Research Article

Throughput Optimization via Cooperative Spectrum Sensing with Novel Frame Structure

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In cognitive radio (CR) networks, cooperation can greatly improve the performance of spectrum sensing. In this paper, we propose a novel cooperative spectrum sensing (CSS) frame structure in which CR users conduct spectrum sensing and data transmission concurrently over two different parts of the primary user (PU) spectrum band. Energy detection sensing scheme is used to prove that there exists an optimal sensing bandwidth which yields the highest throughput for the CR network. Thus, we focus on the optimal sensing settings of the proposed sensing scheme in order to maximize the throughput of the CR network under the conditions of sufficient protection to PUs and required bandwidth for potential CR user data transmission. Some algorithms are also derived to jointly optimize the sensing bandwidth and the final decision threshold. Our simulation results show that optimizing the sensing bandwidth and the final decision threshold together will further increase the throughput of the CR network as compared to that which only optimizes the sensing bandwidth or the final decision threshold.

1. Introduction

Cognitive radio is a promising technology to improve the efficiency of spectrum usage [1]. The CR users should obey the two following etiquettes [2]: (1) they are allowed to use some unoccupied spectrum bands; (2) they must vacate the spectrum bands quickly whenever the PUs return. In order to avoid interference with PUs, a CR user needs to efficiently and effectively detect the presence of the PUs. CR users can use one of several common detection methods [3], such as matched filter, feature detection, and energy detection. Energy detection is the most popular method addressed in the literature [4–8]. Measuring only the received signal power, energy detection has much lower complexity than the other two schemes. Therefore, we consider energy detection for spectrum sensing in this paper.

The detection quality of spectrum sensing easily suffers from the fading and shadowing environment, which can cause hidden terminal problem. To combat these impacts, cooperative spectrum sensing has been introduced to obtain the space diversity in multiuser CR networks [9–12]. Game theory is suitable for analyzing conflict and cooperation

among rational decision makers [13–16]. In [17], the authors focused on dynamical effects of coevolutionary rules on the evolution of cooperation. Game theory has been widely applied to study distributed optimization problems, such as power control, dynamic spectrum access and management [18, 19], and so on. In [20], the authors proposed an evolutionary game framework to answer the question of “how to collaborate” in multiuser decentralized cooperative spectrum sensing. The cooperative spectrum sensing involves sensing, reporting, and decision making steps [21]. In the sensing step, every CR user performs spectrum sensing independently using energy detection method and makes a local decision. In the reporting step, all the local sensing observations are reported to a fusion center (FC). The decision step is made to indicate the absence or the presence of the PU.

In the frame structure of periodic spectrum sensing (PSS), the CR user senses the status of the radio spectrum in the sensing slot and transmits data using the remaining frame duration [22]. Since the CR user must interrupt data transmission during the sensing slot, the CR user transmission delay will be long. Thus, in the case of delay sensitive applications, the QoS (quality of service) will not

FIGURE 1: Novel frame structure for cooperative spectrum sensing.

be guaranteed. In our previous work [23], we propose a novel cooperative spectrum sensing frame structure in which CR users conduct spectrum sensing and data transmission concurrently over two different parts of the primary user spectrum band. In this way, the CR users do not need to interrupt data transmission in the sensing stage, and the QoS can be guaranteed. The optimal multiminislot sensing scheme and the optimal fusion scheme that minimize the CR user transmission delay were analyzed in [23]. However, the performance of throughput with the novel CSS frame structure was not investigated. In this paper, we focus on analyzing the throughput of the CR network.

In the novel frame structure designed for CSS, one part of the PU transmission bandwidth is assigned exclusively to spectrum sensing and sensing results reporting, and the other part is assigned exclusively to potential CR user data transmission. According to this frame structure, under the condition of sufficient protection to PUs, an increase in the sensing bandwidth results in a lower false alarm probability, which leads to higher throughput of the CR network. However, the increase of the sensing bandwidth results in a decrease of the bandwidth assigned to potential CR user data transmission and hence the throughput of the CR network. Therefore, there could exist the optimal sensing bandwidth that maximizes the throughput of the CR network.

In this paper, we study the tradeoff problem for cooperative spectrum sensing with novel frame structure. We are interested in the problem of designing the sensing bandwidth to maximize the throughput of the CR network under the conditions of sufficient protection to PUs and required bandwidth for potential CR user data transmission. When energy detection is utilized for spectrum sensing, we prove that there indeed exists one optimal sensing bandwidth which yields the highest throughput for the CR network.

In the cooperative spectrum sensing with novel frame structure, we employ the counting rule as the fusion rule at the FC since it requires the least communication overhead and is easy to implement. Since CR is originally designed to improve the spectrum efficiency, maximizing the CR users' throughput is one of the most practical interests. Our object is to find the optimal sensing settings to maximize the throughput of the CR network under the conditions of sufficient protection to PUs and required bandwidth for potential CR user data transmission. To achieve that, we propose an efficient search algorithm that jointly optimizes

the sensing bandwidth and the final decision threshold. Computer simulations show that optimizing the sensing bandwidth and the final decision threshold together will further increase the throughput of the CR network as compared to that which only optimizes the sensing bandwidth or the final decision threshold.

