Research Article

Sensitivity Analysis of Deviation Source for Fast Assembly Precision Optimization

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Received 26 December 2013; Accepted 28 February 2014; Published 17 April 2014

Academic Editor: Manyu Xiao

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Assembly precision optimization of complex product has a huge benefit in improving the quality of our products. Due to the impact of a variety of deviation source coupling phenomena, the goal of assembly precision optimization is difficult to be confirmed accurately. In order to achieve optimization of assembly precision accurately and rapidly, sensitivity analysis of deviation source is proposed. First, deviation source sensitivity is defined as the ratio of assembly dimension variation and deviation source dimension variation. Second, according to assembly constraint relations, assembly sequences and locating, deviation transmission paths are established by locating the joints between the adjacent parts, and establishing each part's datum reference frame. Third, assembly multidimensional vector loops are created using deviation transmission paths, and the corresponding scalar equations of each dimension are established. Then, assembly deviation source sensitivity is calculated by using a first-order Taylor expansion and matrix transformation method. Finally, taking assembly precision optimization of wing flap rocker as an example, the effectiveness and efficiency of the deviation source sensitivity analysis method are verified.

1. Introduction

In the aerospace industry, products are more and more complex, and products' precision requirements are also increasing. Precision performance of complex products is mainly guaranteed by the assembly process. The final assembly precision is affected by multiple assembly deviation sources. The degree of influence is called sensitivity. And the different sensitivities lead to the result that the difficulty of precision optimization [1] is different. Therefore, assembly precision can be optimized accurately and rapidly by analyzing each deviation source's sensitivity and reducing tolerance of the deviation source which has large sensitivity.

A typical assembly precision model requires input data including component geometry and tolerance specifications and assembly information (such as assembly sequences, locating, and clamping) to produce the desired dimensional output. Many commercial software packages exist for this purpose, such as vis VSA [2] and CE/TOL6 σ [3], using Monte Carlo simulations. These tools are built on commonly accepted GD&T standards [4] and adopt more recent

research results, as reported in [5–7]. The basic assumption is that the product is comprised of rigid bodies.

Tolerance design in computer aided process planning needs to obtain an appropriate set of manufacturing tolerances for the various manufacturing operations involved, considering process capability of the machines and manufacturing allowance for each operation in succession. Aiming at drawbacks of tolerance design [8–11], a few authors proposed a series of optimization methods [12–15]. Chen and Chung [16] introduced a model to determine the inspection precision and the optimal number of repeated measurements in order to maximize the net expected profit per item. Kannan and Jayabalan [17] used a Genetic Algorithm to solve the problem by generating six partitions for each component (subassembly). One kind of method used small displacements torsor (SDT) to model the process planning [18].

Assembly precision optimization [19–22] is an important means to ensure product quality. Assembly deviation source sensitivity analysis [23–26] provides critical information for assembly precision optimization. ANSELMETTI [27]

proposed the concept of deviation sources. Deviation transmission mechanism was analyzed, but the effect of deviation source sensitivity to cumulative deviation was not considered in the multidimensional environment. Mansuy et al. [28] introduced segmented geometric elements along the three-dimensional vector direction and then calculated the sensitivity and predicted the precision. Laperrière and ElMaraghy [29] proposed a method of tolerance analysis by describing the tolerance vector with six scalar equations in three-dimensional space. But it did not consider the role of assembly datum in the process of deviation transmission.

Because the physical world is predominantly nonlinear, Monte Carlo simulation is naturally the most accurate and sometimes the only method for tolerance analysis for a generic assembly product. However, computational inefficiency from Monte Carlo simulations hinders many advanced features such as assembly precision and sequence optimization, which all require a significant number of analysis iterations.

For most engineering applications, including the wing assembly, the nonlinear kinematic relations can be approximated by linear models through a first-order Taylor series expansion. Therefore, developing an efficient linear deviation source sensitivity analysis method is both imperative and feasible. In multidimensional space, for a comprehensive sensitivity analysis of mutual coupled deviation source, an assembly deviation source sensitivity calculation method is proposed based on multidimensional vector loops. Combining the characteristic of complex product assembly, multidimensional vector loops are built. Based on the loops, the sensitivity of each deviation source is calculated by reduced-order operation.

This paper is organized as follows. Section 2 gives the principle of assembly precision optimization. Section 3 proposes a method of deviation source sensitivity analysis. Section 4 presents a computational example to demonstrate the method. Section 5 comes up with a summary for this paper.

