Research Article

Group Synchronization of Nonlinear Complex Dynamics Networks with Sampled Data

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Based on a nonlinear consensus protocol, this paper considers the group synchronization of complex dynamical networks with sampled data. Using the Lyapunov method, the group synchronization of the nonlinear complex networks is analyzed. All the nodes in each group can converge to their own synchronous state asymptotically, if the sampled period satisfies some matrix inequality conditions. Furthermore, the theoretical results are verified by some simulations.

1. Introduction

Because of the wide application of the complex dynamical networks [1–26], the synchronization problem of the complex networks has become a hot topic recently. A lot of researches concentrate on the continuous information transmission; that is, every node can receive or detect neighbor information of the network all the time [1–3, 5–7, 10]. In [1], the authors considered the group synchronization of the nonlinear complex dynamical network via pinning control. A new criterion for the cluster synchronization of coupling networks with time-varying delays was established in continuous time in [2]. Moreover, Liu et al. [3] considered the adaptive synchronization of complex dynamical networks with switching topology by local Lipschitz nonlinearity. In [5], group consensus of multiagent systems with switching topologies and communication delays was studied in the continuous-time case.

However, in many engineering practices, the information communication between nodes could be interrupted at any time due to some factors, such as the unreliability of communication channels and the limitations of the node detection ability. Hence, it is necessary to consider the information transmission under the discrete state [9, 13–16, 18–21, 23, 26].

Information data on the discrete time can also be regarded as sampled data [11, 25]. In [11], the group consensus of linear multiagent systems with sampled-data was considered. Xiao and Wang further considered a discrete-time model with time delays in [13] and analyzed a consensus problem in the existence of the time delay when agents exchanged information between each other. Gao and Wang [16] studied the continuous-time consensus of multiagent systems with sampled data by time-varying topology. In [18], consensus of multiple dynamic agents with sampled information was considered. In [20], average consensus control of networks with sampled data and measurement noises was studied on continuous time. In this paper, we consider the group synchronization of nonlinear complex dynamical network with sampled data.

The rest of this paper is organized as follows. In Section 2, we consider the group synchronization of a complex dynamical network with sampled data and give some assumptions and lemmas. In Section 3, we analyze group synchronization of the proposed network and give the synchronization condition based on linear matrix inequality (LMI). In Section 4, we give the synchronization condition for a special case. In Section 5, the simulations verify the theoretical results. Conclusion is finally summarized in Section 6.

2. Preliminaries and Problem Statement

Consider a complex dynamical network of $n + m$ nodes with sampled data as follows:

$$\dot{x}_i(t) = u_i(t), \quad (1)$$

with

$$u_i(t) = \begin{cases} f(x_i(t_k)) + c \sum_{j \in N_{1,i}} a_{ij} \Gamma(x_j(t_k) - x_i(t_k)) \\ \quad + c \sum_{j \in N_{2,i}} b_{ij} \Gamma x_j(t_k) + \mu_i(t_k) \\ \quad \forall i \in \ell_1, \quad \forall t \in [t_k, t_{k+1}); \\ f(x_i(t_k)) + c \sum_{j \in N_{2,i}} a_{ij} \Gamma(x_j(t_k) - x_i(t_k)) \\ \quad + c \sum_{j \in N_{1,i}} b_{ij} \Gamma x_j(t_k) + \mu_i(t_k) \\ \quad \forall i \in \ell_2, \quad \forall t \in [t_k, t_{k+1}), \end{cases} \quad (2)$$

