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Research Article

The Extended Multiple (G'/G)-Expansion Method and Its Application to the Caudrey-Dodd-Gibbon Equation

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An extended multiple (G'/G)-expansion method is used to seek the exact solutions of Caudrey-Dodd-Gibbon equation. As a result, plentiful new complexiton solutions consisting of hyperbolic functions, trigonometric functions, rational functions, and their mixture with arbitrary parameters are effectively obtained. When some parameters are properly chosen as special values, the known double solitary-like wave solutions are derived from the double hyperbolic function solutions.

1. Introduction

A trouble and tedious but very important problem is to find exact solutions of nonlinear evolution equations (NLEEs). To solve this problem, a number of powerful and efficient methods have been established for obtaining exact traveling wave solutions, such as the inverse scattering method [1], the Backlund transform method [2, 3], the Darboux transform method [4], the Hirota's bilinear transformation method [5], the Exp-function method [6], the tanh-function method [7], the Weierstrass elliptic function method [8], the Jacobi elliptic function expansion method [9], the simplest equation method [10–12], and the modified method of simplest equation [13, 14]. With the development of computer science, directly searching for exact traveling wave solutions of NLEEs has attracted much attention. This is due to the availability of symbolic computation systems like Mathematica or Maple which enable us to perform the complex and tedious computation on computers. Recently, Wang et al. [15] proposed a new direct method called the (G'/G)-expansion method to look for traveling wave solutions of NLEEs. This method is straightforward, concise, and capable of producing new applications. Moreover, the solutions obtained by this method are of general nature and a number of specific solutions can be deduced by putting conditions on arbitrary constants present in the general solutions. Aslan reported the relationship between the (G'/G)-expansion method and the simplest equation method [16]. He told us that the former one is a specific form of the later one. However, (G'/G)-expansion method has become widely used to search for various exact solutions of NLEEs [17–20]. Lately, the further developed methods named the generalized (G'/G)-expansion method [21], the modified (G'/G)-expansion method [22], the extended (G'/G)-expansion method [23], the improved (G'/G)-expansion method [24], the $(G'/G, 1/G)$-expansion method [25], and the multiple (G'/G)-expansion method [26] have been proposed for constructing exact solutions to NLEEs. The aim of this paper is to find new exact complexiton solutions and double solitary-like wave solutions of the Caudrey-Dodd-Gibbon equation by using the extended multiple (G'/G)-expansion method.

The organization of this paper is as follows. In Section 2, we briefly describe the extended multiple (G'/G)-expansion method. In Section 3, for illustration, we restrict our attention to the Caudrey-Dodd-Gibbon equation and successfully construct many complexiton solutions and double solitary-like wave solutions. In Section 4, some conclusions are given.

2. Description of the Extended Multiple (G'/G)-Expansion Method

In this section, we will give the detailed description of the extended (G'/G)-expansion method for seeking the exact traveling wave solutions of NLEEs.

Suppose that a nonlinear partial differential equation (PDE), say in two independent variables x and t, is given by

$$F\left(u, u_x, u_t, u_{tt}, u_{xt}, u_{xx}, \ldots\right) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function and F is a polynomial with respect to $u(x,t)$ and its partial derivatives which involve the highest order derivatives and the nonlinear terms. In the following, we give the main steps of the extended (G'/G)-expansion method.

Step 1. The traveling wave transformation is

$$u(x,t) = u(\xi_1, \xi_2), \quad \xi_1 = k_1 x + c_1 t + l_1, \quad \xi_2 = k_2 x + c_2 t + l_2, \tag{2}$$

where k_i, c_i, l_i $(i = 1,2)$ are all constants to be determined later.

Step 2. Suppose that the solution of (1) can be written as follows:

$$u(\xi_1, \xi_2) = A_0 + \sum_{k=1}^{n} \sum_{i+j=k} A_{ij} \left(\frac{G_1'(\xi_1)}{G_1(\xi_1)}\right)^i \left(\frac{G_2'(\xi_2)}{G_2(\xi_2)}\right)^j, \quad \sum_{i+j=n} A_{ij} \left(\frac{G_1'(\xi_1)}{G_1(\xi_1)}\right)^i \left(\frac{G_2'(\xi_2)}{G_2(\xi_2)}\right)^j \neq 0, \tag{3}$$

where A_0, A_{ij} $(i, j = 1, 2, \ldots, n)$ are constants to be determined later, n is an undetermined integer, and $G_i(\xi_i)$ $(i = 1,2)$ satisfy the auxiliary linear ordinary differential equations:

$$G_i''(\xi_i) + \lambda_i G_i'(\xi_i) + \mu_i G_i(\xi_i) = 0, \quad (i = 1,2), \tag{4}$$

where $G_i''(\xi_i) = \mathbf{d}^2 G_i(\xi_i)/\mathbf{d}\xi_i^2$, $G_i'(\xi_i) = \mathbf{d}G_i(\xi_i)/\mathbf{d}\xi_i$ $(i = 1,2)$, and λ_i, μ_i $(i = 1,2)$ are constants.

