gamma_2 (h(Ex(t)) - h(E\hat{x}(t))), \quad (10)$$

where

$$\Gamma_1 = \int_a^b \sigma(z) B^T(z) E dz, \quad \Gamma_2 = \int_a^b \sigma(z) b_n(z) dz. \quad (11)$$

Thus, the problem of designing an observer-based fault detection can be described as follows:

- (i) design matrix L such that error system (9) is asymptotically stable;
- (ii) the fault can be detected by residual generator.

2.2. Residual Evaluator. After designing of FD observer, the next task for FD is the evaluation of the generated residual. One of the widely adopted approaches is to use the following logical relationship for FD:

$$\begin{aligned} \|r(t)\| > J_{th} &\implies \text{faults} \implies \text{alarm}, \\ \|r(t)\| \leq J_{th} &\implies \text{no faults}, \end{aligned} \quad (12)$$

where $r(t)$ is the residual evaluation function and $J_{th} > 0$ is the threshold. From the above logical relationship, it is clear that the threshold is important for FD sensitivity performance. In order to detect the fault more sensitively, the optimization techniques for the threshold are preferable. For this purpose, we construct a reference vector as follows:

$$z(t) = C\xi(t) = C\Gamma_1 e(t) + C\Gamma_2 (h(Ex(t)) - h(E\hat{x}(t))), \quad (13)$$

where C is a selected weighting matrix. At this stage, we can consider the suboptimal guaranteed cost problem for $J = \|z(t)\|_2^2$ subjected to error system (9) with $\hat{x}(0) = 0$ and $e(t) = \phi(t)$ ($-h \leq t \leq 0$). In this paper, we choose $\|z(t)\|$ and the bound $J^* = J^{1/2}$ as the residual evaluation function and threshold, respectively.

3. Main Result

In the section, an augmented Lyapunov-Krasovskii functional involving some slack variables and a tuning parameter is constructed and the following criterion is obtained.

Theorem 4. If for the parameters $\lambda_i > 0$ ($i = 1, 2$) and ε , there exist matrices $P_1 > 0$, $Q_1 > 0$, $Q_2 > 0$, $T > 0$, P_i ($i = 2, \dots, 6$), and R such that the following inequality holds:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & -P_4^T & P_2^T G & R\Gamma_2 & -hP_3^T & -hP_4^T & \Gamma_1^T C^T \\ * & \Pi_{22} & \varepsilon P_2^T A_d & 0 & \varepsilon P_2^T G & \varepsilon R\Gamma_2 & 0 & 0 & 0 \\ * & * & \Pi_{33} & -P_6^T & 0 & 0 & -hP_5^T & -hP_6^T & 0 \\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\frac{1}{\lambda_2^2} I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{\lambda_1^2} I & 0 & 0 & \Gamma_2^T C^T \\ * & * & * & * & * & * & -hT & 0 & 0 \\ * & * & * & * & * & * & * & -hT & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (14)$$

where $*$ denotes the symmetric terms in a symmetric matrix and

$$\begin{aligned} \Pi_{11} &= (P_2^T A - R\Gamma_1) + (P_2^T A - R\Gamma_1)^T + \frac{1}{\lambda_1^2} E^T U_1^T U_1 E \\ &\quad + \frac{1}{\lambda_2^2} U_2^T U_2 + Q_1 + Q_2 + P_3^T + P_3, \\ \Pi_{12} &= \varepsilon A^T P_2 - \varepsilon \Gamma_1^T R^T + P_1 - P_2^T, \\ \Pi_{13} &= P_2^T A_d - P_3^T + P_4^T + P_5, \\ \Pi_{22} &= -\varepsilon P_2 - \varepsilon P_2^T + hT, \\ \Pi_{33} &= -P_5 - P_5^T - P_6 - P_6^T - (1 - \mu) Q_1, \end{aligned} \quad (15)$$

then in the absence of $F(t)$, error system (9) with gain $L = P_2^{-\tau} R$ is stable and the reference vector satisfies

$$\begin{aligned} J = \|z(t)\|_2^2 &\leq V(0) := \phi^\tau(0) P_1 \phi(0) \\ &+ \int_{-h}^0 \int_{\theta}^0 \dot{\phi}^\tau(s) T \dot{\phi}(s) ds d\theta \\ &+ \int_{-d(0)}^0 \phi^\tau(s) Q_1 \phi(s) ds + \int_{-h}^0 \phi^\tau(s) Q_2 \phi(s) ds. \end{aligned} \quad (16)$$

Moreover, the fault can be detected when $\|z(t)\| > J^* = V^{1/2}(0)$ for $t \in [-h, \infty)$.