The rest of this paper is organized as follows. The cooperative spectrum sensing with novel frame structure is analyzed in Section 2. The optimization problem of maximizing the throughput with sensing bandwidth and final decision threshold as the optimization variables is formulated in Section 3. Optimal sensing settings of throughput are analyzed in detail in Section 4. Simulation results are presented in Section 5, followed by concluding remarks in Section 6.

2. Cooperative Spectrum Sensing with Novel Frame Structure

We consider a CR network, with a number of K CR users and a fusion center. The size of the secondary network is small compared with the distance between the primary network and the secondary network to ensure the QoS of primary link. Then the path loss of each CR user is almost identical and the primary signals received at the CR users are considered to be independent and identically distributed (i.i.d.) [24]. In cooperative spectrum sensing, local CR users individually sense the channels and then send information to the fusion center, and the fusion center will make the final decision.

Figure 1 illustrates the novel frame structure designed for cooperative spectrum sensing. In the frequency domain, considering the case that the CR users know the PU transmission bandwidth W , W is divided into two parts, namely, W_s and W_t ($W_t = W - W_s$). W_s is assigned exclusively to spectrum sensing and sensing results reporting, which means that the status of PU can be decided by sensing a portion of PU bandwidth, and we do not need to sense the whole PU bandwidth. The other part W_t is assigned exclusively to potential CR user data transmission. The CR users conduct spectrum sensing and data transmission concurrently over two different parts of the primary user spectrum band.

In the time domain, it is assumed that the frame duration is T and the individual reporting duration is T_r . Since each CR user continues the spectrum sensing after sending its sensing result to the fusion center, the sensing duration for each CR user is $T_s = T - T_r$.

The essence of spectrum sensing is a binary hypothesis-testing problem [25]:

$$\begin{aligned} \mathcal{H}_0: & \text{the PU is absent;} \\ \mathcal{H}_1: & \text{the PU is present.} \end{aligned} \quad (1)$$

The received signal $r_i(k)$ at the i th CR user can be formulated as

$$r_i(k) = \begin{cases} \varepsilon_i(k), & \mathcal{H}_0, \\ h_p s(k) + \varepsilon_i(k), & \mathcal{H}_1, \end{cases} \quad (2)$$

where $r_i(k)$ is the received signal at the i th CR user, $\varepsilon_i(k)$ is the noise and $\varepsilon_i(k) \sim \mathcal{CN}(0, \sigma^2)$, $s(k)$ is the signal of PU and $s(k) \sim \mathcal{CN}(0, \sigma_s^2)$, and h_p is the channel gain between PU and the i th CR user. The power spectrum density (PSD) of the noise and the PSD of the primary signal are evenly distributed, with values of S_n and S_p , respectively. The SNR (signal-to-noise ratio) of PU's signal at the i th CR user is $\gamma_i = |h_p|^2 \sigma_s^2 / \sigma^2$.

Since the primary signals received at the CR users are considered to be i.i.d., we can omit the subscript " i ." Suppose the sampling frequency is $2W_s$ and the decision statistic of energy detection at each CR user is given by $\mathcal{R} = (1/\sigma^2) \sum_{k=1}^{2T_s W_s} |r(k)|^2$. It is shown in [26] that when $2T_s W_s$ is large enough, according to central limit theorem, we have

$$\mathcal{R} \sim \begin{cases} \mathcal{N}(2T_s W_s, 2T_s W_s), & \mathcal{H}_0, \\ \mathcal{N}(2T_s W_s(1 + \gamma), 2T_s W_s(1 + \gamma)^2), & \mathcal{H}_1. \end{cases} \quad (3)$$

The probability density function (PDF) of $\mathcal{R}$ can then be written as

$$\begin{aligned} f_{\mathcal{R}}(r) &= \frac{1}{2\sqrt{\pi T_s W_s}} e^{-(r - 2T_s W_s)^2 / 4T_s W_s}, & \mathcal{H}_0. \\ f_{\mathcal{R}}(r) &= \frac{1}{2(1 + \gamma)\sqrt{\pi T_s W_s}} e^{-[r - 2T_s W_s(1 + \gamma)]^2 / 4T_s W_s(1 + \gamma)^2}, & \mathcal{H}_1. \end{aligned} \quad (4)$$

For a nonfading environment, the probability of false alarm and the probability of detection at each CR user can be computed by

$$\begin{aligned} p_f &= P\{\mathcal{R} > \lambda \mid \mathcal{H}_0\} = \int_{\lambda}^{\infty} f_{\mathcal{R}|\mathcal{H}_0}(r) dr \\ &= \mathcal{Q}\left(\frac{\lambda}{\sqrt{2T_s W_s}} - \sqrt{2T_s W_s}\right), \\ p_d &= P\{\mathcal{R} > \lambda \mid \mathcal{H}_1\} = \int_{\lambda}^{\infty} f_{\mathcal{R}|\mathcal{H}_1}(r) dr \\ &= \mathcal{Q}\left(\frac{\lambda}{(1 + \gamma)\sqrt{2T_s W_s}} - \sqrt{2T_s W_s}\right), \end{aligned} \quad (5)$$

where λ denotes the threshold of the energy detection and $\mathcal{Q}(\cdot)$ is the $\mathcal{Q}$ -function defined as $\mathcal{Q}(x) = (1/\sqrt{2\pi}) \int_x^{\infty} e^{-t^2/2} dt$.