Step 3. Determine the positive integer n by balancing the highest order derivatives and nonlinear terms in (1).

Step 4. Substituting (3) along with (4) into (1) yields a partial differential equation. Since terms $(G_1'(\xi_1)/G_1(\xi_1))^i$ $(G_2'(\xi_2)/G_2(\xi_2))^j$ $(i = 0, 1, 2, \ldots; j = 0, 1, 2, \ldots)$ in the partial differential equation are linearly independent, a set of overdetermined ordinary differential equations (ODEs) for A_0, A_{ij} $(i, j = 1, 2, \ldots, n)$, k_1, k_2, c_1, c_2, l_1, and l_2 can be obtained by vanishing all the coefficients of the terms $(G_1'(\xi_1)/G_1(\xi_1))^i$ $(G_2'(\xi_2)/G_2(\xi_2))^j (i = 0, 1, 2, \ldots; j = 0, 1, 2, \ldots)$.

Step 5. Assuming that A_0, A_{ij} $(i, j = 1, 2, \ldots, n)$, $k_{1,2}$, c_1, c_2, l_1, and l_2 can be obtained by solving the ODEs in Step 4, then by substituting them into (3), we can obtain the exact solutions of (1) immediately.

3. Application of the Method to the Caudrey-Dodd-Gibbon Equation

In this section, we apply the extended multiple (G'/G)-expansion method to construct the traveling wave solutions of the Caudrey-Dodd-Gibbon equation which is given by

$$u_t + u_{xxxxx} + 30uu_{xxx} + 30u_x u_{xx} + 180u^2 u_x = 0, \tag{5}$$

where subscripts indicate partial derivatives and u is a real scalar function of the two independent variables x and t.

For simplification, we uniformly denote $\Delta_1 = \lambda_1^2 - 4\mu_1$ and $\Delta_2 = \lambda_2^2 - 4\mu_2$ in the following paper.

According to the homogeneous balance procedure, balancing the highest order nonlinear term $u^2 u_x$ and the highest order derivative term u_{xxxxx} in (5), we get $3n + 1 = n + 5$. Hence we find $n = 2$ and suppose that the solution of (5) is in the form

$$u = A_0 + A_1\left(\frac{G_1'(\xi_1)}{G_1(\xi_1)}\right) + A_2\left(\frac{G_1'(\xi_1)}{G_1(\xi_1)}\right)^2 + B_1\left(\frac{G_2'(\xi_2)}{G_2(\xi_2)}\right) + B_2\left(\frac{G_2'(\xi_2)}{G_2(\xi_2)}\right)^2 + B_0\left(\frac{G_1'(\xi_1)}{G_1(\xi_1)}\right)\left(\frac{G_2'(\xi_2)}{G_2(\xi_2)}\right). \tag{6}$$

Substituting (6) along with (4) into (5) yields a partial differential equation. Then by vanishing all the coefficients of terms $(G_1'(\xi_1)/G_1(\xi_1))^i (G_2'(\xi_2)/G_2(\xi_2))^j$ $(i = 0, 1, 2, \ldots; j = 0, 1, 2, \ldots)$ of the partial differential equation, we obtain 36 algebraic equations for A_i $(i = 0, 1, 2)$, B_j $(j = 0, 1, 2)$, k_1, k_2, c_1, c_2, l_1, and l_2. Solving the system of algebraic equations with the aid of Maple 14, we obtain the following results:

$$\begin{gathered}
k_1 = F_1, \qquad k_2 = F_2, \\
A_0 = \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \frac{1}{4}\left(F_1^2\lambda_1^2 + F_2^2\lambda_2^2\right), \\
B_0 = 0, \\
A_1 = -F_1^2\lambda_1, \qquad A_2 = -F_1^2, \qquad B_1 = -F_2^2\lambda_2, \\
B_2 = -F_2^2, \qquad l_1 = F_3, \qquad l_2 = F_4, \\
c_1 = \frac{F_1}{4}\left(F_1^4\Delta_1^2 - 5F_2^4\Delta_2^2\right), \qquad c_2 = \frac{F_2}{4}\left(F_2^4\Delta_2^2 - 5F_1^4\Delta_1^2\right),
\end{gathered} \tag{7}$$

where F_1, F_2, F_3, and F_4 are arbitrary constants.

