Research Article

Scheduling Simple Linear Deteriorating Jobs with Rejection

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We consider the problems of scheduling deteriorating jobs with release dates on a single machine (parallel machines) and jobs can be rejected by paying penalties. The processing time of a job is a simple linear increasing function of its starting time. For a single machine model, the objective is to minimize the maximum lateness of the accepted jobs plus the total penalty of the rejected jobs. We show that the problem is NP-hard in the strong sense and presents a fully polynomial time approximation scheme to solve it when all jobs have agreeable release dates and due dates. For parallel-machine model, the objective is to minimize the maximum delivery completion time of the accepted jobs plus the total penalty of the rejected jobs. When the jobs have identical release dates, we first propose a fully polynomial time approximation scheme to solve it. Then, we present a heuristic algorithm for the case where all jobs have to be accepted and evaluate its efficiency by computational experiments.

1. Introduction

For most classical scheduling problems, the processing times of jobs are considered to be constant and independent of their starting time. However, this assumption is not appropriate for the modelling of many modern industrial processes; we often encounter situations in which processing time increases over time, when the machines gradually lose efficiency. Such problems are generally known as scheduling with deterioration effects. Scheduling with linear deterioration was first considered by Browne and Yechiali [1] who assumed that the processing times of jobs are nondecreasing and start time dependent linear functions. They derived optimal scheduling policies that minimize the expected makespan. Mosheiov [2] considered simple linear deterioration where jobs have a fixed job-dependent growth rate but no basic processing time. He showed that most commonly applied performance criteria, such as the makespan, the total flow time, the total lateness, the sum of weighted completion time, the maximum lateness, the maximum tardiness, and the number of tardy jobs, remain polynomially solvable. Since then, machine scheduling problems with deteriorating jobs have received increasing attention. An extensive survey of different models and problems was provided by Alidaee and Womer [3].

Cheng et al. [4] presented an updated survey of the results on scheduling problems with time-dependent processing times. Other new results of scheduling with deterioration effect can be found in [5–7].

At the same time, it is always assumed in traditional research that all the jobs have to be processed. In the real world, however, things may be more complicated. For example, due to limited resources or limited available capacity, the decider of a factory can choose only a subset of these tasks to be scheduled, while perhaps incurring some penalty for the rejected jobs. The machine scheduling problem with rejection was first introduced by Bartal et al. [8]. They studied both the offline and online versions of scheduling problem with rejection on identical parallel machines. The objective is to minimize the sum of the makespan of the accepted jobs and the total rejection penalty of the rejected jobs. After that, the machine scheduling problem with rejection received more and more attention. Hoogeveen et al. [9] considered the offline multiprocessor scheduling problem with rejection where preemption is allowed. Zhang et al. [10] studied the single machine scheduling with release dates and rejection to minimize the sum of the maximum completion times of the scheduled jobs and the total penalty of the rejected jobs. Sengupta [11] considered algorithms and approximation

schemes for minimum lateness/tardiness scheduling with rejection. Shabtay et al. [12] recently presented an updated survey on offline scheduling with rejection. In addition, there has been some work on scheduling deteriorating jobs with rejection. For example, Cheng and Sun [13] considered the single machine scheduling problem with deterioration and rejection, in which the processing time of a job is a linear function of its starting time. Li and Yuan [14] studied several parallel-machine scheduling problems in which the processing time of a job is a linear increasing function of its starting time and jobs can be rejected by paying penalties. The objectives are to minimize the makespan and the total weighted completion time plus the total penalty of the rejected jobs. Hsua and Chang [15] considered the unrelated parallel-machine scheduling with deteriorating jobs and rejection. To the best of our knowledge, the problems of scheduling deteriorating jobs with release dates and rejection have not been discussed.

In this paper, we study the single machine (parallel-machine) scheduling of deteriorating jobs with release dates and rejection. The paper is organized as follows. In Section 2, we formulate the problems under consideration and introduce some notations. In Section 3, we first discuss single machine model and show that the problem is strongly NP-hard. Then we propose a fully polynomial time approximation scheme for the case where all jobs have agreeable release dates and due dates. In Section 4, we consider parallel-machine scheduling problem with deterioration and rejection. In Section 5, we conclude the paper and suggest some topics for future research.