By (5), we have

$$p_f = \mathcal{Q}\left((1 + \gamma) \cdot \mathcal{Q}^{-1}(p_d) + \gamma \sqrt{2T_s W_s}\right). \quad (6)$$

In CSS, each CR user makes a "one bit" local decision (1 standing for the presence of PU and 0 standing for the absence of PU) on the primary user activity and then sends the individual decision to the fusion center over a reporting channel. Let Λ denote the number of CR users reporting presence of PU. In the FC, the final decision strategy $\Phi(\cdot)$ is given by [27]

$$\Phi = \begin{cases} \mathcal{H}_0, & \text{if } \Lambda < n, \\ \mathcal{H}_1, & \text{if } \Lambda \geq n, \end{cases} \quad (7)$$

where n is an integer, and $n = 1, 2, \dots, K$ is the final decision threshold at FC. It is seen that OR rule corresponds to the case of $n = 1$, AND rule corresponds to the case of $n = K$, and Majority rule corresponds to the case of $n = \lceil K/2 \rceil$. The final false alarm probability and final detection probability can be calculated as [27]

$$Q_f = \sum_{i=n}^K \binom{K}{i} p_f^i (1 - p_f)^{K-i}, \quad (8)$$

$$Q_d = \sum_{i=n}^K \binom{K}{i} p_d^i (1 - p_d)^{K-i}. \quad (9)$$

Lets define $\mathcal{F}_n(x) = \sum_{i=n}^K \binom{K}{i} x^i (1 - x)^{K-i}$ and $\mathcal{G}_n(x) = d\mathcal{F}_n(x)/dx = K \binom{K-1}{n-1} x^{n-1} (1 - x)^{K-n}$. Then, we have $Q_f = \mathcal{F}_n(p_f)$, $Q_d = \mathcal{F}_n(p_d)$, $dQ_f/dp_f = \mathcal{G}_n(p_f)$, and $dQ_d/dp_d = \mathcal{G}_n(p_d)$.

3. Optimization Problem Formulation

In our proposed novel cooperative spectrum sensing frame structure, the CR user transmits data over the bandwidth W_t only when the PU is sensed to be absent. Suppose that the power spectrum density of the CR user signal is evenly distributed, with the value of S_s . Then, $\sigma_s^2 = S_s W_t$. There are two scenarios for the CR users to transmit data.

(1) *The PU Is Correctly Detected to Be Absent.* The probability of this scenario happening is $p(\mathcal{H}_0)(1 - Q_f)$, where $p(\mathcal{H}_0)$ denotes the prior probability of the absence of the PU. In this case, the achievable throughput of the CR network is

$$\mathbb{T}_0 = W_t \log_2(1 + \gamma_s) p(\mathcal{H}_0)(1 - Q_f), \quad (10)$$

where γ_s is the SNR for the secondary link, and

$$\gamma_s = \frac{|h_s|^2 S_s W_t}{S_n W_t}, \quad (11)$$

where h_s is the channel gain of the secondary link.

(2) *The PU Is Falsely Detected to Be Absent.* The probability of this scenario happening is $p(\mathcal{H}_1)(1 - Q_d)$, where $p(\mathcal{H}_1)$

denotes the prior probability of the presence of the PU. In this case, the primary signal is considered as an interference to the secondary receiver, and the achievable throughput of the CR network is

$$\mathbb{T}_1 = W_t \log_2 (1 + \gamma_{SI}) p(\mathcal{H}_1) (1 - Q_d), \quad (12)$$

where γ_{SI} is the signal-to-noise and interference ratio for the secondary link, and

$$\gamma_{SI} = \frac{|h_s|^2 S_s W_t}{(S_p |h_p|^2 + S_n) W_t} = \frac{\gamma_s}{1 + \gamma}, \quad (13)$$

where h_p is the channel gain between the PU and the CR user.

Then, the achievable throughput of the CR network can be computed by

$$\begin{aligned} \mathbb{T} &= \mathbb{T}_0 + \mathbb{T}_1 \\ &= W_t \log_2 (1 + \gamma_s) p(\mathcal{H}_0) (1 - Q_f) \\ &\quad + W_t \log_2 \left(1 + \frac{\gamma_s}{1 + \gamma} \right) p(\mathcal{H}_1) (1 - Q_d). \end{aligned} \quad (14)$$

Considering the fact that the priority of a CR system is the protection of the QoS of the primary link, a high probability of detection is required to ensure that no harmful interference is caused by the CR network. Our object is to find the optimal sensing settings to maximize the CR users' throughput under the condition of sufficient protection to PUs. To satisfy the required bandwidth for potential CR user data transmission, we set $W_t^{\text{th}} \leq W_t < W$; namely, $0 < W_s \leq W - W_t^{\text{th}}$. Mathematically, the optimization problem can be stated as

$$\max \quad \mathbb{T} \quad (15)$$

$$\begin{aligned} \text{s.t.} \quad & Q_d \geq Q_d^{\text{th}}, \\ & 0 < W_s \leq W - W_t^{\text{th}}, \\ & 1 \leq n \leq K, \end{aligned} \quad (16)$$