where $x_i = (x_i^1(t), x_i^2(t), \dots, x_i^n(t))^T \in R^n$ represents the state vectors of the node i at time t , $f(\cdot) \in R^n$ is continuously differentiable, c is coupling strength, and $\Gamma = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_n) \in R^{n \times n}$ is an inner-coupling matrix, if the node links through its i th state with its neighbors $\gamma_i > 0$; otherwise $\gamma_i = 0$. In this protocol, let $X_1 = \{x_1, x_2, \dots, x_n\}$, and let $X_2 = \{x_{n+1}, x_{n+2}, \dots, x_{n+m}\}$, where $X = X_1 \cup X_2$. $\ell_1 = 1, 2, \dots, n$, $\ell_2 = n + 1, n + 2, \dots, n + m$, with $\ell = \ell_1 \cup \ell_2$. N_i is the neighbors of the node i , $N_i \in N_{1i} \cup N_{2i}$, where $N_{1i} = \{x_j \in X_1 : a_{ij} > 0, j \in \ell_1\}$, $N_{2i} = \{x_j \in X_2 : a_{ij} > 0, j \in \ell_2\}$.

In protocol (2), $\mu_i(t)$ is the control input:

$$\mu_i(t) = \begin{cases} -ch_i \Gamma(x_i(t_k) - \bar{x}_1(t_k)), & i \in \ell_1 \\ -ch_i \Gamma(x_i(t_k) - \bar{x}_2(t_k)), & i \in \ell_2, \end{cases} \quad (3)$$

where h_i is an on-off control; if the system is sampling the data, then $h_i = 1$; otherwise $h_i = 0$. If node i can get information from node j in the same group, then $a_{ij} > 0$; otherwise $a_{ij} = 0$; if node i can get information from node j between different groups, then $b_{ij} \neq 0$; otherwise $b_{ij} = 0$. Thus the coupling configuration matrix $A \in R^{(n+m) \times (n+m)}$ can be written as

$$A = \begin{bmatrix} cA_{11}^{n \times n} & cB_{12}^{n \times m} \\ cB_{21}^{m \times n} & cA_{22}^{m \times m} \end{bmatrix}, \quad (4)$$

where

$$\begin{aligned} ca_{ii} &= - \sum_{j=1, j \neq i}^n ca_{ij}, \quad i = 1, 2, \dots, n, \\ ca_{ii} &= - \sum_{j=n+1, j \neq i}^{n+m} ca_{ij}, \quad i = n + 1, n + 2, \dots, n + m. \end{aligned} \quad (5)$$

$cA_{11} = ca_{ij} \in R^{n \times n}$ and $cA_{22} = ca_{ij} \in R^{m \times m}$ represent the coupling configuration of the subgroups, respectively.

Network (1) is said to group synchronization if

$$\begin{aligned} \lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| &= 0, \quad \forall i, j \in \ell_1, \\ \lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| &= 0, \quad \forall i, j \in \ell_2, \end{aligned} \quad (6)$$

where the nonlinear function satisfies

$$\begin{aligned} f(\bar{x}_1(t), t) &= \dot{\bar{x}}_1(t), \\ f(\bar{x}_2(t), t) &= \dot{\bar{x}}_2(t), \end{aligned} \quad (7)$$

and $\bar{x}_1(t), \bar{x}_2(t) \in R^n$ are the synchronous states.

Given a positive real number α and a sampled period T , we suppose that

$$t_{i+1} - t_i = \alpha T_i, \quad \forall i = 0, 1, 2, \dots, \quad (8)$$

where $t_0 < t_1 < \dots$ shows the discrete time that the node j can obtain information from its neighbors and positive integer T_i is a sampled time about the i th time ($\forall i = 0, 1, 2, \dots$), and it satisfies $T_i \leq T$. Under this condition, a linear consensus protocol based on a linear estimation-based sampling period is designed as follows [11]:

$$u_i(t_k + \alpha) = u_i(t_k) - \frac{1}{T} u_i(t_k) = \left(1 - \frac{1}{T}\right) u_i(t_k),$$

$$u_i(t_k + 2\alpha) = u_i(t_k + \alpha) + (u_i(t_k + \alpha) - u_i(t_k))$$

$$= \left(1 - \frac{2}{T}\right) u_i(t_k),$$

$\vdots$

$$u_i(t_{k+1} - \alpha) = u_i(t_{k+1} - 2\alpha) + (u_i(t_k - 2\alpha) - u_i(t_{k+1} - 3\alpha))$$

$$= \left(1 - \frac{t_{k+1} - t_k - 1}{T}\right) u_i(t_k)$$

$$= \left(1 - \frac{T_k - 1}{T}\right) u_i(t_k).$$

(9)

