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Research Article

Output-Feedback Controller Design of a Wireless Networked Control System with Packet Loss and Time Delay

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This paper investigates the problem of modeling and stabilization of a wireless network control system (NCS) with both time-varying delay and packet-dropout. And the time-varying delay can be more or less than one sampling period. The wireless NCS is modeled as an asynchronous dynamic system (ADS) with three subsystems. Sufficient condition of the closed-loop NCS to be stable is obtained by using the ADS approach. A numerical example is presented to demonstrate the effectiveness of the proposed result.

1. Introduction

In modern control systems, the sensors, controllers, and controlled process are often connected by a real-time network medium. Such systems are called network control systems (NCSs) [1]. In the last decade, the general theory for NCSs has been widely investigated. It is widely used at almost all levels of operation and information processing in various areas, including manufacturing plants, automobiles, aircraft, remote operation, and teleautonomy [2–8]. As an alternative item for the wired network, wireless NCSs are becoming fundamental components of modern control systems due to their flexibility, ease of deployment, and low cost [9–12]. Thus, wireless NCS is considered in this paper.

One of the main issues in the NCS is the effect of network-induced delay that occurs when sensors, actuators, and controllers exchange data across the shared network. Without considering the delay, it not only degrades the performance of the control system but also even destabilizes the system. On the other hand, packet-dropout results from the network traffic congestion and the limited network reliability. When a data packet is dropped, the complete information of the NCS becomes unavailable. In this case, the controller or actuator has to decide what control signal is output with incomplete information.

Recently, many researchers have tried to solve the above problems with network-induced delay and packet-dropout in wireless NCS. For the problem with delay, for example, Hu and Yuan [2] introduce a finite sum equality based on quadratic terms to H_∞ output-feedback control for switched linear discrete-time systems. Together with a Lyapunov sequence, a novel delay-dependent condition is implemented for H_∞ without ignoring the useful terms. It can obtain the suboptimal H_∞ static and dynamic output-feedback controllers at last, when a procedure involving a modified iterative algorithm is performed. For the problems with packet-dropout, for instance, the sufficient conditions for the exponential stability of the closed-loop NCSs using the average dwell time method are proposed in [5]. Furthermore, the relation between the packet-dropout rate and the stability of the closed-loop NCSs is also explicitly established in order to prove the effectiveness.

Unlike separately considered these two issues of the delay and the packet-dropout, this paper intends to deal with the modelling, analysis, and synthesis for the wireless NCS with both delay and packet-dropout as shown in Figure 1. An asynchronous dynamic system (ADS) approach is presented to stabilize the wireless NCS. Firstly, a switched system with time-varying delay model is presented to describe the wireless NCS. In [4], a new switched linear system model is proposed to describe NCS while the delay is assumed

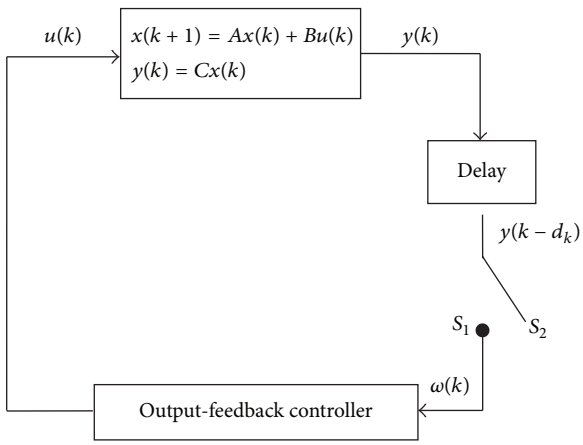

Figure 1: Diagram of network control system with network-induced delay and packet-dropout.

to be less than one sampling period with the state feedback controller. And stochastic optimal control method is used by an adaptive estimator (AE) and ideas from Q-learning to solve the infinite horizon optimal regulation of unknown wireless NCS with time-varying system matrices in [13]. Nevertheless, the computational complexity of the controller will increase when the delay bound is increased in NCS in [13]. Furthermore, the network system identification problem and estimation problem are also studied in [14, 15] based on the Matlab/Simulink simulator TrueTime and orthogonal projection principle, respectively, which aim at identifying mathematical models required in network control/estimation/filtering systems while stabilization is not considered. Similarly, we can also get the new ideas and future applications from the related papers [16, 17], respectively.