Substituting (7) into (6), the general form of solution of (5) can be expressed by

$$\begin{aligned} u = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \frac{1}{4}\left(F_1^2\lambda_1^2 + F_2^2\lambda_2^2\right) \\ & - F_1^2\lambda_1\left(\frac{G_1'(\xi_1)}{G_1(\xi_1)}\right) - F_1^2\left(\frac{G_1'(\xi_1)}{G_1(\xi_1)}\right)^2 \\ & - F_2^2\lambda_2\left(\frac{G_2'(\xi_2)}{G_2(\xi_2)}\right) - F_2^2\left(\frac{G_2'(\xi_2)}{G_2(\xi_2)}\right)^2, \end{aligned} \tag{8}$$

with

$$\begin{aligned} \xi_1 &= F_1 x + \frac{F_1}{4}\left(F_1^4\Delta_1^2 - 5F_2^4\Delta_2^2\right)t + F_3, \\ \xi_2 &= F_2 x + \frac{F_2}{4}\left(F_2^4\Delta_2^2 - 5F_1^4\Delta_1^2\right)t + F_4, \end{aligned} \tag{9}$$

where F_1, F_2, F_3, and F_4 are arbitrary constants.

From the general solutions of (4), which depend on different choices of λ_1, λ_2, μ_1, and μ_2, some complexiton solutions of (5) can be derived immediately.

Case 1. Setting $\Delta_1 > 0$ and $\Delta_2 > 0$, the complexiton solutions consisting of hyperbolic functions of (5) can be derived as

$$\begin{aligned} u_1 = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \frac{F_1^2\Delta_1}{4} \\ & \times\left(\frac{M_1\sinh\left(\left(\sqrt{\Delta_1}/2\right)\xi_1\right) + N_1\cosh\left(\left(\sqrt{\Delta_1}/2\right)\xi_1\right)}{M_1\cosh\left(\left(\sqrt{\Delta_1}/2\right)\xi_1\right) + N_1\sinh\left(\left(\sqrt{\Delta_1}/2\right)\xi_1\right)}\right)^2 \\ & - \frac{F_2^2\Delta_2}{4} \\ & \times\left(\frac{M_2\sinh\left(\left(\sqrt{\Delta_2}/2\right)\xi_2\right) + N_2\cosh\left(\left(\sqrt{\Delta_2}/2\right)\xi_2\right)}{M_2\cosh\left(\left(\sqrt{\Delta_2}/2\right)\xi_2\right) + N_2\sinh\left(\left(\sqrt{\Delta_2}/2\right)\xi_2\right)}\right)^2, \end{aligned} \tag{10}$$

where F_1, F_2, M_1, M_2, N_1, and N_2 are arbitrary constants and ξ_1 and ξ_2 are determined by using (9). In order to show the properties of the double solitary wave solution visually, as an example, we plot the 3D graphs of solution (10) for some fixed parameters, which are shown in Figure 1.

It is easy to see that the double solitary-like wave solution can be obtained at $M_1^2 > N_1^2$, $M_2^2 > N_2^2$ and $M_1 \neq 0$, $M_2 \neq 0$ as follows:

$$\begin{aligned} u_1 = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \frac{F_1^2\Delta_1}{4}\tanh^2\left(\frac{\sqrt{\Delta_1}}{2}\xi_1 + \xi_{10}\right) \\ & - \frac{F_2^2\Delta_2}{4}\tanh^2\left(\frac{\sqrt{\Delta_2}}{2}\xi_2 + \xi_{20}\right), \end{aligned} \tag{11}$$

where $\xi_{10} = \tanh^{-1}(N_1/M_1)$ and $\xi_{20} = \tanh^{-1}(N_2/M_2)$.