2. Principle of Assembly Precision Optimization

Tolerance analysis methods have the worst case and RSS method, shown as follows [21]:

$$t_{wc} = \sum_{i=1}^n |a_i t_i|, \quad (1)$$

$$t_{rss} = \sqrt{\sum_{i=1}^n (a_i t_i)^2}. \quad (2)$$

Assembly precision optimization is a process of solving the minimum value of assembly deviation value. Another way to determine value of sensitivity, a_i , is developed for fast assembly precision optimization:

$$\begin{aligned} \min t_{wc} &= \min \sum_{i=1}^n |a_i t_i|, \\ \min t_{rss} &= \sqrt{\min \sum_{i=1}^n (a_i t_i)^2}. \end{aligned} \quad (3)$$

According to (3), a_i determines the influence of t_i to t_{wc} and t_{rss} . In process of assembly precision optimization, by determining the a_i , the deviation source, which needs to be optimized, can be quickly determined; then $\min t_{wc}$ and $\min t_{rss}$ can be obtained. So the most important thing of assembly precision optimization is calculating each of the deviation source sensitivities.

In one-dimensional space, a_i is defined as ± 1 (increases ring $a_i = +1$, decreases ring $a_i = -1$). In multidimensional space, because of the uncertainty of deviation vector direction, a_i can be dramatically magnified. So every deviation source sensitivity a_i should be calculated. Determining which a_i is bigger, the corresponding deviation source is the one which needs to be reduced. By decreasing value of deviation source, assembly precision optimization is realized effectively.

The result of assembly deviation accumulation expresses the variable between actual assembly dimension and design dimension. The sensitivities are solved by taking partial derivatives with respect to each variable.

Assume that $\{y_1, y_2, \dots, y_n\}$ are the corresponding part's dimensions of deviation sources, and a function $\Theta(y_1, y_2, \dots, y_n)$ expresses value of assembly dimension. The sensitivity of Θ with respect to y_i is

$$a_i = \left. \frac{\partial \Theta}{\partial y_i} \right|_{\text{Nominal Values}}. \quad (4)$$

The partial derivative at the nominal values of each variable is evaluated. And the nominal value for each variable is the center of a tolerance range, or the value of the dimension when the tolerances are equal bilaterally.

3. Deviation Source Sensitivity Analysis Method

In the process of complicated product assembly, a variety of joint types are required to describe the mating parts' contact points. Different joint types form different kinds of deviation source couplings. And different couplings lead to different results of deviation transmission. Now deviation transmission path, vector loop equations, and sensitivity calculation method are discussed, respectively.

3.1. Deviation Transmission Path Based on Datum Reference Frames. Assembly parts are located through the locating datum in the process of product assembly. In the process of forming deviation transmission path, part's locating datum is called datum reference frame (DRF). DRF is mainly used for locating every needed feature of a part. Datum path (DP) is a path from the joint to the DRF which is connected with nominal dimension vectors. A deviation transmission path of two constrained parts is along a DP and through a joint.

It must obey certain modeling rules as it passes through a part. It must

- (1) enter a part through a joint;
- (2) follow the DP to the DRF in the part;
- (3) follow a second DP leading to another joint;
- (4) exit to the next adjacent part from the joint in the assembly.

The deviation transmission path is illustrated in Figure 1. J ($Part1$, $Part2$) is the joint of $Part1$ and $Part2$. There are four DPs in $Part1$ and $Part2$ (DP_{11} , DP_{12} , DP_{21} , and DP_{22}). They are created by dimension vectors (a , b , c , d , e , f , g , and h). All the dimension vectors are important for sensitivity analysis.

3.2. Dimension Vector Loops and Vector Equations. According to product's assembly sequence and locating, a dimension vector loop is formed by connecting all the deviation transmission paths. Dimension vector loop is divided into closed loop and opened loop. Closed loop describes relations between nominal dimensions of parts and assembly dimensions. Opened loop describes the influence of parts' nominal dimensions to the key characteristics in the assembly. Modeling rules for dimension vector loops include the following.

- (1) Loops must pass through every part and every joint in the assembly.
- (2) A single vector loop passes through the same joint no more than once, but it may start and end in the same part.
- (3) If a vector loop includes the exact same dimension twice, in opposite directions, the dimension must be omitted.
- (4) For an assembly, the number of closed loop L can be expressed as

$$L = J - P + 1, \quad (5)$$

where J is the number of joints and P is the number of parts.