Proof. Define $h = h(x(t))$, $\hat{h} = h(\hat{x}(t))$, $g = g(x(t))$, and $\hat{g} = g(\hat{x}(t))$ and denote the Lyapunov function candidate as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad (17)$$

where

$$\begin{aligned} V_1(t) &= \xi_1^\tau(t) M P \xi_1(t), \\ V_2(t) &= \int_{-h}^0 \int_{t+\theta}^t \dot{e}^\tau(s) T \dot{e}(s) ds d\theta, \\ V_3(t) &= \int_{t-d(t)}^t e^\tau(s) Q_1 e(s) ds + \int_{t-h}^t e^\tau(s) Q_2 e(s) ds, \\ V_4(t) &= \frac{1}{\lambda_1^2} \int_0^t [\|U_1 E e(s)\|^2 - \|h - \hat{h}\|^2] ds \\ &+ \frac{1}{\lambda_2^2} \int_0^t [\|U_2 e(s)\|^2 - \|g - \hat{g}\|^2] ds \end{aligned} \quad (18)$$

with

$$\begin{aligned} M &= \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & 0 & 0 \\ P_2 & \varepsilon P_2 & 0 \\ P_3 & 0 & P_5 \\ P_4 & 0 & P_6 \end{bmatrix}, \\ \xi_1(t) &= \begin{bmatrix} e(t) \\ \dot{e}(t) \\ e(t-d(t)) \end{bmatrix}, \end{aligned} \quad (19)$$

and with $P_1 > 0$, $T > 0$, $Q_1 > 0$, $Q_2 > 0$. It is noted that $V_1(t)$ is actually $e^\tau(t) P_1 e(t)$. Then following (5) and (6) gives $V(t) \geq 0$. On the other hand, from the Leibniz-Newton formula, the following equations are true:

$$\begin{aligned} \alpha_1 &:= e(t) - e(t-d(t)) - \int_{t-d(t)}^t \dot{e}(s) ds = 0, \\ \alpha_2 &:= e(t-d(t)) - e(t-h) - \int_{t-h}^{t-d(t)} \dot{e}(s) ds = 0. \end{aligned} \quad (20)$$

Hence, calculating the time derivative of $V_1(t)$ along the solution of (9) in the absence of $F(t)$ and using (20) yields

$$\begin{aligned} \dot{V}_1(t) &= 2\xi_1^\tau(t) P^\tau \\ &\times \begin{bmatrix} \dot{e}(t) \\ -\dot{e}(t) + (A - L\Gamma_1)e(t) + A_d e(t-d(t)) \\ + G(g - \hat{g}) - L\Gamma_2(h - \hat{h}) \\ \alpha_1 \\ \alpha_2 \end{bmatrix} \\ &= 2e^\tau(t) P_1 \dot{e}(t) - 2e^\tau(t) P_2^\tau \dot{e}(t) \\ &+ e^\tau(t) [(P_2^\tau A - R\Gamma_1) + (P_2^\tau A - R\Gamma_1)^\tau] e(t) \\ &+ 2e^\tau(t) P_2^\tau A_d e(t-d(t)) + 2e^\tau(t) P_2^\tau G(g - \hat{g}) \\ &- 2e^\tau(t) R\Gamma_2(h - \hat{h}) - 2\varepsilon \dot{e}^\tau(t) P_2^\tau \dot{e}(t) \\ &+ 2\varepsilon \dot{e}^\tau(t) (P_2^\tau A - R\Gamma_1) e(t) \\ &+ 2\varepsilon \dot{e}^\tau(t) P_2^\tau A_d e(t-d(t)) + 2\varepsilon \dot{e}^\tau(t) P_2^\tau G(g - \hat{g}) \\ &- 2\varepsilon \dot{e}^\tau(t) R\Gamma_2(h - \hat{h}) + 2e^\tau(t) P_3^\tau e(t) \\ &- 2e^\tau(t) P_3^\tau e(t-d(t)) - 2e^\tau(t) P_3^\tau \int_{t-d(t)}^t \dot{e}(s) ds \\ &+ 2e^\tau(t-d(t)) P_5^\tau e(t) \\ &- 2e^\tau(t-d(t)) P_5^\tau e(t-d(t)) \\ &- 2e^\tau(t-d(t)) P_5^\tau \int_{t-d(t)}^t \dot{e}(s) ds \\ &+ 2e^\tau(t) P_4^\tau e(t-d(t)) - 2e^\tau(t) P_4^\tau e(t-h) \\ &- 2e^\tau(t) P_4^\tau \int_{t-h}^{t-d(t)} \dot{e}(s) ds \\ &+ 2e^\tau(t-d(t)) P_6^\tau e(t-d(t)) \\ &- 2e^\tau(t-d(t)) P_6^\tau e(t-h) \\ &- 2e^\tau(t-d(t)) P_6^\tau \int_{t-h}^{t-d(t)} \dot{e}(s) ds \\ &= \eta^\tau(t) \Theta_1 \eta(t), \end{aligned} \quad (21)$$