Summarizing the aforementioned discussion, in this paper, we aim to investigate the problem of modeling and stabilization of a wireless NCS with both time-varying delay and packet-dropout, where the wireless NCS is modeled as an ADS with three subsystems, and sufficient condition of the closed-loop NCS is obtained to be stable using this ADS method. The main contribution of this paper is highlighted as follows: to obtain the observer-based output-feedback controller, particularly, when the output of the sensor is dropped and delay appeared to the controller input, the ADS approach is presented to stabilize the considered wireless NCS with delay and packet-dropout.

Notations. Throughout this paper, R denotes the set of real numbers, R^n denotes the n-dimensional Euclidean space, and $R^{n\times m}$ refers to the set of all $n\times m$ real matrices. A^T represents the transpose of the matrix A, while A^{-1} denotes the inverse of A. For real symmetric matrices, X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X - Y$ is positive semidefinite (resp., positive-definite). I is the identity matrix with appropriate dimensions. $\|x\|$ refers to the Euclidean norm of the vector x; that is, $\|x\| = \sqrt{x^T x}$. For a symmetric matrix, $*$ denotes the matrix entries implied by symmetry.

2. Problem Formulation and NCS Modeling

The NCS with packet-dropout and possible delay is illustrated in Figure 1, where the plant is described by the following model denoted by Ω:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the system state, $u(k) \in R^m$ is the control input, and $y(k) \in R^r$ is the measured output. And the model of an observer-based output-feedback controller is described as follows:

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + L\left[\omega(k) - \hat{y}(k)\right], \\ \hat{y}(k) &= C\hat{x}(k), \\ u(k) &= K\hat{x}(k), \end{aligned} \tag{2}$$

where $\hat{x}(k) \in R^n$ is the estimated state of the system (1) and $\hat{y}(k) \in R^r$ is the estimated output. $L \in R^{n\times r}$ and $K \in R^{m\times n}$ are the observer and controller gain, respectively. It is also assumed that the pairs (A, B) are controllable and (C, A) are observable. In Figure 1, we can use a switch to denote the packet loss of the states in the network channel. If the switch is closed, the data packet is successfully transmitted. And we have $\omega(k) = y(k)$ without network delay or $\omega(k) = y(k-d_k)$ with network delay. When the switch is open, the previous value of the switch output will be used in the controller (2) and a packet is dropped. Then we have $\omega(k) = \omega(k-1)$ in this case.

Under consideration for the NCS, without loss of generality, we give the following assumptions, which will be useful in our main results.

Assumption 1. (1) The sensors and controllers are all time-driven and synchronized.

(2) Time-stamping of measurements is necessary to reorder data packet at the observer side since they can arrive out of order. And the controller can get the delay of each data packet.

(3) The maximum delay in the network is d_M that is a known integer.

Define the estimation error by $e(k) = x(k) - \hat{x}(k)$ and let

$$z(k) = \left[x^T(k) \quad e^T(k) \quad w^T(k-1)\right]^T. \tag{3}$$

Then the dynamics of the closed-loop system can be described by the following three subsystems.

(S1) There is packet loss and the corresponding controller gain is K_1. Then the closed-loop NCS is described as

$$\begin{aligned} \Omega_1 &: z(k+1) = A_1 z(k), \\ A1 &= \begin{bmatrix} A + BK_1 & -BK_1 & 0 \\ LC & A - LC & -L \\ 0 & 0 & I \end{bmatrix}. \end{aligned} \tag{4}$$

(S2) The data packet is transmitted successfully without network delay and the corresponding controller gain is K_2 in this case. Then the closed-loop NCS can be described as

$$\Omega_2 : z(k+1) = A_2 z(k),$$
$$A2 = \begin{bmatrix} A+BK_2 & -BK_2 & 0 \\ 0 & A-LC & 0 \\ C & 0 & 0 \end{bmatrix}. \tag{5}$$

(S3) The data packet is transmitted successfully with network delay d_k and the corresponding controller gain becomes K_{3,d_k} here. Then the closed-loop NCS is as follows:

$$\Omega_3 : z(k+1) = A_3 z(k) + A_{d3} z(k-d_k),$$
$$A3 = \begin{bmatrix} A+BK_{3,d_k} & -BK_{3,d_k} & 0 \\ LC & A-LC & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{6}$$
$$A_{d3} = \begin{bmatrix} 0 & 0 & 0 \\ -LC & 0 & 0 \\ C & 0 & 0 \end{bmatrix}.$$