where Q_d^{th} is the target detection probability with which the PUs are defined as being sufficiently protected. In the second scenario, the primary signal is considered as an interference with the secondary receiver, and we have $\log_2(1 + \gamma_s) > \log_2(1 + \gamma_{SI})$. Suppose that the prior probability of the presence of the PU is small; say less than 0.3; thus it is economically advisable to explore the secondary usage for PU spectrum band. Since $\mathcal{Q}(x)$ is a decreasing function of x , according to (5), we have $p_f < p_d$. Furthermore, since $\mathcal{G}_n(x) \geq 0$ for $0 \leq x \leq 1$, $\mathcal{F}_n(x)$ is an increasing function of x for $0 \leq x \leq 1$; thus $\mathcal{F}_n(p_f) < \mathcal{F}_n(p_d)$ and $Q_f < Q_d$. Therefore, the optimization problem can be approximated by maximizing $\mathbb{T}_0$ subject to (16).

Theorem 1. $\mathbb{T}_0$ is a decreasing function of Q_d .

Proof. For a given W_s and n , we have

$$\frac{d\mathbb{T}_0}{dQ_d} = -p(\mathcal{H}_0) (W - W_s) \log_2 (1 + \gamma_s) \frac{dQ_f}{dQ_d}, \quad (17)$$

where

$$\begin{aligned} \frac{dQ_f}{dQ_d} &= \frac{(dQ_f)/(dp_d)}{(dQ_d)/(dp_d)} = \frac{\mathcal{G}_n(p_f)}{\mathcal{G}_n(p_d)} \cdot \frac{dp_f}{dp_d}, \\ \frac{dp_f}{dp_d} &= \frac{(dp_f)/(d\lambda)}{(dp_d)/(d\lambda)} \\ &= (1 + \gamma) \exp \left\{ \frac{\lambda \gamma}{1 + \gamma} - \frac{\lambda^2 \gamma (\gamma + 2)}{4 T_s W_s (1 + \gamma)^2} \right\} > 0. \end{aligned} \quad (18)$$

Since $\mathcal{G}_n(p_f) > 0$ and $\mathcal{G}_n(p_d) > 0$, it is derived that $(d\mathbb{T}_0)/dQ_d < 0$. Thus, $\mathbb{T}_0$ is a decreasing function of Q_d . Theorem 1 is proved. $\square$

Therefore, the optimal solution must occur when the following equation stands:

$$Q_d = \sum_{i=n}^K \binom{K}{i} p_d^i (1 - p_d)^{K-i} = Q_d^{\text{th}}. \quad (19)$$

To get the optimal solution of (15), we need to calculate the root of $\Psi(p_d) = Q_d - Q_d^{\text{th}} = 0$ numerically. Newton-Raphson algorithm can be employed to find the root of $\Psi(p_d) = Q_d - Q_d^{\text{th}} = 0$ [28]. The process of the Newton-Raphson algorithm is stated as follows.

- (1) Choose tolerance δ and initial guess $p_{d,1}$; let $j = 1$.
- (2) If $|\Psi(p_{d,j})| < \delta$, stop; otherwise, go to step (3).
- (3) Let $p_{d,j+1} = p_{d,j} - \Psi(p_{d,j})/\mathcal{G}_n(p_{d,j})$; let $j = j + 1$; go to step (2).

The optimization problem is reduced to

$$\max \quad \mathbb{T}_0 \quad (20)$$

$$\begin{aligned} \text{s.t.} \quad & Q_d = Q_d^{\text{th}}, \\ & 0 < W_s \leq W - W_t^{\text{th}}, \\ & 1 \leq n \leq K. \end{aligned} \quad (21)$$

In the next section, we will find the optimal solutions of W_s and n to maximize the throughput of the CR network.

4. Optimal Sensing Settings of Throughput

First we will prove that, for any target detection probability Q_d^{th} and the final decision threshold n , there exists an optimal value of W_s that maximizes the throughput of the CR network for the CSS with novel frame structure. The derivative of $\mathbb{T}_0$ with respect to W_s is

$$\begin{aligned} \frac{d\mathbb{T}_0}{dW_s} &= -p(\mathcal{H}_0) (1 - Q_f) \log_2 (1 + \gamma_s) \\ &\quad - p(\mathcal{H}_0) \frac{dQ_f}{dW_s} (W - W_s) \log_2 (1 + \gamma_s), \end{aligned} \quad (22)$$

where

$$\begin{aligned} \frac{dQ_f}{dW_s} &= \frac{dQ_f}{dp_f} \cdot \frac{dp_f}{dW_s} = \mathcal{G}_n(p_f) \frac{dp_f}{dW_s}, \\ \frac{dp_f}{dW_s} &= -\frac{\gamma}{2} \sqrt{\frac{T_s}{\pi W_s}} \exp \left\{ -\frac{1}{2} \left[(1+\gamma) \mathcal{Q}^{-1}(p_d) \right. \right. \\ &\quad \left. \left. + \gamma \sqrt{2T_s W_s} \right]^2 \right\}. \end{aligned} \quad (23)$$

Obviously,

$$\begin{aligned} \lim_{W_s \rightarrow 0} \frac{d\mathbb{T}_0}{dW_s} &= \infty, \\ \lim_{W_s \rightarrow W} \frac{d\mathbb{T}_0}{dW_s} &= -p(\mathcal{H}_0) (1 - Q_f) \log_2(1 + \gamma_s) < 0. \end{aligned} \quad (24)$$