2. Problem Description and Notation

The problems considered in this paper can be formally described as follows. There are an independent job set $\mathcal{J} = \{J_1, \dots, J_n\}$ and a single machine ($m \geq 2$ identical parallel machines $\{M_1, \dots, M_m\}$). Each job J_j ($j = 1, \dots, n$) has a release date r_j , a due date d_j (a delivery time q_j), a deteriorating rate $b_j > 0$, and a rejection penalty e_j . Each job J_j ($j = 1, \dots, n$) is either rejected with a rejection penalty e_j having to be paid or accepted and processed on the single machine (one of parallel machines). The actual processing time of job J_j is given by $p_j = b_j t$, where $t \geq r_j$ is the starting time of J_j . Denote by R and $\bar{R} = \mathcal{J} \setminus R$ the set of rejected jobs and the set of accepted jobs, respectively. Denote by C_j the completion time of the accepted job J_j . In the single machine model, assume $\min\{r_j \mid j = 1, \dots, n\} \geq t_0 > 0$; the lateness of job J_j is defined as $L_j = C_j - d_j$. The objective is to minimize the maximum lateness (i.e., $L_{\max} = \max\{L_j : J_j \in \bar{R}\}$) plus the total penalty of the rejected jobs. In parallel-machine model, assume $\min\{r_j \mid j = 1, \dots, n\} = t_0 > 0$. We consider that each job J_j has a delivery time $q_j \geq 0$ rather than a due date d_j . The delivery of a job begins immediately after it completes processing, and job deliveries are allowed to overlap. We define the delivery completion time as $D_j = C_j + q_j$; the objective is to minimize the maximum delivery completion time (i.e., $D_{\max} = \max\{D_j : J_j \in \bar{R}\}$) plus the total penalty of the rejected jobs. Using the three-field

notation of Graham et al. [16], the problems are denoted by $1 \mid r_j, p_j = b_j t, \text{rej} \mid L_{\max}(\bar{R}) + \sum_{J_j \in R} e_j$ and $Pm \mid r_j = t_0, p_j = b_j t, \text{rej} \mid D_{\max}(\bar{R}) + \sum_{J_j \in R} e_j$.

3. The Single Machine Problem

3.1. NP-Hardness Proof. In this subsection, we first discuss the complexity of the problem $1 \mid r_j, p_j = b_j t \mid L_{\max}$ where rejection is not considered.

Theorem 1. *The problem $1 \mid r_j, p_j = b_j t \mid L_{\max}$ is strongly NP-hard.*

Proof. The decision version of the scheduling problem is clearly in NP. We use a reduction from the 4-product problem (4-P), which is strongly NP-hard (see [17]).

An instance of the 4-product problem is formulated as follows. Given $D \in \mathbb{Q}^+$, a finite set $N = \{1, 2, \dots, 4k\}$ for some natural number k , a size $y_j \in \mathbb{Q}^+$ and $D^{1/5} < y_j < D^{1/3}$ for every $j \in N$, $\prod_{j \in N} y_j = D^k$. Does there exist disjoint subsets $N_1, \dots, N_k$ such that $\bigcup_{i=1}^k N_i = N$ and the product of sizes of the elements in N_i satisfies $\prod_{j \in N_i} y_j = D$ for $1 \leq i \leq k$?

Given an arbitrary instance I of 4-product problem, we construct an instance II of the scheduling problem as follows.

- (i) There are $n = 5k - 1$ jobs.
- (ii) For $1 \leq j \leq 4k$, $r_j = 1$, $b_j = y_j - 1$, and $d_j = 2^{k-1} D^k$.
- (iii) For $4k + 1 \leq j \leq 5k - 1$, $r_j = 2^{j-4k+1} D^{j-4k}$, $b_j = 1$, and $d_j = 2^{j-4k} D^{j-4k}$.
- (iv) The threshold value is defined by $Y = 0$.

The decision version asks whether there is a schedule π for problem $1 \mid r_j, p_j = b_j t \mid L_{\max}$ such that $L_{\max} \leq Y$.

It is clear that the reduction can be done in polynomial time. Now we prove that instance I has a solution if and only if instance II has a schedule π with $L_{\max} \leq Y$.

Assume that 4-product problem has a solution; that is, there exist disjoint subsets $N_1, \dots, N_k$ such that $\bigcup_{i=1}^k N_i = N$ and the product of sizes of the elements in N_i satisfies $\prod_{j \in N_i} y_j = D$ for $1 \leq i \leq k$. For convenience, we denote the set of jobs $\{J_j : j \in N_i\}$ ($1 \leq i \leq k$) by J_{N_i} . The jobs are scheduled without idle times as follows. First, the set of jobs J_{N_i} are scheduled in the interval $[2^{l-1} D^{l-1}, 2^{l-1} D^l]$ for $l = 1, \dots, k$. Next, each job J_j is scheduled at time $r_j = 2^{j-4k+1} D^{j-4k}$ for $j = 4k + 1, \dots, 5k - 1$. Therefore, the maximum lateness of this schedule is $L_{\max} = \max\{C_j - d_j : 1 \leq j \leq 5k - 1\} = 0$.