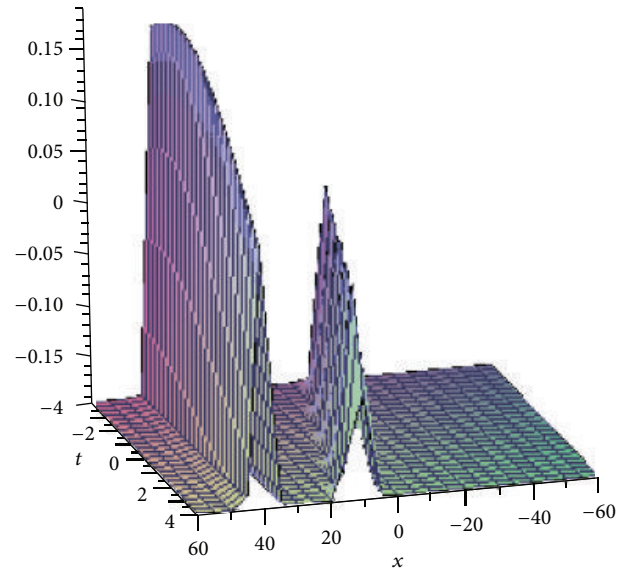

Figure 1: The 3D graphs of profiles of the exact solution (10) for fixed parameters $F_1 = -0.2$, $F_2 = 0.1$, $\lambda_1 = 6$, $\mu_1 = 4$, $\lambda_2 = 12$, $\mu_2 = -3, l1 = 0.2$, $l2 = -4$, $M_1 = -4$, $N_1 = 1$, $M_2 = -5$, and $N_2 = 1$.

Case 2. Setting $\Delta_1 > 0$ and $\Delta_2 < 0$, the complexiton solutions consisting of hyperbolic functions and trigonometric functions of (5) can be derived as

$$\begin{aligned} u_2 = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \frac{F_1^2\Delta_1}{4} \\ & \times\left(\frac{M_1\sinh\left(\left(\sqrt{\Delta_1}/2\right)\xi_1\right) + N_1\cosh\left(\left(\sqrt{\Delta_1}/2\right)\xi_1\right)}{M_1\cosh\left(\left(\sqrt{\Delta_1}/2\right)\xi_1\right) + N_1\sinh\left(\left(\sqrt{\Delta_1}/2\right)\xi_1\right)}\right)^2 \\ & + \frac{F_2^2\Delta_2}{4} \\ & \times\left(\frac{M_2\sin\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right) - N_2\cos\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right)}{M_2\cos\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right) + N_2\sin\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right)}\right)^2, \end{aligned} \tag{12}$$

where F_1, F_2, M_1, M_2, N_1, and N_2, are arbitrary constants and ξ_1 and ξ_2 are determined by using (9).

The profiles of (12) are shown in Figure 2.

It is easy to see that the complexiton solutions can be rewritten at $M_1^2 > N_1^2$, $M_2^2 > N_2^2$ and $M_1 \neq 0$, $M_2 \neq 0$ as follows:

$$\begin{aligned} u_2 = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \frac{F_1^2\Delta_1}{4}\tanh^2\left(\frac{\sqrt{\Delta_1}}{2}\xi_1 + \xi_{10}\right) \\ & + \frac{F_2^2\Delta_2}{4}\tan^2\left(\frac{\sqrt{-\Delta_2}}{2}\xi_2 - \xi_{20}\right), \end{aligned} \tag{13}$$

where $\xi_{10} = \tanh^{-1}(N_1/M_1)$ and $\xi_{20} = \tan^{-1}(N_1/M_2)$.

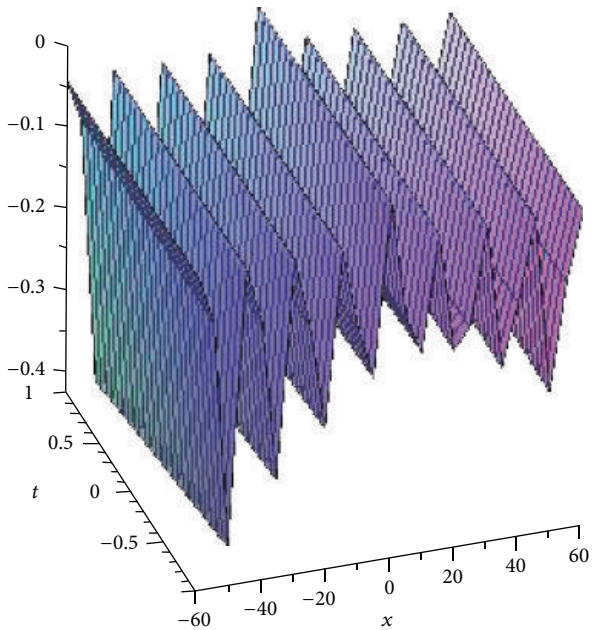

Figure 2: The 3D graphs of profiles of the exact solution (12) for fixed parameters $F_1 = -0.1$, $F_2 = 0.15$, $\lambda_1 = 6$, $\mu_1 = 4$, $\lambda_2 = -4$, $\mu_2 = 6, l1 = 0.2$, $l2 = -4$, $M_1 = -4$, $N_1 = 1$, $M_2 = -5$, and $N_2 = -4$.