Thus, we have

$$u_i(t) = \begin{cases} \left(1 - \frac{T_k - 1}{T}\right) \\ \times \left[f(x_i(t_k)) + c \sum_{j \in N_{1,i}} a_{ij} \Gamma(x_j(t_k) - x_i(t_k)) \right. \\ \left. + c \sum_{j \in N_{2,i}} b_{ij} \Gamma x_j(t_k) + \mu_i(t_k) \right] \\ \forall i \in \ell_1, \quad \forall t \in [t_k, t_{k+1}); \\ \left(1 - \frac{T_k - 1}{T}\right) \\ \times \left[f(x_i(t_k)) + c \sum_{j \in N_{2,i}} a_{ij} \Gamma(x_j(t_k) - x_i(t_k)) \right. \\ \left. + c \sum_{j \in N_{1,i}} b_{ij} \Gamma x_j(t_k) + \mu_i(t_k) \right] \\ \forall i \in \ell_2, \quad \forall t \in [t_k, t_{k+1}]. \end{cases} \quad (10)$$

Then the complex dynamical network (1) in every sampled period becomes

$$\dot{x}_i(t) = u_i(t), \quad (11)$$

with

$$u_i(t) = \begin{cases} \left(1 - \frac{h}{T}\right) \\ \times \left[f(x_i(t_k)) + c \sum_{j \in N_{1,i}} a_{ij} \Gamma(x_j(t_k) - x_i(t_k)) \right. \\ \left. + c \sum_{j \in N_{2,i}} b_{ij} \Gamma x_j(t_k) + \mu_i(t_k) \right] \\ \forall i \in \ell_1, \quad \forall t \in [t_k + h\alpha, t_k + (h+1)\alpha], \\ h = 0, 1, \dots, T_k - 1; \\ \left(1 - \frac{h}{T}\right) \\ \times \left[f(x_i(t_k)) + c \sum_{j \in N_{2,i}} a_{ij} \Gamma(x_j(t_k) - x_i(t_k)) \right. \\ \left. + c \sum_{j \in N_{1,i}} b_{ij} \Gamma x_j(t_k) + \mu_i(t_k) \right] \\ \forall i \in \ell_2, \quad \forall t \in [t_k + h\alpha, t_k + (h+1)\alpha], \\ h = 0, 1, \dots, T_k - 1. \end{cases} \quad (12)$$

Some assumptions and lemmas are needed.

Assumption 1 (see [3]). If each f_i of the nonlinear function $f(x_i(t)) = (f_1(x_i(t)), f_2(x_i(t)), \dots, f_n(x_i(t)))^T$ in network (1)

satisfies the local Lipschitz condition, for any compact set $S \in \mathbb{R}^m$, there exists a positive constant matrix $\eta(S)$, such that

$$(x - y)^T [f(x) - f(y)] \leq (x - y)^T K \Gamma (x - y) \leq \eta(s) (x - y)^T \Gamma (x - y), \quad (13) \\ \forall x, y \in S, \quad \eta(s) = \|K\|_1.$$

Assumption 2 (see [1]). Assume that protocol (1) satisfies the balance of effectiveness between the subgroups

$$\sum_{j=n+1}^{n+m} b_{ij} = 0, \quad i \in \ell_1, \\ \sum_{j=1}^n b_{ij} = 0, \quad i \in \ell_2. \quad (14)$$

Lemma 3 (see [1]). Define $d_0 = (1/2) \sum_{i=1}^N (x_i(0) - x_0)^2$, and construct a closed space

$$B(\sigma d_0, x_0) = \left\{ x \in \mathbb{R}^{Nn} \mid \frac{1}{2} \sum_{i=1}^N \|x_i(0) - x_0\|^2 \leq \sigma d_0 \right\}, \quad (15)$$

where $x = (x_1^T, x_2^T, \dots, x_N^T)^T$ and $\sigma > 1$ is a constant. For $\forall x \in B(\sigma d_0, x_0)$, there exists a constant $\eta(\sigma, x_0)$ such that

$$\sum_{i=1}^N (x_i - x_0) (f(x_i) - f(x_0)) \leq \eta(\sigma, x_0) \|x_i - x_0\|^2. \quad (16)$$