Assuming that an assembly contains part dimensions $\{x_1, x_2, \dots, x_n\}$ and assembly dimensions $\{U_1, U_2, \dots, U_m\}$, multidimensional vector closed loops can be expressed as

$$h_D = D(x_1, x_2, \dots, x_n, U_1, U_2, \dots, U_m) = 0, \quad (6)$$

where D is the vector direction of dimensions; for example, in three-dimensional space, $D = (u, v, w, \alpha, \beta, \gamma)$, and, in two-dimensional space, $D = (x, y, \theta)$. The parameters (u, v, w, x , and y) are location parameters of a vector. The parameters (α, β, γ , and θ) are direction parameters of a vector.

Multidimensional vector opened loops can be expressed as

$$\text{Gap} = \Gamma(x_1, x_2, \dots, x_n, U_1, U_2, \dots, U_m), \quad (7)$$

where Γ is the vector direction along the opened loop distance.

By (4), the explicit expression of x and U should be calculated. But (6) and (7) are nonlinear and implicit. They contain products and trigonometric functions of the variables. So certain mathematical methods are needed for sensitivity analysis.

3.3. Order Reduction of Vector Loop Equations. The assembly deviation describes a small variable of dimension. For explicit expression of x and U , first-order Taylor's series expansion [30] of (6) and (7) is used. The following equations show the explicit expression for closed loop and opened loop:

$$\begin{aligned} \delta h_D &= \frac{\partial h_D}{\partial x_1} \delta x_1 + \dots + \frac{\partial h_D}{\partial x_n} \delta x_n \\ &+ \frac{\partial h_D}{\partial U_1} \delta U_1 + \dots + \frac{\partial h_D}{\partial U_n} \delta U_n = 0, \end{aligned} \quad (8)$$

$$\begin{aligned} \delta \text{Gap} &= \frac{\partial \text{Gap}}{\partial x_1} \delta x_1 + \dots + \frac{\partial \text{Gap}}{\partial x_n} \delta x_n \\ &+ \frac{\partial \text{Gap}}{\partial U_1} \delta U_1 + \dots + \frac{\partial \text{Gap}}{\partial U_n} \delta U_n. \end{aligned} \quad (9)$$

Equations (8) and (9) may be written in matrix form and solved for the deviation source sensitivities by matrix algebra. Equation (8) can be expressed in matrix form as follows:

$$[A] [\delta x] + [B] [\delta U] = 0. \quad (10)$$

The multidimensional vector opened loop scalar equation (9) can be expressed in matrix form as follows:

$$[\delta \text{Gap}] = [C] [\delta x] + [E] [\delta U], \quad (11)$$

where $[A]$ is the partial derivative matrix of a closed loop scalar equation to the part dimension variables; $[B]$ is the partial derivative matrix of a closed loop scalar equation to the assembly dimension variables; $[\delta x]$ is the vector of small variations in the part dimensions; $[\delta U]$ is the vector of small variations in the assembly dimensions; $[C]$ is the partial derivative matrix of an opened loop scalar equation to the part dimension variables; $[E]$ is the partial derivative matrix of an opened loop scalar equation to the assembly dimension variables; and $[\delta \text{Gap}]$ is the vector of variations in the assembly key characteristics.

By matrix transformation, (10) and (11) are expressed in the following form:

$$\begin{aligned} [\delta U] &= -[B^{-1}A] [\delta x], \\ [\delta \text{Gap}] &= [C - EB^{-1}A] [\delta x]. \end{aligned} \quad (12)$$

The matrix $-[B^{-1}A]$ is the sensitivity matrix of assembly dimension variables with respect to deviation source dimension variables. The matrix $[C - EB^{-1}A]$ is the sensitivity matrix of assembly key characteristics dimension variables with respect to deviation source dimension variables.

4. Computational Experiment

Taking assembly deviation source sensitivity analysis of wing flap rocker as a research example, the deviation transmission paths and dimension vector loops are established. The main influence factors of assembly precision optimization are analyzed based on calculating deviation source sensitivity.

FIGURE 1: Deviation transmission path formed by four kinds of joint types.

FIGURE 2: A profile of wing flap rocker assembly.

Wing flap rocker mainly contains five components: $P_{wing\ body}$, $P_{front\ connector}$, $P_{back\ connector}$, P_{rocker} , and P_{pin} , shown in Figure 2. The deviation sources are $x_1, x_2, \dots, x_{13}$ and $\perp | 0.2 | A$. All the deviation sources dimensions' nominal values and tolerances are shown in Table 1.

The process of wing flap rocker assembly deviation source sensitivity analysis based on dimension vector loops is illustrated in Figure 3.