Conversely, assume that there is a schedule π with $L_{\max} \leq 0$. We are ready to prove that instance I has a solution.

First, each job J_j ($4k + 1 \leq j \leq 5k - 1$) must start exactly at its release date r_j and finish at its due date d_j , since $L_{\max} \leq Y = 0$. Second, the rest $4k$ jobs are scheduled as the k intervals from time $t = 1$ to time $t = 2^{k-1} D^k$. Without loss of generality, let J_{N_l} ($1 \leq l \leq k$) be the set of jobs scheduled in the interval $[2^{l-1} D^{l-1}, 2^{l-1} D^l]$.

Therefore, from the above discussion, we know that $\prod_{j \in N_l} y_j \leq D$ for $1 \leq l \leq k$. Note that $\prod_{l=1}^k \prod_{j \in N_l} y_j = \prod_{j=1}^{4k} y_j = D^k$. Thus, we have $\prod_{j \in N_l} y_j = D$ for $1 \leq l \leq k$. That is, instance I has a solution. This completes the proof. $\square$

Corollary 2. *The problem $1 \mid r_j, p_j = b_j t, rej \mid L_{\max}(\bar{R}) + \sum_{J_j \in R} e_j$ is strongly NP-hard, so it has no fully polynomial time approximation scheme unless $P = NP$.*

3.2. An FPTAS for One Special Case. An algorithm $\mathcal{A}$ is a ρ -approximation ($\rho \geq 1$) algorithm for a minimization problem if it produces a solution that is at most ρ times than the optimal one (ρ is also referred to as the worst-case ratio). A family of algorithms $\{\mathcal{A}_\epsilon : \epsilon > 0\}$ is called a fully polynomial time approximation scheme (FPTAS), if for each $\epsilon > 0$, the algorithm $\mathcal{A}_\epsilon$ is a $(1 + \epsilon)$ approximation algorithm running in polynomial time in the input size and $1/\epsilon$.

In this subsection, we consider the special case where all jobs have agreeable release dates and due dates and assume that the jobs have been indexed such that $r_1 \leq \dots \leq r_n$ and $d_1 \leq \dots \leq d_n$. The problem can be denoted by $1 \mid r_j, p_j = b_j t, rej, agreeable \mid L_{\max}(\bar{R}) + \sum_{J_j \in R} e_j$.

We design an FPTAS for this problem by considering the modified deteriorating rates, the modified release dates, the modified due dates, and the inflated rejection penalty. The definition of the modified deteriorating rates and the modified release dates involves a geometric rounding technique developed by Afrati et al. [18]. The definition of the inflated rejection penalty is stated by Sengupta [11]. The rounding technique and the inflated rejection penalty are stated, respectively, as follows.

For any $\epsilon' > 0$ and $x \geq 1$, if $(1 + \epsilon')^{k-1} < x < (1 + \epsilon')^k$, then we define $\lceil x \rceil_{\epsilon'} = (1 + \epsilon')^k$, $\lfloor x \rfloor_{\epsilon'} = (1 + \epsilon')^{k-1}$. If x is an exact power of $(1 + \epsilon')$, then $\lceil x \rceil_{\epsilon'} = \lfloor x \rfloor_{\epsilon'} = x$. Note that $\lceil x \rceil_{\epsilon'} \leq (1 + \epsilon')x$ for any $x \geq 1$.

For any $\epsilon > 0$, we define the modified deteriorating rate of job J_j as $b'_j = \lceil 1 + b_j \rceil_{\epsilon'} - 1$, the modified release date of job J_j as $r'_j = \lceil r_j \rceil_{\epsilon'}$, and the modified due date of job J_j as $d'_j = (1 + \epsilon)d_j$, where $\epsilon' = \epsilon/2(n + 1)$. Let L_j denote the exponent of $1 + b'_j$; that is, $1 + b'_j = \lceil 1 + b_j \rceil_{\epsilon'} = (1 + \epsilon')^{L_j}$; then $L_j = \log \lceil 1 + b_j \rceil_{\epsilon'} / \log(1 + \epsilon') = O(n \log(1 + b_j)/\epsilon)$; let l_j denote the exponent of r'_j ; that is, $r'_j = \lceil r_j \rceil_{\epsilon'} = (1 + \epsilon')^{l_j}$; then $l_j = \log \lceil r_j \rceil_{\epsilon'} / \log(1 + \epsilon') = O(n \log r_j / \epsilon)$.