Equation (24) means that $\mathbb{T}_0$ increases when W_s is small and decreases when W_s approaches W . Hence, there must be a maximum point of $\mathbb{T}_0$ for $W_s \in (0, W)$. Next we will prove that the maximum point of $\mathbb{T}_0$ is unique in this range; that is, there is a unique W_s^* where $W_s^* \in (0, W)$ such that $d\mathbb{T}_0/dW_s|_{W_s=W_s^*} = 0$. Setting $d\mathbb{T}_0/dW_s = 0$ and after some algebraic manipulations, it is derived that $\mathcal{A}(W_s) = \mathcal{B}(W_s)$, where

$$\begin{aligned} \mathcal{A}(W_s) &= -2 \ln \left[\frac{2(n-1)!(K-n)!}{\gamma K!} \sqrt{\frac{\pi}{T_s}} \frac{\sqrt{W_s}}{W - W_s} \right. \\ &\quad \left. \times \sum_{i=0}^{n-1} \binom{K}{i} p_f^{i-n+1} (1-p_f)^{n-i} \right], \end{aligned} \quad (25)$$

$$\mathcal{B}(W_s) = \left[(1+\gamma) \mathcal{Q}^{-1}(p_d) + \gamma \sqrt{2T_s W_s} \right]^2.$$

If functions $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$ intersect each other only once for $W_s \in (0, W)$, we can conclude that the root of $d\mathbb{T}_0/dW_s = 0$ is unique. The derivative of $\mathcal{A}(W_s)$ with respect to W_s is

$$\begin{aligned} \frac{d\mathcal{A}(W_s)}{dW_s} &= -\frac{W + W_s}{W_s(W - W_s)} + \frac{2}{\sum_{i=0}^{n-1} \binom{K}{i} p_f^{i-n+1} (1-p_f)^{n-i}} \\ &\quad \times \sum_{i=0}^{n-1} \binom{K}{i} \left[(n-1-i) p_f^{i-n} (1-p_f)^{n-i} \right. \\ &\quad \left. + (n-i) p_f^{i-n+1} (1-p_f)^{n-i-1} \right] \frac{dp_f}{dW_s}. \end{aligned} \quad (26)$$

Since $n-1-i \geq 0$ and $n-i > 0$ for $i = 0, 1, \dots, n-1$, and $dp_f/dW_s < 0$, we can obtain that $d\mathcal{A}(W_s)/dW_s < 0$ for $W_s \in (0, W)$. Thus, $\mathcal{A}(W_s)$ is a decreasing function of W_s for $W_s \in (0, W)$. The derivative of $\mathcal{B}(W_s)$ with respect to W_s is

$$\frac{d\mathcal{B}(W_s)}{dW_s} = \gamma \sqrt{\frac{2T_s}{W_s}} \left[(1+\gamma) \mathcal{Q}^{-1}(p_d) + \gamma \sqrt{2T_s W_s} \right]. \quad (27)$$

Case 1. If $\mathcal{Q}^{-1}(p_d) \geq 0$, we have $(1+\gamma)\mathcal{Q}^{-1}(p_d) + \gamma\sqrt{2T_s W_s} > 0$ for $W_s \in (0, W)$. According to (27), we can obtain that $d\mathcal{B}(W_s)/dW_s > 0$. In this case, $\mathcal{B}(W_s)$ is an increasing function of W_s for $W_s \in (0, W)$. Thus, $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$ can intersect at most once. Therefore, there is only one intersection between $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$ for $W_s \in (0, W)$. The root of $d\mathbb{T}_0/dW_s = 0$ is unique in this case.

Case 2. If $\mathcal{Q}^{-1}(p_d) < 0$, when $0 < W_s < [(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2 / 2T_s\gamma^2$, we have $(1+\gamma)\mathcal{Q}^{-1}(p_d) + \gamma\sqrt{2T_s W_s} < 0$ and $d\mathcal{B}(W_s)/dW_s < 0$. Thus, $\mathcal{B}(W_s)$ is a decreasing function of W_s for $W_s \in (0, [(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2 / 2T_s\gamma^2)$. When $[(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2 / 2T_s\gamma^2 \leq W_s < W$, we have $(1+\gamma)\mathcal{Q}^{-1}(p_d) + \gamma\sqrt{2T_s W_s} \geq 0$ and $d\mathcal{B}(W_s)/dW_s \geq 0$. Thus, $\mathcal{B}(W_s)$ is an increasing function of W_s for $W_s \in [[(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2 / 2T_s\gamma^2, W)$. In this case, we will prove that $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$ can intersect each other only once for $W_s \in (0, W)$.

Theorem 2. $d\mathcal{A}(W_s)/dW_s < d\mathcal{B}(W_s)/dW_s$ for $W_s \in (0, W)$.

Proof. For $W_s \in [[(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2 / 2T_s\gamma^2, W)$, we have $d\mathcal{A}(W_s)/dW_s < 0$ and $d\mathcal{B}(W_s)/dW_s \geq 0$. Thus, $d\mathcal{A}(W_s)/dW_s < d\mathcal{B}(W_s)/dW_s$.