First of all, DPs are established. According to assembly constraints, the connection types and joints ($J(P_{rocker}, P_{pin})$, $J(P_{pin}, P_{front\ connector})$, $J(P_{front\ connector}, P_{wing\ body})$, $J(P_{wing\ body}, P_{back\ connector})$, and $J(P_{back\ connector}, P_{rocker})$) in the assembly are located; DRFs (DRF_{rocker} , DRF_{pin} , $DRF_{front\ connector}$, $DRF_{wing\ body}$, and $DRF_{back\ connector}$) are defined based on factors such as component design references and assembly locating datum; DPs from joints to DRFs are created along the nominal dimension vector directions, as shown in Figure 4.

Then, the deviation transmission paths and multidimensional vector loops are created based on the DPs, as shown in Figure 5. Figure 5(a) shows assembly deviation transmission vector closed loop. Because of the $\perp | 0.2 | A$, a_{14} becomes a deviation source. Figure 5(b) shows assembly deviation

transmission vector opened loop. Gap means the distance from $DRF_{wing\ body}$ to x_1 .

Finally, (13) is generated according to the dimensionality $D = \{x, y, \theta\}$. The parameters δx , δU , A, B, C , and E are solved by first-order Taylor's series expansion. The parameters are shown in (14). Sensitivities are solved by matrix operation, as shown in Table 2:

$$\begin{aligned}
 h_x &= x_1 \cos(0) + x_3 \cos(x_{14}) + x_4 \cos(0) + x_8 \cos(-U_3) \\
 &\quad + x_9 \cos(90) + x_{10} \cos(90) + U_1 \cos(-x_{12}) \\
 &\quad + x_{13} \cos(-U_2 - 90) + x_7 \cos(90) \\
 &\quad + x_5 \cos(-180 + x_6) + x_2 \cos(-180) = 0, \\
 h_y &= x_1 \sin(0) + x_3 \sin(a_{14}) + x_4 \sin(0) + x_8 \sin(-U_3) \\
 &\quad + x_9 \sin(90) + x_{10} \sin(90) + U_1 \sin(-x_{12}) \\
 &\quad + x_{13} \sin(-U_2 - 90) + x_7 \sin(90) \\
 &\quad + x_5 \sin(-180 + a_6) + x_2 \sin(-180) = 0, \\
 h_\theta &= 0 + x_{14} - 90 - U_3 + 180 - (90 + x_{12}) \\
 &\quad - 90 - (180 - U_2) + 90 - x_6 + 180 = 0,
 \end{aligned}$$

TABLE 1: The deviation sources dimensions' nominal values and tolerances.

Dimension parameter	x_1/mm	x_2/mm	x_3/mm	x_4/mm	x_5/mm	x_6/deg	x_7/mm	x_8/mm	x_9/mm
Nominal value	260	320	42	20	221	9	28	10	30
Tolerance	± 0.3	± 0.3	± 0.3	± 0.3	± 0.3	± 0.5	± 0.3	± 0.3	± 0.3
Dimension parameter	x_{10}/mm	x_{11}/mm	x_{12}/deg	x_{13}/mm	x_{14}/deg	U_1/mm	U_2/deg	U_3/deg	
Nominal value	10	150	6	80	90	251	15	90	
Tolerance	± 0.3	± 0.3	± 0.5	± 0.3	± 0.2	± 0.5	± 1	± 1

FIGURE 3: The process of assembly deviation source sensitivity analysis.

$$\begin{aligned} \text{Gap} = & (x_{11} + U_1) \sin(-x_{12}) + x_{13} \sin(-U_2 - 90) \\ & + x_7 \sin(90) + x_5 \sin(-180 + x_6). \end{aligned} \quad (13)$$