For any $\epsilon > 0$, let $\epsilon' = \epsilon/2(n + 1)$ and $R = \{J_{i_1}, J_{i_2}, \dots, J_{i_m}\}$ be the set of the rejected jobs, where $i_1 < i_2 < \dots < i_m$. The inflated rejection penalty of the set R is defined as

$$\phi_{\epsilon'}(R) = \begin{cases} \lceil e_{i_m} + \phi_{\epsilon'}(R - J_{i_m}) \rceil_{\epsilon'}, & \text{if } m \geq 1, \\ 0, & \text{if } R = \emptyset. \end{cases} \quad (1)$$

Lemma 3 (see [11]). *For any set R of jobs and any $\epsilon' > 0$, $\phi_{\epsilon'}(R) \leq (1 + \epsilon')^{|R|} \sum_{J_j \in R} e_j$.*

Lemma 4 (see [19]). *For any $0 < x \leq 1$ and for any integer $n \geq 1$, $(1 + x/n)^n \leq 1 + 2x$ holds.*

Theorem 5. *For any $0 < \epsilon \leq 2$, the optimal objective function value for $1 \mid r_j, p_j = b_j t, rej, agreeable \mid L_{\max}(\bar{R}) + \phi_{\epsilon'}(R)$ with the modified deteriorating rates, the modified release dates, and the modified due dates is at most $(1 + \epsilon)$ times the optimal objective function value for $1 \mid r_j, p_j = b_j t, rej, agreeable \mid L_{\max}(\bar{R}) + \sum_{J_j \in R} e_j$.*

Proof. Assume that jobs in the set $\bar{R}$ are scheduled in the order of $[1], \dots, [\lceil \bar{R} \rceil]$. Let C'_j denote the completion time of job J_j under the modified deteriorating rates and release dates; we have $C'_j = \max_{1 \leq i \leq j} \{r'_{[i]} \prod_{k=i}^j (1 + b'_{[k]})\}$, then

$$\begin{aligned} L'_j &= C'_j - d'_j = \max_{1 \leq i \leq j} \left\{ r'_{[i]} \prod_{k=i}^j (1 + b'_{[k]}) \right\} - d'_{[j]} \\ &\leq (1 + \epsilon')^{j+1} \max_{1 \leq i \leq j} \left\{ r_{[i]} \prod_{k=i}^j (1 + b_{[k]}) \right\} - (1 + \epsilon) d_{[j]} \quad (2) \\ &\leq (1 + \epsilon')^{n+1} C_j - (1 + \epsilon) d_j \\ &\leq (1 + \epsilon) L_j. \end{aligned}$$

The objective function is

$$\begin{aligned} L'_{\max}(\bar{R}) + \phi_{\epsilon'}(R) &\leq \max \{L'_j\} + (1 + \epsilon')^{|\bar{R}|} \sum_{J_j \in R} e_j \\ &\leq (1 + \epsilon) \left(L_{\max}(\bar{R}) + \sum_{J_j \in R} e_j \right). \end{aligned} \quad (3)$$

Since the above inequality is valid for any $\bar{R} \subseteq \mathcal{J}$, the results hold.

Assume that the jobs have been reindexed such that $d_1 \leq \dots \leq d_n$ and define

$$\tau_k = \begin{cases} (1 + \epsilon')^k, & \text{if } k \neq -1, \\ 0, & \text{if } k = -1. \end{cases} \quad (4)$$

Let $f(j, \lambda, k)$ denote the minimum value of the maximum lateness satisfying the following conditions.

- (i) The jobs in consideration are $J_1, \dots, J_j$.
- (ii) The maximum completion time of the accepted jobs is $(1 + \epsilon')^\lambda$.
- (iii) The inflated rejection penalty of the rejected jobs is τ_k .

In any such schedule, there are two possible cases: either job J_j is rejected or J_j is accepted and processed on the machine. $\square$

Case 1 (Job J_j is rejected). Let $(1 + \epsilon')^{k'}$ be the inflated rejection penalty of the rejected jobs among $J_1, \dots, J_{j-1}$. From the definition of inflated rejection penalty, the largest value of k' is given by $(1 + \epsilon')^{k'} = \lfloor (1 + \epsilon')^k - e_j \rfloor_{\epsilon'}$. Hence, the value of the maximum lateness for the optimal schedule is $\min_{-1 \leq k' \leq k} f(j - 1, \lambda, k')$.