For $W_s \in (0, [(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2 / 2T_s\gamma^2)$, we have $d\mathcal{A}(W_s)/dW_s < 0$ and $d\mathcal{B}(W_s)/dW_s < 0$. We first prove that, when $n = 1$, $d\mathcal{A}(W_s)/dW_s < d\mathcal{B}(W_s)/dW_s$. According to (26), when $n = 1$, we have

$$\left. \frac{d\mathcal{A}(W_s)}{dW_s} \right|_{n=1} = -\frac{W + W_s}{W_s(W - W_s)} + \frac{2}{1 - p_f} \cdot \frac{dp_f}{dW_s}. \quad (28)$$

According to (27) and (28), $d\mathcal{A}(W_s)/dW_s|_{n=1} < d\mathcal{B}(W_s)/dW_s$ is given as

$$\begin{aligned} & -\frac{W + W_s}{W_s(W - W_s)} \\ & - \frac{\gamma}{1 - \mathcal{Q}((1+\gamma) \cdot \mathcal{Q}^{-1}(p_d) + \gamma\sqrt{2T_s W_s})} \sqrt{\frac{T_s}{\pi W_s}} \\ & \times \exp \left\{ -\frac{1}{2} \left[(1+\gamma) \mathcal{Q}^{-1}(p_d) + \gamma\sqrt{2T_s W_s} \right]^2 \right\} \\ & < \gamma \sqrt{\frac{2T_s}{W_s}} \left[(1+\gamma) \mathcal{Q}^{-1}(p_d) + \gamma\sqrt{2T_s W_s} \right]. \end{aligned} \quad (29)$$

For $W_s \in (0, [(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2 / 2T_s\gamma^2)$, $(1+\gamma)\mathcal{Q}^{-1}(p_d) + \gamma\sqrt{2T_s W_s} < 0$. Let $x = -[(1+\gamma)\mathcal{Q}^{-1}(p_d) + \gamma\sqrt{2T_s W_s}] > 0$. Substituting x into (29), it is derived that

$$\frac{W + W_s}{\gamma(W - W_s) \sqrt{2T_s W_s}} + \frac{1}{\sqrt{2\pi} \mathcal{Q}(x)} e^{-x^2/2} > x. \quad (30)$$

Since $(1/\sqrt{2\pi} \mathcal{Q}(x))e^{-x^2/2} > x$ for $x \geq 0$ [29], the inequality at (30) is verified. Thus, $d\mathcal{A}(W_s)/dW_s|_{n=1} < d\mathcal{B}(W_s)/dW_s$ has been proved. Next we will prove that $d\mathcal{A}(W_s)/dW_s <$

$d\mathcal{B}(W_s)/dW_s$ for $n = 2, 3, \dots, K$. Since $n - i \geq 1$ for $i = 0, 1, \dots, n - 1$, we have

$$\begin{aligned} & \frac{2}{\sum_{i=0}^{n-1} \binom{K}{i} p_f^{i-n+1} (1-p_f)^{n-i}} \\ & \times \sum_{i=0}^{n-1} \binom{K}{i} \left[(n-1-i) p_f^{i-n} (1-p_f)^{n-i} \right. \\ & \quad \left. + (n-i) p_f^{i-n+1} (1-p_f)^{n-i-1} \right] \\ & > 2 \frac{\sum_{i=0}^{n-1} \binom{K}{i} (n-i) p_f^{i-n+1} (1-p_f)^{n-i-1}}{\sum_{i=0}^{n-1} \binom{K}{i} p_f^{i-n+1} (1-p_f)^{n-i-1}} \cdot \frac{1}{1-p_f} \\ & > \frac{2}{1-p_f}. \end{aligned} \quad (31)$$

According to (26), (28), and (31), we obtain $d\mathcal{A}(W_s)/dW_s|_{n=2,3,\dots,K} < d\mathcal{A}(W_s)/dW_s|_{n=1}$. Therefore, $d\mathcal{A}(W_s)/dW_s < d\mathcal{B}(W_s)/dW_s$ for $W_s \in (0, W)$. Theorem 2 is proved. $\square$

In Case 2, there are two possible scenarios for the intersection between $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$.

- (i) $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$ intersect each other in the region of $W_s \in (0, [(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2)$. Since $d\mathcal{A}(W_s)/dW_s < d\mathcal{B}(W_s)/dW_s$, it is impossible for them to intersect more than once because $\mathcal{A}(W_s)$ decreases at a faster rate than $\mathcal{B}(W_s)$ for $W_s \in (0, [(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2)$. For $W_s \in [[(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2, W)$, $\mathcal{B}(W_s)$ is always larger than $\mathcal{A}(W_s)$ since $\mathcal{B}(W_s)$ is an increasing function of W_s while $\mathcal{A}(W_s)$ is a decreasing function of W_s . It is impossible for them to intersect each other for $W_s \in [[(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2, W)$. Therefore, in this scenario, there is only one intersection between $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$, and it occurs in the region of $W_s \in (0, [(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2)$.
- (ii) $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$ do not intersect each other in the region of $W_s \in (0, [(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2)$. In this scenario, they must intersect in the region of $W_s \in [[(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2, W)$ since there must be at least one intersection in the entire range of $0 < W_s < W$. For $W_s \in [[(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2, W)$, $\mathcal{A}(W_s)$ is a decreasing function of W_s and $\mathcal{B}(W_s)$ is an increasing function of W_s . Thus, they can intersect each other at most once. Therefore, in this scenario, there is only one intersection between $\mathcal{A}(W_s)$ and $\mathcal{B}(W_s)$, and it occurs in the region of $W_s \in [[(1+\gamma)\mathcal{Q}^{-1}(p_d)]^2/2T_s\gamma^2, W)$.