Lemma 4 (see [1]). Suppose that $a \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$ are vectors, and in matrix M , the following inequality holds:

$$2x^T y \leq x^T M x + y^T M^{-1} y. \quad (17)$$

In this paper, $A > 0$ ($\geq, <, \leq$) means that A is a positive (or semipositive, negative, or seminegative) definite matrix; $\|A\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}|$, $\|A\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^n |a_{ij}|$.

3. Group Synchronization Analysis in Complex Network with Sampled Data

In this section, we consider group synchronization problem of the complex networks with sampled data. We have the following theorem.

Theorem 5. For network (1) with protocol (12) of $n + m$ nodes, under Assumptions 1-2 and Lemmas 3-4, if h, T satisfy

$$\frac{h}{1-T} \left\{ [\eta(\sigma, \bar{x}_1) I_n + c(A_{11} - H_1)] \otimes \Gamma \right. \\ \left. + \frac{c}{2} (\|B_{12}\|_1 I_n + \|B_{21}\|_\infty I_n) \otimes \Gamma \right\} < 0, \\ h = 0, 1, \dots, T_k - 1;$$

$$\begin{aligned} & \frac{h}{1-T} \left\{ [\eta(\sigma, \bar{x}_2) I_m + c(A_{22} - H_2)] \otimes \Gamma \right. \\ & \quad \left. + \frac{c}{2} (\|B_{21}\|_1 I_m + \|B_{12}\|_\infty I_m) \otimes \Gamma \right\} < 0, \\ & h = 0, 1, \dots, T_k - 1, \end{aligned} \quad (18)$$

then all the nodes in each group can converge to their own synchronous state asymptotically, where

$$\begin{aligned} H_1 &= \text{diag}\{h_1, h_2, \dots, h_l, 0, \dots, 0\}, \\ H_2 &= \text{diag}\{h_{n+1}, h_{n+2}, \dots, h_{n+l}, 0, \dots, 0\}. \end{aligned} \quad (19)$$

Proof. Construct a Lyapunov function as follows:

$$V(t_k) = V_1(t_k) + V_2(t_k), \quad (20)$$

where

$$V_1(t_k) = \frac{1}{2} \sum_{i=1}^n (x_i(t_k) - \bar{x}_1(t_k))^T (x_i(t_k) - \bar{x}_1(t_k)), \quad (21)$$

$$V_2(t_k) = \frac{1}{2} \sum_{i=n+1}^{n+m} (x_i(t_k) - \bar{x}_2(t_k))^T (x_i(t_k) - \bar{x}_2(t_k)).$$

Define

$$\begin{aligned} e_{i,1}(t_k) &= x_i(t_k) - \bar{x}_1(t_k), \quad i = 1, 2, \dots, n, \\ e_{i,2}(t_k) &= x_i(t_k) - \bar{x}_2(t_k), \quad i = n+1, n+2, \dots, n+m, \\ e_1(t_k) &= (e_{1,1}(t_k), e_{2,1}(t_k), \dots, e_{n,1}(t_k))^T, \\ e_2(t_k) &= (e_{n+1,2}(t_k), e_{n+2,2}(t_k), \dots, e_{n+m,2}(t_k))^T. \end{aligned} \quad (22)$$

Then,

$$\begin{aligned} V_1(t_k) &= \frac{1}{2} \sum_{i=1}^n e_{i,1}(t_k)^T e_{i,1}(t_k), \\ V_2(t_k) &= \frac{1}{2} \sum_{i=n+1}^{n+m} e_{i,2}(t_k)^T e_{i,2}(t_k). \end{aligned} \quad (23)$$