where

$$\Theta_1 = \begin{bmatrix} \Theta_{11} & \Pi_{12} & \Pi_{13} & -P_4^\tau & P_2^\tau G & R\Gamma_2 & -P_3^\tau & -P_4^\tau \\ * & \Theta_{22} & \varepsilon P_2^\tau A_d & 0 & \varepsilon P_2^\tau G & \varepsilon R\Gamma_2 & 0 & 0 \\ * & * & \Theta_{33} & -P_6^\tau & 0 & 0 & -P_5^\tau & -P_6^\tau \\ * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\eta(t) = \begin{bmatrix} e(t) \\ \dot{e}(t) \\ e(t-d(t)) \\ e(t-h) \\ (g-\hat{g}) \\ (h-\hat{h}) \\ \int_{t-d(t)}^t \dot{e} ds \\ \int_{t-h}^{t-d(t)} \dot{e} ds \end{bmatrix}, \quad (22)$$

where

$$\begin{aligned} \Theta_{11} &= (P_2^T A - R\Gamma_1) + (P_2^T A - R\Gamma_1)^T + P_3 + P_3^T, \\ \Theta_{22} &= -\varepsilon P_2 - \varepsilon P_2^T, \\ \Theta_{33} &= -P_5 - P_5^T - P_6 - P_6^T, \end{aligned} \quad (23)$$

with $R = P_2^T L$ and Π_{12}, Π_{13} being defined in (14). It follows from Jensen integral inequality with $d(t) \leq h$ that one can obtain

$$\begin{aligned} \dot{V}_2(t) &= h\dot{e}^T T \dot{e} - \int_{t-h}^t \dot{e}^T T \dot{e} ds \\ &= h\dot{e}^T T \dot{e} - \int_{t-h}^{t-d(t)} \dot{e}^T T \dot{e} ds - \int_{t-d(t)}^t \dot{e}^T T \dot{e} ds \\ &\leq h\dot{e}^T(t) T \dot{e}(t) \\ &\quad - \frac{1}{h} \left(\int_{t-h}^{t-d(t)} \dot{e}(s) ds \right)^T T \left(\int_{t-h}^{t-d(t)} \dot{e}(s) ds \right) \\ &\quad - \frac{1}{h} \left(\int_{t-d(t)}^t \dot{e}(s) ds \right)^T T \left(\int_{t-d(t)}^t \dot{e}(s) ds \right). \end{aligned} \quad (24)$$

On the other hand, it can be verified that

$$\begin{aligned} \dot{V}_3(t) &= e^T(t) (Q_1 + Q_2) e(t) - (1-\mu) e^T(t-d(t)) \\ &\quad \times Q_1 e(t-d(t)) - e^T(t-h) Q_2 e(t-h), \\ \dot{V}_4(t) &= \frac{1}{\lambda_1^2} E^T U_1^T U_1 E + \frac{1}{\lambda_2^2} U_2^T U_2 \\ &\quad - \frac{1}{\lambda_1^2} (h-\hat{h})^T (h-\hat{h}) - \frac{1}{\lambda_2^2} (g-\hat{g})^T (g-\hat{g}). \end{aligned} \quad (25)$$

It then follows from (21)–(25) that

$$\dot{V}(t) \leq \eta^T(t) \Xi_0 \eta(t), \quad (26)$$

where $\Xi_0 = \Theta_1 + \text{diag}\{Q_1 + Q_2 + (1/\lambda_1^2)E^T U_1^T U_1 E + (1/\lambda_2^2)U_2^T U_2, hT, -(1-\mu)Q_1, -Q_2, -(1/\lambda_2^2)I, -(1/\lambda_1^2)I, -(1/h)T, -(1/h)T\}$. Applying the Schur complement to (14) gives $\Xi_0 < 0$, which implies $\dot{V}(t) < 0$ and error system (9) is asymptotically stable.