From the above analysis, we can conclude that there are three different cases which may appear during every sampling period. So the closed-loop NCS can be described as a discrete-time switched system within three subsystems Ω_1 to Ω_3. In subsystem Ω_3, when $d_k = 0$, Ω_3 turns to Ω_2. And the system matrices Ω_1 and Ω_2 are similar. So subsystem Ω_3 in case 3 includes cases 1 and 2 by appropriately choosing the value of the matrices. Then the wireless NCS can be represented by the following switched system with time-varying delay:

$$z(k+1) = A_i z(k) + A_{di} z(k-d_k), \quad i = 1,2,3, \tag{7}$$

where $A_{d1} = 0$, $A_{d2} = 0$, and $d_k = 1,2,\ldots,d_M$.

To end this section, the following definition and lemma are introduced to obtain our main results.

Definition 2. For any given initial conditions $(k_0, \phi) \in \mathfrak{R}^+ \times C^n$, (7) is globally exponentially stable if the solutions of (10) satisfy

$$\|x(k)\| \le a\lambda^{-(k-k_0)} \|x(k_0)\|, \quad \forall k \ge k_0, \tag{8}$$

where $a > 0$ is a constant and $\lambda > 1$ is the decay rate.

Lemma 3 (see [2]). *For any appropriately dimensional matrices $R = R^T > 0$, N, X, $\eta(l) \triangleq x(l+1) - x(l)$ and two positive integer time-varying $d(k_1)$, $d(k_2)$ satisfying $d(k_1)+1 \le d(k_2) \le d_M$, the following equality holds:*

$$\begin{aligned} &-\sum_{l=k-d(k_2)}^{k-d(k_1)-1} \eta^T(l) R \eta(l) \\ &\quad = 2\xi^T(k) N [x(k-d(k_1)) - x(k-d(k_2))] \\ &\qquad + (d(k_2) - d(k_1)) \xi^T(k) X \xi(k) \\ &\qquad - \sum_{l=k-d(k_2)}^{k-d(k_1)-1} \begin{bmatrix} \xi(k) \\ \eta(l) \end{bmatrix}^T \begin{bmatrix} X & N \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \eta(l) \end{bmatrix}. \end{aligned} \tag{9}$$

3. Stability Analysis of the Wireless NCS

More generally, we consider the following discrete-time switched system with time-varying delay:

$$z(k+1) = A_i z(k) + A_{di} z(k-d_k), \quad i = 1,2,\ldots,N, \quad d_k = 1,2,\ldots,d_M, \tag{10}$$

where N is the number of the subsystems. Suppose that the event rates of the described subsystems S_i are defined as $r_1, r_2, \ldots, r_N$. The time interval $[0, kT]$ will be simplified $[0, k]$ in the following text. Let n_i, $i = 1,2,\ldots,N$ denote the times in which the subsystems S_i are activated on the interval $[0, k]$. Then we can obtain

$$k = \sum_{i=1}^{N} n_i; \quad r_i = \frac{n_i}{k}, \quad i = 1,2,\ldots,N; \quad \sum_{i=1}^{N} r_i = 1. \tag{11}$$

The following theorem gives a criterion to guarantee that the Lyapunov function $V(k)$ exponentially decays along state trajectory of system (10).

Theorem 4. *Given scalar $\lambda > 0$ and any delay satisfying $0 \le d_k \le d_M$, if there exist appropriate dimensional symmetric positive-definite matrices P, Q_1, Q_2, R, symmetric matrix $X = \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \ge 0$, and matrices $G, N = [N_1^T \ N_2^T]^T$ and $M = [M_1^T \ M_2^T]^T$, such that the following matrix inequalities hold:*

$$\Phi(d_k) = \begin{bmatrix} \Phi_{11} & \Phi_{12} & -N_1 & P + (\widehat{A}_i^T - I) G^T - G \\ * & \Phi_{22} & -N_2 & \widehat{A}_{di}^T G^T \\ * & * & -Q_2 & 0 \\ * & * & * & P + d_M R - G - G^T \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} X & N \\ * & R \end{bmatrix} > 0, \qquad \begin{bmatrix} X & M \\ * & R \end{bmatrix} > 0, \tag{13}$$

where $d_k = 0,\ldots,d_M$, $\widehat{A}_i = \lambda A_i$, $\widehat{A}_{di} = \lambda^{1+d_k} A_{di}$, and