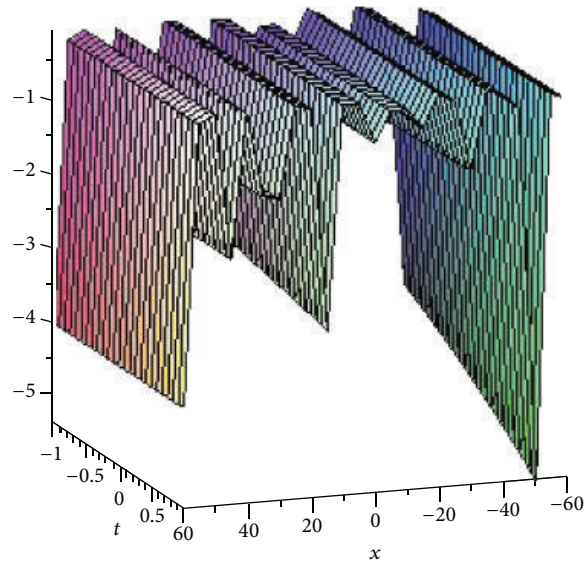

Figure 3: The 3D graphs of profiles of the exact solution (14) for fixed parameters $F_1 = -0.11$, $F_2 = 0.15$, $\lambda_1 = -3$, $\mu_1 = 4$, $\lambda_2 = 2$, $\mu_2 = 3$ $l1 = 0.2$, $l2 = -4$, $M_1 = -4$, $N_1 = 1$, $M_2 = 5$, and $N_2 = -4$.

Case 3. Setting $\Delta_1 < 0$ and $\Delta_2 < 0$, the complexiton solutions consisting of trigonometric functions of (5) can be derived as

$$\begin{aligned} u_3 = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) + \frac{F_1^2\Delta_1}{4} \\ & \times\left(\frac{M_1\sin\left(\left(\sqrt{-\Delta_1}/2\right)\xi_1\right) - N_1\cos\left(\left(\sqrt{-\Delta_1}/2\right)\xi_1\right)}{M_1\cos\left(\left(\sqrt{-\Delta_1}/2\right)\xi_1\right) + N_1\sin\left(\left(\sqrt{-\Delta_1}/2\right)\xi_1\right)}\right)^2 \\ & + \frac{F_2^2\Delta_2}{4} \\ & \times\left(\frac{M_2\sin\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right) - N_2\cos\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right)}{M_2\cos\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right) + N_2\sin\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right)}\right)^2, \end{aligned} \tag{14}$$

where F_1, F_2, M_1, M_2, N_1, and N_2 are arbitrary constants and ξ_1 and ξ_2 are determined by using (9).

The profiles of (14) are shown in Figure 3.

It is easy to see that the complexiton solutions can be rewritten at $M_2^2 > M_1^2$, $N_2^2 > N_1^2$ and $M_2 \neq 0$, $N_2 \neq 0$ as follows:

$$\begin{aligned} u_3 = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) + \frac{F_1^2\Delta_1}{4}\tan^2\left(\frac{\sqrt{-\Delta_1}}{2}\xi_1 - \xi_{10}\right) \\ & + \frac{F_2^2\Delta_2}{4}\tan^2\left(\frac{\sqrt{-\Delta_2}}{2}\xi_2 - \xi_{20}\right), \end{aligned} \tag{15}$$

where $\xi_{10} = \tan^{-1}(N_1/M_1)$ and $\xi_{20} = \tan^{-1}(N_2/M_2)$.

Case 4. Setting $\Delta_1 = 0$ and $\Delta_2 > 0$, the complexiton solutions consisting of rational functions and hyperbolic functions of (5) can be derived as

$$\begin{aligned} u_4 = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \left(\frac{F_1N_1}{M_1 + N_1\xi_1}\right)^2 - \frac{F_2^2\Delta_2}{4} \\ & \times\left(\frac{M_2\sinh\left(\left(\sqrt{\Delta_2}/2\right)\xi_2\right) + N_2\cosh\left(\left(\sqrt{\Delta_2}/2\right)\xi_2\right)}{M_2\cosh\left(\left(\sqrt{\Delta_2}/2\right)\xi_2\right) + N_2\sinh\left(\left(\sqrt{\Delta_2}/2\right)\xi_2\right)}\right)^2, \end{aligned} \tag{16}$$

where F_1, F_2, M_1, M_2, N_1, and N_2 are arbitrary constants and ξ_1 and ξ_2 are determined by using (9)

The profiles of (16) are shown in Figure 4.