Case 2 (Job J_j is accepted). Consider the following.

Case 2.1 ($\lambda > l_j + L_j$). In this case, the maximum completion time of accepted jobs among $J_1, \dots, J_{j-1}$ is $(1 + \epsilon')^\lambda / (1 + \epsilon')^{L_j}$. Therefore, $f(j, \lambda, k) = \max\{f(j-1, \lambda - L_j, k), (1 + \epsilon')^\lambda - d'_j\}$.

Case 2.2 ($\lambda = l_j + L_j$). In this case, let $(1 + \epsilon')^{\lambda'}$ be the maximum completion time of accepted jobs among $J_1, \dots, J_{j-1}$. Since the starting time of job J_j is r'_j , we have $\lambda' \leq l_j$. Hence, $f(j, \lambda, k) = \max\{\min_{0 \leq \lambda' \leq l_j} f(j-1, \lambda', k), (1 + \epsilon')^\lambda - d'_j\}$.

Combining the above two cases, the dynamic programming algorithm is stated as follows.

Algorithm DPI

Step 1 (initialization).

$$f(1, \lambda, k) = \begin{cases} r'_1[1 + b_1]_{\epsilon'} - d' & \lambda = l_1 + L_1, k = -1, \\ -\infty & \lambda = 0, \tau_k = [e_1]_{\epsilon'}, \\ +\infty & \text{otherwise.} \end{cases} \quad (5)$$

Step 2 (recursion). If $\lambda < l_j + L_j$, $f(j, \lambda, k) = \min_{-1 \leq k' \leq \tilde{k}} f(j-1, \lambda, k')$,

If $\lambda > l_j + L_j$,

$$f(j, \lambda, k) = \min \left\{ \max \left\{ f(j-1, \lambda - L_j, k), (1 + \epsilon')^\lambda - d'_j \right\}, \min_{-1 \leq k' \leq \tilde{k}} f(j-1, \lambda, k') \right\}. \quad (6)$$

If $\lambda = l_j + L_j$,

$$f(j, \lambda, k) = \min \left\{ \max \left\{ \min_{0 \leq \lambda' \leq l_j} f(j-1, \lambda', k), (1 + \epsilon')^\lambda - d'_j \right\}, \min_{-1 \leq k' \leq \tilde{k}} f(j-1, \lambda, k') \right\}, \quad (7)$$

where $\tilde{k}$ is given by $(1 + \epsilon')^{\tilde{k}} = \lfloor (1 + \epsilon')^k - e_j \rfloor_{\epsilon'}$.

Step 3 (output). The optimal value is determined by $\min\{f(n, \lambda, k) + \tau_k : 0 \leq \lambda \leq \max_{1 \leq i \leq n} \{l_i\} + \sum_{j=1}^n L_j, -1 \leq k \leq K\}$, where K is the smallest integer such that $(1 + \epsilon')^K \geq (1 + \epsilon')^n \sum_{j=1}^n e_j$.

Let $L = \log \max\{r_{\max}, 1 + b_{\max}, \sum_{j=1}^n e_j\}$, where $r_{\max} = \max_{1 \leq j \leq n} r_j$, $b_{\max} = \max_{1 \leq j \leq n} b_j$. We have the following theorem.

Theorem 6. *There exists an FPTAS for problem 1 | $r_j, p_j = b_j t, \text{rej}$, agreeable | $L_{\max}(\bar{R}) + \sum_{j \in R} e_j$ which runs in $O(n^7 L^4 / \epsilon^4)$ time.*

Proof. We need to compute exactly $O(n(\max_{1 \leq i \leq n} \{l_i\} + \sum_{j=1}^n L_j + 1)(K + 2))$ values $f(j, \lambda, k)$ and computation of each value takes at most $O((\max_{1 \leq i \leq n} \{l_i\} + \sum_{j=1}^n L_j)K)$ time, where $K = O((n/\epsilon) \log \sum_{j=1}^n e_j)$. Hence, the running time of Algorithm DPI is

$$\begin{aligned} & O \left(n \left(\max_{1 \leq i \leq n} \{l_i\} + \sum_{j=1}^n L_j \right)^2 K^2 \right) \\ &= O \left(n \left(\frac{n \log r_{\max}}{\epsilon} + \frac{n^2 \log(1 + b_{\max})}{\epsilon} \right)^2 \right. \\ & \quad \left. \times \left(\frac{n \log \sum_{j=1}^n e_j}{\epsilon} \right)^2 \right) \\ &= O \left(\frac{n^7 L^4}{\epsilon^4} \right). \end{aligned} \quad (8)$$

