Output-Feedback Controller Design of a Wireless Networked Control System with Packet Loss and Time Delay

Zhen Hong, JinFeng Gao, and Ning Wang

1 Faculty of Mechanical Engineering & Automation, Zhejiang Sci-Tech University, Hangzhou 310018, China
2 College of Business Administration, Zhejiang University of Finance & Economics, Hangzhou 310018, China

Correspondence should be addressed to Zhen Hong; zhong@zstu.edu.cn

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This paper investigates the problem of modeling and stabilization of a wireless network control system (NCS) with both time-varying delay and packet-dropout. And the time-varying delay can be more or less than one sampling period. The wireless NCS is modeled as an asynchronous dynamic system (ADS) with three subsystems. Sufficient condition of the closed-loop NCS to be stable is obtained by using the ADS approach. A numerical example is presented to demonstrate the effectiveness of the proposed result.

1. Introduction

In modern control systems, the sensors, controllers, and controlled process are often connected by a real-time network medium. Such systems are called network control systems (NCSs) [1]. In the last decade, the general theory for NCSs has been widely investigated. It is widely used at almost all levels of operation and information processing in various areas, including manufacturing plants, automobiles, aircraft, remote operation, and teleautonomy [2–8]. As an alternative item for the wired network, wireless NCSs are becoming fundamental components of modern control systems due to their flexibility, ease of deployment, and low cost [9–12]. Thus, wireless NCS is considered in this paper.

One of the main issues in the NCS is the effect of network-induced delay that occurs when sensors, actuators, and controllers exchange data across the shared network. Without considering the delay, it not only degrades the performance of the control system but also even destabilizes the system. On the other hand, packet-dropout results from the network traffic congestion and the limited network reliability. When a data packet is dropped, the complete information of the NCS becomes unavailable. In this case, the controller or actuator has to decide what control signal is output with incomplete information.

Recently, many researchers have tried to solve the above problems with network-induced delay and packet-dropout in wireless NCS. For the problem with delay, for example, Hu and Yuan [2] introduce a finite sum equality based on quadratic terms to $H_{\infty}$ output-feedback control for switched linear discrete-time systems. Together with a Lyapunov sequence, a novel delay-dependent condition is implemented for $H_{\infty}$ without ignoring the useful terms. It can obtain the suboptimal $H_{\infty}$ static and dynamic output-feedback controllers at last, when a procedure involving a modified iterative algorithm is performed. For the problems with packet-dropout, for instance, the sufficient conditions for the exponential stability of the closed-loop NCSs using the average dwell time method are proposed in [5]. Furthermore, the relation between the packet-dropout rate and the stability of the closed-loop NCSs is also explicitly established in order to prove the effectiveness.

Unlike separately considered these two issues of the delay and the packet-dropout, this paper intends to deal with the modelling, analysis, and synthesis for the wireless NCS with both delay and packet-dropout as shown in Figure 1. An asynchronous dynamic system (ADS) approach is presented to stabilize the wireless NCS. Firstly, a switched system with time-varying delay model is presented to describe the wireless NCS. In [4], a new switched linear system model is proposed to describe NCS while the delay is assumed...
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\[ x(k + 1) = Ax(k) + Bu(k), \]
\[ y(k) = Cx(k), \]

where \( x(k) \in \mathbb{R}^n \) is the system state, \( u(k) \in \mathbb{R}^m \) is the control input, and \( y(k) \in \mathbb{R}^r \) is the measured output. And the model of an observer-based output-feedback controller is described as follows:

\[ \tilde{x}(k + 1) = A\tilde{x}(k) + Bu(k) + L[\omega(k) - \tilde{y}(k)], \]
\[ \tilde{y}(k) = C\tilde{x}(k), \]
\[ u(k) = K\tilde{x}(k), \]

where \( \tilde{x}(k) \in \mathbb{R}^n \) is the estimated state of the system (1) and \( \tilde{y}(k) \in \mathbb{R}^r \) is the estimated output. \( L \in \mathbb{R}^{m \times r} \) and \( K \in \mathbb{R}^{n \times r} \) are the observer and controller gain, respectively. It is also assumed that the pairs \( (A, B) \) are controllable and \( (C, A) \) are observable. In Figure 1, we can use a switch to denote the packet loss of the states in the network channel. If the switch is closed, the data packet is successfully transmitted. And we have \( \omega(k) = y(k) \) without network delay or \( \omega(k) = y(k - d_k) \) with network delay. When the switch is open, the previous value of the switch output will be used in the controller (2) and a packet is dropped. Then we have \( \omega(k) = \omega(k - 1) \) in this case.