From the analysis above, one can conclude that there is a unique W_s^* where $W_s^* \in (0, W)$ such that $d\mathbb{T}_0/dW_s|_{W_s=W_s^*} = 0$. Bisection method can be used to find the root of $d\mathbb{T}_0/dW_s = 0$ [28]. The process of bisection method is stated as follows.

- (1) Choose lower guess W_s^l and upper guess W_s^u for the root such that the function changes sign over the interval. This can be checked by ensuring that $d\mathbb{T}_0/dW_s|_{W_s=W_s^l} \cdot d\mathbb{T}_0/dW_s|_{W_s=W_s^u} < 0$.
- (2) An estimate of the root W_s^r is determined by $W_s^r = (W_s^l + W_s^u)/2$.
- (3) Make the following evaluations to determine in which subinterval the root lies.
 - (a) If $d\mathbb{T}_0/dW_s|_{W_s=W_s^l} \cdot d\mathbb{T}_0/dW_s|_{W_s=W_s^r} < 0$, the root lies in the lower subinterval. Therefore, set $W_s^u = W_s^r$ and return to step (2).
 - (b) If $d\mathbb{T}_0/dW_s|_{W_s=W_s^l} \cdot d\mathbb{T}_0/dW_s|_{W_s=W_s^r} > 0$, the root lies in the upper subinterval. Therefore, set $W_s^l = W_s^r$ and return to step (2).
 - (c) If $d\mathbb{T}_0/dW_s|_{W_s=W_s^l} \cdot d\mathbb{T}_0/dW_s|_{W_s=W_s^r} = 0$, the root equals W_s^r ; terminate the computation.

Note that the second constraint of the optimization problem is $0 < W_s \leq W - W_t^{\text{th}}$. If $W_s^* \geq W - W_t^{\text{th}}$, $\mathbb{T}_0$ is monotonically increasing in the range of $0 < W_s \leq W - W_t^{\text{th}}$. In this case, choose $W_s = W - W_t^{\text{th}}$; the optimization is achieved. If $0 < W_s^* < W - W_t^{\text{th}}$, $\mathbb{T}_0$ is monotonically increasing in the range of $0 < W_s \leq W_s^*$ and is monotonically decreasing in the range of $W_s^* < W_s < W - W_t^{\text{th}}$. In this case, choose $W_s = W_s^*$; the optimization is achieved.

For the third constraint of the optimization problem, $1 \leq n \leq K$, no closed-form solution for the optimal n^* is available. However, since n is an integer, it is not computationally expensive to search through n from 1 to K to obtain the optimal n^* that maximizes (20). Therefore, the process to achieve the maximum throughput for the CSS with novel frame structure is listed as follows.

- (1) For each n ($1 \leq n \leq K$), calculate the root $p_{d,n}$ of $\Psi(p_{d,n}) = 0$ by using the Newton-Raphson algorithm.
- (2) For each $p_{d,n}$, calculate the root $W_{s,n}^*$ of $d\mathbb{T}_0/dW_s = 0$ by using the bisection method. If $W_{s,n}^* \geq W - W_t^{\text{th}}$ is satisfied, choose $W_{s,n}^{\text{opt}} = W - W_t^{\text{th}}$; otherwise, choose $W_{s,n}^{\text{opt}} = W_{s,n}^*$.
- (3) For each $W_{s,n}^{\text{opt}}$, calculate the corresponding individual false alarm probability $p_{f,n}$ by (6), the final false alarm probability $Q_{f,n}$ by (8), and the throughput $\mathbb{T}_{0,n}$ by (10).
- (4) Compare $\mathbb{T}_{0,n}$ and choose the maximum one.

5. Simulation Results

To get insight into the effectiveness of the proposed sensing methods and validate some related theorems, computer simulations have been conducted to evaluate the performance of throughput for various spectrum sensing schemes. In the simulations, we set the bandwidth of PU as $W = 20000$ Hz; the frame duration is $T = 20$ ms; the individual reporting duration is $T_r = 1$ ms; the target detection probability is

FIGURE 2: The throughput of the CR network versus the sensing bandwidth W_s for various sensing schemes.

$Q_d^{\text{th}} = 99\%$; the SNR for the secondary link is $\gamma_s = 20$ dB; the prior probability of the absence of PU is $p(\mathcal{H}_0) = 0.8$ and $p(\mathcal{H}_1) = 1 - p(\mathcal{H}_0) = 0.2$. To satisfy the required bandwidth for potential CR user data transmission, we set $W_t^{\text{th}} = W/5$.