$$\begin{aligned} \Phi_{11} &= d_M Q_1 + Q_2 + d_M X_1 + \mathrm{Sym}(M_1 + G(\widehat{A}_i - I)), \\ \Phi_{12} &= d_M X_2 + N_1 - M_1 + M_2^T + G\widehat{A}_{di}, \\ \Phi_{22} &= d_M X_3 - Q_1 + \mathrm{Sym}(N_2 - M_2), \end{aligned} \tag{14}$$

then

$$V(k) < \lambda^{-2(k-k_0)} V(k_0). \tag{15}$$

Proof. The following expression of $V(k)$ is the Lyapunov function of system (10):

$$\begin{aligned} V_1(k) &= z^T(k) P z(k), \\ V_2(k) &= \sum_{l=k-d_k}^{k-1} \lambda^{2(l-k)} z^T(l) Q_1 z(l) \\ &\quad + \sum_{m=-d_M+2}^{0} \sum_{l=k+m-1}^{k-1} \lambda^{2(l-k)} z^T(l) Q_1 z(l), \\ V_3(k) &= \sum_{l=k-d_M}^{k-1} \lambda^{2(l-k)} z^T(l) Q_2 z(l), \\ V_4(k) &= \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \lambda^{2(l-k)} (\lambda z(l+1) - z(l))^T \\ &\quad \times R(\lambda z(l+1) - z(l)). \end{aligned} \tag{16}$$

And defining $\xi(k) = \lambda^{k-k_0} z(k)$, $\lambda > 0$, $\widehat{A}_i = \lambda A_i$, $\widehat{A}_{di} = \lambda^{1+d_k} A_{di}$, combined with (10), we have

$$\begin{aligned} &\xi(k+1) = \widehat{A}_i \xi(k) + \widehat{A}_{di} \xi(k-d_k), \\ &i = 1, 2, \ldots, N, \quad d_k = 1, 2, \ldots, d_M. \end{aligned} \tag{17}$$

Choose the following Lyapunov function of system (17):

$$\begin{aligned} W(k) &= \xi^T(k) P \xi(k) + \sum_{l=k-d_k}^{k-1} \xi^T(l) Q_1 \xi(l) \\ &\quad + \sum_{m=-d_M+2}^{0} \sum_{l=k+m-1}^{k-1} \xi^T(l) Q_1 \xi(l) \\ &\quad + \sum_{l=k-d_M}^{k-1} \xi^T(l) Q_2 \xi(l) \\ &\quad + \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \delta^T(l) R \delta(l), \end{aligned} \tag{18}$$

where $\delta(l) = \xi(l+1) - \xi(l)$. Then the forward difference for $W(k)$ along any trajectory of system (17) is given by

$$\begin{aligned} \Delta W(k) &= W(k+1) - W(k) \\ &= \xi^T(k+1) P \xi(k+1) - \xi^T(k) P \xi(k) \\ &\quad + d_M \xi^T(k) Q_1 \xi(k) + \sum_{l=k+1-d_{k+1}}^{k-1} \xi^T(l) Q_1 \xi(l) \\ &\quad - \sum_{l=k-d_M+1}^{k-1} \xi^T(l) Q_1 \xi(l) - \sum_{l=k-d_k}^{k-1} \xi^T(l) Q_1 \xi(l) \\ &\quad + \xi^T(k) Q_2 \xi(k) - \xi^T(k-d_M) Q_2 \xi(k-d_M) \\ &\quad + d_M \delta^T(k) R \delta(k) \\ &\quad + d_M \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T X \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix} \\ &\quad + 2 \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \\ &\quad \times (\xi(k-d_k) - \xi(k-d_M)) \\ &\quad + 2 \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} (\xi(k) - \xi(k-d_k)) \\ &\quad - \sum_{l=k-d_M}^{k-d_k-1} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix}^T \begin{bmatrix} X & N \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix} \\ &\quad - \sum_{l=k-d_k}^{k-1} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix}^T \begin{bmatrix} X & M \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix} \\ &\quad + 2 \left(\xi^T(k) + \delta^T(k) \right) G \\ &\quad \times \left[(A_i - I) \xi^T(k) + A_{di} \xi(k-d_k) - \delta(k) \right] \\ &< \theta^T(k) \Phi \theta(k) < 0, \end{aligned} \tag{19}$$