Furthermore, we denote an auxiliary cost function as

$$J = \int_0^\infty [z^T(\tau) z(\tau) + \dot{V}(\tau)] d\tau. \quad (27)$$

Note that

$$\begin{aligned} z^T(t) z(t) &= e^T(t) \Gamma_1^T C^T C \Gamma_1 e(t) + 2e^T(t) \Gamma_1^T C^T C \Gamma_2 (h-\hat{h}) \\ &\quad + (h-\hat{h})^T \Gamma_2^T C^T C \Gamma_2 (h-\hat{h}). \end{aligned} \quad (28)$$

From (14), $J < 0$ holds. On the other hand, it can be shown that

$$\begin{aligned} J &= \int_0^\infty z^T(\tau) z(\tau) d\tau + \lim_{T \rightarrow \infty} (V(T) - V(0)) \\ &\leq \|z(t)\|^2 - V(0), \end{aligned} \quad (29)$$

where $V(0)$ is defined in (16). Therefore (16) is satisfied. This completes the proof. $\square$

Remark 5. Theorem 4 is based on a newly proposed augmented Lyapunov functional of form (17), which contains a structure more general than the traditional ones as those in [19, 20] for involving $\dot{e}(t)$ and $e(t-d(t))$ in the first term of (17). In addition, the importance of augmented Lyapunov functional is that it separates Lyapunov function matrix P_1 from A, A_d ; that is, there are no terms containing the product of P_1 and any of them, which makes the design of the fault detection observer more easier.

Remark 6. Recently, Li et al. [21] provided a fault detection observer design criterion by using PDFs for stochastic system with constant delay. Compared with the criterion in [21], the advantage in our paper is that time-varying delay is considered and the design criterion includes more delay information. Moreover, it is noted that setting $P_3 = Y, P_5 = W, Q_2 = 0$, and $P_4 = P_6 = 0$ in Theorem 4 yields precisely Theorem 1 in [21]. So, Theorem 4 in this paper is an extension of the result in [21].

In order to improve fault detection sensitivity, the following result provides an optimization algorithm in order to make the threshold J^* smaller.

Theorem 7. Consider error system (9) with cost function (27). For given parameters λ_i ($i = 1, 2$) and ε , if the following optimization problem:

$$\min \{\alpha + \text{tr}(\Omega_1) + \text{tr}(\Omega_2) + \text{tr}(\Omega_3)\} \quad (30)$$

subject to

$$\begin{aligned} &\text{(i) LMI (14),} \\ &\text{(ii) } \begin{bmatrix} -\alpha & \phi^T(0) P_1 \\ P_1 \phi(0) & -P_1 \end{bmatrix} < 0, \\ &\text{(iii) } \begin{bmatrix} -\Omega_1 & Y_1^T Q_1 \\ Q_1 Y_1 & -Q_1 \end{bmatrix} < 0, \\ &\text{(iv) } \begin{bmatrix} -\Omega_2 & Y_2^T Q_2 \\ Q_2 Y_2 & -Q_2 \end{bmatrix} < 0, \\ &\text{(v) } \begin{bmatrix} -\Omega_3 & Y_3^T T \\ T Y_3 & -T \end{bmatrix} < 0, \end{aligned} \quad (31)$$

where

$$\begin{aligned} \int_{-d(0)}^0 \phi(s) \phi^\tau(s) ds &= Y_1 Y_1^\tau, \\ \int_{-h}^0 \phi(s) \phi^\tau(s) ds &= Y_2 Y_2^\tau, \\ \int_{-h}^0 \int_\theta^0 \dot{\phi}(s) \dot{\phi}^\tau(s) ds d\theta &= Y_3 Y_3^\tau, \end{aligned} \quad (32)$$

has solutions with positive definite matrices $P_1, Q_1, Q_2, T, \Omega_1, \Omega_2$, and Ω_3 ; matrices P_i ($i = 2, \dots, 6$) and R ; and positive scalar α , then in the absence of $F(t)$, error system (9) with gain $L = P_2^{-\tau} R$ is stable and the smallest threshold satisfies

$$J^* = \{\alpha + \text{tr}(\Omega_1) + \text{tr}(\Omega_2) + \text{tr}(\Omega_3)\}^{1/2}, \quad (33)$$

for $t \in [-h, \infty)$.