Under consideration for the NCS, without loss of generality, we give the following assumptions, which will be useful in our main results.

**Assumption 1.**

(1) The sensors and controllers are all time-driven and synchronized.

(2) Time-stamping of measurements is necessary to reorder data packet at the observer side since they can arrive out of order. And the controller can get the delay of each data packet.

(3) The maximum delay in the network is \( d_M \) that is a known integer.

Define the estimation error by \( e(k) = x(k) - \tilde{x}(k) \) and let

\[ z(k) = [x^T(k) \ e^T(k) \ w^T(k - 1)]^T. \]

Then the dynamics of the closed-loop system can be described by the following three subsystems.

(S1) There is packet loss and the corresponding controller gain is \( K_1 \). Then the closed-loop NCS is described as

\[ \Omega_1 : z(k + 1) = A_1 z(k), \]
\[ A_1 = \begin{bmatrix} A + BK_1 & -BK_1 & 0 \\ LC & A - LC & -L \end{bmatrix}. \]
The data packet is transmitted successfully without network delay and the corresponding controller gain is $K_2$ in this case. Then the closed-loop NCS can be described as

$$\Omega_2 : z (k + 1) = A_2 z (k),$$

$$A_2 = \begin{bmatrix} A + BK_2 & -BK_2 & 0 \\ 0 & A - LC & 0 \\ C & 0 & 0 \end{bmatrix}.$$(5)

(S3) The data packet is transmitted successfully with network delay $d_k$ and the corresponding controller gain becomes $K_{3, d_k}$ here. Then the closed-loop NCS is as follows:

$$\Omega_3 : z (k + 1) = A_3 z (k) + A_{d, 3} z (k - d_k),$$

$$A_3 = \begin{bmatrix} A + BK_{3, d_k} & -BK_{3, d_k} & 0 \\ LC & A - LC & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A_{d, 3} = \begin{bmatrix} 0 & 0 & 0 \\ -LC & 0 & 0 \\ C & 0 & 0 \end{bmatrix}.$$(6)

From the above analysis, we can conclude that there are three different cases which may appear during every sampling period. So the closed-loop NCS can be described as a discrete-time switched system within three subsystems $\Omega_1$ to $\Omega_3$. In subsystem $\Omega_3$, when $d_k = 0$, $\Omega_3$ turns to $\Omega_2$. And the system matrices $\Omega_1$ and $\Omega_2$ are similar. So subsystem $\Omega_1$ in case 3 includes cases 1 and 2 by appropriately choosing the value of the matrices. Then the wireless NCS can be represented by the following switched system with time-varying delay:

$$z (k + 1) = A_i z (k) + A_{d, i} z (k - d_k), \quad i = 1, 2, 3,$$(7)

where $A_{d, 1} = 0, A_{d, 2} = 0$, and $d_k = 1, 2, \ldots, d_M$.

To end this section, the following definition and lemma are introduced to obtain our main results.

**Definition 2.** For any given initial conditions $(k_0, \phi) \in \mathbb{R}_+^n \times C^n$, (7) is globally exponentially stable if the solutions of (10) satisfy

$$\|x(k)\| \leq a\lambda^{-(k-k_0)}\|x(k_0)\|, \quad \forall k \geq k_0,$$(8)

where $a > 0$ is a constant and $\lambda > 1$ is the decay rate.

**Lemma 3** (see [2]). For any appropriately dimensional matrices $R = R^T > 0$, $N$, $X$, $\eta(l) \triangleq x(l + 1) - x(l)$ and two positive integer time-varying $d(k_1), d(k_2)$ satisfying $d(k_1) + 1 \leq d(k_2) \leq d_M$, the following equality holds:

$$\sum_{l=k-d(k_2)}^{k-d(k_1)-1} \eta^T(l) R \eta(l) = 2 \xi^T(k) N \left[ x(k - d(k_1)) - x(k - d(k_2)) \right]$$

+ $(d(k_2) - d(k_1))\xi^T(k)X \xi(k)$

$$- \sum_{l=k-d(k_2)}^{k-d(k_1)-1} \eta^T(l) \begin{bmatrix} X & N \end{bmatrix} \begin{bmatrix} \xi(l) \\ \xi(l) \end{bmatrix}.$$(9)