where $\theta^T(k) = [\xi^T(k) \;\; \xi^T(k-d_k) \;\; \xi^T(k-d_M) \;\; \delta^T(k)]$, which means that $W(k) < W(k_0)$ for any $k \geq k_0$. Furthermore,

$$\begin{aligned} V_1(k) &= z^T(k) P z(k) = \lambda^{-2(k-k_0)} \xi^T(k) P \xi(k) \\ V_2(k) &= \sum_{l=k-d_k}^{k-1} \lambda^{2(l-k)} \lambda^{-2(l-k_0)} \xi^T(l) Q_1 \xi(l) \\ &\quad + \sum_{m=-d_M+2}^{0} \sum_{l=k+m-1}^{k-1} \lambda^{2(l-k)} \lambda^{-2(l-k_0)} \xi^T(l) Q_1 \xi(l) \\ V_3(k) &= \sum_{l=k-d_M}^{k-1} \lambda^{2(l-k)} \lambda^{-2(l-k_0)} \xi^T(l) Q_2 \xi(l) \\ V_4(k) &= \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \lambda^{2(l-k)} \lambda^{-2(l-k_0)} \delta^T(l) R \delta(l) \\ V(k) &= V_1(k) + V_2(k) + V_3(k) + V_4(k) = \lambda^{-2(k-k_0)} W(k). \end{aligned} \tag{20}$$

As $W(k_0) = V(k_0)$, it is easy to verify that $V(k) < \lambda^{-2(k-k_0)} V(k_0)$. This completes the proof. □

The following theorem gives a sufficient condition for the closed-loop NCS (10) to be exponentially stable.

Theorem 5. *The discrete-time switched system* (10) *is stable if there exist Lyapunov function* $V(k)$, r_i *defined in* (11), *and some*

positive scalars μ_i, $i = 1, 2, \ldots, N$ which correspond to each subsystem such that the following inequalities hold:

$$V(k) < \mu_i^{-2(k-k_0)} V(k_0),$$
$$\prod_{i=1}^{N} \mu_i^{r_i} < 1. \tag{21}$$

Proof. Defining the transition time of the subsystems to be $t_1 = 0, t_2, \ldots, t_k = k$, then

$$\begin{aligned} V(k) &< \mu_{t_k}^{t_k - t_{k-1}} V(t_{k-1}) < \mu_{t_k}^{t_k - t_{k-1}} \mu_{t_{k-1}}^{t_{k-1} - t_{k-2}} V(t_{k-2}) \\ &< \cdots < \prod_{i=1}^{N} \mu_i^{n_i} V(0) = \left(\prod_{i=1}^{N} \mu_i^{r_i} \right)^k V(0). \end{aligned} \tag{22}$$

So the system is stable if $\prod_{i=1}^{N} \mu_i^{r_i} < 1$. This completes the proof. □

4. Output-Feedback Controller Design

An algorithm to design the observer-based output-feedback controller of the wireless NCS is presented in this section. Since the data packets are time-stamped, the packet loss rate and time delay are known to the controller, which is designed to depend on both the packet loss rate and time delay. Firstly, A_i can be rewritten as $A_i = A_{0i} + B_{00} D_i C_{0i}$, $i = 1, 2, 3$, where

$$\begin{gathered} A_{01} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & I \end{bmatrix}, \qquad B_{00} = \begin{bmatrix} B & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}, \\ D_i = \begin{bmatrix} K_i & 0 \\ 0 & L \end{bmatrix}, \qquad C_{01} = \begin{bmatrix} I & -I & 0 \\ C & -C & -I \end{bmatrix} \\ A_{02} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ C & 0 & 0 \end{bmatrix}, \qquad C_{02} = \begin{bmatrix} I & -I & 0 \\ 0 & C & 0 \end{bmatrix} \\ A_{03} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad C_{03} = \begin{bmatrix} I & -I & 0 \\ C & -C & 0 \end{bmatrix} \end{gathered} \tag{23}$$