$$\begin{aligned} [A] = & \begin{bmatrix} \frac{\partial h_x}{\partial x_1} & \frac{\partial h_x}{\partial x_2} & \dots & \frac{\partial h_x}{\partial x_{14}} \\ \frac{\partial h_y}{\partial x_1} & \frac{\partial h_y}{\partial x_2} & \dots & \frac{\partial h_y}{\partial x_{14}} \\ \frac{\partial h_\theta}{\partial x_1} & \frac{\partial h_\theta}{\partial x_2} & \dots & \frac{\partial h_\theta}{\partial x_{14}} \end{bmatrix}, \\ [B] = & \begin{bmatrix} \frac{\partial h_x}{\partial U_1} & \frac{\partial h_x}{\partial U_2} & \frac{\partial h_x}{\partial U_3} \\ \frac{\partial h_y}{\partial U_1} & \frac{\partial h_y}{\partial U_2} & \frac{\partial h_y}{\partial U_3} \\ \frac{\partial h_\theta}{\partial U_1} & \frac{\partial h_\theta}{\partial U_2} & \frac{\partial h_\theta}{\partial U_3} \end{bmatrix}, \\ [C] = & \begin{bmatrix} \frac{\partial \text{Gap}}{\partial x_1} & \frac{\partial \text{Gap}}{\partial x_2} & \dots & \frac{\partial \text{Gap}}{\partial x_{14}} \end{bmatrix}, \\ [E] = & \begin{bmatrix} \frac{\partial \text{Gap}}{\partial U_1} & \frac{\partial \text{Gap}}{\partial U_2} & \frac{\partial \text{Gap}}{\partial U_3} \end{bmatrix} \\ [\delta x] = & [\delta x_1 \quad \delta x_2 \quad \dots \quad \delta x_{14}]^T, \\ [\delta U] = & [\delta U_1 \quad \delta U_2 \quad \delta U_3]^T. \end{aligned} \quad (14)$$

As shown in Table 2, the greatest impact on the assembly precision is the deviation sources x_6 , x_{12} , and x_{14} . The angle dimensions x_6 , x_{12} , and x_{14} correspond to length dimensions

x_5 , U_1 , and x_3 . It can be seen that the larger the length dimension, the greater the sensitivity of the corresponding angle.

According to (1), assembly precision $\delta U_2 = 0.5394$.

The first assembly precision optimization method is that the tolerances of deviation sources x_1 , x_2 , x_3 , x_4 , x_5 , x_7 , x_8 , x_9 , x_{10} , x_{11} , x_{13} , and x_{14} are reduced by 50%. An optimized assembly precision is obtained; $\delta U_2 = 0.3940$.

The second assembly precision optimization method is that the tolerances of deviation sources x_6 and x_{12} are reduced by 50%. An optimized assembly precision is obtained; $\delta U_2 = 0.2533$. So the goal of assembly precision optimization is the tolerances of deviation sources x_6 and x_{12} .

The results indicate that it would be more conducive to optimize the assembly precision by reducing the deviations which have large sensitivity.

5. Conclusions

This paper presents an approach for fast assembly precision optimization of complex products based on deviation sources sensitivities analysis. The joints between the adjacent parts and each part's datum reference frame are defined for creating deviation transmission paths and multidimensional dimension vector loops. Sensitivity calculations of assembly deviation source are established by linearizing all the multidimensional vector loop scalar equations which can be gotten using first-order Taylor's series expansion and matrix algebra.

In practice, we find that the sensitivity of deviation source is not always +1 and -1. In the multidimensional space, sensitivity of deviation source is enlarged dramatically under certain conditions. If a list of deviation sources has the same vector directions, the sensitivities of an assembly dimension to the deviation sources are the same. If the vector direction

FIGURE 4: DRFs and DPs.

FIGURE 5: Deviation transmission vector loops.

TABLE 2: The value of deviation source sensitivities.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
Sensitivity of U_1	-1.8	1.8	-7.9	-1.8	3.1	1644	-7.9	7.9	-7.9	-7.9	0	2005	8.1	99.3
Sensitivity of U_2	0	0	-0.1	0	0	18.8	-0.1	0.1	-0.1	-0.1	0	22.2	0.1	0.5
Sensitivity of U_3	0	0	-0.1	0	0	17.8	-0.1	0.1	-0.1	-0.1	0	21.2	0.1	1.5
Sensitivity of Gap	0	0	-1	0	0	0	0	1	-1	-1	-0.1045	-149	0	0

of an assembly dimension and a deviation source is the same, the sensitivity of the assembly dimension to the deviation source is +1. If the vector direction of an assembly dimension and a deviation source is opposite, the sensitivity of the assembly dimension to the deviation source is -1. If the vector direction of an assembly dimension and a deviation source is perpendicular, the sensitivity of the assembly dimension to the deviation source is 0.

Deviation source sensitivity is an important indicator of assembly precision optimization in the aerospace industry. To further improve the flexibility of our approach, four kinds of joint types and multidimensional vector loops are used. Considering the deviation source sensitivity analysis of complex product assembly, our future work also includes improving the algorithm by providing a more traceable and robust method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant no. 51105313 and the Doctorate Foundation of Northwestern Polytechnical University under Grant no. CX201313.

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