It is easy to see that the complexiton solutions can be rewritten at $N_2^2 > N_1^2$ and $N_2 \neq 0$ as follows:

$$\begin{aligned} u_4 = {} & \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \left(\frac{F_1N_1}{M_1 + N_1\xi_1}\right)^2 \\ & - \frac{F_2^2\Delta_2}{4}\tanh^2\left(\frac{\sqrt{\Delta_2}}{2}\xi_2 + \xi_{20}\right), \end{aligned} \tag{17}$$

where $\xi_{20} = \tanh^{-1}(N_2/M_2)$.

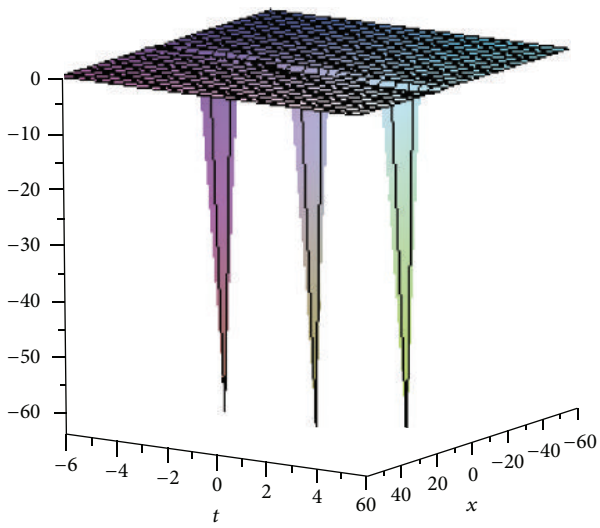

Figure 4: The 3D graphs of profiles of the exact solution (16) for fixed parameters $F_1 = 0.4$, $F_2 = 0.5$, $\lambda_1 = 6$, $\mu_1 = 9$, $\lambda_2 = 4$, $\mu_2 = 3$, $l1 = 0.2$, $l2 = -4$, $M_1 = -4$, $N_1 = 1$, $M_2 = -5$, and $N_2 = 1$.

Case 5. Setting $\Delta_1 = 0$ and $\Delta_2 < 0$, the complexiton solutions consisting of rational functions and trigonometric functions of (5) can be derived as

$$u_5 = \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \left(\frac{F_1N_1}{M_1 + N_1\xi_1}\right)^2 + \frac{F_2^2\Delta_2}{4} \times \left(\frac{M_2\sin\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right) - N_2\cos\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right)}{M_2\cos\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right) + N_2\sin\left(\left(\sqrt{-\Delta_2}/2\right)\xi_2\right)}\right)^2, \tag{18}$$

where F_1, F_2, M_1, M_2, N_1, and N_2 are arbitrary constants and ξ_1 and ξ_2 are determined by using (9)

It is easy to see that the complexiton solutions can be rewritten at $N_2^2 > N_1^2$ and $N_2 \neq 0$ as follows:

$$u_5 = \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \left(\frac{F_1N_1}{M_1 + N_1\xi_1}\right)^2 + \frac{F_2^2\Delta_2}{4}\tan^2\left(\frac{\sqrt{-\Delta_2}}{2}\xi_2 - \xi_{20}\right), \tag{19}$$

where $\xi_{20} = \tan^{-1}(N_2/M_2)$.

Case 6. Setting $\Delta_1 = 0$ and $\Delta_2 = 0$, the complexiton solutions consisting of rational functions of (5) can be derived as

$$u_6 = \frac{1}{6}\left(F_1^2\Delta_1 + F_2^2\Delta_2\right) - \left(\frac{F_1N_1}{M_1 + N_1\xi_1}\right)^2 - \left(\frac{F_2N_2}{M_2 + N_2\xi_2}\right)^2, \tag{20}$$

where F_1, F_2, M_1, M_2, N_1, and N_2 are arbitrary constants and ξ_1 and ξ_2 are determined by using (9)

Similarly, when setting ($\Delta_1 < 0$ and $\Delta_2 > 0$), ($\Delta_1 > 0$ and $\Delta_2 = 0$), and ($\Delta_1 < 0$ and $\Delta_2 = 0$) some other complexiton solutions of (5) can be obtained. We omit them here for convenience.

Remark 1. In this method, two independent variables ξ_1 and ξ_2 were introduced, and, therefore, double solitary-like wave solutions and some other complexiton solutions consisting of hyperbolic functions, trigonometric functions, rational functions, and their mixture with arbitrary parameters for the Caudrey-Dodd-Gibbon equation can be obtained.

Remark 2. Comparing our results with Naher et al.'s results in [20], it can be seen that the solutions we obtained are new and more plentiful.