Theorem 6. *The system (7) is stable if there exist positive scalars μ_i, $i = 1, 2, 3$ satisfying $\prod_{i=1}^{3} \mu_i^{r_i} < 1$ and appropriate dimensional matrices P, Q_1, Q_2, R, X_1, X_3, matrices M_1, M_2, N_1, N_2, X_2 and matrices G_1, G_2, G_3, and matrices K_1, K_2, K_{3,d_k} such that the following matrix inequality holds:*

$$\begin{gathered} \Phi_0 + \Phi_1 < 0, \\ \Phi_0 + \Phi_2 < 0, \\ \Phi_0 + \Phi_3(d_k) < 0, \quad d_k = 1, 2, \ldots, d_M, \\ \begin{bmatrix} X & N \\ * & R \end{bmatrix} > 0, \qquad \begin{bmatrix} X & M \\ * & R \end{bmatrix} > 0, \end{gathered} \tag{24}$$

where

$$\Phi_0 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & -N_1 & P - G^T - G \\ * & \Sigma_{22} & -N_2 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & P + d_M R - G - G^T \end{bmatrix},$$

$$\Sigma_{11} = d_M Q_1 + Q_2 + d_M X_1 + \mathrm{Sym}(M_1 - G),$$

$$\Sigma_{12} = d_M X_2 + N_1 - M_1 + M_2^T,$$

$$\Sigma_{22} = d_M X_3 - Q_1 + \mathrm{Sym}(N_2 - M_2),$$

$$G = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix}, \qquad \Phi_1 = \begin{bmatrix} \overline{\phi}_{11} & 0 & 0 & \overline{\phi}_{14} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},$$

$$\overline{\phi}_{11} = \begin{bmatrix} \overline{\psi}_{11} & \overline{\psi}_{12} & 0 \\ * & \overline{\psi}_{22} & -\mu_1 G_2 L \\ * & * & \mu_1 G_3 + \mu_1 G_3^T \end{bmatrix},$$

$$\overline{\psi}_{11} = \mathrm{Sym}(\mu_1 G_1 A + \mu_1 G_1 B K_1),$$

$$\overline{\psi}_{12} = -\mu_1 G_1 B K_1 + \mu_1 C^T L^T G_2^T,$$

$$\overline{\psi}_{22} = \mathrm{Sym}(\mu_1 G_2 A - \mu_1 G_2 L C),$$

$$\overline{\phi}_{14} = \begin{bmatrix} \overline{\varphi}_{11} & -\mu_1 C^T L^T G_2^T & 0 \\ -\mu_1 K_1^T B^T G_1^T & \overline{\varphi}_{22} & 0 \\ 0 & -\mu_1 L^T G_2^T & \mu_1 G_3^T \end{bmatrix},$$

$$\overline{\varphi}_{11} = \mu_1 A^T G_1^T + \mu_1 K_1^T B^T G_1^T,$$

$$\overline{\varphi}_{22} = \mu_1 A^T G_2^T - \mu_1 C^T L^T G_2^T,$$

$$\Phi_2 = \begin{bmatrix} \widehat{\phi}_{11} & 0 & 0 & \widehat{\phi}_{14} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},$$

$$\widehat{\phi}_{11} = \begin{bmatrix} \widehat{\psi}_{11} & \widehat{\psi}_{12} & 0 \\ * & \widehat{\psi}_{22} & 0 \\ * & * & 0 \end{bmatrix},$$

$$\widehat{\psi}_{11} = \mathrm{Sym}(\mu_2 G_1 A + \mu_2 G_1 B K_2),$$

$$\widehat{\psi}_{12} = -\mu_2 G_1 B K_2,$$

$$\widehat{\psi}_{22} = \mathrm{Sym}(\mu_2 G_2 A - \mu_2 G_2 L C),$$

$$\widehat{\phi}_{14} = \begin{bmatrix} \widehat{\varphi}_{11} & 0 & \mu_2 C^T G_3^T \\ -\mu_2 K_2^T B^T G_1^T & \widehat{\varphi}_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\widehat{\varphi}_{11} = \mu_2 A^T G_1^T + \mu_2 K_2^T B^T G_1^T,$$

$$\widehat{\varphi}_{22} = \mu_2 A^T G_2^T - \mu_2 C^T L^T G_2^T,$$

$$\Phi_3 = \begin{bmatrix} \tilde{\phi}_{11} & \tilde{\phi}_{12} & 0 & \tilde{\phi}_{14} \\ * & 0 & 0 & \tilde{\phi}_{24} \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},$$

$$\tilde{\phi}_{11} = \begin{bmatrix} \tilde{\psi}_{11} & \tilde{\psi}_{12} & 0 \\ * & \tilde{\psi}_{22} & 0 \\ * & * & 0 \end{bmatrix},$$

$$\tilde{\psi}_{11} = \mathrm{Sym}\left(\mu_3 G_1 A + \mu_3 G_1 B K_{3,d_K}\right),$$