Under Assumption 2, we can know

$$\begin{aligned} \dot{V}_1(t_k) &= \frac{h}{1-T} \\ & \times \sum_{i=1}^n (x_i(t_k) - \bar{x}_1(t_k))^T \\ & \times \left[f(x_i(t_k)) - f(\bar{x}_1(t_k)) \right. \end{aligned}$$

$$\begin{aligned} & + c \sum_{j \in N_{1,i}} a_{ij} \Gamma \left((x_j(t_k) - \bar{x}_1(t_k)) \right. \\ & \quad \left. - (x_i(t_k) - \bar{x}_1(t_k)) \right) \\ & + c \sum_{j \in N_{2,i}} b_{ij} \Gamma (x_j(t_k) - \bar{x}_2(t_k)) \\ & \left. + c \sum_{j \in N_{2,i}} b_{ij} \bar{x}_2(t_k) - c h_i \Gamma (x_i(t_k) - \bar{x}_1(t_k)) \right] \\ & = \frac{h}{1-T} \sum_{i=1}^n e_{i,1}^T(t_k) \\ & \times \left[f(x_i(t_k)) - f(\bar{x}_1(t_k)) \right. \\ & \quad + c \sum_{j \in N_{1,i}} a_{ij} \Gamma (e_{j,1}(t_k) - e_{i,1}(t_k)) \\ & \quad \left. + c \sum_{j \in N_{2,i}} b_{ij} \Gamma e_{j,2}(t_k) - c h_i \Gamma e_{i,1}(t_k) \right]. \end{aligned} \quad (24)$$

Under Assumption 1 and Lemma 3, we can have

$$\begin{aligned} \dot{V}_1(t_k) &\leq \frac{h}{1-T} \\ & \times \sum_{i=1}^n e_{i,1}^T(t_k) \left[\eta(\sigma, \bar{x}_1) \Gamma e_{i,1}(t_k) \right. \\ & \quad + c \sum_{j \in N_{1,i}} a_{ij} \Gamma (e_{j,1}(t_k) - e_{i,1}(t_k)) \\ & \quad \left. + c \sum_{j \in N_{2,i}} b_{ij} \Gamma e_{j,2}(t_k) - c h_i \Gamma e_{i,1}(t_k) \right] \\ & = \frac{h}{1-T} \left[e_1^T(t_k) (\eta(\sigma, \bar{x}_1) I_n \otimes \Gamma) e_1(t_k) \right. \\ & \quad + e_1^T(t_k) (c(A_{11} - H_1) \otimes \Gamma) e_1(t_k) \\ & \quad \left. + c \sum_{i=1}^n e_{i,1}^T(t_k) \sum_{j \in N_{2,i}} b_{ij} \Gamma e_{j,2}(t_k) \right]. \end{aligned} \quad (25)$$

Then, we can get

$$\begin{aligned} \dot{V}_2(t_k) &\leq \frac{h}{1-T} \\ & \times \sum_{i=n+1}^{n+m} e_{i,2}^T(t_k) \left[\eta(\sigma, \bar{x}_2) \Gamma e_{i,2}(t_k) \right. \end{aligned}$$

$$\begin{aligned}
& + c \sum_{j \in N_{2,i}} a_{ij} \Gamma (e_{j,2}(t_k) - e_{i,2}(t_k)) \\
& + c \sum_{j \in N_{1,i}} b_{ij} \Gamma e_{j,1}(t_k) - c h_i \Gamma e_{i,2}(t_k) \Bigg] \\
& = \frac{h}{1-T} \left[e_2^T(t_k) (\eta(\sigma, \bar{x}_2) I_m \otimes \Gamma) e_2(t_k) \right. \\
& \quad + e_2^T(t_k) (c(A_{22} - H_2) \otimes \Gamma) e_2(t_k) \\
& \quad \left. + c \sum_{i=n+1}^{n+m} e_{i,2}^T(t_k) \sum_{j \in N_{1,i}} b_{ij} \Gamma e_{j,1}(t_k) \right]. \quad (26)
\end{aligned}$$