Proof. By Theorem 4, (i) is clear. Also, it follows from the Schur complement that (ii), (iii), (iv), and (v) are equivalent to

$$\begin{aligned} \phi^\tau(0) P_1 \phi(0) &< \alpha, \\ Y_1^\tau Q_1 Y_1 &< \Omega_1, \\ Y_2^\tau Q_2 Y_2 &< \Omega_2, \\ Y_3^\tau T Y_3 &< \Omega_3, \end{aligned} \quad (34)$$

respectively. On the other hand

$$\begin{aligned} &\int_{-d(0)}^0 \phi^\tau(s) Q_1 \phi(s) ds \\ &= \int_{-d(0)}^0 \text{tr}(\phi^\tau(s) Q_1 \phi(s)) ds \\ &= \text{tr}\left(Q_1^{1/2} \int_{-d(0)}^0 \phi(s) \phi^\tau(s) ds Q_1^{1/2}\right) \\ &= \text{tr}\left(Q_1^{1/2} Y_1 Y_1^\tau Q_1^{1/2}\right) = \text{tr}(Y_1^\tau Q_1 Y_1) < \text{tr}(\Omega_1), \\ &\int_{-h}^0 \phi^\tau(s) Q_2 \phi(s) ds \\ &= \int_{-h}^0 \text{tr}(\phi^\tau(s) Q_2 \phi(s)) ds \\ &= \text{tr}\left(Q_2^{1/2} \int_{-h}^0 \phi(s) \phi^\tau(s) ds Q_2^{1/2}\right) \\ &= \text{tr}\left(Q_2^{1/2} Y_2 Y_2^\tau Q_2^{1/2}\right) = \text{tr}(Y_2^\tau Q_2 Y_2) < \text{tr}(\Omega_2), \end{aligned}$$

FIGURE 2: 3D mesh plot of the measured output PDFs.

$$\begin{aligned} &\int_h^0 \int_\theta^0 \dot{\phi}^\tau(s) T \dot{\phi}(s) ds d\theta \\ &= \int_h^0 \int_\theta^0 \text{tr}(\dot{\phi}^\tau(s) T \dot{\phi}(s)) ds d\theta \\ &= \text{tr}\left(T^{1/2} \int_h^0 \int_\theta^0 \dot{\phi}(s) \dot{\phi}^\tau(s) ds d\theta T^{1/2}\right) \\ &= \text{tr}\left(T^{1/2} Y_3 Y_3^\tau T^{1/2}\right) = \text{tr}(Y_3^\tau T Y_3) < \text{tr}(\Omega_3). \end{aligned} \quad (35)$$

Hence, it follows from (17) that

$$J < \alpha + \text{tr}(\Omega_1) + \text{tr}(\Omega_2) + \text{tr}(\Omega_3). \quad (36)$$

Thus, the minimization of J implies the minimization of the threshold values $J^* = J^{1/2}$. $\square$

4. Simulations

An application of paper-making process is given to demonstrate the applicability of our proposed approach. The basis functions are selected in a similar way to [20] as follows:

$$\begin{aligned} B(z) &= [b_1(z), b_2(z), \dots, b_9(z)]^\tau, \\ b_i(z) &= \exp(-(z - z_i)^2 \sigma_i^{-2}), \quad (i = 1, 2, \dots, 10), \\ z_i &= 0.003 + 0.006(i - 1), \quad \sigma_i = 0.003, \quad (i = 1, 2, \dots, 10). \end{aligned} \quad (37)$$

Consider the following weighting system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t - d(t)) + Gg(x(t)) \\ &\quad + Hu(t) + J_f(t), \\ \bar{v}(t) &= x(t), \end{aligned} \quad (38)$$

FIGURE 3: The response of residual signal.