Remark 3. The validity of the solutions we obtained is verified by using the mathematical software Maple 14.

Remark 4. The ansatz (6) can be expressed as another form:

$$\begin{aligned} u = {} & A_0 + A_1\left(\frac{Y_1'(\xi_1)}{Y_1(\xi_1)} - \frac{\lambda_1}{2}\right) + A_2\left(\frac{Y_1'(\xi_1)}{Y_1(\xi_1)} - \frac{\lambda_1}{2}\right)^2 \\ & + B_1\left(\frac{Y_2'(\xi_2)}{Y_2(\xi_2)} - \frac{\lambda_2}{2}\right) + B_2\left(\frac{Y_2'(\xi_2)}{Y_2(\xi_2)} - \frac{\lambda_2}{2}\right)^2 \\ & + B_0\left(\frac{Y_1'(\xi_1)}{Y_1(\xi_1)} - \frac{\lambda_1}{2}\right)\left(\frac{Y_2'(\xi_2)}{Y_2(\xi_2)} - \frac{\lambda_2}{2}\right), \end{aligned} \tag{21}$$

where $Y(\xi_i)$ $(i = 1, 2)$ satisfy the simplest equations:

$$Y''(\xi_i) + \frac{4\mu_i - \lambda_i^2}{4}Y(\xi_i) = 0, \quad (i = 1, 2). \tag{22}$$

4. Conclusions

In the present work, an extended multiple (G'/G)-expansion method was applied to the Caudrey-Dodd-Gibbon equation, and we successfully obtained some exact complexiton solutions expressed by hyperbolic functions, the trigonometric functions, the rational functions, and their mixture. To the best of our knowledge, these solutions with arbitrary parameters are new. The results of [20] have been enriched.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] M. J. Ablowitz and P. A. Clarkson, *Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform*, vol. 149, Cambridge University Press, Cambridge, UK, 1991.

[2] C. Rogers and W. F. Shadwick, *Backlund Transformations*, Academic Press, New York, NY, USA, 1982.

[3] Q.-X. Qu, B. Tian, K. Sun, and Y. Jiang, "Bäcklund transformation, Lax pair, and solutions for the Caudrey-Dodd-Gibbon equation," *Journal of Mathematical Physics*, vol. 52, no. 1, Article ID 013511, 2011.

[4] C. Gu, H. Hu, and Z. Zhou, *Darboux Transformations in Integrable Systems Theory and Their Applications to Geometry*, vol. 26 of *Mathematical Physics Studies*, Springer, Dordrecht, The Netherlands, 2005.

[5] R. Hirota, "Exact solution of the korteweg-de vries equation for multiple Collisions of solitons," *Physical Review Letters*, vol. 27, no. 18, pp. 1192–1194, 1971.

[6] S. Zhang, "Application of Exp-function method to Riccati equation and new exact solutions with three arbitrary functions of Broer-Kaup-Kupershmidt equations," *Physics Letters A*, vol. 372, no. 11, pp. 1873–1880, 2008.

[7] M. A. Abdou, "The extended tanh method and its applications for solving nonlinear physical models," *Applied Mathematics and Computation*, vol. 190, no. 1, pp. 988–996, 2007.

[8] N. A. Kudryashov, "Exact solutions of the generalized Kuramoto-Sivashinsky equation," *Physics Letters A*, vol. 147, no. 5-6, pp. 287–291, 1990.

[9] S. Liu, Z. Fu, S. Liu, and Q. Zhao, "Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations," *Physics Letters A*, vol. 289, no. 1-2, pp. 69–74, 2001.

[10] N. A. Kudryashov, "Simplest equation method to look for exact solutions of nonlinear differential equations," *Chaos, Solitons and Fractals*, vol. 24, no. 5, pp. 1217–1231, 2005.

[11] N. A. Kudryashov, "Exact solitary waves of the Fisher equation," *Physics Letters A*, vol. 342, no. 1-2, pp. 99–106, 2005.

[12] N. K. Vitanov and Z. I. Dimitrova, "Application of the method of simplest equation for obtaining exact traveling-wave solutions for two classes of model PDEs from ecology and population dynamics," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 10, pp. 2836–2845, 2010.

[13] N. K. Vitanov, "Modified method of simplest equation: powerful tool for obtaining exact and approximate traveling-wave solutions of nonlinear PDEs," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 3, pp. 1176–1185, 2011.

[14] N. K. Vitanov, Z. I. Dimitrova, and H. Kantz, "Modified method of simplest equation and its application to nonlinear PDEs," *Applied Mathematics and Computation*, vol. 216, no. 9, pp. 2587–2595, 2010.