Using Lemma 4, we can have

$$\begin{aligned}
& c \sum_{i=1}^n e_{i,1}^T(t_k) \sum_{j \in N_{2,i}} b_{ij} \Gamma e_{j,2}(t_k) \\
& \leq \frac{c}{2} \sum_{i=1}^n \sum_{j \in N_{2,i}} b_{ij} (e_{i,1}^T(t_k) \Gamma e_{i,1}(t_k) + e_{j,2}^T(t_k) \Gamma e_{j,2}(t_k)) \\
& \leq \frac{c}{2} \|B_{12}\|_1 \sum_{i=1}^n e_{i,1}^T(t_k) \Gamma e_{i,1}(t_k) \\
& \quad + \frac{c}{2} \|B_{12}\|_\infty \sum_{j \in N_{2,i}} e_{j,2}^T(t_k) \Gamma e_{j,2}(t_k) \\
& = e_{i,1}^T(t_k) \left[\frac{c}{2} \|B_{12}\|_1 I_n \otimes \Gamma \right] e_{i,1}(t_k) \\
& \quad + e_{i,2}^T(t_k) \left[\frac{c}{2} \|B_{12}\|_\infty I_m \otimes \Gamma \right] e_{i,2}(t_k), \\
& c \sum_{i=n+1}^{n+m} e_{i,2}^T(t_k) \sum_{j \in N_{1,i}} b_{ij} \Gamma e_{j,1}(t_k) \\
& \leq \frac{c}{2} \sum_{i=n+1}^{n+m} \sum_{j \in N_{1,i}} b_{ij} (e_{i,2}^T(t_k) \Gamma e_{i,2}(t_k) + e_{j,1}^T(t_k) \Gamma e_{j,1}(t_k)) \\
& \leq \frac{c}{2} \|B_{21}\|_1 \sum_{i=n+1}^{n+m} e_{i,2}^T(t_k) \Gamma e_{i,2}(t_k) \\
& \quad + \frac{c}{2} \|B_{21}\|_\infty \sum_{j \in N_{1,i}} e_{j,1}^T(t_k) \Gamma e_{j,1}(t_k) \\
& = e_{i,2}^T(t_k) \left[\frac{c}{2} \|B_{21}\|_1 I_m \otimes \Gamma \right] e_{i,2}(t_k) \\
& \quad + e_{i,1}^T(t_k) \left[\frac{c}{2} \|B_{21}\|_\infty I_n \otimes \Gamma \right] e_{i,1}(t_k). \quad (27)
\end{aligned}$$

Thus, we can obtain

$$\begin{aligned}
\dot{V}(t_k) & = \dot{V}_1(t_k) + \dot{V}_2(t_k) \\
& \leq \frac{h}{1-T} \left\{ e_1^T(t_k) (\eta(\sigma, \bar{x}_1) I_n \otimes \Gamma) e_1(t_k) \right. \\
& \quad + e_1^T(t_k) (c(A_{11} - H_1) \otimes \Gamma) e_1(t_k) \\
& \quad + e_2^T(t_k) (\eta(\sigma, \bar{x}_2) I_m \otimes \Gamma) e_2(t_k) \\
& \quad + e_2^T(t_k) (c(A_{22} - H_2) \otimes \Gamma) e_2(t_k) \\
& \quad + e_{i,1}^T(t_k) \left[\frac{c}{2} \|B_{12}\|_1 I_n \otimes \Gamma \right] e_{i,1}(t_k) \\
& \quad + e_{i,2}^T(t_k) \left[\frac{c}{2} \|B_{12}\|_\infty I_m \otimes \Gamma \right] e_{i,2}(t_k) \\
& \quad + e_{i,2}^T(t_k) \left[\frac{c}{2} \|B_{21}\|_1 I_m \otimes \Gamma \right] e_{i,2}(t_k) \\
& \quad \left. + e_{i,1}^T(t_k) \left[\frac{c}{2} \|B_{21}\|_\infty I_n \otimes \Gamma \right] e_{i,1}(t_k) \right\} \\
& = e_1^T(t_k) \left\{ \frac{h}{1-T} [\eta(\sigma, \bar{x}_1) I_n + c(A_{11} - H_1)] \otimes \Gamma \right. \\
& \quad \left. + \frac{c}{2} (\|B_{12}\|_1 I_n + \|B_{21}\|_\infty I_n) \otimes \Gamma \right\} e_1(t_k) \\
& \quad + e_2^T(t_k) \left\{ \frac{h}{1-T} [\eta(\sigma, \bar{x}_2) I_m + c(A_{22} - H_2)] \otimes \Gamma \right. \\
& \quad \left. + \frac{c}{2} (\|B_{21}\|_1 I_m + \|B_{12}\|_\infty I_m) \otimes \Gamma \right\} e_2(t_k). \quad (28)
\end{aligned}$$