Figure 2 compares the throughput of the CR network when the novel CSS frame structure is employed to the case when the previous frame structure is employed. For the previous frame structure, we can refer to Figure 1 in [25]; for the novel CSS scheme, the number of CR users is $K = 9$; the SNR of the PU's signal at the CR user is $\gamma = -10$ dB. It is seen that using the novel CSS frame structure can achieve a much higher throughput than that using the previous frame structure, especially when the final decision threshold n is optimized. For the previous frame structure, the maximum achievable throughput is approximately 1.55×10^4 bits/s. However, when our proposed novel CSS frame structure is employed and n is optimized, the maximum achievable throughput is approximately 6.4×10^4 bits/s. From this figure, it can be seen that, for a given sensing bandwidth, the optimal sensing settings improve the throughput of the CR network. When $W_s \geq 1.6$ kHz, Majority rule ($n = 5$) is suboptimal. However, OR rule ($n = 1$) outperforms Majority rule when $0 < W_s < 1.6$ kHz. It is also observed that there exists an optimal sensing bandwidth which yields the highest throughput for the secondary network. The optimal sensing bandwidth varies with different values of n .

In Figure 3, it is also observed that the throughput of the novel CSS scheme is much higher than that of the previous scheme. For the novel CSS scheme, optimal n values are used for each fixed W_s . The optimal sensing settings can achieve a higher throughput than that using fixed sensing bandwidth. In addition, one can clearly see that the disadvantage with a fixed sensing bandwidth is that, at high SNR levels where the PU can be easily detected, the throughput of the CR network

FIGURE 3: The throughput of the CR network versus SNR for various values of W_s ; optimal n values are used for each fixed W_s in the novel CSS scheme.

is bounded by the percentage of the total PU transmission bandwidth that is assigned to spectrum sensing. For the novel CSS scheme, by comparing the curves of $W_s = W/5$ and $W_s = W/4$, we can see that, in low SNR region, $-20 \sim -9$ dB, the throughput of the former scheme ($W_s = W/5$) is lower than that of the latter scheme ($W_s = W/4$). However, when SNR is larger than -9 dB, the two curves approach constants and the throughput of the former scheme is larger than that of the latter scheme. This indicates that no fixed W_s is optimal for all SNR values. Thus, there is a need to optimize the value of W_s to enhance the throughput of the CR network.

Figure 4 illustrates the maximum throughput of the CR network versus the average SNR for the novel CSS scheme with various counting rules. Optimal sensing bandwidth is used in each of the counting rules. One can see clearly that the optimal sensing settings can achieve a higher throughput than that using uniform thresholds. It is seen that, when $-15 \text{ dB} \leq \text{SNR} \leq 0 \text{ dB}$, Majority rule ($n = 5$) is suboptimal and AND rule ($n = 9$) performs the worst. In particular, at -10 dB SNR, the optimal sensing settings can achieve almost 1.5 times throughput than that when AND rule is used. However, when SNR is lower than -15 dB, AND rule outperforms Majority rule and OR rule ($n = 1$). And it achieves the same throughput as the optimal setting when SNR is extremely weak. Therefore, there is no single decision threshold n that is optimal for all cases. To maximize the throughput of the CR network, different SNR levels require different optimal n values.

Figure 5 is simulated to show the maximum throughput of the CR network versus number of CR users K for various sensing schemes with different values of W_s , in which optimal n values are used for each fixed W_s . The SNR of PU's signal

FIGURE 4: The maximum throughput versus SNR for various counting rules; optimal sensing bandwidth is used in each of the counting rules.

FIGURE 5: The maximum throughput versus the number of CR users for various values of W_s ; optimal n values are used for each fixed W_s .

at the CR user is $\gamma = -8$ dB. It has been shown that the throughput with optimal sensing settings is larger than that using fixed sensing bandwidth and grows as the number of CR users increases. For the sensing scheme with fixed sensing bandwidth, the throughput of the CR network approaches constants when the number of CR users is extremely large and is bounded by the percentage of the total PU transmission bandwidth that is assigned to spectrum sensing. By comparing the curves of the sensing schemes with different values of W_s , we can also find that no fixed W_s is optimal for all cases. To enhance the throughput of the CR network, the value of W_s needs to be optimized.

FIGURE 6: The maximum throughput versus the number of CR users for various counting rules; optimal sensing bandwidth is used in each of the counting rules.

In Figure 6, the maximum throughput of the CR network is presented versus the number of CR users K for the novel CSS scheme with various counting rules. Optimal sensing bandwidth is used in each of the counting rules. The SNR of PU's signal at the CR user is $\gamma = -8$ dB. It is observed that the optimal sensing settings can achieve a higher throughput than that using uniform thresholds. Majority rule ($n = 5$) is suboptimal and AND rule ($n = 9$) always performs the worst. It is also seen that the throughput of the CR network increases as the number of CR users increases. However, the complexity of our proposed algorithm grows approximately linearly with the number of cooperating CR users.

6. Conclusions

In this paper, we propose a novel frame structure for cooperative spectrum sensing. It has been proved that there exists an optimal sensing bandwidth which yields the highest throughput for the CR network. The optimal sensing settings to maximize the throughput of the CR network under the conditions of sufficient protection to PUs and required bandwidth for potential CR user data transmission have been proposed. The proposed optimal sensing settings are analyzed and calculated in detail by using some simple but reliable methods. Computer simulations have shown that significant improvement in the throughput of the CR network has been achieved when the sensing bandwidth and the final decision threshold are jointly optimized.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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