[15] M. Wang, X. Li, and J. Zhang, "The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics," *Physics Letters A*, vol. 372, no. 4, pp. 417–423, 2008.

[16] I. Aslan, "A note on the (G'/G)-expansion method again," *Applied Mathematics and Computation*, vol. 217, no. 2, pp. 937–938, 2010.

[17] A. Malik, F. Chand, and S. C. Mishra, "Exact travelling wave solutions of some nonlinear equations by (G'/G)-expansion method," *Applied Mathematics and Computation*, vol. 216, no. 9, pp. 2596–2612, 2010.

[18] A. Bekir, "Application of the (G'/G)-expansion method for nonlinear evolution equations," *Physics Letters A*, vol. 372, no. 19, pp. 3400–3406, 2008.

[19] E. M. E. Zayed, "New traveling wave solutions for higher dimensional nonlinear evolution equations using a generalized (G'/G)-expansion method," *Journal of Applied Mathematics and Informatics*, vol. 28, no. 1-2, pp. 383–395, 2010.

[20] H. Naher, F. A. Abdullah, and M. Ali Akbar, "The (G'/G)-expansion method for abundant traveling wave solutions of Caudrey-Dodd-Gibbon equation," *Mathematical Problems in Engineering*, vol. 2011, Article ID 218216, 11 pages, 2011.

[21] E. M. E. Zayed, "New traveling wave solutions for higher dimensional nonlinear evolution equations using a generalized (G'/G)-expansion method," *Journal of Physics A: Mathematical and Theoretical*, vol. 42, no. 19, Article ID 195202, p. 13, 2009.

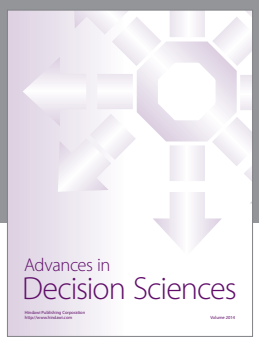
[22] Y.-B. Zhou and C. Li, "Application of modified (G'/G)-expansion method to traveling wave solutions for Whitham-Broer-Kaup-like equations," *Communications in Theoretical Physics*, vol. 51, no. 4, pp. 664–670, 2009.

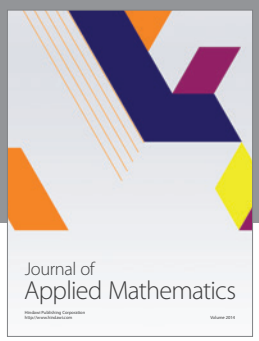
[23] S. Guo and Y. Zhou, "The extended (G'/G)-expansion method and its applications to the Whitham-Broer-Kaup-like equations and coupled Hirota-Satsuma KdV equations," *Applied Mathematics and Computation*, vol. 215, no. 9, pp. 3214–3221, 2010.

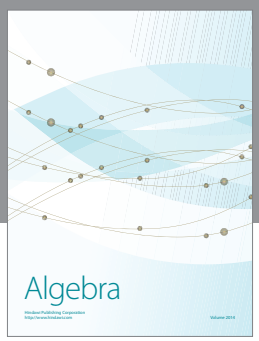
[24] S. Guo, Y. Zhou, and C. Zhao, "The improved (G'/G)-expansion method and its applications to the Broer-Kaup equations and approximate long water wave equations," *Applied Mathematics and Computation*, vol. 216, no. 7, pp. 1965–1971, 2010.

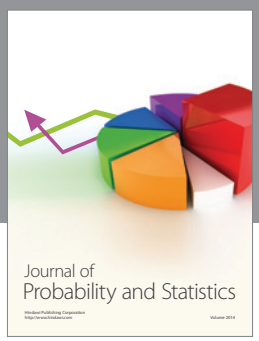
[25] L.-x. Li, E.-q. Li, and M.-l. Wang, "The (G'/G)-expansion method and its application to travelling wave solutions of the Zakharov equations," *Applied Mathematics*, vol. 25, no. 4, pp. 454–462, 2010.

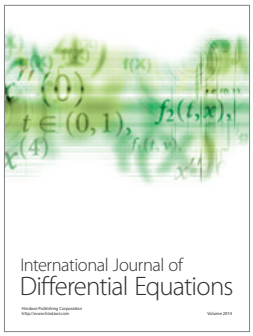
[26] J. Chen and B. Li, "Multiple (G'/G)-expansion method and its applications to nonlinear evolution equations in mathematical physics," *Pramana: Journal of Physics*, vol. 78, no. 3, pp. 375–388, 2012.











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