If

$$\begin{aligned}
& \frac{h}{1-T} \left\{ [\eta(\sigma, \bar{x}_1) I_n + c(A_{11} - H_1)] \otimes \Gamma \right. \\
& \quad \left. + \frac{c}{2} (\|B_{12}\|_1 I_n + \|B_{21}\|_\infty I_n) \otimes \Gamma \right\} < 0, \\
& \frac{h}{1-T} \left\{ [\eta(\sigma, \bar{x}_2) I_m + c(A_{22} - H_2)] \otimes \Gamma \right. \\
& \quad \left. + \frac{c}{2} (\|B_{21}\|_1 I_m + \|B_{12}\|_\infty I_m) \otimes \Gamma \right\} < 0, \quad (29)
\end{aligned}$$

then

$$\dot{V}(t_k) < 0. \quad (30)$$

Therefore, $V(t_k)$ decreases on an interval $\forall t \in [t_k + h\alpha, t_k + (h+1)\alpha]$, $h = 0, 1, \dots, T_k - 1$, and $V(t_k) \leq V(t_0)$. From the above discussion, we have

$$\begin{aligned}
\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| & = 0, \quad \forall i, j \in \ell_1, \\
\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| & = 0, \quad \forall i, j \in \ell_2. \quad (31)
\end{aligned}$$

Then, network (1) with sampled data is group-synchronized under protocol (12). $\square$

4. A Special Case

In this section, we consider a special case about network (1). For convenience, we let $\bar{A}_{11} = A_{11} - H_1$ and $\bar{A}_{22} = A_{22} - H_2$.

Lemma 6 (see [1]). If $A = (a_{ij}) \in \mathbb{R}^{N \times N}$ is a symmetric irreducible matrix with $a_{ii} = -\sum_{j=1, j \neq i}^N a_{ij}$ and $a_{ij} = a_{ji} \geq 0 (i \neq j)$, then for any matrix $E = \text{diag}(e, 0, \dots, 0)$ with $e \geq 0$ all eigenvalues of the matrix $(A - E)$ are negative.

In networks, when the information transmission is the same between the nodes i and j , the coupling configuration matrix A is symmetric. From Lemma 6, when A is symmetric, $\bar{A}_{11}$ and $\bar{A}_{22}$ are negative. Hence, all of their eigenvalues are strictly negative; one denotes them as

$$\begin{aligned} \lambda_n(\bar{A}_{11}) &\leq \dots \leq \lambda_2(\bar{A}_{11}) \leq \lambda_1(\bar{A}_{11}) < 0, \\ \lambda_m(\bar{A}_{22}) &\leq \dots \leq \lambda_2(\bar{A}_{22}) \leq \lambda_1(\bar{A}_{22}) < 0. \end{aligned} \quad (32)$$

Then, using Lemma 6, we can obtain the following result when the coupling configuration matrix is symmetric.