### 3. Stability Analysis of the Wireless NCS

More generally, we consider the following discrete-time switched system with time-varying delay:

$$z (k + 1) = A_i z (k) + A_{d, i} z (k - d_k), \quad i = 1, 2, \ldots, N,$$

$$d_k = 1, 2, \ldots, d_M,$$(10)

where $N$ is the number of the subsystems. Suppose that the event rates of the described subsystems $S_i$ are defined as $r_1, r_2, \ldots, r_N$. The time interval $[0, K_T]$ will be simplified $[0, k]$ in the following text. Let $r_i, i = 1, 2, \ldots, N$ denote the times in which the subsystems $S_i$ are activated on the interval $[0, k]$. Then we can obtain

$$k = \sum_{i=1}^{N} r_i; \quad r_i = \frac{n_i}{k}, \quad i = 1, 2, \ldots, N; \quad \sum_{i=1}^{N} n_i = 1.$$$(11)

The following theorem gives a criterion to guarantee that the Lyapunov function $V(k)$ exponentially decays along state trajectory of system (10).

**Theorem 4.** Given scalar $\lambda > 0$ and any delay satisfying $0 \leq d_k \leq d_M$, if there exist appropriate dimension symmetric positive-definite matrices $P, Q_1, Q_2, R$, symmetric matrix $X = \begin{bmatrix} X_1 & X_2 \\ X_2 & X_3 \end{bmatrix} \geq 0$, and matrices $G_1, G_2 = [N_1^T, N_2^T]^T$ and $M = [M_1^T, M_2^T]^T$, such that the following matrix inequalities hold:

$$\Phi (d_k) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} \begin{bmatrix} - \phi_{N_1} & P + (\hat{A}^T_1 - I) G^T - G \\ * & - \phi_{N_2} \\ * & * \end{bmatrix} \begin{bmatrix} \hat{A}^T_1 G \\ 0 \end{bmatrix} \begin{bmatrix} P + d_M R \end{bmatrix} - G^{-T} \begin{bmatrix} X \ N \\ * \ R \end{bmatrix} > 0,$$$(12)

$$\begin{bmatrix} X \ M \\ * \ R \end{bmatrix} > 0,$$$(13)

where $d_k = 0, \ldots, d_M$, $\hat{A}_1 = \lambda A_1, \hat{A}_{d_1} = \lambda^{1+d_1} A_{d_1}$, and

$$\Phi_{11} = d_M Q_1 + Q_2 + d_M X_1 + \text{Sym} \left( M_1 + G (\hat{A}_1 - I) \right),$$

$$\Phi_{12} = d_M X_2 + N_1 - M_1 - M_2 + G \hat{A}_{d_1},$$

$$\Phi_{22} = d_M X_3 - Q_1 + \text{Sym} (N_2 - M_2),$$

then

$$V (k) < \lambda^{-2(k-k_0)} V (k_0).$$$(15)
Proof. The following expression of $V(k)$ is the Lyapunov function of system (10):

\[
V_1(k) = z^T(k) Pz(k),
\]

\[
V_2(k) = \sum_{l=k-d_k}^{k-1} \lambda^{2(l-k)} z^T(l) Q_1 z(l),
\]

\[
V_3(k) = \sum_{m=-d_M+1}^{0} \sum_{l=k+m}^{k-1} \lambda^{2(l-k)} z^T(l) Q_2 z(l),
\]

\[
V_4(k) = \sum_{m=-d_M+1}^{0} \sum_{l=k+m}^{k-1} \delta^T(l) R \delta(l),
\]

(16)

where $\delta(l) = \xi(l+1) - \xi(l)$. Then the forward difference for $W(k)$ along any trajectory of system (17) is given by

\[
\Delta W(k) = W(k+1) - W(k) = \xi^T(k+1) P \xi(k+1) - \xi^T(k) P \xi(k)
\]

\[
+ d_M \xi^T(k) Q_1 \xi(k) + \sum_{l=k-d_k}^{k-1} \xi^T(l) Q_1 \xi(l)
\]

\[
- \sum_{l=k-d_k+1}^{k-1} \xi^T(l) Q_1 \xi(l) - \sum_{l=k-d_k}^{k-1} \xi^T(l) Q_1 \xi(l)
\]

\[
+ \xi^T(k) Q_2 \xi(k) - \xi^T(k-d_M) Q_2 \xi(k-d_M)
\]

\[
+ d_M \delta^T(k) R \delta(k),
\]