□

4. Parallel-Machine Problem

4.1. An FPTAS. In this subsection, we focus our attention on identical parallel machines problem with deterioration and rejection. For convenience of discussion, we consider a scheduling model in which each job J_j has a delivery time $q_j \geq 0$ rather than a due date d_j . When all jobs have identical release dates, the problem $Pm \mid r_j = t_0, p_j = b_j t, \text{rej} \mid D_{\max}(\bar{R}) + \sum_{j \in R} e_j$ is at least ordinarily NP-hard according to Kononov [20]. We give a fully polynomial time approximation scheme to solve it by considering the modified deteriorating rates, the modified delivery times, and the inflated rejection penalty.

For any $\epsilon > 0$, we define the modified deteriorating rate and delivery time of job J_j as $b'_j = \lceil 1 + b_j \rceil_{\epsilon'} - 1$, $q'_j = (1 + \epsilon')q_j$, where $\epsilon' = \epsilon/2n$. Let L_j denote the exponent of $1 + b'_j$; that is, $1 + b'_j = \lceil 1 + b_j \rceil_{\epsilon'} = (1 + \epsilon')^{L_j}$; then $L_j = \log \lceil 1 + b_j \rceil_{\epsilon'} / \log(1 + \epsilon') = O(n \log(1 + b_j) / \epsilon)$.

For any $\epsilon > 0$, let $\epsilon' = \epsilon/2n$ and $R = \{J_{i_1}, J_{i_2}, \dots, J_{i_m}\}$ be the set of the rejected jobs, where $i_1 < i_2 < \dots < i_m$. The inflated rejection penalty of the set R is defined as

$$\phi_{\epsilon'}(R) = \begin{cases} \lceil e_{i_m} + \phi_{\epsilon'}(R - J_{i_m}) \rceil_{\epsilon'}, & \text{if } m \geq 1, \\ 0, & \text{if } R = \emptyset. \end{cases} \quad (9)$$

Lemma 7. *For any $0 < \epsilon \leq 2$, the optimal objective function value for $Pm \mid r_j = t_0, p_j = b_j t, \text{rej} \mid D_{\max}(\bar{R}) + \phi_{\epsilon'}(R)$ with the modified deteriorating rates and the modified delivery times is at most $(1 + \epsilon)$ times the optimal objective function value for*

$$Pm \mid r_j = t_0, p_j = b_j t, \text{rej} \mid D_{\max}(\bar{R}) + \sum_{j \in R} e_j. \quad (10)$$

Lemma 8. *There exists an optimal job sequence for $Pm \mid r_j = t_0, p_j = b_j t, \text{rej} \mid D_{\max}(\bar{R}) + \sum_{j \in R} e_j$ such that on each machine the accepted jobs are sequenced in nonincreasing order of q_j .*

Based on Lemmas 7 and 8, we design a dynamic programming algorithm for $Pm \mid r_j = t_0, p_j = b_j t, \text{rej} \mid D_{\max}(\bar{R}) + \phi_{\epsilon'}(R)$ with the modified deteriorating rates and delivery times. Let us index the jobs such that $q_1 \geq \dots \geq q_n$ and define

$$\tau_k = \begin{cases} (1 + \epsilon')^k, & \text{if } k \neq -1, \\ 0, & \text{if } k = -1. \end{cases} \quad (11)$$

Let $F(j, \lambda_1, \dots, \lambda_m, k)$ denote the optimal value of the maximum delivery completion time satisfying the following conditions.

- (i) The jobs in consideration are $J_1, \dots, J_j$.
- (ii) The maximum completion time of the accepted jobs on machine M_i is $t_0(1 + \epsilon')^{\lambda_i}$ for $i = 1, \dots, m$.
- (iii) The inflated rejection penalty of the rejected jobs is τ_k .

The dynamic programming algorithm is stated as follows.