FIGURE 4: Threshold and the response of evaluation function.

where

$$\begin{aligned}
 A &= \text{diag}\{-0.83, -0.83, \dots, -0.83\} \in R^{9 \times 9}, \\
 A_d &= \text{diag}\{0.2, 0.2, \dots, 0.2\} \in R^{9 \times 9}, \\
 G &= \text{diag}\{1, 1, \dots, 1\} \in R^{9 \times 9}, \\
 H &= [0 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0]^T, \\
 J_f &= \text{diag}\{1.18, 0, 1.56, 0, 1.56, 0, 1.56, 1.56, 1.56\},
 \end{aligned} \tag{39}$$

$g(x(t)) = \sin(x(t))$, and $u(t)$ is random number. By setting $\sigma(z) = 1$, then the following parametric matrices related to the B-spline approximation can be obtained:

$$\Lambda_0 = 10^{-5}$$

$$\times \begin{bmatrix} 2.400 & 0.956 & 0.048 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.956 & 2.607 & 0.959 & 0.047 & 0 & 0 & 0 & 0 & 0 \\ 0.048 & 0.959 & 2.605 & 0.956 & 0.049 & 0.001 & 0 & 0 & 0 \\ 0 & 0.047 & 0.956 & 2.601 & 0.960 & 0.048 & 0 & 0 & 0 \\ 0 & 0 & 0.049 & 0.960 & 2.605 & 0.958 & 0.048 & 0.001 & 0 \\ 0 & 0 & 0.001 & 0.048 & 0.958 & 2.605 & 0.960 & 0.049 & 0 \\ 0 & 0 & 0 & 0 & 0.048 & 0.960 & 2.601 & 0.956 & 0.047 \\ 0 & 0 & 0 & 0 & 0.001 & 0.049 & 0.956 & 2.604 & 0.939 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.047 & 0.939 & 2.226 \end{bmatrix},$$

$$\Lambda_1 = 10^{-3} \times \begin{bmatrix} 4.9 & 2 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 5.3 & 2 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 2 & 5.3 & 2 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 2 & 5.3 & 2 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 2 & 5.3 & 2 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 2 & 5.3 & 2 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 2 & 5.3 & 2 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 2 & 5.3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 2 & 5.3 \end{bmatrix},$$

$$\Lambda_2 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.0001 \ 0.0020],$$

$$\Lambda_3 = 0.0049.$$

(40)

It can be verified that $U_1 = \text{diag}\{0.05, 0.05, \dots, 0.05\} \in R^{9 \times 9}$ and it can be supposed that $U_2 = \text{diag}\{1, 1, \dots, 1\} \in R^{9 \times 9}$. Corresponding to (11), it can be calculated that $\Gamma_1 = 10^{-2} \times [6.3 \ 7.5 \ 7.5 \ 7.5 \ 7.5 \ 7.5 \ 7.5 \ 7.4 \ 5.0]$, $\Gamma_2 = 0.0063$. In this case, $d(t) = 0.5 \sin(t)$, $\mu = 0.5$, $h = 0.5$, and the initial value of observer (7) is selected as $\hat{x}(t) = 0 \in R^9$ for all $0 \leq t \leq +\infty$, while the initial value of (4) is selected as $x_1(t) = 2.05 + \exp(t-2)$, $x_2(t) = 1.05 + \exp(t-2)$, $x_5(t) = -2.05 + \exp(t-2)$, $x_9(t) = 3.0 + \exp(t-2)$, and $x_3(t) = x_4(t) = x_6(t) = x_7(t) = x_8(t) = 0$, $(-0.5 \leq t \leq 0)$. The fault is supposed to be

$$F(t)$$

$$= \begin{cases} 0, & t < 20, \\ \begin{bmatrix} 2 & 0.2 & 2 & 0.2 & 0.2 & 2 & 0.2 & 2 & 0.2 \end{bmatrix}, & 20 \leq t \leq 40, \\ 0, & t \geq 40. \end{cases} \tag{41}$$

Figure 2 shows the changes of the output PDF when the fault occurs. With the detection observer, Figure 3 demonstrates the responses of the residual signal. The threshold can be computed to give $J^* = 0.1137$ by using Theorem 7 and the residual satisfies $z(t) \geq 0.1143 > J^*$ for $t > 26$. However, it is infeasible by using Theorem 2 in [21]. Thus, a better FD algorithm may more rapidly detect the occurrence of faults. From Figure 4, we can see that the fault can be detected 6 s after its occurrence.

5. Conclusion

The FD problem is studied for a class of non-Gaussian stochastic systems with time-varying delay using augmented

Lyapunov functional approach, where only the output PDFs can be measured rather than an output signal. A new delay-dependent FD observer design criterion is obtained. And the fault can be detected with an optimal threshold, where the guaranteed cost optimization algorithm is applied to minimize it. Further work will focus on the fault diagnosis and fault tolerant for these systems.

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