$$\tilde{\psi}_{12} = -\mu_3\left(G_1 B K_{3,d_K} + C^T L^T G_2^T\right),$$

$$\tilde{\psi}_{22} = \mathrm{Sym}\left(\mu_3 G_2 A - \mu_3 G_2 L C\right),$$

$$\tilde{\phi}_{12} = \begin{bmatrix} 0 & 0 & 0 \\ -\mu^{1+d_k} G_2 L C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \tilde{\phi}_{24} = \tilde{\phi}_{12}^T,$$

$$\tilde{\phi}_{14} = \begin{bmatrix} \tilde{\varphi}_{11} & \mu_3 C^T L^T G_2^T & 0 \\ -\mu_3 K_{3,d_k}^T B^T G_1^T & \tilde{\varphi}_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\varphi}_{11} = \mu_3 A^T G_1^T + \mu_3 K_{3,d_k}^T B^T G_1^T,$$

$$\tilde{\varphi}_{22} = \mu_3 A^T G_2^T - \mu_3 C^T L^T G_2^T. \tag{25}$$

Replacing the system matrices of (12) by (7), it is easy to obtain the above results according to Theorems 4 and 5. Furthermore, let $G_2 L = \overline{L}$, $G_1 B K_i = \overline{K}_i$, $i = 1, 2$, $G_1 B K_3 = \overline{K}_{3,d_k}$, and the controllers gain matrices K_i, L can be gained by solving the corresponding linear matrix inequalities.

5. Simulation Results

A numerical example of output-feedback stabilization of wireless NCS is evaluated in this section.

Consider the following discrete-time system:

$$A = \begin{bmatrix} 0.6852 & 0.3358 & 0.5832 \\ 0.8098 & 0.7896 & 0.4389 \\ 0.2065 & 0.6633 & 0.8528 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}^T. \tag{26}$$

The eigenvalues of A are 1.7183 and $0.3142 \pm 0.2595i$ and the system is unstable without control. Let the event rates of the packet loss and time delay be $r_1 = 0.5$ and $r_3 = 0.2$, respectively, and let the maximum delay be $d_M = 3$. Choose $\mu_1 = 1.8$, $\mu_2 = 0.5$, and $\mu_3 = 0.6$. So

$$\mu_1^{r_1} \mu_2^{r_2} \mu_3^{r_3} = 1.8^{0.5} \times 0.5^{0.3} \times 0.6^{0.2} = 0.9839 < 1. \tag{27}$$

By using Theorem 6, a suitable controller gain matrix can be obtained. The simulation result is shown in Figure 2. The above subgraph depicts the packet loss and the time delay of the wireless NCS in Figure 2. When the delay value is −1, it means that this packet is lost. And the below graph is to describe the state trajectories of closed-loop wireless NCS. From Figure 2, the states of the system diverge at the case when the data packets are dropped, but they converge to zero finally. Therefore, the example illustrates the effectiveness of the proposed method.

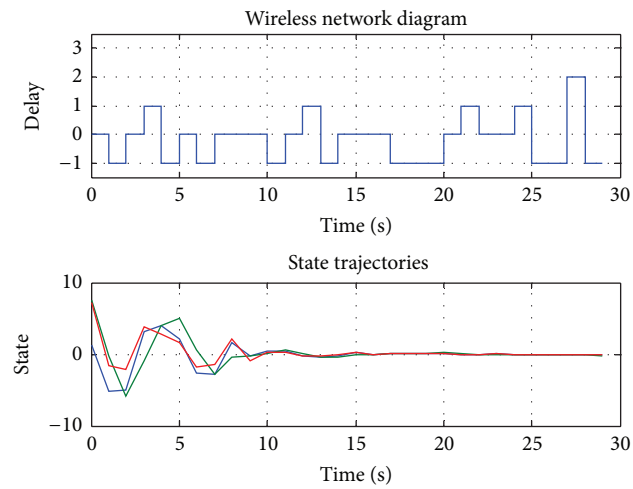

Figure 2: State trajectories of NCS with delay and packet loss.

6. Conclusions

The problem of modeling and stabilization of wireless NCS with both packet loss and time-varying delay is discussed in this paper. The output-feedback controller based on state observer is designed to stabilize the closed-loop wireless NCS by using ADS approach. And the numerical example is presented to demonstrate the effectiveness of the proposed result.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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