Algorithm DP2

Step 1 (initialization).

$$F(1, \lambda_1, \dots, \lambda_m, k)$$

$$= \begin{cases} t_0(1 + \epsilon')^{L_1} + q'_1, & (\lambda_1, \dots, \lambda_i, \dots, \lambda_m, k) \\ & = (0, \dots, L_1, \dots, 0, -1), \\ -\infty, & (\lambda_1, \dots, \lambda_i, \dots, \lambda_m) \\ & = (0, \dots, 0, \dots, 0), \\ +\infty, & \tau_k = \lceil e_1 \rceil_{\epsilon'} \\ & \text{otherwise.} \end{cases} \quad (12)$$

Step 2 (recursion). If job J_j is rejected,

$$F(j, \lambda_1, \dots, \lambda_m, k) = \min_{-1 \leq k' \leq \bar{k}} F(j-1, \lambda_1, \dots, \lambda_m, k'). \quad (13)$$

If job J_j is accepted,

$$\begin{aligned} & F(j, \lambda_1, \dots, \lambda_m, k) \\ &= \min_{1 \leq i \leq m} \left\{ \max \left\{ F(j-1, \lambda_1, \dots, \lambda_i - L_i, \dots, \lambda_m, k), \right. \right. \\ & \quad \left. \left. t_0(1 + \epsilon')^{\lambda_i} + q'_j \right\} \right\}. \end{aligned} \quad (14)$$

Combining the two cases, we have

$$\begin{aligned} & F(j, \lambda_1, \dots, \lambda_m, k) \\ &= \min \left\{ \min_{1 \leq i \leq m} \left\{ \max \left\{ F(j-1, \lambda_1, \dots, \lambda_i - L_i, \dots, \lambda_m, k), \right. \right. \right. \\ & \quad \left. \left. t_0(1 + \epsilon')^{\lambda_i} + q'_j \right\} \right\}, \\ & \quad \left. \min_{-1 \leq k' \leq \bar{k}} F(j-1, \lambda_1, \dots, \lambda_m, k') \right\}, \end{aligned} \quad (15)$$

where $\bar{k}$ is given by $(1 + \epsilon')^{\bar{k}} = \lfloor (1 + \epsilon')^k - e_j \rfloor_{\epsilon'}$.

Step 3 (output). The optimal value is determined by

$$\min \left\{ F(n, \lambda_1, \dots, \lambda_m, k) + \tau_k : 0 \leq \lambda_i \leq \sum_{j=1}^n L_j, \right. \\ \left. 1 \leq i \leq m, -1 \leq k \leq K \right\}, \quad (16)$$

where K is the smallest integer such that $(1 + \epsilon')^K \geq (1 + \epsilon')^n \sum_{j=1}^n e_j$.

Theorem 9. *There exists an FPTAS for problem $Pm \mid r_j = t_0, p_j = b_j t, \text{rej} \mid D_{\max}(\bar{R}) + \sum_{J_j \in R} e_j$ which runs in $O((n^{m+3}/\epsilon^{m+2})(\sum_{j=1}^n \log(1 + b_j))^m \log^2(\sum_{j=1}^n e_j))$ time.*

Proof. We need to compute $n(\sum_{j=1}^n L_j + 1)^m(K + 2)$ values $F(j, \lambda_1, \dots, \lambda_m, k)$. Computation of each such value takes $O(K)$ time, where $K = O((n/\epsilon) \log \sum_{j=1}^n e_j)$. Therefore the overall time of the algorithm is $O(n(\sum_{j=1}^n L_j)^m K^2) = O((n^{m+3}/\epsilon^{m+2})(\sum_{j=1}^n \log(1 + b_j))^m \log^2(\sum_{j=1}^n e_j))$. $\square$

4.2. A Heuristic Algorithm for a Special Case. In this subsection, it is assumed that all jobs have to be processed; that is, rejection is not allowed. This problem can be denoted by $Pm \mid r_j = t_0, p_j = b_j t \mid D_{\max}$. We present a heuristic algorithm and give its worst case analysis.

Heuristic DLS. Consider the following steps.

Step 1. Place all jobs in a list ordered by nonincreasing delivery times. Set $S_i = t_0$ for $1 \leq i \leq m$, and S_i denotes the total completion time of jobs assigned to machine M_i .

Step 2. Select a machine M_i for which S_i is as small as possible. Choose the first unscheduled job J_j of the list and assign it to machine M_i . Set $S_i = S_i(1 + b_j)$.

Step 3. Repeat Step 2 until all jobs are scheduled.