(18)

where $\theta^T(k) = [\xi^T(k) \xi^T(k-d_k) \xi^T(k-d_M)] \delta^T(k)$, which means that $W(k) < W(k_0)$ for any $k \geq k_0$. Furthermore,

\[
V_1(k) = z^T(k) Pz(k) = \lambda^{-2(k-k_0)} \xi^T(k) P \xi(k)
\]

\[
V_2(k) = \sum_{l=k-d_k}^{k-1} \lambda^{2(l-k)} \lambda^{-2(l-k_0)} \xi^T(l) Q_1 \xi(l)
\]

\[
+ \sum_{m=-d_M+1}^{0} \sum_{l=k+m}^{k-1} \lambda^{2(l-k)} \lambda^{-2(l-k_0)} \xi^T(l) Q_2 \xi(l)
\]

\[
V_3(k) = \sum_{m=-d_M+1}^{0} \sum_{l=k+m}^{k-1} \lambda^{2(l-k)} \lambda^{-2(l-k_0)} \delta^T(l) R \delta(l)
\]

\[
V_4(k) = \sum_{m=-d_M+1}^{0} \sum_{l=k+m}^{k-1} \lambda^{2(l-k)} \lambda^{-2(l-k_0)} \delta^T(l) R \delta(l)
\]

\[
V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) = \lambda^{-2(k-k_0)} W(k).
\]

(20)

As $W(k_0) = V(k_0)$, it is easy to verify that $V(k) < \lambda^{-2(k-k_0)} V(k_0)$. This completes the proof. □

The following theorem gives a sufficient condition for the closed-loop NCS (10) to be exponentially stable.

**Theorem 5.** The discrete-time switched system (10) is stable if there exist Lyapunov function $V(k)$, $r_i$ defined in (11), and some
positive scalars $\mu_i, i = 1, 2, \ldots, N$ which correspond to each subsystem such that the following inequalities hold:

$$V(k) < \mu_k^{2(k-k_0)} V(k_0), \quad \prod_{i=1}^{N} \mu_i^k < 1. \quad (21)$$

Proof. Defining the transition time of the subsystems to be $t_1 = 0, t_2, \ldots, t_k = k$, then

$$V(k) < \mu_k^{k-k_0} V(t_{k-1}) < \mu_k^{k-k_1} \mu_k^{k_1-k_2} V(t_{k-2}) < \cdots < \prod_{i=1}^{N} \mu_i^k V(0) = \left( \prod_{i=1}^{N} \mu_i^k \right) V(0). \quad (22)$$

So the system is stable if $\prod_{i=1}^{N} \mu_i^k < 1$. This completes the proof. \hfill \square

4. Output-Feedback Controller Design

An algorithm to design the observer-based output-feedback controller of the wireless NCS is presented in this section. Since the data packets are time-stamped, the packet loss rate and time delay are known to the controller, which is designed to depend on both the packet loss rate and time delay. Firstly, $A_i$ can be rewritten as $A_i = A_{i0} + B_{i0}D_tC_{i0}, i = 1, 2, 3,$ where

$$A_{01} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & I \end{bmatrix}, \quad B_{00} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix},$$

$$D_t = \begin{bmatrix} K_t \\ 0 \\ L \end{bmatrix}, \quad C_{01} = \begin{bmatrix} I & -I \\ C & -C \end{bmatrix} \begin{bmatrix} 0 \\ I \end{bmatrix},$$

$$A_{02} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ C & 0 & 0 \end{bmatrix}, \quad C_{02} = \begin{bmatrix} I & -I \\ 0 & C \end{bmatrix} \begin{bmatrix} 0 \\ I \end{bmatrix},$$

$$A_{03} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C_{03} = \begin{bmatrix} I & -I \\ C & -C \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (23)$$

Theorem 6. The system (7) is stable if there exist positive scalars $\mu_i, i = 1, 2, 3$ satisfying $\prod_{i=1}^{N} \mu_i^k < 1$ and appropriate dimensional matrices $P, Q_1, Q_2, R, X_1, X_3, X_2, X_5, M_1, M_2, N_1, N_2, X_3,$ and matrices $G_1, G_2, G_3,$ and matrices $K_1, K_2, K_3$ such that the following matrix inequality holds:

$$\Phi_0 + \Phi_1 < 0,$$

$$\phi_0 + \phi_2 < 0,$$

$$\Phi_0 + \phi_3 (d_k) < 0, \quad d_k = 1, 2, \ldots, d_M, \quad (24)$$

where

$$\Phi_0 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} - N_1 & P - G^T - G \\ * & -Q_2 & 0 \\ * & * & -P + d_M R - G - G^T \end{bmatrix},$$