Theorem 7. For network (1) with protocol (12) of $n + m$ nodes, when the coupling configuration matrix is symmetric, under Assumptions 1-2 and Lemmas 3-6, if h, T satisfy

$$\begin{aligned} &\frac{h}{1-T} \left\{ \left[\eta(\sigma, \bar{x}_1) I_n + c \lambda_1(\bar{A}_{11}) \right] \otimes \Gamma \right. \\ &\quad \left. + \frac{c}{2} (\|B_{12}\|_1 I_n + \|B_{21}\|_\infty I_n) \otimes \Gamma \right\} < 0, \\ &h = 0, 1, \dots, T_k - 1; \\ &\frac{h}{1-T} \left\{ \left[\eta(\sigma, \bar{x}_2) I_m + c \lambda_1(\bar{A}_{22}) \right] \otimes \Gamma \right. \\ &\quad \left. + \frac{c}{2} (\|B_{21}\|_1 I_m + \|B_{12}\|_\infty I_m) \otimes \Gamma \right\} < 0, \\ &h = 0, 1, \dots, T_k - 1, \end{aligned} \quad (33)$$

then all the nodes in each group can converge to their own synchronous state asymptotically, where $\bar{A}_{11} = A_{11} - H_1$ and $\bar{A}_{22} = A_{22} - H_2$.

The proof of Theorem 7 is similar to that of Theorem 5 and here is omitted.

5. Simulations

In this section, we give some simulation results of the above discussions. For convenience, let $n = 5, m = 4, c = 12, a_{ij} \neq a_{ji}$ for all i, j , and $H_1 = \text{diag}\{0, 1, 1, 1, 0\}$, $H_2 = \text{diag}\{0, 0, 0, 1\}$. We consider group synchronization of complex dynamical network with sampled data in the time interval $[0, 6]$. The curves in the graphs are the locations of $5 + 4$ nodes and the synchronous targets in the network.

From Figures 1, 2, and 3, the synchronous states are $\bar{x}_1(0) = [0, -3.4 + 80 \cos x]$, $\bar{x}_2(0) = [2, 30 + 20 \cos(20x)]$. From these figures, we can see that all the nodes in each subgroup can converge to their own synchronous targets. But,

FIGURE 1: The positions of the nodes of network (1) on interval $t \in [0, 6]$.

FIGURE 2: The positions of the nodes of network (1) on interval $t \in [0, 6]$, sampled period $T = 1$.

in Figures 4, 5, and 6, all of the nodes can asymptotically converge to a synchronous state, when the synchronous state is the only one $\bar{x}_1(0) = \bar{x}_2(0) = [0, 30 + 20 \cos(20x)]$.

In Figures 1 and 4, we simulate synchronization of the network including two subgroups without sampled data. In Figures 2 and 5, we chose $T = 1$ as the sampled period. That is, the complex network is sampling at the moment $t = k$, for $k = 0, 1, \dots$. In Figures 3 and 6, we chose $T = 2.5$ as the sampled period; that is, the complex network is sampling at the moment $t = 2.5k$, for $k = 0, 1, \dots$.

In these simulation results, we can find that the nodes of the system keep their state value in the first nT time until before the moment $(n + 1)T$, keeping their state value in $(n + 1)T$ time until the moment $(n + 2)T$ and so on. By comparing, when choosing the sampled data on every period, the rate of convergence of the nodes in the complex dynamics network is slow. Moreover, the bigger the sampled period is,

FIGURE 3: The positions of the nodes of network (1) on interval $t \in [0, 6]$, sampled period $T = 2.5$.

FIGURE 5: The nodes of network (1) on interval $t \in [0, 6]$ converge to the same target, sampled period $T = 1$.

FIGURE 4: The nodes of network (1) on interval $t \in [0, 6]$ converge to the same target.

FIGURE 6: The nodes of network (1) on interval $t \in [0, 6]$ converge to the same target, sampled period $T = 2.5$.

the slower the rate of convergence of the nodes in the complex dynamics network is.

6. Conclusion

In this paper, we have investigated the group synchronization problem of a complex dynamical network with sampled data. We prove that the nodes of the network arrive at synchronization in two subgroups if the sampling period satisfies the condition based on the linear matrix inequality (LMI). In addition, we have given some simulation results about the proposed complex network.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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