Theorem 10. *The worst case performance ratio of heuristic DLS is $(1 + b_{\max})^{1-1/m}$ and this bound is tight.*

Proof. Let $D_{\max}^{\text{DLS}}$ be the maximum delivery completion time according to heuristic DLS and let $D_{\max}^*$ be the corresponding time in an optimal schedule. Assume that there exists a counterexample with $D_{\max}^{\text{DLS}}/D_{\max}^* > (1 + b_{\max})^{1-1/m}$. Let J_k be the first job in the sorted list with $C_k + q_k > (1 + b_{\max})^{1-1/m} D_{\max}^*$, remove all the following jobs from the list, and produce a minimum counterexample with $D_{\max}^{\text{DLS}} = C_k + q_k$, where J_k is the job with smallest delivery time. Next, we transform the counterexample by subtracting q_k from all delivery times. Obviously, this decreases both $D_{\max}^{\text{DLS}}$ and $D_{\max}^*$ by q_k . Since

$$\frac{D_{\max}^{\text{DLS}} - q_k}{D_{\max}^* - q_k} \geq \frac{D_{\max}^{\text{DLS}}}{D_{\max}^*} > (1 + b_{\max})^{1-1/m} \quad (17)$$

TABLE 1: Summary of the computational results.

m	n	$L(D_{\max}^*)$	$D_{\max}^{\text{DLS}}$	$\frac{D_{\max}^{\text{DLS}}}{L(D_{\max}^*)}$	$\frac{ D_{\max}^{\text{DLS}} - L(D_{\max}^*) }{L(D_{\max}^*)}$
3	10	240	559	2.329	1.329
4	12	208	577	2.774	1.774
4	12	168	485	2.887	1.887
4	15	380	520	1.368	0.368
4	15	477	931	1.952	0.952
5	10	33	60	1.818	0.818
5	10	57	115	2.018	1.018
5	18	611	1688	2.762	1.763

holds, the transformation yields another counterexample. Now the last job has delivery times zero and $D_{\max}^{\text{DLS}} = C_k$. At the time before job J_k is assigned to its machine M_i ,

$$S_i \leq \frac{(\prod_{l=1}^k (1 + b_l))^{1/m}}{(1 + b_k)^{1/m}} \leq \frac{D_{\max}^*}{(1 + b_k)^{1/m}}. \quad (18)$$

This implies

$$\begin{aligned} D_{\max}^{\text{DLS}} &= C_k = S_i (1 + b_k) \leq (1 + b_k)^{1-1/m} D_{\max}^* \\ &\leq (1 + b_{\max})^{1-1/m} D_{\max}^*. \end{aligned} \quad (19)$$

To prove that the bound is tight, consider the following example with $n = m(m - 1) + 1$ jobs, all jobs have delivery time zero, the first $m(m - 1)$ of them has deteriorating rates $b_j = 1$, $j = 1, 2, \dots, m(m - 1)$, and the last job's deteriorating rate is $b_n = 2^m - 1$. Then $b_{\max} = b_n$ and $(1 + b_{\max})^{1-1/m} = 2^{m-1}$. By DLS, $D_{\max}^{\text{DLS}} = t_0(1 + b_j)^{m-1}(1 + b_n) = t_0 2^{m-1} 2^m = t_0 2^{2m-1}$. In an optimal schedule, J_n is assigned to one machine and all the previous $m(m - 1)$ jobs are equally assigned to the other $m - 1$ machines. This yields to $D_{\max}^* = t_0 2^{2m}$. Thus,

$$\frac{D_{\max}^{\text{DLS}}}{D_{\max}^*} = (1 + b_{\max})^{1-1/m} = 2^{m-1}. \quad (20)$$

This completes the proof. $\square$

4.3. Numerical Experiment. This section conducts some numerical experiments to evaluate the performance of heuristic DLS. Without loss of generality, we set $t_0 = 1$. The delivery time q_j , $1 \leq j \leq n$ and the deteriorating rate b_j , $1 \leq j \leq n$ are generated from $U[0, 50]$ and $U[1, 10]$, respectively. The lower bound of optimal value is $L(D_{\max}^*) = (\prod_{l=1}^n (1 + b_l))^{1/m}$. The numerical results are summarized in Table 1. The program was coded in Matlab. The results of numerical experiments show that the algorithm has good performance. The proposed algorithm is acceptable for the NP-hard problem.

5. Conclusion

In this paper, we focus on several scheduling problems with deterioration, release date, and rejection. For the single

machine model, we show that this problem is strongly NP-hard and provides a fully polynomial time approximation scheme when all jobs have agreeable release dates and due dates. For parallel machines model, we propose a fully polynomial time approximation scheme for $Pm \mid r_j = t_0, p_j = b_j t, \text{rej} \mid D_{\max}(\bar{R}) + \sum_{J_j \in R} e_j$ and a heuristic algorithm for a special case. The computational results show that the heuristics is efficient in obtaining near-optimal solutions. For future research, it would be interesting to focus on scheduling deteriorating jobs and rejection with other objectives. Furthermore, investigation of the online version of this scheduling model is also an interesting research direction.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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