$$\Sigma_{11} = d_M Q_1 + Q_2 + d_M X_1 + \text{Sym}(M_1 - G),$$

$$\Sigma_{12} = d_M X_2 + N_1 - M_1 + M_2^T,$$

$$\Sigma_{22} = d_M X_3 - Q_1 + \text{Sym}(N_2 - M_2),$$

$$G = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix}, \quad \Phi_1 = \begin{bmatrix} \phi_{11} & 0 & 0 & \phi_{14} \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} \phi_{11} & 0 & 0 & \phi_{14} \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix},$$

$$\Phi_3 = \begin{bmatrix} \phi_{11} & 0 & 0 & \phi_{14} \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix},$$

$$\phi_{11} = \text{Sym}(\mu_1 G_1 A + \mu_1 G_1 B K_1),$$

$$\phi_{12} = -\mu_1 G_1 B K_1 + \mu_1 C^T L^T G_2,$$

$$\phi_{22} = \text{Sym}(\mu_1 G_2 A - \mu_1 G_2 L C),$$

$$\phi_{14} = \begin{bmatrix} \phi_{11} & 0 & 0 & \phi_{14} \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix},$$

$$\phi_{11} = \text{Sym}(\mu_2 G_2 A + \mu_2 G_1 B K_2),$$

$$\phi_{12} = -\mu_2 G_1 B K_2,$$

$$\phi_{22} = \text{Sym}(\mu_2 G_2 A - \mu_2 G_2 L C),$$

$$\phi_{14} = \begin{bmatrix} \phi_{11} & 0 & 0 & \phi_{14} \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix},$$

$$\phi_{11} = \text{Sym}(\mu_3 G_3 A + \mu_3 G_1 B K_3),$$

$$\phi_{12} = -\mu_3 G_2 B K_3,$$

$$\phi_{22} = \text{Sym}(\mu_3 G_3 A - \mu_3 G_2 L C),$$

$$\phi_{14} = \begin{bmatrix} \phi_{11} & 0 & 0 & \phi_{14} \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix}.$$
Furthermore, let to obtain the above results according to Theorems 4 and 5. The simulation result is shown in Figure 2. The above subgraph depicts the packet loss and the time delay be obtained. The simulation result is shown in Figure 2. The above subgraph depicts the packet loss and the time delay be unstable without control. Let the event rates respectively, and let the maximum delay be . Choose . So

\[
\mu_1 = 1.8, \quad \mu_2 = 0.5, \quad \mu_3 = 0.6. \quad (27)
\]

By using Theorem 6, a suitable controller gain matrix can be obtained. The simulation result is shown in Figure 2. The above subgraph depicts the packet loss and the time delay of the wireless NCS in Figure 2. When the delay value is , it means that this packet is lost. And the below graph is to describe the state trajectories of closed-loop wireless NCS. From Figure 2, the states of the system diverge at the case when the data packets are dropped, but they converge to zero finally. Therefore, the example illustrates the effectiveness of the proposed method.

5. Simulation Results

A numerical example of output-feedback stabilization of wireless NCS is evaluated in this section.

Consider the following discrete-time system:

\[
A = \begin{bmatrix}
0.6852 & 0.3358 & 0.5832 \\
0.8098 & 0.7896 & 0.4389 \\
0.2065 & 0.6633 & 0.8528
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 \\
1 \\
1
\end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}^T.
\]

The eigenvalues of are 1.7183 and 0.3142 ± 0.2595i and the system is unstable without control. Let the event rates of the packet loss and time delay be and respectively, and let the maximum delay be . Choose . So

\[
\mu_1 = 1.8, \mu_2 = 0.5, \mu_3 = 0.6. \quad (27)
\]

By using Theorem 6, a suitable controller gain matrix can be obtained. The simulation result is shown in Figure 2. The above subgraph depicts the packet loss and the time delay of the wireless NCS in Figure 2. When the delay value is , it means that this packet is lost. And the below graph is to describe the state trajectories of closed-loop wireless NCS. From Figure 2, the states of the system diverge at the case when the data packets are dropped, but they converge to zero finally. Therefore, the example illustrates the effectiveness of the proposed method.

6. Conclusions

The problem of modeling and stabilization of wireless NCS with both packet loss and time-varying delay is discussed in this paper. The output-feedback controller based on state observer is designed to stabilize the closed-loop wireless NCS by using ADS approach. And the numerical example is presented to demonstrate the effectiveness of the proposed result.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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